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FUZZY ZERO DIVISOR GRAPH IN A COMMUTATIVE RING

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Abstract. Let R be a commutative ring and let $\Gamma(Z_n)$ be the zero divisor graph of a commutative ring R, whose vertices are non-zero zero divisors of Z_n, and such that the two vertices u, v are adjacent if n divides uv. In this paper, we introduce the concept of fuzzy zero zivisor graph in a commutative ring and also discuss the some special cases of $\Gamma_f(Z_{2p})$, $\Gamma_f(Z_{3p})$, $\Gamma_f(Z_{5p})$, $\Gamma_f(Z_{7p})$ and $\Gamma_f(Z_{pq})$. Throughout this paper we denote the Fuzzy Zero Divisor Graph(FZDG) by $\Gamma_f(Z_n)$.

Keywords: Fuzzy graph, Zero divisor graph, Fuzzy zero divisor graph(FZDG).
AMS Subject Classification: 05C25, 05C69.

1. Introduction

The zero divisor graph is very useful to find the algebraic structures and properties of rings. The idea of a zero divisor graph of a commutative ring was introduced by I. Beck's in [2]. Given a ring R, let G(R) denote the graph whose vertex set is R, such that distinct vertices r and s are adjacent provided that rs = 0. I.Beck's main interest was the chromatic number $\chi(G(R))$ of the graph G(R).

Rosenfeld [7] defined a fuzzy graph as a graph that consists of vertices and edges with membership value in the interval [0,1]. More specifically, he defined a fuzzy graph as a pair $G = (\sigma, \mu)$ of functions $\sigma : S \to [0, 1]$ and $\mu : S \times S \to [0, 1]$ where for all $x, y \in S$ we have $\mu(x, y) = \mu(y, x)$ and $\mu(x, y) = \sigma(x) \wedge \sigma(y)$ with $\wedge$ denoting the minimum. The general terminology, notation everything based on the papers [[1], [3] $-$ [6]].

In this paper, an attempt to combine the two concepts: Fuzzy graph theory and zero divisor graph of a commutative ring together by introducing a new concept called fuzzy zero divisor graph of commutative ring. Finally, we discuss some specal cases of $\Gamma_f(Z_{2p})$, $\Gamma_f(Z_{3p})$, $\Gamma_f(Z_{5p})$, $\Gamma_f(Z_{7p})$ and $\Gamma_f(Z_{pq})$.

2. Preliminaries

Definition 2.1. *[8] A fuzzy graph as a pair $G = (\sigma, \mu)$ of functions $\sigma : S \to [0, 1]$ and $\mu : S \times S \to [0, 1]$ where for all $x, y \in S$ we have $\mu(x, y) = \mu(y, x)$ and $\mu(x, y) = \sigma(x) \wedge \sigma(y)$ with $\wedge$ denoting the minimum.*

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Definition 2.2. *[1] Let R be a commutative ring (with 1) and let Z(R) be its set of zero-divisors. We associate a (simple) graph $\Gamma(R)$ to R with vertices $Z(R)^* = Z(R) - 0$, the set of nonzero zero-divisor of R, and for distinct $x, y \in Z(R)^*$ the vertices x and y are adjacent if and only if $xy = 0$. Thus $\Gamma(R)$ is the empty graph if and only if R is an integral domain.*

3. Fuzzy Zero Divisor Graph

Definition 3.1. *An fuzzy zero divisor graph(FZDG) is of the form $G =< V, V_f, E_f >$ then the vertex set of non-zero zero divisor graph is*

$$\Gamma(Z_n) = V = \{(u, v) : uv = 0 \ (multiplication\ modulo\ n), \forall u, v \in V\} \tag{1}$$

Let p and q are any prime numbers with $p < q$. Then

$$V_f = \left\{V_1^f \cup V_2^f \mid \forall\ V_1^f, V_2^f \in (0,1)\right\} \text{ such that } V_f : V \to (0,1) \tag{2}$$

where $V_1^f = \left\{\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p}\right\}$ *and* $V_2^f = \left\{\frac{1}{q}, \frac{2}{q}, \frac{3}{q}, ..., \frac{q-1}{q}\right\}$, $0 < V_f < 1$.

