A PREDICTION REGARDING THE WEAKENING OF THE BLUE SHIFT OF LIGHT FROM GEOSYNCHRONOUS SATELLITES

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ABSTRACT. We base the present approach, on an alternative theory of gravitation, consisting essentially on the law of energy conservation broadened to embody the mass \( k \) energy equivalence of the Special Theory of Relativity, and remedying, known problems and incompatibilities, associated with the actually reigning conception. The mere rotation problem of say, a sphere, can well be undertaken, along the same idea. Accordingly, we consider the problem of gravity created by a rotating celestial body. Finally we apply our results to the case of a geosynchronous satellite, which is, schematically speaking, nothing but a clock placed on a considerably high tower. The approach ironically furnishes the Newton’s law of motion, which however we derive, based on just static forces, and not an acceleration, governing a motion. (There is anyway no motion for a geosynchronous satellite, when observed from Earth.) We predict accordingly that, the blue shift of light from a geosynchronous satellite on an orbit of radius \( r_{\text{Gs}} \) should be softened as much as \( \left( \frac{\omega^2}{2c^2} \right)(r_{\text{Gs}}^2 - R^2) \) as compared to what is expected classically; here \( \omega \) Earth’s self rotation angular momentum, \( R \) Earth’s radius, and \( c \) the speed of light in empty space. We hope, the validity of this unforeseen prediction, can soon be checked out.

Keywords: Geosynchronous satellites, blue shift, red shift, general theory of relativity, Corrected Kundig Effect.

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1. INTRODUCTION

The attempt presented herein is triggered by anomalies recently reported about spacecrafts flybys of Earth [1]. The related results, so far, remain unexplained. Before we frame an explanation of these results, we find essential to undertake the gravitational effect of a rotating celestial body on an object, first, rotating on a geosynchronous orbit (GsO), in the light of recent, few publications [2-6]. Even before that, it becomes important to understand what happens to an object residing on, or in, a rotating celestial body, at rest,
as referred to that body. Classically speaking, the rotation affects the object, due to just the tangential instantaneous velocity, the object is carried along with, through the displacement in question [7]. In other words, classically, no specific attribution is specifically made to the acceleration.

At this point we hate not to refer in abundance, to the widely adopted approaches, and by doing so, we would like to request the patience of the conservative reader. It is that, we hope, the approach we will present herein, will constitute an opportunity to circumvent difficulties associated with the established understanding. And we find it simpler to go ahead directly, instead of losing time, also direction, amongst futile comparisons and efforts toward the justification of our approach. If open-mindedness is offered, then we hope equations and experimental results, will efficiently speak for themselves.

Thus consider two clocks placed respectively, on top of say Eiffel tower, and on the entrance level of this. The only difference on the ticking rates of these clocks, as referred to an observer on Earth, is classically speaking, due to the difference of altitudes they are located at. In other words, the rotation of Earth (classically speaking), does not bring in, any additional effect (as regards to a fixed observer on Earth). According to the approach considered herein, this is not, however, so.

Below we first summarize our approach in question, vis-a-vis gravitation, omitting at first the rotation of the source. Next we undertake, along with the same idea, the mere rotation problem, this time, omitting any additional effect, and specifically gravitation. We then consider the problem of gravity created by a rotating celestial body. Finally, we apply our results to the case of a geosynchronous satellite, which is, schematically speaking, nothing but a clock placed on a considerably high tower, planted on Earth.

2. Sketch of the Presently Undertaken, Theory of Gravitation

Suppose we have two celestial bodies interacting with each other, such as Earth, for instance, and a satellite, in motion around Earth. We can thus assume that, one of them is very massive, as compared to the other. Let $M$ the mass of the massive one, i.e. for instance Earth, and $m_{0\infty}$ the mass of the light body, but this, at infinity, where the gravitational effect vanishes. (We suppose, there is nothing else, around.)

The restriction $M \gg m_{0\infty}$, we just framed for the masses, is not a necessity for the approach we will sketch below [3,8]; it is only a convenience. It makes that, when $m_{0\infty}$ is in motion around $M$, as regards to the distant observer; $M$ always stays in place. Furthermore, the case we will handle herein, well fits in such a frame. As a first approach, we overlook $M$’s possible self rotation; but we will take it into account, pretty soon.

Now consider that the tiny body, say the satellite of concern, is engaged in a given motion around Earth (assumed temporarily, free of self rotation); the motion of the satellite can be conceived as made of the two following steps:

i) Bring the Satellite (which we shall call $S$) quasistatically, from infinity to a given location $r$, on its orbit, around Earth, but hold it there, at rest.
ii) Deliver to it, at the mentioned location, its motion on the orbit, in consideration.

The first step, owing to the law of energy conservation, yet broadened to embody the mass and energy equivalence of the Special Theory of Relativity (STR), yields a decrease in the mass of $m_{0\infty}$ as much as the static binding energy $B(r_0)$ coming into play [2-4]; $m_{0\infty}$ becomes $m(r)$ so that

$$m(r)c^2 = m_{0\infty}c^2 - B(r) \quad (1)$$

*(Decrease of the mass of the object in hand, in accordance with the law of energy conservation)*

where $c$ is the speed of light in empty space.

