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$m-$POLAR NEUTROSOPHIC GRAPHS

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Abstract. The concept of $m-$Polar antipodal single valued neutrosophic graph ($m-$PASVNG), eccentric $m-$PSVNG, self centered $m-$PSVNG and self median $m-$PSVNG of the given $m-$PSVNG are introduced here. We also investigate different types of isomorphism properties of antipodal $m-$PSVNG, eccentric $m-$PSVNG and self centered $m-$PSVNG.

Keywords: Radius, diameter in $m-$PSVNG, antipodal $m-$PSVNG, eccentric $m-$PSVNG, self centered $m-$PSVNG and self median $m-$PSVNG.

AMS Subject Classification: 99A00.

1. Introduction

Neutrosopic sets were introduced by Smarandache [10], which are the generalization of fuzzy sets and intuitionistic fuzzy sets. The Neutrosophic sets have many applications in medical, management sciences, life sciences, engineering, graph theory, robotics, automata theory and computer science. The single valued neutrosophic graphs and isolated SVNGs were introduced by Broumi, Talea, Bakali and Smarandache [1, 2]. Also recently in [8, 9, 3] proposed some algorithms dealt with shortest path problem in a network (graph) where edge weights are characterized by a neutrosophic numbers including single valued neutrosophic numbers, bipolar neutrosophic numbers and interval valued neutrosophic numbers. Nasir in [7] also contributed on neutrosophic graphs.
Malik and Hassan in [4] defined the concept of classes of some single valued neutrosophic graphs and studied of their properties. Later on, the concept of single valued neutrosophic hyper-graphs has generalized by Hassan et Malik in [5, 6]. The SVNGs have also many applications in path problems, networks and computer science. The concept of antipodal fuzzy graphs introduced by Gani and Malarvizhi [11]. The self centered intuitionistic fuzzy graphs were introduced by Karunambigai [12], the complete intuitionistic fuzzy graph to be a self centered intuitionistic fuzzy graph and its properties discussed, also the necessary and sufficient condition to be a self centered intuitionistic fuzzy graph were discussed. In

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this paper, we introduce new classes of SVNGs, antipodal SVNGs, eccentric SVNGs, self centered and self median SVNGs.

2. Preliminary

In this section we recall some basic concepts on SVNG and let G denotes SVNG and and $G^* = (V, E)$ denotes underlying crisp graph.

Definition 2.1. [10] *Let X be a space of points (objects) with generic elements in X denoted by x; then the neutrosophic set A (NS A) is an object having the form $A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in X\}$, where the functions $T, I, F : X \to]^-0, 1^+[$ define respectively the truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element $x \in X$ to the set A with the condition $-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$. The functions $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$.*

Since it is difficult to apply NSs to practical problems, Wang et al. introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

Definition 2.2. [10] *Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. For each point $x \in X$ $T_A(x), I_A(x), F_A(x) \in [0, 1]$. A SVNS A can be written as $A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in X\}$. And for every $x \in X$; $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.*

Definition 2.3. [1] *The single valued neutrosophic graph (SVNG) is a pair $G = (C, D)$ of $G^* = (V, E)$, where C is SVNS on V and D is SVNS on E such that*

$$T_D(\alpha, \beta) \leq \min(T_C(\alpha), T_C(\beta))$$

$$I_D(\alpha, \beta) \geq \max(I_C(\alpha), I_C(\beta))$$

$$F_D(\alpha, \beta) \geq \max(F_C(\alpha), F_C(\beta))$$

whenever

$$0 \leq T_D(\alpha, \beta) + I_D(\alpha, \beta) + F_D(\alpha, \beta) \leq 3$$

$\forall\ \alpha, \beta \in V$. The SVNG $G = (C, D)$ is said to be complete (strong) SVNG, if

$$T_D(x, y) = \min(T_C(x), T_C(y))$$

$$I_D(x, y) = \max(I_C(x), I_C(y))$$

$$F_D(x, y) = \max(F_C(x), F_C(y))$$

$\forall\ x, y \in V (\forall (x, y) \in E)$. The order of G, which is denoted by $O(G)$, is defined by

$$O(G) = (O_T(G), O_I(G), O_F(G)),$$

where

$$O_T(G) = \sum_{\alpha \in V} T_C(\alpha),\ O_I(G) = \sum_{\alpha \in V} I_C(\alpha),\ O_F(G) = \sum_{\alpha \in V} F_C(\alpha).$$

