TWMS J. App. and Eng. Math. V.10, N.4, 2020, pp. 909-917

ZERO FORCING AND POWER DOMINATION FOR LEXICOGRAPHIC PRODUCT OF TWO FUZZY SOFT GRAPHS

ZAHRA SADRI IRANI[1], ASEFEH KARBASIOUN[2], §

Abstract. Zero forcing and power domination are iterative processes on graphs where an initial set of vertices are observed, and additional vertices become observed based on some rules. In both cases, the goal is to eventually observe the entire graph using the fewest number of initial vertices. In this paper, we combine the study of zero forcing and power domination and compute upper bound for zero forcing for lexicographic product of two fuzzy soft graphs.
Keywords: Power domination, Zero forcing, Fuzzy soft graph, Lexicographic product.
AMS Subject Classification: (2010):05C78.

1. Introduction

We know that a graph [2, 15] is a symmetric binary relation on a nonempty set V. Similarly, a fuzzy graph is a symmetric binary fuzzy relation on a fuzzy subset. The concept of fuzzy sets and fuzzy relations was introduced by L. A. Zadeh in 1965 [22]. Rosenfeld [19] introduced the concept of fuzzy graph theory. After that fuzzy graph theory becomes a vast research area. Soft set theory has potential applications in many different fields including the smoothness of functions, game theory, operational research, perron integration, probability theory, and measurement theory [12, 14, 16]. Research works on soft sets are very active and progressing rapidly in these years. Maji et al. [11] defined theoretical study on the theory of soft sets. In 2015, Mohinta and Samanta [13] introduced the notions of fuzzy soft graphs, union, intersection of two fuzzy soft graphs with a few properties related to finite union and intersection of fuzzy soft graphs. The notion of zero forcing set, as well as the associated zero forcing number of a simple graph was introduced in [12] to bound of the minimum rank of associated matrices for numerous families of graphs. Let each vertex of a graph G be given one of two colors, "black" and "white" by convention. Let S denote the initial set of black vertices of G. The color-change rule converts the color of a vertex u_2 from white to black if the white vertex u_2 is the only white neighbor of a black vertex u_1; we say that u_1 forces u_2, which we denote by $u_1 \to u_2$. And a sequence, $u_1 \to u_2 \cdots \to u_i \to u_{i+1} \to \cdots \to u_t$, obtained through

[1]Department of Chemistry, Faculty of Science, Falavarjan Branch, Islamic Azad University, Iran.
e-mail: sadri@iaufala.ac.ir; ORCID: https://orcid.org/0000-0002-2890-8585.
[2]Department of Mathematics, Payam Noor University, (PNU), P.O.Box 19395-3697, Tehran, Iran.
e-mail: a.karbasioun@student.pnu.ac.ir; ORCID: https://orcid.org/0000-0003-2971-806X.
§ Manuscript received: February 19, 2019; accepted: August 24, 2019.

iterative applications of the color change rule is called a forcing chain. The set S is said to be a zero forcing set of G if all the vertices of G will be turned black after finitely many applications of the color-change rule. The zero forcing number of G, denoted by $Z(G)$, is the minimum of $|S|$ over all zero forcing sets $S \subseteq V(G)$. Independently, Amos et al. defined k-forcing in [5] to generalize zero forcing. The k-forcing number of G, denoted $Z_k(G)$, is the minimum number of vertices that need to be initially colored so that all vertices eventually become colored during the discrete dynamical process described by the following rule. Starting from an initial set of colored vertices and stopping when all vertices are colored: if a colored vertex has at most k non-colored neighbors, then each of its non-colored neighbors becomes colored. When $k = 1$, this is equivalent to the zero forcing number, usually denoted with $Z(G)$, a recently introduced invariant that gives an upper bound on the maximum nullity of a graph[9, 20]. In graph theory, a dominating set for a graph $G = (V, E)$ is a subset D of V such that every vertex not in D is adjacent to at least one member of D. The domination number $\gamma(G)$ is the number of vertices in a smallest dominating set for G. Power domination was introduced by Haynes et al. in [8]when using graph models to study the monitoring process of electrical power networks. When a power network is modeled by a graph, a power dominating set provides the locations where monitoring devices can be placed in order to monitor the power network.