For the conservatives, perplexed by Eq. (1), we are ready to consider it as a law, we adopt, though, again, it is in fact nothing else, but the relativistic law of energy conservation.

**Law 1:** The rest mass of an object bound to a celestial body, in fact any given body, it may interact with, amounts less than its rest mass measured in empty space, and this as much as its static binding energy vis-à-vis the gravitational field of concern.

Recall that we have considered the host object infinitely more massive than $m_{0\infty}$.

Thus here, we request the conservatives to please hold back their reactions, if any, and accord us the favor, to make, in the worse case, based on their value judgment, such an assumption (which is again, nothing else, but the relativistic law of energy conservation).

Now, if one moves $m(r)$ quasistatically, as much as $dr$, he has to work against the gravitational attraction force, $M$ exerts on $m(r)$; this then, owing to Law 1, will yield an increase in $m(r)$, as much as $dm(r)$, in such as way that

$$dm(r)c^2 = G\frac{m(r)M}{r^2} dr \quad (2)$$

$G$ is the universal gravitational constant. We would like to recall that, $G$ is not Lorentz invariant, though classified as a universal constant [3]. $Gm(r)M$ however is, given that it bears the dimensions of the square of an electric charge intensity, and electric charges are well Lorentz invariant.

Here, the force term we make use of, is nothing else, but the usual Newton Gravitational Attraction Force. Thus once again, we would like to demand the tolerance of the conservative reader.

In any case, we would like to stress the fact that, Newton Gravitational Attraction Force’s $1/r^2$ dependency, is a requirement imposed by the STR. In other words, we do not really have to postulate the Newton Gravitational Attraction Force. It can well be derived based on the STR, if the spatial dependency of the force term is postulated to behave as $1/r^n$; thereby $n$, necessarily, takes the value of 2, in order to cope with the Lorentz transformations, were the static mass dipole composed of $m(r)$ and $M$ (in between which the static force $Gm(r)M/r^n$ reigns), brought to a uniform, translational motion (cf. Reference 3).

To emphasize this occurrence, more importantly to smooth down conservative reactions, against Newton’s Law’s, when judged with regards to the widely accepted existing conception, we would like to state it as our next law.
Law 2: Were the spatial dependency \(1/r^n\) of the Newton’s gravitational attraction force \(Gm(r)M/r^n\), reigning between two masses \(m(r)\) and \(M\), strictly at rest, vis-a-vis each other, postulated to behave as such, along with an unknown \(n\); this exponent, necessarily, takes the value of 2, in order to cope with the Lorentz transformations, in case the static mass dipole composed of \(m(r)\) and \(M\) brought to a uniform, translational motion.

We have to emphasize that, if the masses \(m(r)\) and \(M\) are not at rest, with respect to each other, then, as Newton suspected, the law of force of the attraction, between these masses, is not anymore given by \(Gm(r)M/r^2\) (cf. Reference 3).

The integration of Eq. (2), yields \(m(r)\):

\[
m(r) = m_0\infty e^{-\alpha(r)}
\]

(Rest mass of the bound object)

where \(\alpha\) is a definition, i.e.

\[
\alpha = \frac{GM}{rc^2}.
\]

The comparison of Eqs. (1) and (3) furnishes, the static binding energy \(B(r)\), at the location \(r\):

\[
B(r) = m_0\infty c^2(1 - e^{-\alpha})
\]

(Binding energy of the object at the given location)

Now, along with the second step, we proposed right above, we bring \(S\) (the Satellite) (already moved quasistatically, from infinity, to \(r\)), to its orbital motion of concern, of velocity \(v\), it is supposed to delineate at \(r\). This yields the Lorentz dilation of the rest mass \(m(r)\) at \(r\), so that the overall relativistic mass \(m_\gamma(r)\), or the same, the overall relativistic energy of the object, which should for an isolated system, stays constant throughout, in orbit, becomes

\[
m_\gamma(r)c^2 = m_0\infty c^2 \frac{1 - \frac{B(r)}{m_0\infty c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = m_0\infty c^2 \frac{e^{-\alpha}}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

(Overall relativistic energy of the object on the given orbit)

This equation is, in fact written as assessed by the distant observer (who is not affected by the gravitational field). In effect, right here, already \(G\) is assessed by the distant observer, and this is precisely why we have operated based on the orbital distance \(r\) of the object to the center of Earth, as referred to the distant observer. For the purpose of the present paper, we will not bother with such aspects. At any rate, in the present approach, the orbital velocity \(v\), is the same, either for the distant observer, or the local observer, on board of \(S\), or that on Earth. Such peculiarities are presented in Reference 3. The same essentially holds for the speed of light. It remains as a universal constant in our approach. This property is however not an assumption, but is well yield by the quantum mechanical aspects of the present approach, and the reason is as follows. Within our framework, as shown below, both distances and lengths are affected in exactly the same way, as referred, say to the distant observer, while in the widely accepted existing conception (GTR), briefly speaking, distances are contracted, and periods of time are
stretched [7]. Note further that, the constant that appears in Eq.(6), becomes $m_0c^2$, if the motion were a free fall [2,3]. Thus based on Eq.(6), $\frac{\text{Constant}}{m_0c^2}$, usually, is very near to unity; but it is the difference coming into play that determines, how the orbit in question, will look like.