The size of G, which is denoted $S(G)$, is defined by

$$S(G) = (S_T(G), S_I(G), S_F(G)),$$

where

$$S_T(G) = \sum_{\substack{(\alpha,\beta)\in E\\ \alpha\neq\beta}} T_D(\alpha,\beta),\ S_I(G) = \sum_{\substack{(\alpha,\beta)\in E\\ \alpha\neq\beta}} I_D(\alpha,\beta),\ S_F(G) = \sum_{\substack{(\alpha,\beta)\in E\\ \alpha\neq\beta}} F_D(\alpha,\beta).$$

The degree of a vertex α in G, which is denoted by $d_G(\alpha)$, is defined by

$$d_G(\alpha) = (d_T(\alpha), d_I(\alpha), d_F(\alpha)),$$

where

$$d_T(\alpha) = \sum_{\substack{(\alpha,\beta)\in E\\ \alpha\neq\beta}} T_D(\alpha,\beta),\ d_I(\alpha) = \sum_{\substack{(\alpha,\beta)\in E\\ \alpha\neq\beta}} I_D(\alpha,\beta),\ d_F(\alpha) = \sum_{\substack{(\alpha,\beta)\in E\\ \alpha\neq\beta}} F_D(\alpha,\beta).$$

Definition 2.4. [1] *The Partial single valued neutrosophic subgraph of SVNG $G = (C, D)$ on $G^* = (V, E)$ is a SVNG $H = (C^{'}, D^{'})$, if*
(**1**) $C^{'} \subseteq C$, *that is* $\forall x \in V$

$$T_{C'}(x) \leq T_C(x),\ I_{C'}(x) \geq I_C(x),\ F_{C'}(x) \geq F_C(x).$$

(**2**) $D^{'} \subseteq D$, *that is* $\forall(\alpha,\beta) \in E$

$$T_{D'}(\alpha,\beta) \leq T_D(\alpha,\beta),\ I_{D'}(\alpha,\beta) \geq I_D(\alpha,\beta),\ F_{D'}(\alpha,\beta) \geq F_D(\alpha,\beta).$$

The single valued neutrosophic subgraph of SVNG $G = (C, D)$ of $G^* = (V, E)$ is a SVNG $H = (C^{'}, D^{'})$ on a $H^* = (V^{'}, E^{'})$, such that
(**1**) $C^{'} = C$, *that is* $\forall x \in V^{'} \subseteq V$, *with*

$$T_{C'}(x) = T_C(x),\ I_{C'}(x) = I_C(x),\ F_{C'}(x) = F_C(x).$$

(**2**) $D^{'} = D$, *that is* $\forall(\alpha,\beta) \in E^{'} \subseteq E$, *with*

$$T_{D'}(\alpha,\beta) = T_D(\alpha,\beta),\ I_{D'}(\alpha,\beta) = I_D(\alpha,\beta),\ F_{D'}(\alpha,\beta) = F_D(\alpha,\beta).$$

Definition 2.5. [1] *A path P in a SVNG $G = (C, D)$ is $P : v_1, v_2, v_3, \ldots, v_n$ such that $T_D(v_i, v_{i+1}) > 0, I_D(v_i, v_{i+1}) > 0, F_D(v_i, v_{i+1}) > 0$ for $1 \leq i \leq n$. The SVNG G is said to be a connected, if there is at least one path between every pair of vertices, else G is disconnected.*

3. The classes of SVNGs or 1-PSVNGs

In this section we discuss the antipodal SVNGs, eccentric SVNGs, self centered SVNGs and self median SVNGs.

Definition 3.1. *Let $G_1 = (C_1, D_1)$ and $G_2 = (C_2, D_2)$ be two SVNGs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. The homomorphism $\chi : V_1 \to V_2$ is a mapping from V_1 into V_2 satisfying following conditions*

$$T_{C_1}(p) \leq T_{C_2}(\chi(p)),\ I_{C_1}(p) \geq I_{C_2}(\chi(p)),\ F_{C_1}(p) \geq F_{C_2}(\chi(p))$$

$\forall p \in V_1$.

$$T_{D_1}(p,q) \leq T_{D_2}(\chi(p),\chi(q)),\ I_{D_1}(p,q) \geq I_{D_2}(\chi(p),\chi(q)),\ F_{D_1}(p,q) \geq F_{D_2}(\chi(p),\chi(q))$$