Chang et al. defined k-power domination in [5]using sets of observed vertices. Given a graph G and a set of vertices S, initially all vertices in S and their neighbors are observed; all other vertices are unobserved. Iteratively apply the following propagation rule: if there exists an observed vertex u that has k or fewer unobserved neighbors, then all the neighbors of u are observed. Once this rule does not produce any additional observed vertices, if all vertices of G are observed, S is a k-power dominating set of G.

Although k-forcing and k-power domination have been studied independently, an in-depth analysis of k-power domination leads to the study of k-forcing [7]. Indeed, after the initial step in which a set observes itself and its neighbors, the observation process in k-power domination proceeds exactly as the color changing process in k-forcing. Recently Javid and coauthors studied on zero forcing for some product of two graphs[9]. Lately authores expand domination to fuzzy graphs [3, 4, 18, 21].

The aim of this paper is compute upper bound for zero forcing of the lexicographic product of two fuzzy soft graphs and then we get a relation between zero forcing and power domination for lexicographic product of two fuzzy soft graphs.

2. Definitions and notation

Let V be a nonempty finite set and $\rho : V \to [0, 1]$ and let $\mu : V \times V \to [0, 1]$ such that $\mu(u, v) \leq \rho(u) \wedge \rho(v)$ then the pair $G = (\rho, \mu)$ is called a fuzzy graph. The order of G is $ordG = \sum_{u_i \in V} \rho(u_i)$. Two vertices u and v are adjacent or neighbors in G if $\mu(u, v) \leq \rho(u) \wedge \rho(v)$. The (open) neighborhood of a vertex v is the set $N_G(v) = \{u \in V : \{u, v\} \in E\}$, and the closed neighborhood of v is the set $N_G[v] = N_G(v) \cup \{v\}$. Similarly, for any set of vertices S, $N_G(S) = \cup_{v \in S} N_G(v)$ and $N_G[S] = \cup_{v \in S} N_G[v]$. The degree of a vertex v is $deg_G(v) = \sum_{uv \in E(G)} \mu(u, v)$. The maximum and minimum degree of G are $\Delta(G) = \max\{deg_G(v) : v \in V\}$ and $\delta(G) = \min\{deg_G(v) : v \in V\}$, respectively; a graph G is regular if $\delta(G) = \Delta(G)$. We will omit the subscript G when the graph G is clear from the context.

Fuzzy soft graph over a graph (V, E) which is a triple (ρ, μ, A) where:

(1) A is a nonempty set of parameters.
(2) (ρ, A) is a fuzzy soft set over V.

(3) (μ, A) is a fuzzy soft set over E.
(4) $(\rho(a), A(a))$ is a fuzzy(sub)graph on (V, E) for all $a \in A$. That is,

$$\mu(a)(xy) \leq \min(\rho(a)(x), \rho(a)(y))$$

for every $a \in A$ and $x, y \in V$

Note that the fuzzy soft graph $G = (\rho(a), \mu(a))$ will be denoted by $H(a)$ Also, it is evident that a fuzzy soft graph is a parametrized family of fuzzy graphs. Let k be a nonnegative integer. The definition of k-power domination on a graph G will be given in terms of a family of sets, $(P^i_{G,k}(S))_i \geq 0$, associated to each set of vertices S in G.

$$P^0_{G,k}(S) = N[S]$$
$$P^{i+1}_{G,k}(S) = P^i_{G,k}(S) \cup \{N(v) : v \in P^i_{G,k}(S) \text{ and } 1 \leq |N_G(v) \backslash P^i_{G,k}(S)| \leq k\} \text{for } i \geq 0.$$

A set $S \subseteq V$ is a k-power dominating set of G if there is an integer l such that $P^i_{G,k}(S) = V$. A minimum k-power dominating set is a k-power dominating set of minimum cardinality. The k-power domination number is the cardinality of a minimum k-power dominating set and is denoted by $\gamma_{P,k}(G)$. We say that a set $S \subset V(G)$ is a power dominating set of a graph G if at the end of the process above $PD(S) = V(G)$. A minimum power dominating set is a power dominating set of minimum cardinality, and the power domination number, $\gamma_P(G)$ of G is the cardinality of a minimum power dominating set.