Let us now recall that our distances and periods of time are altered via just quantum mechanics [2,3]. It is that the rest mass, or the same, rest total energy decreases, via Eq.(3) of the object in consideration in a field, it interacts with (thus, not solely a gravitational field), through its quantum mechanical description, leads to a size increase, as well as period of time stretching. Thus this latter, in fact, is nothing else, but a weakening of the internal energy of the object in hand. Note also that in the GTR, the mass (owing to the principle of equivalence, as originally considered by this theory, between the effect of acceleration and the effect of gravitation), increases in the gravitational field, whereas the energy decreases, which leads to the breaking of the law of energy conservation [9].

3. The Mere Rotation Problem

At this stage, we would like to summarize briefly, the results we obtained along the line, presented right above, but now as regards to a rotating solid sphere [5]. Thus consider a rotating sphere. We propose to understand what happens to an object, which we call "$m$", attached to the sphere in rotation, as the object is moved quasistatically, along a given radial tunnel, existing beforehand, within the sphere.

The object is monitored by an observer situated at the center of it. We will denote the center of the sphere by $O_{\text{Sphere}}$, and the edge of the sphere, where our radial direction, intercepts the surface of the sphere, by $E_{\text{Sphere}}$. Note that here we do not consider, what happens as referred to an observer located outside of the sphere, when we move $m$, within the rotating sphere. Thus we consider the rotating sphere as a closed world, isolated from the surrounding. Such a consideration in fact is a necessity allowing us to draw an analogy between the closed sphere world and a corresponding gravitational world [5].

In the closed sphere world, via possible measurements, it is straightforward to estimate the strength of the acceleration $\gamma_{\text{Disc}}(r)$, at a given location $P$, whose distance to $O_{\text{Sphere}}$ is $r$. The acceleration will be measured to be proportional to $r$:

$$\gamma_{\text{Sphere}}(r) = kr$$

(Acceleration as measured by the observer in the closed disc world at the given location)

It can be shown that, $r$ does not point to the same distance, if for instance, assessed by an observer situated at the location $P$ (defined by $r$) [5]. Then, Eq.(7), should be altered; the dependence of $\gamma_{\text{Sphere}}(r)$ to $r$, becomes more complicated. But here, we do not really have to get into such complexities. In any case, the outside observer (who does not belong to the closed sphere world), can well measure that

\footnote{And this is exactly why the speed of light is, according to the present approach, not altered near a celestial body (though light will still take a longer period of time to graze, the body of concern, as referred to the distant observer, due to the stretching of lengths).}
\[ k = \omega^2 \] (8)

\( \omega \) being the angular velocity of the rotating sphere, as viewed from the outside.

Since we have considered the sphere as a closed world, the energy in this world, must be conserved. For the sake of clarity, this should be stated as a law drawn by an observer living in the closed sphere world.

**Law 3:** In the closed, isolated spinning sphere world, energy must be conserved, so to allow the drawing of an analogy between the closed sphere world, and a corresponding gravitational world.

Suppose then, the object \( m \) is situated at \( E_{\text{Sphere}} \).

Let us make the following definitions.

- \( m_0 \): rest mass of the object in free space, thus the rest mass of it, at the center of the sphere.
- \( m_{0\text{Sphere}}(r) \): rest mass of the object at the location \( r \).

It should be emphasized that, this latter mass, owing to the relativistic law of energy conservation, as will be detailed, must be subject to a change.

If now, one wants to carry \( m \), quasistatically, from \( E_{\text{Sphere}} \) up to the location \( O_{\text{Sphere}} \), he has to furnish to it, a given amount of energy, while working against the outward (centrifugal) force

\[ F(r) = m_{0\text{Sphere}}(r) \gamma_{\text{Sphere}}(r) = m_{0\text{Sphere}}(r)kr \] (9)

(Outward force's strength at the given location)

On the other hand, it may be noted that, we have borrowed this static law of force, from Newton. Yet the observer in the closed sphere world, can well make experiments, the way Newton conceived, and can establish, as we anticipate, the following law [10], to be precise, for a static mass, exclusively:

**Law 4:** In the Rotating Sphere World, a static mass is submitted to a static force, given by

\[ \text{Force} = [\text{Local Rest Mass}] \times [\text{Local Centrifugal Acceleration}] \]

This law, corresponds to Newton’s law of gravitation (i.e. gravitational force = rest mass \( \times \) local gravitational acceleration), with respect to a gravitation world, but the way we consider, strictly for static masses, as framed by Law 2, stated above. On the other hand, the law of conservation of energy, broadened to embody the mass \& energy equivalence of the STR, requires that the rest mass of the object (fully engaged to the centrifugal force), should be increased, when carried (against the centrifugal force in question), from the periphery, toward \( O_{\text{Sphere}} \). Thus, we can further stress the following law, in fact nothing else, but once again, the law of energy conservation, where though, energy and mass are essentially not different from each other [2].

**Law 5:** The rest mass of an object bound to a location in the closed sphere world’s accelerational field, amounts, less than its rest mass, measured at the center of the sphere, and this, as much as its binding energy, vis-a-vis the location of concern.