$$E_{pq}^f = \left\{E_q^f, E_{2q}^f, E_{3q}^f, ..., E_{pq}^f \mid \forall\ E_{jq}^f \in (0,1]\right\} \text{ such that } E_f : V \to (0,1] \tag{3}$$

where $j = 1, 2, 3, ..., p$, $0 < E_f \leq 1$.
If any one of the condition is not satisfied, then the graph G is not an FZDG.

Definition 3.2. *A fuzzy graph $G : (V_f, E_f)$ is said to be a fuzzy star graph if $\Gamma(Z_n)$ is a zero divisor graph where n=2p and $p > 2$.*

Theorem 3.1. *[4] For (Z_{2p}), where p is any prime number then $\gamma_c(\Gamma(Z_{2p})) = 1$. Also, if n=8,9 then $\gamma_c(\Gamma(Z_n)) = 1$.*

Theorem 3.2. *[4] In (Z_{3p}) where p is any prime with $p > 3$, then $\gamma_c(\Gamma(Z_{3p})) = 2$.*

Theorem 3.3. *[4] If $p > 5$ is any prime, then $\gamma_c(\Gamma(Z_{5p})) = 2$.*

Theorem 3.4. *[4] For any graph (Z_{7p}) where p is any prime with $p > 7$, then $\gamma_c(\Gamma(Z_{7p})) = 2$.*

Theorem 3.5. *[4] If p and q are distinct primes and $q > p$, then $\gamma_c(\Gamma(Z_{pq})) = 2$.*

Theorem 3.6. *If $n = 2p$ where p is any prime and $p > 2$ then $\Gamma_f(Z_{2p})$ be the non-zero fuzzy zero divisor graph is $K_{1,p-1}^f$ fuzzy star graph.*

Proof. Let $\Gamma(Z_{2p})$ be a non-zero zero divisor graph. Then the vertex set of non-zero zero divisor graph $V = \{(u, v) : uv = 0 \ (multiplication\ modulo\ n), \forall u, v \in V\}$. Take two distinct vertex sets V_1 and V_2 in $\Gamma(Z_{2p})$ where $V_1 = \{2\}$ and $V_2 = \{2, 4, 6, ..., 2(p-1)\}$ then clearly we know that a vertex $2 \in V_1$ adjacent to all the vertices in V_2.

Clearly, $\Gamma(Z_{2p})$ is isomorphic with $K_{1,p-1}$.

Case(i): If V_f is fuzzy vertex set. The vertex set of fuzzy zero divisor graph V_f is partition in to two vertex subsets namely, V_{f_1} and V_{f_2}. Let

$$V_{f_1} = \left\{\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p}\right\}$$

where p is any prime number with $p > 2$.
That is,

$$V_{f_1} = \left\{ \frac{N}{p} \mid N = 1, 2, 3, ..., p-1 \right\}$$

with $V_{f_1} : V \to (0, 1)$. Let $V_{f_2} = \left\{ \frac{1}{2} \right\}$, with $V_{f_2} : V \to (0, 1)$
Since,

$$\begin{aligned} V_f &= V_{f_1} \cup V_{f_2} \\ V_f &= \left\{ \frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p} \right\} \cup \left\{ \frac{1}{2} \right\} \\ &= \left\{ \frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p}, \frac{1}{2} \right\} \\ &= \{ V_{f_1} \cup V_{f_2} \} : V \to (0, 1) \\ V_f &=: V \to (0, 1) \end{aligned}$$

Hence V_f be the fuzzy vertex set.