Then if $t = \{v_1, v_2, \ldots, v_n\} \subseteq V$ is a power dominating set for fuzzy graph $G = (\rho, \mu)$, then

$$\gamma_P(G) = \sum_{i=1}^{n} \rho(x_i)$$

The concept of zero forcing can be explained via a coloring game on the vertices of G. The color change rule is: If u is a black vertex and exactly one neighbor w of u is white, then change the color of w to black. We say u forces w and denote this by $u \to w$. A zero forcing set for G is a subset of vertices B such that when the vertices in B are colored blue and the remaining vertices are colored white initially, repeated application of the color change rule can color all vertices of G black. A minimum zero forcing set is a zero forcing set of minimum cardinality, and the zero forcing number $Z(G)$ of G is the cardinality of a minimum zero forcing set. Let T denote the set of black vertices. The color changing process in k-forcing can be formally described by associating to T the family of sets recursively defined by the following rules.

$$F^0_{G,k}(T) = T,$$
$$F^{i+1}_{G,k}(T) = F^i_{G,k}(T) \cup \{N(v) : v \in F^i_{G,k}(T) \text{ and } 1 \leq |N_G(v) \backslash F^i_{G,k}(T)| \leq k\} \text{for } i \geq 0.$$

A set $T \subseteq V$ is a k-forcing set of G if there is an integer t such that $F^t_{G,k}(T) = V$. A minimum k-forcing set is a k-forcing set of minimum cardinality. The k-forcing number of G is the cardinality of a minimum k-forcing set and is denoted by $Z_k(G)$. If $v \in F^i_{G,k}(T)$ and $|N(v) \backslash F^i_{G,k}(T)| \leq k$ then v is said to k-force (or simply force if k is clear from the context) every vertex in $N(v) \backslash F^i_{G,k}(T)$. Then if $t = \{v_1, v_2, \ldots, v_n\} \subseteq V$ is a zero forcing set for fuzzy graph $G = (\rho, \mu)$, then $Z(G) = \sum_{i=1}^{n} \rho(x_i)$. Let G and H be two graphs. The lexicographic product of G and H, denoted by $G_1 \circ G_2$, is the graph with vertex set $V(G) \times V(H) = \{(a, v) | a \in V(G) \text{ and } v \in V(H)\}$, where (a, v) is adjacent to (b, w) whenever $ab \in E(G)$ or $a = b$ and $uw \in E(H)$. For any vertex $a \in V(G)$ and $b \in V(H)$, we define the vertex set $H(a) = \{(a, v) \in V(G_1 \circ G_2) | v \in V(H)\}$ and $G(b) = \{(v, b) \in V(G_1 \circ G_2) | v \in V(G)\}$. It is clear that the graph induced by $H(a)$

called a layer $H(a)$, is isomorphic to H and the graph induced by $G(b)$, called a layer $G(b)$ is isomorphic to G, denoted by $H(a) \cong H$ and $G(b) \cong G$ respectively. We write $H(a) \cong H(b)$ when each vertex of $H(a)$ is adjacent to all vertices of $H(b)$ and vice versa, and $H(a) \not\sim H(b)$ means that no vertex of $H(a)$ is adjacent to any vertex of $H(b)$ and vice versa.

3. Preliminaries

The following observations follow directly from the definitions of k-power domination and k-forcing, and provide the initial connection between both concepts. In any graph G, if T is a k-forcing set, all sets are k-forcing sets of G; if S is a k-power dominating set of G, the sets are also k-forcing sets of G. In any graph G, if T is a k-forcing set of G then T is also a k-power dominating set. The converse is not necessarily true, but S is a k-power dominating set if and only if $N[S]$ is a k-forcing set. The proofs of the next results are omitted.