To avoid conservative reactions, it may be worth to discuss a little bit, this latter assertion. Is it a postulate, or a law? To us, just like we argued at the level of Law 1, it is nothing else but the relativistic law of energy conservation. But the conservative reader
would argue that the rest mass is a universal property of the particle or object in question. Thereby, it should not be altered, by the centrifugal field, or by any means. Then gladly, we would call the above statement a postulate about of the conservation of the relativistic energy.

The issue though, comes to the fundamental question of "What is rest mass?", and "Wouldn’t really change?". Let us consider an electron. If we think we understand what the electric charge of the electron is, then we can go ahead, and define its rest mass as the internal energy of the charge in question.

Normally the rest mass of the electron via the equalities of the expressions for energy $E$, established by Planck [11] and Einstein [12], i.e. $E = h\nu$ and $E = m_0c^2$, more specifically $m_0c^2 = h\nu$, tells us that if the rest mass $m_0$ were totally annihilated, then an electromagnetic energy of frequency $\nu$ would be created.

The rest mass thus is, somewhat, this energy, if we take $c = 1$. When the electron is statically bound \(^2\) to, say, a proton (which can be for simplicity, assumed to be infinitely more massive than the electron), then its internal energy, according to our visualization (cf. Law 1), must get decreased, as much as the static binding energy coming into pay. Henceforth, the rest mass when bound, is not a universal characteristic of the particle at hand.

The same occurs for a particle bound to a centrifugal field, in fact let us stress, any field, the object in consideration interacts with. Thus, as assessed by the observer situated at $O_{\text{sphere}}$ \(^3\), one can write

$$-dm_{0\text{sphere}}(r)c^2 = F(r)dr = m_{0\text{sphere}}(r)krdr,$$

(10)

where $c$ is the speed of light. Via integration, we can write

$$m_{0\text{sphere}}(r) = m_0\exp\left(\frac{\int_0^r \gamma_{\text{sphere}}(r')dr'}{c^2}\right) = m_0\exp\left(\frac{-kr^2}{2c^2}\right).$$

(11)

(Rest mass of the object at $r$, in the closed, sphere world as referred to the center of the sphere)

This equation is to be compared with Eq.(3), written for gravitation. The similarity is evident. The result is that, in any accelerational field, the rest mass of the given object, is decreased as much as the integrated strength of the field, over the depth. Note that this result holds in the closed sphere world, exclusively. In other terms, we did not yet taken

\(^2\)One can argue how a proton and an electron can be statically bound. If they cannot be, then he would argue, our reasoning is false. Thus conceive instead a water molecule. It is known that it works as an electric dipole made of $Q$ and $q$. In water molecule, the oxygen atom ($O$) attracts, respectively the two binding electrons of the hydrogen ($H$) atoms, delineating an angle HOH of about $105^\circ$. This makes that, the hydrogen atoms get charged positively, and the oxygen atom, negatively. Thus, water molecule can indeed be described as dipole, made of $-2e$ situated nearby the oxygen atom, and $+2e$ situated on the median of the triangle HOH, in between the hydrogen atoms; $e$ is the electron’s charge intensity. Let us call $d$ the distance between the two representative charges $+2e$ and $-2e$. Thus in a water molecule at rest, we well have an electric charge $Q=+2e$ and an other one $q=-2e$, statically bound to each other, being situated at a distance $d$ from each other.

\(^3\)We tacitly assume that the object $m$ is very small as compared to the sphere’s mass, so that moving it within the sphere, does not alter the angular momentum, thus the rotational speed, of the sphere.
into account the effect of the tangential displacement process of the rotating sphere, as
assessed by the outside observer, although it is the rotation, which causes the acceleration.

At any rate, according to our approach the overall relativistic mass \( m_{\text{OverAll}}(R) \) at \( R \),
due to the rotation of the sphere, becomes

\[
m_{\text{OverAll}}(R) = m_0 \exp\left(-\frac{\omega^2 R^2}{2c^2}\right) \approx m_0\left(1 + \frac{\omega^4 R^4}{4c^4}\right) \approx m_0. \tag{12}
\]

(Overall relativistic mass, due to the rotational motion, as viewed by the outside observer)

This result is of course non-conform, with the classical prediction consisting, in just the
Lorentz dilation, due to the displacement, and neglecting any specific effect due to acceleration [7].
Nonetheless an error in processing the data collected based on a rotating clock [13] is recently reported [14].
Furthermore recent measurements did put firmly at stake the classical belief [15]. All the more that the present theory is well capable to predict the bound muon decay rate retardation, with an unequal success, up to now [16]. We thus anticipate that the prediction framed by Eq.(12), must now, be seriously considered.

4. Quantum Mechanical Assertions

The first author has previously established the following general quantum mechanical assertion [6,17,18]:

**Assertion 1:** Consider a relativistic or non-relativistic quantum mechanical description
of a given object, depending on whichever, may be appropriate. This description points to
an internal dynamics which consists in a "clock motion", achieved in a "clock space", along
with a "unit period of time". The object is supposed to embody \( K \) particles, altogether.
If then, different masses \( m_{k0}, k = 1, \ldots, K \), involved by this description of the object at
rest, are over all multiplied by the arbitrary number \( \chi \), the following two general results
are conjointly obtained: 1) The total energy \( E_0 \) associated with the given clock’s motion
of the object is increased as much, or the same, the unit period of time \( T_0 \), of the motion
associated with this energy, is decreased as much. 2) The characteristic length, or the size
\( R_0 \) to be associated with the given clock’s motion of concern, contracts as much.