Case(ii): If E_f is fuzzy edge set. Let as take any two vertices $u, v \in V(\Gamma(Z_{2p}))$ and

$$\Gamma(Z_{2p}) = \{(u, v) : uv = 0 \; multiplication \; modulo \; n, \forall u, v \in V\}.$$

Let $u = 2(p-1)$ and $v = p$ then $uv = 2(p-1).p = 2p(p-1)$. Clearly, 2p must divides $2p(p-1)$, then there exist a edge connect between u and v.
Let E_f be a collection of edges.

$$E_f = \left\{ 1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{p-1} \right\},$$

where p is any prime number with $p > 2$.

$$E_f = \left\{ \frac{1}{N} \mid N = 1, 2, 3, ..., p-1 \right\}.$$

Thus, clearly $E_f : V \to (0, 1]$.
Clearly we know that every vertex in V_{f_1} is adjacent to all the vertices in V_{f_2}. Hence the graph $K^f_{1,p-1}$ is fuzzy star graph.

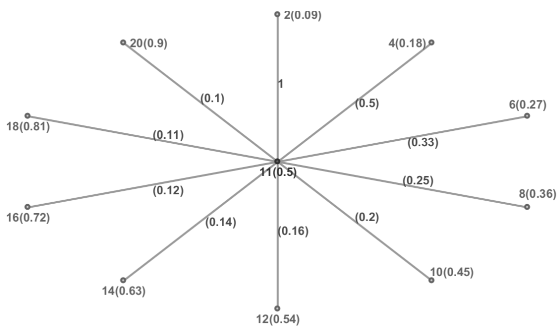

FIGURE 1. $\Gamma_f(Z_{22})$

□

Theorem 3.7. *If $n = 3p$ where p is any prime and $p > 3$ then $\Gamma_f(Z_{3p})$ be the non-zero fuzzy zero divisor graph is $K^f_{2,p-1}$ fuzzy complete bipartite graph.*

Proof. Let p is any prime number with greater than 3. Then the vertex set of $\Gamma(Z_{3p})$ is $V = \{(u, v) : uv = 0 \ \textit{multiplication modulo } n, \forall u, v \in V\}$. Take two distinct vertex sets V_1 and V_2 in $\Gamma(Z_{3p})$ where $V_1 = \{p, 2p\}$ and $V_2 = \{3, 6, 9, ..., 3(p-1)\}$. Every vertex in V_1 is adjacent to all the vertices in V_2. Clearly, $\Gamma(Z_{3p})$ is a complete bipartite graph, namely $K_{2,p-1}$.
Let V_f be a fuzzy vertex set. Let us show that $V_f : V \to (0, 1)$.
Fuzzy vertex set V_f is partition in to two vertex subsets, namely V_{f_1} and V_{f_2}.
Let

$$V_{f_1} = \left\{\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p}\right\},$$

where p is any prime with $p > 3$.
That is,

$$V_{f_1} = \left\{\frac{N}{p} \mid N = 1, 2, 3, ..., p-1\right\},$$

with $V_{f_1} : V \to (0, 1)$.

Let $V_{f_2} = \left\{\frac{1}{3}, \frac{2}{3}\right\}$ with $V_{f_2} \to (0, 1)$
Clearly,

$$\begin{aligned} V_f &= V_{f_1} \cup V_{f_2} \\ &= \left\{\left(\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p}\right) \cup \left(\frac{1}{3}, \frac{2}{3}\right)\right\} \\ &= \left\{\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p}, \frac{1}{3}, \frac{2}{3}\right\} \end{aligned}$$

where p is any prime with $p > 3$.

Thus, $V_f = \{V_{f_1} \cup V_{f_2}\} : V \to (0, 1)$ which implies that $V_f : V \to (0, 1)$
Hence, V_f is a fuzzy vertex set.

Let E_f be a fuzzy edge set. Let as show that $E_f : V \to (0, 1]$.
Let as take any two vertices $u, v \in V(\Gamma(Z_{3p}))$. Edge set E(G) defined by
$E(G) = \{(u, v) : uv = 0 \ (\textit{multiplication modulo } n), \forall u, v \in V\}$. Let u = p, v = 2p or u = 2p, v = p. Then $uv = 2p \times p = 2p^2$ which does not divide by 3p.