Lemma 3.1. *Let T be a k-forcing set of a graph G. Let $A \subseteq T$.*

1. *If A is k-forcing set of T in G, then A is a k-forcing set of G;*
2. *If A is k-power dominating set of T in G, then A is a k-power dominating set of G.*

Lemma 3.2. [10] *Let S be a k-power dominating set of a graph G. Let $A \subseteq S$.*

1. *If A is k-forcing set of $N[S]$ in G, then A is a k-forcing set of G.*
2. *If A is k-power dominating set of $N[S]$ in G, then A is a k-power dominating set of G*

Lemma 3.3. [10] *Let G be a graph and $X \subseteq V$ such that $G[X]$ is connected and every vertex $x \in X$ adjoin by at least $k+1$. Let u be an arbitrary vertex in X. Then $\{u\}$ is a (minimum) k-power dominating set of $N[x]$ in G. In addition if x adjoin by at least k vertices, then $\{u\}$ is also a (minimum) k-forcing set of $N[X]$ in G.*

Corollary 3.1. [10] *Let G if there is $u \in V(G)$ that adjoin by $k+1$ vertices, then k-power domination set has one vertices be a connected graph. If in addition there is a vertex by at least k neighbors then k-forcing set has one vertices.*

Corollary 3.2. [10] *Let G be a connected graph, $X \subseteq V$ and $u_j \in V(G[X]_j)$ for every $j = 1, \ldots, c(G[X])$. Let $S = \{u_1, \ldots, u_{c(G[X])}\}$. If for every $x \in X$, x adjoin by at most $k+1$ vertices then S is a minimum k-power dominating set of $N[X]$ in G; if in addition for $j = 1, \ldots, c(G[X])$, u_j adjoin by at most k vertices, then S is a minimum k-forcing set of $N[X]$ in G.*

Lemma 3.4. [6] *If G is connected there is $u \in V(G)$ by at least $k+2$ neighbors, then there exists a minimum k-power dominating set S such that for all $v \in S$, v adjoin by at least $k+2$ vertices.*

Lemma 3.5. [10] *Let G be a connected graph and let $X \subseteq V(G)$. There exists $S \subseteq X$ such that S is a minimum k-power dominating set of $\hat{X}$.*

Lemma 3.6. [10] *Let G be a connected graph and let $X \subseteq V(G)$. If $S \subseteq X$ is a minimum k-power dominating set of $\hat{X}$, then S is a k-power dominating set of $N_G[X]$ in G.*

Theorem 3.1. [10] *Let G be a connected graph and let $P_1, P_2, \ldots, P_r$ be a partition of G. Then*

$$\gamma_{P,k}(G) \leq \sum_{i=1}^{r} \gamma_{P,k}(\hat{P_l})$$

Lemma 3.7. [20] *Let $\{u_1, u_2, \dots, u_t\}$ be a power dominating set for a graph G with no isolated vertices. Then*

$$Z(G) \leq \sum_{i=1}^{t} d(u_i)$$

Theorem 3.2. [10] *Let G be a connected graph and let $P_1, P_2, \dots, P_r$ be a partition of V. If $\hat{P_l}$ has a minimum k-power dominating set in P_i for every $i = 1, 2, \dots, r$ then*

$$Z_k(G) \leq \sum_{i=1}^{r} Z_k(\hat{P_l})$$

Lemma 3.8. [7] *In every connected graph G with there are some vertex adjoin by at least $k+2$ vertices, there exists a minimum k-power dominating set S in which every vertex has at least $k+1$ S-private neighbors.*

Lemma 3.9. [10] *In every connected graph G with there are some vertex adjoin by at least $k+2$ vertices there exists a minimum k-power dominating set S in which every vertex has at least $k+1$ external S-private neighbors.*

4. Main result

Operations on graphs such as union, intersection, composition and etc. are methods to extend them. S. Mohinta in [13] showed that the union and intersection of fuzzy soft graphs is again fuzzy soft graph. In this section we show lexicographic product of two fuzzy soft graphs is fuzzy soft graphs and compute upper bound for zero forcing of the lexicographic product of two fuzzy soft graphs and then we get a relation between zero forcing and power domination for lexicographic product of two fuzzy soft graphs.