In mathematical words this is:

\[
[(m_{k0}, k = 1, \ldots, K) \to (\chi m_{k0}, k = 1, \ldots, K)] \Rightarrow [(E_0 \to \chi E_0) \text{ or } (T_0 \to \frac{T_0}{\chi}), \text{ and } (R_0 \to \frac{R_0}{\chi})].
\]

**Assertion 2:** A clock interacting with any accelerating field, electric, nuclear, gravitational,
or else (without loosing its identity), retards as imposed by its quantum mechanical
description, due to the rest mass deficiency, which amounts to the equivalent of the binding
energy it displays in the field in consideration; at the same time, and for the same
reason, the space size in which it is installed, stretches just as much.

This can further be grasped rather easily, as follows. The mass deficiency, the object
displays in the accelerating field, weakens its internal dynamics as much, which makes
it slow down, and this is nothing but the same as the red shift, predicted by the GTR
(yet here deduced through just the law of energy conservation and quantum mechanics).
Thence, one arrives at the principal results, stated above.
This leads, for the total energy $E_{0\text{Sphere}}(r)$ to be associated with the internal dynamics of the object in hand, assumed at rest, at the location $r$ of the accelerational field of the sphere world,

$$E_{0\text{Sphere}}(r) = E_0 \exp\left(-\frac{\omega^2 r^2}{2c^2}\right), \quad (13a)$$

*(Total energy delineated by the internal dynamics of the object at $r$, within the sphere, as referred to the center of the sphere)*

where, to simplify our notation, we wrote straight $\gamma$, instead of $\gamma_{\text{Sphere}}(r)$. Similarly, we have for the period of time $T_{0\text{Sphere}}(r)$, and the unit length $R_{0\text{Sphere}}(r)$, to be associated with the internal dynamics of it, at $r$, respectively,

$$T_{0\text{Sphere}}(r) = T_0 \exp\left(\frac{\omega^2 r^2}{2c^2}\right), \quad (14a)$$

*(Period of time delineated by the internal dynamics of the object at $r$, within the rotating sphere, as referred to the center of the sphere)*

$$R_{0\text{Sphere}}(r) = R_0 \exp\left(\frac{\omega^2 r^2}{2c^2}\right). \quad (15a)$$

*(Unit length delineated by the internal dynamics of the object at $r$, within the rotating sphere, as referred to the center of the sphere)*

Thus (neglecting any other possible occurrence), in the closed sphere world, the unit period of time and the unit length stretch, just as much, and this uniformly (i.e. there is no directional dependency).

Note that the application of the foregoing calculation to gravitation is immediate, and contrary to what the Principle of Equivalence of the GTR delineates, does not require any boosting analogy. All we have to do (as we will soon detail), is to replace the static centrifugal force by the static Newton gravitational attraction force, and that is all. Thus in our approach, the gravitational red shift is nothing else, but a quantum mechanical occurrence, due to the rest mass decrease of the object, yield by the binding process of it, with regard to the celestial body of concern. Thereby, in order to predict the gravitational energy decrease, period stretching and unit length stretching, all we have to do is to replace the acceleration term in Eq.(11), by the static gravitational acceleration. This, with the familiar notation we have adopted herein, yields to

$$E(r) = E_0 \exp\left(-\frac{GM}{rc^2}\right), \quad (13b)$$

*(Total energy delineated by the internal dynamics of the object at a distance $r$, from the center of the celestial body of mass $M$)*

$$T(r) = T_0 \exp\left(\frac{GM}{rc^2}\right), \quad (14b)$$

*(Period of time delineated by the internal dynamics of the object at a distance $r$, from the center of the celestial body of mass $M$)*
\[ R(r) = R_0 \exp \left( \frac{GM}{rc^2} \right). \]  

(15b)

(Unit length delineated by the internal dynamics of the object at a distance \( r \), from the center of the celestial body of mass \( M \))

5. Superimposed Effect of Both Gravitation and Rotation on an Object Situated on a Rotating Celestial Body

From the above discussion it becomes clear that solely, the acceleration due to rotation, already, causes a red shift.

Thus consider an object \( S \) of mass \( m_\infty \), at infinity. Suppose we bring it, onto a celestial body of mass \( M \), and radius \( R \), originally at rest. Next, suppose we deliver to \( M \) a rotational motion of angular velocity \( \omega \). A distant observer, under the given circumstances, will assess the overall relativistic mass \( m_{Overall}(R) \) of now the rotating object \( S \), fixed to gravitational source of mass \( M \), as [cf. Eq.(3), (6), (11), and (12)]

\[ m_{Overall}(R) = m_\infty \exp[-\alpha(R)] \exp[-\omega^2 R^2/(2c_0^2)] \exp[-\omega^2 R^2/(2c_0^2)] \]  

\[ \sqrt{1 - \omega^2 R^2/c_0^2} \]  

(16)

Note yet that, the construction of Eq.(3), involved the process of moving \( S \) from infinity to the vicinity of \( M \), whereas the construction of Eq.(11), involved the process of moving \( S \) from the center of the rotating sphere to its edge. On the other hand, bringing an object from the center of a rotating object to the edge of this, does not seem to constitute a process identical to bring the object residing at the edge of a sphere originally at rest, to a rotational motion along with the sphere. At any rate, any term such as \( \exp(-\omega^2 R^2/(2c_0^2)) \) makes that, Eq.(16), constitutes an unforeseen prediction, up to now. This is worth to be stated as a assertion.