Therefore u and v are non-adjacent vertices in $\Gamma(Z_{3p})$. Let x be any other vertex in $\Gamma(Z_{3p})$ such that $ux = vx = 0$. That is the remaining vertices in $\Gamma(Z_{3p})$ are adjacent to both u and v.

Let E_p^f, $E_{2p}^f \in E_f$, where E_p^f and E_{2p}^f are collection of fuzzy edges from p and 2p respectively. since vertex p and 2p are the adjacent to all the vertices in $\Gamma(Z_{3p})$.

$$E_p^f = \left\{1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{p-1}\right\}$$

$$E_{2p}^f = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, ..., \frac{1}{2(p-1))}\right\}$$

$$E_f = \left\{E_p^f, E_{2p}^f\right\}$$

$$E_f = \left[\left\{1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{p-1}\right\}, \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, ..., \frac{1}{2(p-1))}\right\}\right]$$

where p is any prime number with $p > 3$.
which implies $E_f : V \to (0,1)$. Hence E_f is a fuzzy edge set. Clearly, $\Gamma_f(Z_{3p}) = (V, V_f, E_f)$ be a fuzzy zero divisor graph in $\Gamma_f(Z_{3p})$ and we know that every vertex in V_{f_1} is adjacent to all the vertices in V_{f_2}. Hence that the graph $K_{2,p-1}^f$ is fuzzy complete bipartite graph. □

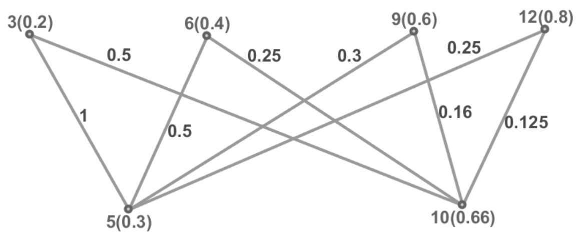

FIGURE 2. $\Gamma_f(Z_{15})$

Theorem 3.8. *If $n = 5p$ where p is any prime and $p > 5$ then $\Gamma_f(Z_{5p})$ be the non-zero fuzzy zero divisor graph is $K_{4,p-1}^f$ fuzzy complete bipartite graph.*

Proof. Let p is any prime number, greater than 5. Then the vertex set of $\Gamma(Z_{5p})$ is $V = \{(u,v) : uv = 0 \ multiplication\ modulo\ n, \forall u, v \in V\}$ where n=3p. Take two distinct vertex sets V_1 and V_2 in $\Gamma(Z_{5p})$ where $V_1 = \{p, 2p, 3p, 4p\}$ and $V_2 = \{5, 10, 15, ..., 5(p-1)\}$. Every vertex in V_1 is adjacent to all the vertices in V_2. Clearly, $\Gamma(Z_{5p})$ complete bipartite graph, namely $K_{4,p-1}$.

Let V_f be a fuzzy vertex set. Let as show that $V_f : V \to (0,1)$. Fuzzy vertex set V_f is partition in to two vertex subsets, namely V_{f_1} and V_{f_2}.
Let

$$V_{f_1} = \left\{\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p}\right\},$$

where p is any prime number with $p > 5$.
That is,

$$V_{f_1} = \left\{\frac{N}{p} \mid N = 1, 2, 3, ..., p-1\right\},$$

with $V_{f_1} : V \to (0,1)$.

Let $V_{f_2} = \left\{\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}\right\}$ with $V_{f_2} \to (0, 1)$
Clearly,

$$\begin{aligned} V_f &= V_{f_1} \cup V_{f_2} \\ &= \left\{ \left(\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p} \right) \cup \left(\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5} \right) \right\} \\ &= \left\{ \frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5} \right\} \end{aligned}$$

where p is any prime number with $p > 3$.