Definition 4.1. *Let $G = (V_1 \times V_2, E)$ denotes the lexicographic product of graph $G_1 = (V_1, E)$with graph $G_2 = (V_2, E)$, where*

$$E = E^{'} \cup \{(u_1u_2, v_1v_2) \in E_1, u_2 \neq v_2\}$$

and where $E^{'}$ is defined as in

$$E^{'} = \{(uu_2, vv_2) | v \in V_1, u_2v_2 \in E_2\} \cup \{(u_1w, v_1w) | w \in V_2, u_1v_1 \in E_1\}$$

Consider the fuzzy soft graphs $G_i = (\rho_i, \mu_i, A_i)$ on (V_i, E_i), $i = 1, 2$. The lexicographic product of two fuzzy soft graphs $G_1 = (\rho_1, \mu_1, A_1)$ and $G_2 = (\rho_2, \mu_2, A_2)$ denoted by $G_1 \circ G_2 = (\rho, \mu, A_1 \times A_2)$ where:

$$\begin{cases} \rho = (\rho_1 \circ \rho_2) : A_1 \times A_2 \to FS(V_1 \times V_1) \\ \rho = (\rho_1 \circ \rho_2)(a, b)(u_1, u_2) := (\rho_1\rho_2)(a, b)(u_1, u_2) \end{cases}$$

and

$$\mu = (\mu_1 \circ \mu_2) : A_1 \times A_2 \to E$$

$$\mu = (\mu_1 \circ \mu_2)(a, b) := \begin{cases} \mu_1\mu_2(a, b), (u_1u_2, v_1v_2) \in E^{'} \\ \min\{\mu_2(b)(u_2), \mu_2(b)(v_2), \mu_1(a)(u_1v_1)\}, (u_1, u_2v_1v_2) \in E - E^{'} \end{cases}$$

We can generalize the concept of zero forcing for fuzzy graph.

Definition 4.2. *Suppose $G = (\rho, \mu)$ is a fuzzy graph, and S is a zero forcing in crisp graph. We define fuzzy zero forcing number $Z(G) = \wedge \sum_{v \in S} \rho(v)$.*

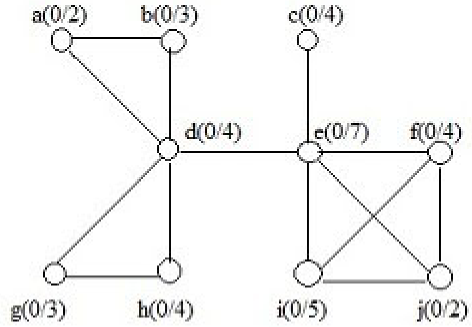

FIGURE 1. A fuzzy graph

Example 4.1. *In figure 1 $A = \{a, b, g, f, j\}$ and $B = \{h, d, a, i, j\}$ are two zero forcing sets by $\sum_{v \in S} \rho(v) = 1.4$ and $\sum_{v \in S} \rho(v) = 1.9$, then by defintion of zero forcing number for fuzzy graphs, $Z(G) = 1.4$.*

Proposition 4.1. *Consider $G = (V_1 \times V_2, E)$ be the lexicographic product of graph $G_1 = (V_1, E)$with graph $G_2 = (V_2, E)$. Let G_i be a fuzzy soft graph on $G_i = (V_i, \mu_i)$, $i = 1, 2$. Then $G_1 \circ G_2 = (\rho_1 \circ \rho_2, \mu_1 \circ \mu_2, A_1 \times A_2)$ is a fuzzy soft graph on $G = (V_1 \times V_2, E)$*

Proof. For all $a \in A_1, b \in A_2, (u_1u_2, v_1v_2) \in E - E'$:

$$\begin{aligned}(\mu_1 \circ \mu_2)(a, b)(u_1u_2, v_1v_2) &= \min\{\rho_2(b)(u_2), \rho_2(b)(v_2), \mu_1(a)(u_1v_1)\}\\ &\leq \min\{\rho_2(b)(u_2), \rho_2(b)(v_2), \min(\rho_1(a)(u_1), \rho_1(b)(v_2))\}\\ &= \min\{\min(\rho_1(a)(u_1), \rho_2(b)(u_2), \min(\rho_1(a)(v_1), \rho_2(b)(v_2)))\}\\ &= \min\{(\rho_1 \circ \rho_2)(a, b)(u_1, u_2), (\rho_1 \circ \rho_2)(a, b)(v_1, v_2)\}\end{aligned}$$