**Assertion:** Any rotating celestial object, as referred to a distant observer, next to the gravitational red shift, also the Lorentz time dilation - or the same red shift - due to the rotational instantaneous displacement, must as well, display a further red shift due to the rotational acceleration.

How can we make sure of this prediction? We elaborate on this right below.

6. The Weakening of the Blueshift of Light From Geosynchronous Satellites

Let us consider \( S \), the Satellite of concern, initially on the surface of the rotating Earth, just the way, it is originally, before launch. Let us call its rest mass, \( m_{Ground} = m(R) \), as referred to an observer on Earth, at the location Q. Suppose at first, for simplicity, that Q is situated on the equatorial plane. Imagine we have there, a very tall tower, planted on Earth vertically. We now propose to carry \( S \), along this tower upward, as Earth normally rotates. The rest mass of \( S \) will change, as referred to Q (its original location on Earth), as Eqs.(1) and (2) suggest. It becomes \( m(r) \) at the altitude \( r \), as measured from the center of Earth. We have thus, to work, against the gravitational force \( F_G \).
\[ F_G = G \frac{Mm(r)}{r^2}. \]  

(17)

Let us emphasize that \( m(r) \), is the increased rest mass of \( S \) at \( r \), as referred to \( Q \). Note that, we have perfectly, the right, to make use of the above force law [Eq.(17)] (cf. Law 2), for \( m(r) \) and \( M \) are well at rest, with respect to each other (regardless the self rotation of Earth). Recall, we happen to have proven that, Eq.(17) is a requirement imposed by the STR [3], were the spatial dependency of it, assumed to behave as \( 1/r^n \). (Then, \( n \) turns to be identical to 2.) The gravitational attraction force, while carrying \( S \) upward, though, is weakened due to the centrifugal force \( F_{\text{Centrifugal}} \) created by Earth’s rotation. This latter force is upward [cf. Eqs. (7), (8) and (9)]. Its strength is (cf. Law 4):

\[ F_{\text{Centrifugal}} = m(r)\gamma(r) = m(r)\omega^2 r \]  

(18)

Here again, we have perfectly the right of writing this force law, for in the first place, it appears of the same nature as that of Newton’s gravitational attraction law [Eq.(17)] (cf. Law 2); the only difference is that, in Eq.(18), the centrifugal acceleration takes place, instead of the gravitational acceleration.

We would like to stress that, both forces [Eq.(17) and (18)], delineate ”static forces”. And within the frame of the process of quasistatic move, we have chosen, we have no motion, what so ever. The strength of the net static force \( F_{\text{Net}}(r) \) on \( S \), at \( r \), along our tall tower, becomes

\[ F_{\text{Net}}(r) = G \frac{Mm(r)}{r^2} - m(r)\omega^2 r. \]  

(19)

As we move \( S \), quasistatically upward, as much as \( dr \), we deliver to it, the infinitely small amount of energy \( c^2 dm(r) \), so that

\[ \int_r^R \left( G \frac{Mm(r)}{r^2} - m(r)\omega^2 r \right) dr = c^2 dm(r). \]  

(20)

The system made of [Rotating Earth + Satellite Rotating With Earth], must pile up this energy, and assuming that Earth is not influenced by our act (since it remains in place, practically, throughout), it is the Satellite (\( S \)), which will acquire the energy defined by Eq.(20). Indeed, if set free, at \( r + dr \), while falling up to \( r \), \( S \) would gain a kinetic energy, exactly equal to the LHS of Eq.(20).

Via integration of this equation, one obtains

\[ \int_r^R \left( G \frac{M}{r^2} - \omega^2 r \right) dr = c^2 \int_r^R \frac{dm(r)}{m(r)}. \]  

(21)

or

\[ m(r)c^2 = m(R)c^2 exp\left( G \frac{M}{Rc^2} - G \frac{M}{r^2c^2} + \frac{\omega^2 R^2}{2c^2} - \frac{\omega^2 r^2}{2c^2} \right). \]  

(22)

(Overall energy of the satellite on the geosynchronous orbit, as referred to the observer on Earth, according to the present approach)

It is that, as we ascend, on the one hand, the mass of \( S \) increases, due to the weakening in the strength of the (static) gravitational attraction force; but on the other hand, the
mass of $S$ decreases due to the enhancement of the (static) centrifugal force. Thus, the

derivative of the RHS of this equation with respect to $r$, leads to zero, for the altitude $r_{Gs}$

$$G\frac{Mm(r_{Gs})}{r^2} = m(r_{Gs})\omega^2 r_{Gs}. \quad (23)$$

*(Orbit equation for the geosynchronous satellite, as referred to the observer on Earth)*

This is not just a triviality, anyway framed by the equality of the strengths of the

gravitational and centrifugal forces.