Thus, $V_f = \{V_{f_1} \cup V_{f_2}\} : V \to (0, 1)$ which implies that $V_f : V \to (0, 1)$
Hence, V_f is a fuzzy vertex set.

Let E_f is fuzzy edge set. Let as show that $E_f : V \to (0, 1]$. Let as take any two vertices $u, v \in V(\Gamma(Z_5 p))$. Let $u = 2p$ and $v = 3p$ in $\Gamma(Z_{5p})$ then 5p does not divides $uv = 6p^2$, which implies that no two vertices of V_1 and V_2 are adjacent.

Let E_p^f, E_{2p}^f, E_{3p}^f, $E_{4p}^f \in E_f$, where E_p^f, E_{2p}^f, E_{3p}^f and E_{4p}^f are collection of fuzzy edges from p, 2p, 3p and 4p respectively. Since vertex p, 2p, 3p and 4p are the adjacent to all the vertices in $V_2(\Gamma(Z_{5p}))$.

$$\begin{aligned} E_p^f &= \left\{ 1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{p-1} \right\} \\ E_{2p}^f &= \left\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, ..., \frac{1}{2(p-1)} \right\} \\ E_{3p} &= \left\{ \frac{1}{3}, \frac{1}{6}, \frac{1}{9}, ..., \frac{1}{3(p-1)} \right\} \\ E_{4p}^f &= \left\{ \frac{1}{4}, \frac{1}{8}, \frac{1}{12}, ..., \frac{1}{4(p-1)} \right\} \\ E_f &= [E_p^f, E_{2p}^f, E_{3p}^f, E_{4p}^f] \\ E_f &= \left[\left\{ 1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{p-1} \right\}, \left\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, ..., \frac{1}{2(p-1)} \right\}, \left\{ \frac{1}{3}, \frac{1}{6}, \frac{1}{9}, ..., \frac{1}{3(p-1)} \right\}, \right. \\ & \left. \left\{ \frac{1}{4}, \frac{1}{8}, \frac{1}{12}, ..., \frac{1}{4(p-1)} \right\} \right] \end{aligned}$$

where p is any prime with $p > 5$, which implies $E_f : V \to (0, 1]$. Hence, E_f is a fuzzy edge set. Clearly, $\Gamma_f(Z_{5p}) = (V, V_f, E_f)$ be a fuzzy zero divisor graph in $\Gamma_f(Z_{5p})$. Every vertex in V_{f_1} is adjacent to all the vertices in V_{f_2}. Hence that the graph $K_{4,p-1}^f$ is fuzzy complete bipartite graph.

□

Theorem 3.9. *If $n = 7p$ where p is any prime and $p > 7$ then $\Gamma_f(Z_{7p})$ be the non-zero fuzzy zero divisor graph is $K_{6,p-1}^f$ fuzzy complete bipartite graph*

Proof. From the theorem 3.6, theorem 3.7 and theorem 3.8, $\Gamma_f(Z_{7p})$ is a fuzzy complete bipartite graph namely $K_{6,p-1}^f$.

□

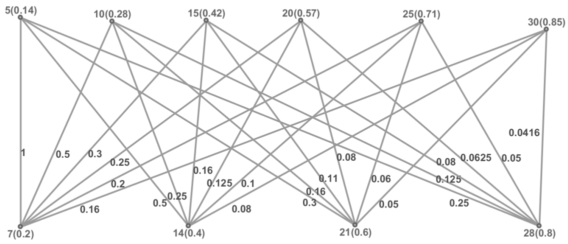

FIGURE 3. $\Gamma_f(Z_{35})$

Theorem 3.10. *If $n = pq$ where p and q are distinct prime numbers and $q > p$ then $\Gamma_f(Z_{pq})$ be the non-zero fuzzy zero divisor graph is $K^f_{p-1,q-1}$ fuzzy complete bipartite graph.*

Proof. Let p and q are any prime numbers and $p < q$. Then the vertex set of $\Gamma(Z_{pq})$ is $V = \{(u, v) : uv = 0 \ multiplication \ modulo \ n, \forall u, v \in V\}$ where n=pq. Take two distinct vertex sets V_1 and V_2 in $\Gamma(Z_{pq})$ where $V_1 = \{p, 2p, 3p, ..., p(q-1)\}$ and $V_2 = \{q, 2q, 3q, ..., (p-1)q\}$.