□

If $G_1 = (\rho_1, \mu_1, A_1)$ and $G_2 = (\rho_2, \mu_2, A_2)$ are nontrivial graphs and S is zero forcing set for $G_1 \circ G_2 = (\rho_1 \circ \rho_2, \mu_1 \circ \mu_2, A_1 \times A_2)$, then $|S| \geq 2$.

Theorem 4.1. *Let $G = (\rho_1, \mu_1, A_1)$ be a connected fuzzy soft graph and $H = (\rho_2, \mu_2, A_2)$ be an arbitrary fuzzy soft graph containing $k \geq 1$ components $H_1, H_2, \dots, H_k$ and $m_i \geq 2$. Let Z be a zero forcing set of $G_1 \circ G_2$ and T_i is a zero forcing set for H_i, if $Z_i(u) = Z \cap H_i(u)$ then*

$$|Z_i(u)| \geq |T_i|$$

Proof. Suppose that $|Z_i(u)| < |T_i|$ and $Z_i(u) = \{(u, v_1), (u, v_2), \dots, (u, v_t)\}$ for some zero forcing basis T_i of H_i, where $\{v_1, v_2, \dots, v_t\} \subseteq V(H_i)$. Then, each black vertex in $H_i(u)$ has more than one white neighbors and no vertex of $H_i(u)$ can be forced by any vertex in $H_i(w)$ for any $w \in V(G)$, $i \neq j$. Hence

$$|Z_i(u)| \geq |T_i|$$

□

Theorem 4.2. *Let $G = (\rho_1, \mu_1, A_1)$ be a connected fuzzy soft graph and $H = (\rho_2, \mu_2, A_2)$ be an arbitrary graph containing $k \geq 1$ components $H_1, H_2, \dots, H_k$ and $m_i \geq 2$, let $G_1 \circ G_2 =$*

$(\rho_1 \circ \rho_2, \mu_1 \circ \mu_2, A_1 \times A_2)$ and $N = \{(u_c, v_d^i)|1 \leq i \leq k\}$, such that u_c, v_d^i are arbitrary vertexes of G, H respectively. Then

$$Z(G_1 \circ G_2) \leq (\sum_{i=1}^{n}\sum_{j=1}^{k}\sum_{l=1}^{m_j} \rho(u_i, v_l^j) - \sum_{j=1}^{k} \rho(u_c, v_d^i)$$

Proof. Let $V(G) = \{u_1, u_2, \ldots, u_n\}$ and $V(H_i) = \{v_1^i, v_2^i, \ldots, v_{m_i}^i\}$ for $1 \leq i \leq k$. Suppose $Z = V(G_1 \circ G_2)\backslash N$. We claim that Z is a zero forcing set of $G_1 \circ G_2$. To prove the claim, assume that Z is initially colored black. Clearly for any $vp^i \nsim v_q^j$, in hence $(u_r, vp^i) \nsim (u_r, v_q^j)$, $1 \leq r \leq n$, is in $G_1 \circ G_2$. Since H_i is connected for $1 \leq i \leq k$, so there exists at least one vertex v_l^i such that $v_d^i \sim v_l^i$ in H_i and hence in $(u_r, v_d^i) \sim (u_r, v_l^i)$ in $G_1 \circ G_2$. Therefore $(u_c, v_l^i) \to (u_c, v_d^i)$, for $1 \leq i \leq k$. Hence,

$$Z(G_1 \circ G_2) \leq (\sum_{i=1}^{n}\sum_{j=1}^{k}\sum_{l=1}^{m_j} \rho(u_i, v_l^j) - \sum_{j=1}^{k} \rho(u_c, v_d^i)$$

$\square$

Example 4.2. *In figure 2, it was shown zero forcing set for lexicographic product of two fuzzy graphs and zero forcing number is $\sum_{v \in S} \rho(v) = 1.4$.*