In effect, the value of $r_{Gs}$ makes the RHS of Eq.(22) maximum. One can indeed check

that the second derivative of $m(r)$ with respect to $r$ is always negative.

A maximum in $m(r)$, means on the other hand, a minimum rest mass decrease in the

original rest mass $m_{0\infty}$ of the object in hand. Thus $r_{Gs}$ is the altitude where the object

gives away the least of its internal energy.

The following points should be cautiously remembered.

- Eq.(23), no matter how ironical it may be, is indeed an equation written by Newton, some three hundred years ago.
- But for an observer on Earth, watching a geosynchronous satellite, it is, relativistically speaking, rigorous. And we considered it, only for such an observer.
- The masses involved by the LHS of Eq.(23), no matter what they cancel out, depend on the altitude. In other words, they do not constitute a universally constant property of the object at hand.
- Eq.(23), beyond its classical significance, tells us, profoundly that, it is the condition for which the changing rest mass draws a maximum, with respect to the altitude. It decreases more upward due to the centrifugal acceleration, and it decreases more downward due to gravitation, and all that, at rest, with regards to an observer, say on the Rotating Earth (for whom the geosynchronous satellite is obviously at rest).
- In any case, Eq.(20) is a relativistic equation, and naturally is, out of the classical Newtonian Mechanics scope.

We will discuss the outcome in question, further, below.

Regarding the geosynchronous orbit ($GsO$), as assessed by the observer on Earth, based on Eq.(23), we have anyway

$$m(r_{Gs})c^2 = m(R)c^2 \exp \left( \frac{GM}{Rc^2} + \frac{\omega^2 R^2}{2c^2} - \frac{3}{2} \frac{GM}{r_{Gs}c^2} \right)$$

$$\cong m(R)c^2 \left( 1 + \frac{GM}{Rc^2} + \frac{\omega^2 R^2}{2c^2} - \frac{3}{2} \frac{GM}{r_{Gs}c^2} \right) \quad (24)$$

*(Overall energy of the satellite on the GsO, as referred to the observer on the rotating Earth, according to the present approach)*

One can write, a similar relationship on the basis of the GTR [19]. The geosynchronous distant observer (i.e. the observer at rest as referred to the rotating Earth), measures the energy $m(R)c^2$ of $S$, on Earth, as

$$m(R)c^2 = m_{0\infty}c^2 \sqrt{1 - \frac{GM}{Rc^2}} \quad (25)$$
recall that here, $R$ denominates, Earth’s radius.

The geosynchronous distant observer, on the other hand, assesses the energy of $S$, situated at the altitude $r_{Gs}$ on the $GsO$, as

$$m(r_{Gs})c^2 = m_{\infty}c^2\sqrt{1 - 2\frac{GM}{r_{Gs}c^2}}.$$  

(26)

Via combining the last two equations, according to the GTR, we land at

$$m(r_{Gs})c^2 = m(R)c^2\sqrt{1 - 2\frac{GM}{r_{Gs}c^2}} \approx m(R)c^2(1 + \frac{GM}{Rc^2} - \frac{GM}{r_{Gs}c^2}).$$  

(27)

(Overall energy of the satellite on the $GsO$, as referred to the observer on the rotating Earth, according to the GTR)

The relative energy discrepancy $D$, between the classical prediction and the present prediction, thus as regards to light issued from $S$, via Eqs. (22) and (27), amounts to

$$|D| = \frac{1}{2} \frac{GM}{r_{Gs}c^2} - \frac{\omega^2 R^2}{2c^2} = \frac{1}{2} \frac{\omega^2}{c^2}(r_{Gs}^2 - R^2).$$  

(28)

(The relative frequency discrepancy $D$, between the classical prediction and the present prediction, as referred to $S$, on the $GsO$)

In fact, classically speaking, there is practically no contribution due to the rotation of Earth, on the frequency shift of light from $S$. Thence, the blue shift coefficient $B$, we expect, as referred to $S$ on the $GsO$, is given by Eq.(24):

$$B = \frac{GM}{Rc^2} + \frac{\omega^2 R^2}{2c^2} - \frac{3}{2} \frac{GM}{r_{Gs}c^2}.$$  

(29)

(The blue shift term expected based on the present theory as referred to $S$, situated on the $GsO$)

When measured from Earth, evidently [the energy delineated by the gravitational fall of the photon issued from $S/c^2$], i.e. $[GM/(Rc^2) - GM/(r_{Gs}c^2)]$, should be added to $B$, to yield the shift coefficient $B'$, one is to measure on Earth, in reference to the frequency of light issued from the same sample residing on Earth:

$$B' = \frac{GM}{Rc^2} + \frac{\omega^2 R^2}{2c^2} - \frac{3}{2} \frac{GM}{r_{Gs}c^2} + \frac{GM}{Rc^2} - \frac{GM}{r_{Gs}c^2} = 2\frac{GM}{Rc^2} + \frac{\omega^2 R^2}{2c^2} - \frac{5}{2} \frac{GM}{r_{Gs}c^2}.$$  

(30)

(The blue shift term delineated by the Satellite’s light, based on the present theory, if measured on Earth)