Every vertex in V_1 adjacent to all the vertices in V_2. Clearly, $\Gamma(Z_{pq})$ complete bipartite graph, namely $K_{p-1,q-1}$.

Let V_f is fuzzy vertex set. Let as show that $V_f : V \to (0, 1)$. Fuzzy vertex set V_f partition in to two sets, namely V_{f_1} and V_{f_2}.

Let

$$V_{f_1} = \left\{\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{(p-1)}{p}\right\}$$

where p and q are any prime numbers with $p < q$. That is,

$$V_{f_1} = \left\{\frac{N}{p} \mid N = 1, 2, 3, ..., p-1\right\}$$

with $V_{f_1} : V \to (0, 1)$. Let

$$V_{f_2} = \left\{\frac{1}{q}, \frac{2}{q}, \frac{3}{q}, ..., \frac{(q-1)}{q}\right\}$$

with $V_{f_2} : V \to (0, 1)$. where p and q are any prime numbers with $p < q$.

$$\begin{aligned} V_f &= V_{f_1} \cup V_{f_2} \\ &= \left\{\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{(p-1)}{p}\right\} \cup \left\{\frac{1}{q}, \frac{2}{q}, \frac{3}{q}, ..., \frac{(q-1)}{q}\right\} \\ &= \left\{\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{(p-1)}{p}, \frac{1}{q}, \frac{2}{q}, \frac{3}{q}, ..., \frac{(q-1)}{q}\right\} \end{aligned}$$

where p and q are any prime numbers with $p < q$. Thus, $V_f = \{V_{f_1} \cup V_{f_2}\} : V \to (0, 1)$ which implies that $V_f : V \to (0, 1)$. Hence V_f is a fuzzy vertex set.

Let E_f is fuzzy edge set. Let as show that Let

$$E^f_p = \left\{1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{q-1}\right\}$$

$$E^f_{2p} = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, ..., \frac{1}{2(q-1)}\right\}$$

$$E_{3p} = \left\{\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, ..., \frac{1}{p(q-1)}\right\}$$

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$$E^f_{pq} = \left\{\frac{1}{p}, \frac{1}{2p}, \frac{1}{3p}, ..., \frac{1}{p(q-1)}\right\}$$

$$E_f = [E^f_p, E^f_{2p}, E^f_{3p}, ..., E^f_{pq}]$$

$$E_f = \left[\left\{1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{q-1}\right\}, \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, ..., \frac{1}{2(q-1)}\right\}, \left\{\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, ..., \frac{1}{p(q-1)}\right\}, \right.$$

$$\left. ..., \left\{\frac{1}{p}, \frac{1}{2p}, \frac{1}{3p}, ..., \frac{1}{p(q-1)}\right\}\right]$$

where p and q are any prime numbers with $p < q$. which implies that $E_f : V \to (0, 1]$. Hence E_f is a fuzzy edge set.
Clearly, $\Gamma_f(Z_{pq}) = (V, V_f, E_f)$ is called fuzzy zero divisor graph. Therefore every vertex in V_{f_1} is adjacent to all the vertices in V_{f_2}. Hence that the graph $K^f_{p-1,q-1}$ is fuzzy complete bipartite graph. □

4. Conclusion

In this paper, we have defined the Fuzzy Zero Divisor Graph of a commutative ring. Also established some special cases of $\Gamma_f(Z_{2p})$, $\Gamma_f(Z_{3p})$, $\Gamma_f(Z_{5p})$, $\Gamma_f(Z_{7p})$ and $\Gamma_f(Z_{pq})$. In future we will study some more properties and applications of FZDG.

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