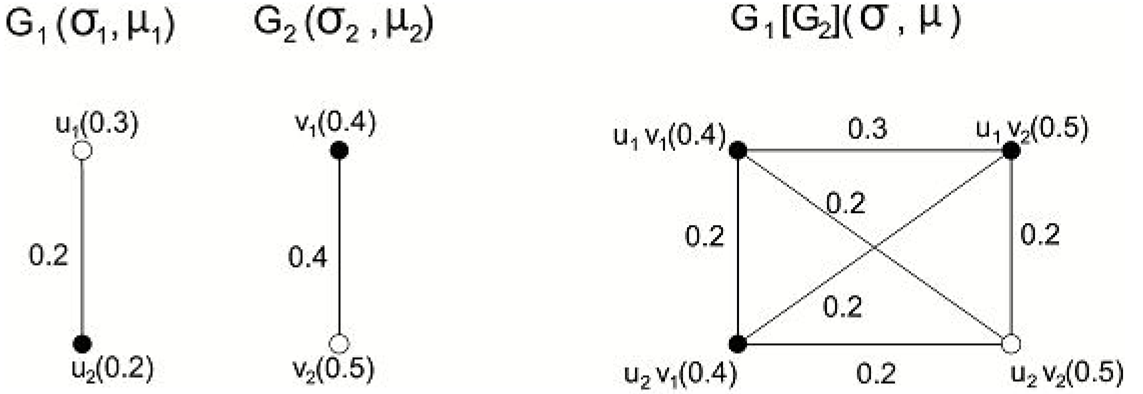

FIGURE 2. A fuzzy graph

Theorem 4.3. *If $G = (\rho, \mu, A)$ is a lexicographic product of two connected fuzzy soft graphs and some vertex in G adjoin by at least $k + 2$ vertices and $S = \{u_1, u_2, \ldots, u_t\}$ be a minimum k-power dominating set of G in which every vertex has at least $k + 1$ external S-private neighbors, then*

$$Z_k(G) \leq (\gamma_{P,k} + t\Delta(G))$$

Proof. By hypothesis, for each $i = 1, \ldots, t$ there exists a set $\{X_1^i, X_2^i, \ldots, X_k^i\}$ of external S-private neighbors of u_i. We prove that $T : \bigcup_{i=1}^{t}(N[u_i]\backslash\{x_1^i, x_2^i, \ldots, x_k^i\})$ is a k-forcing set of G. Since $\{x_1^i, x_2^i, \ldots, x_k^i\}$ are external S-private neighbors of u_i then $\{x_1^i, x_2^i, \ldots, x_k^i\} \cap S = \emptyset$ which implies $u_i \in T$, for every $i = 1, \ldots, t$. In the first step of the k-forcing process each vertex u_i forces $x_1^i, x_2^i, \ldots, x_k^i$ so T is a k-forcing set of $N[S]$ in G. Since S is a k-power dominating set of G, by observation 3.2 $N[S]$ is a k-forcing set of G. Then T is a k-forcing set of G. So,

$$Z_k(G) \leq |T| \leq (\gamma_{P,k} + t\Delta(G)).$$

$\square$

5. Conclusion

In this paper basic definitions of k-power domination and k-forcing extended for fuzzy soft graphs and compute upper bound for zero forcing of the lexicographic product of two fuzzy soft graphs and then we get a relation between zero forcing and power domination for lexicographic product of two fuzzy soft graphs.