Recall that, the relative energy discrepancy $D$, between the classical prediction and the present prediction, is still given by Eq.(28). Thus as referred to the classical prediction, a weakening in the blue shift comes into play (because of the extra, unexpected red shift, due to the centrifugal effect), and this, precisely amounts to $(\omega^2/2c^2)(r_{Gs}^2 - R^2)$. This

\footnote{As referred to the distant observer at rest, with regards to remote stars, we should (supposing that the velocities are measured locally), write (cf. Chapter 10, Reference 17)

$$m(r_{Gs})c^2 = m_{\infty}c^2\sqrt{1 - 2\frac{GM}{r_{Gs}c^2}}\sqrt{1 - \frac{\omega^2 R^2}{c^4}}.$$}
unforeseen prediction is on the relative order of $10^{-10}$, and may well be checked out. One can, based on Eqs. (6), (23), and (26), further, provide an estimate on the correction to be brought to the velocity, at the geosynchronous orbit of radius $r_{Gs}$ as we switch from one approach to the other (these equations point to); the change in question, turns out to be on the order of $10^{-4}$ mm/s (for a geosynchronous orbit around Earth).

7. Discussion

Herein we have considered the effect of self rotation of Earth, on mainly, geosynchronous satellites. The effect of self rotation, on the determination of a geosynchronous orbit, has thus been disclosed, specifically.

Classically speaking, the orbit is determined by [cf. footnote in conjunction with Eq.(26), above, based on Chapter 10 of Reference 19]

$$m(r_{Gs})c^2 = m_0\infty c^2 \sqrt{1 - \frac{2GM}{r_{Gs}c^2}}/\sqrt{1 - \omega^2 r_{Gs}^2/c^2}, \quad (31)$$

supposing that the velocities are measured locally. According to the gravitational approach, used herein, but omitting the self rotation of Earth, the orbit would be determined by Eq.(6). The difference between the two theories then, would reside in merely, the difference between the $\exp[-GM/(r_{Gs}c^2)]$, and $\sqrt{1 - 2(GM/(r_{Gs}c^2))}$ terms, coming into play. Strikingly enough, in the approach presented herein, taking care of the self rotation of Earth, the orbit is rigorously determined, via just Newton Equation of Motion, i.e. Eq.(23). Yet it is important to emphasize that we did not write Eq.(23), via [8]

Gravitational Force = Mass × Acceleration of the Motion \hspace{1cm} (32)

Quite on the contrary, in the rotating frame we have picked up, once we get embarked in it, we have no motion at all. Instead, both gravitation and self rotation, affect the rest mass of the object, situated in the corresponding gravitational and centrifugal fields, owing to the law of energy conservation embodying the mass & energy equivalence of the STR (cf. Laws 1, 3 and 5), no matter whether mass cancels out of Eq.(23); this mass anyhow, is a rest mass, and it varies with the altitude.

Thence, Eq.(23) is fully relativistic.

Note that our approach, of course diverges from the classical prediction consisting, in just the Lorentz dilation, due to the tangential displacement, and neglecting any specific effect due to the acceleration, as such [7]. Nonetheless, a serious error in processing the data collected based on a rotating clock [13] is recently reported [14]. Furthermore, recent measurements did put firmly at stake, the classical belief [15]. All the more that, the present theory is well capable to predict the bound muon decay rate retardation, with an unequal success, up to now [16]. We thus anticipate that the approach presented herein, deserves to be critically considered.

Recall further that the $1/r^2$ dependency of the Newton gravitational attraction law, through for strictly static masses, is a straight requirement imposed by the STR (cf. Law

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\[5\] The differentiation of Eq.(6), for a circular orbit leads to [2,3] $v^2/c^2 = \alpha/(1 + \alpha)$, whereas the Newtonian approach yields $v^2/c^2 = \alpha$. The difference of these two expressions, for a given altitude, leads to a change $dv$ in $v$, i.e. $\Delta v = c^2 \alpha^2/(2v)$, or $\Delta v = \alpha^{3/2}$. For a geosynchronous orbit, $\alpha$ roughly is, $10^{-10}$. This then yields, $\Delta v \approx 10^{-4}$ mm/s.
2). Thus, what we have presented herein, is in full harmony with the law of energy conservation, and the STR, just the way our original approach leading to Eq.(6), is.

The validity of the approach we have presented herein may be checked by measuring the prediction we made about the weakening of the blue shift of light from a geosynchronous satellite. This remains on the relative order of $10^{-10}$ [cf. Eq.(30)]. Sardonically, the relativistically rigorous result on a geosynchronous orbit, is the Newtonian outcome, no matter what, we arrived at it, through a totally different mean than that delineated by the Newton Equation of Motion [cf. Eq.(32)].

Any derivation taking into account self rotation, should better be based on the approach we have presented herein, i.e. i) one should first ascend from the surface, with the payload, along the vertical tower, we have considered throughout, rotating with the celestial body, ii) then, at the appropriate altitude, deliver to the payload, its orbital motion, which is what we will undertake next.

In the foregoing derivation, we did not take into account the quantum mechanical stretching of lengths [cf. Eqs. (15-a) and (15-b)], for we did not need, the related precision.

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