References

[1] Amos, D., Caro, Y., Davila, R., Pepper, R., (2015), Upper bounds on the k-forcing number of a graph, Disccrete Applied Math, 181, pp 1-10.
[2] Bollobas, B., (2005), Modern Graph Theory, Springer Verlag,
[3] Borzooei, R.A. and Rashmanlou, H., (2015), Dominating in vague graph and its applications, Journal of Intelligent and Fuzzy Systems, 29, pp 1933-1940.
[4] Borzooei, R. A. And Rashmanlou, H., (2016), Semi global domination sets in vague graphs with application, Journal of Intelligent and Fuzzy Systems, 30, no.6, pp 3645-3652.
[5] Chang, G.J., Dorbec, P., Montassier, M., Raspaud, A., (2012), Generalized power domination in graphs, Discrete Applied Math., 160, pp 1691-1698.
[6] Dean, N., Ilic, A., Ramirez, I., Shen, J., Tian, K., (2011), On the Power Dominating Sets of Hypercubes, IEEE International Conference on Computational Science and Engineering, IEEE Xplore: 01 November, (CSE), pp. 488-491.
[7] Ferrero, D., Hogben, L., Young, M., (2018), The relationship between k-forcing and k-power domination, Discrete Mathematics, 341, no. 6, pp 1789-1797.
[8] Haynes, T.W., Hedetniemi, S.M., Hedetniemi, S.T., Henning, M.A., (2002), Domination in graphs applied to electric power networks, SIAM J. Discrete Math., 15, pp 519-529.
[9] Javid, I., Irshad, I., Batool, M., Raza, Z., (2016), On the zero forcing number of corona and Lexicographic product of graphs, arXiv:1607.04071v1 [math.CO].
[10] Koh, K.M., Soh, K.W, (2016), Power domination of the Cartesian product of graphs, International journal of Graphs and Combinatorics, 13, no.1, pp 22-30.
[11] Maji, P. K., Biswas, R. and Roy, A. R, (2003), Soft set theory, Computers and Mathematics with Applications, 45, no. 4-5, pp 555-562.
[12] Barioli, F., Barrett, W., Butler, S. M., Cioabă, S.M., Cvetković, D., Fallat, S.M., Godsil, C., Haemers, W., Hogben, L., Mikkelson, R., Narayan, S., Pryporova, O., Sciriha, I., So, W., Stevanović, D., van derHolst, H., Vander Meulen, K. And Wehe A. W., (2008), Zero forcing sets and the minimum rank of graphs, Linear Algebra Appl., 428, no.7, pp 1628 - 1648.
[13] Mohinta, S. and Samanta, T.K., (2015), An introduction to fuzzy soft graphs, Mathematica Moravica, 19, no.2, pp 35-48.
[14] Molodtsov, D., (1999), Soft set theory—frst results, Computers and Mathematics with Applications, 37, no.4-5, pp 19-31.
[15] Mordeson, J. N., Nair, P. S., (2000), Fuzzy Graphs and Fuzzy Hypergraphs, Physica. Verlag.
[16] Molodtsov, D., (2004), The Theory of Soft Sets (in Russian), URSS Publishers, Moscow.
[17] Pouhassani, A., Doostie, H., (2017), On the properties of fuzzy soft graphs, Journal of Information Optimization Sciences, 38, no.3-4, pp 541-557.
[18] Rashmanlou, H., Samanta, S., Pal, M., Borzooei, R.A, (2015), A study on bipolar fuzzy graphs, Journal of Intelligent Fuzzy Systems, 28(2), pp 571-580.
[19] Rosenfeld. A., (1975), Fuzzy graphs, in :L.A. Zadeh, K.S. Fu, K. Tanaka, M. Shimura(Eds.), Fuzzy Sets and Their Applications to Cognitive and Decision Process, Academic Press, New York , pp 75-95.
[20] Row, D.D., (2012), A technique for computing the zero forcing number of a graph with a cut– vertex, Linear Algebra and its Applications, 436, pp 4423-4432.
[21] Talebi, A.A., Zameni, M. and Rashmanlo, H., (2017), Vertex Domination Critical in Circulant Graphs, Intern. J. Fuzzy Mathematical Archive, 12, pp 55-66.
[22] Zadeh, L. A., (1965), Fuzzy sets, Information and Control 8(3), pp 338-353.

Zahra Sadri Irani is faculty member of Islamic Azad University of Falavarjan. She has published 3 research articles in international journals of mathematical and chemistry science. She has present of 2 research papers at national conferences in Iran. Her research interests is mathematical chemistry.

Asefeh Karbasioun is PhD student of Payame Noor University of Tehran, Iran. Her research interests are Discrete Mathematics and Fuzzy Graph and Nano Computation. She has published 9 research papers in various international journals. She has presented 5 research papers at national conferences in Iran.