$\forall(p, q) \in E_1$. *The weak isomorphism $\upsilon : V_1 \to V_2$ is a bijective homomorphism from V_1 into V_2 satisfying following conditions*

$$T_{C_1}(p) = T_{C_2}(\upsilon(p)),\ I_{C_1}(p) = I_{C_2}(\upsilon(p)),\ F_{C_1}(p) = F_{C_2}(\upsilon(p))$$

$\forall p \in V_1$. The co-weak isomorphism $\kappa : V_1 \to V_2$ is a bijective homomorphism from V_1 into V_2 satisfying following conditions

$$T_{D_1}(p,q) = T_{D_2}(\kappa(p),\kappa(q)),\ I_{D_1}(p,q) = I_{D_2}(\kappa(p),\kappa(q)),\ F_{D_1}(p,q) = F_{D_2}(\kappa(p),\kappa(q))$$

$\forall (p,q) \in E_1$. An isomorphism $\psi : V_1 \to V_2$ is a bijective homomorphism from V_1 into V_2 satisfying following conditions

$$T_{C_1}(p) = T_{C_2}(\psi(p)), I_{C_1}(p) = I_{C_2}(v(p)), F_{C_1}(p) = F_{C_2}(\psi(p))$$

$\forall p \in V_1$.

$$T_{D_1}(p,q) = T_{D_2}(\psi(p),\psi(q)),\ I_{D_1}(p,q) = I_{D_2}(\psi(p),\psi(q)),\ F_{D_1}(p,q) = F_{D_2}(\psi(p),\psi(q))$$

$\forall (p,q) \in E_1$.

Remark 3.1. *One can see the following.*

(1) *The weak isomorphism between two SVNGs preserves the orders.*
(2) *The weak isomorphism between SVNGs is a partial order relation.*
(3) *The co-weak isomorphism between two SVNGs preserves the sizes.*
(4) *The co-weak isomorphism between SVNGs is a partial order relation.*
(5) *The isomorphism between two SVNGs is an equivalence relation.*
(6) *The isomorphism between two SVNGs preserves the orders and sizes.*
(7) *The isomorphism between two SVNGs preserves the degrees of their vertices's.*

Definition 3.2. *Let $G = (C, D)$ be a SVNG of G^*. The strength of connectedness between x and y in V, which is denoted by $S^\infty_D(x, y)$, is defined by*

$$S^\infty_D(x,y) = (T^\infty_D(x,y), I^\infty_D(x,y), F^\infty_D(x,y)),$$

where

$$T^\infty_D(x,y) = \sup\{T^k_D(x,y) : k = 1,2,\dots,n\},$$

$$T^\infty_D(x,y) = \sup\{T_D(x,v_1) \wedge \dots \wedge T_D(v_{k-1},y) : x, v_1, v_2, \dots, v_{k-1}, y \in V, k = 1,2,\dots,n\},$$

$$I^\infty_D(x,y) = \inf\{I^k_D(x,y) : k = 1,2,\dots,n\},$$

$$I^\infty_D(x,y) = \inf\{I_D(x,v_1) \vee \dots \vee I_D(v_{k-1},y) : x, v_1, v_2, \dots, v_{k-1}, y \in V, k = 1,2,\dots,n\}$$

$$F^\infty_D(x,y) = \inf\{F^k_D(x,y) : k = 1,2,\dots,n\},$$

$$F^\infty_D(x,y) = \inf\{F_D(x,v_1) \vee \dots \vee F_D(v_{k-1},y) : x, v_1, v_2, \dots, v_{k-1}, y \in V, k = 1,2,\dots,n\},$$

Here $T^\infty_D(x,y), I^\infty_D(x,y)$ and $F^\infty_D(x,y)$ are called T-strength, I-strength and F-strength between vertices x and y in V, respectively. The length of path $P : v_1, v_2, \dots, v_n$, which is denoted by $l(P)$, is defined by $l(P) = (l_T(P), l_I(P), l_F(P))$, where

$$l_T(P) = \sum_{i=1}^{n-1} \frac{1}{T_D(v_i,v_{i+1})},\ l_I(P) = \sum_{i=1}^{n-1} \frac{1}{I_D(v_i,v_{i+1})},\ l_F(P) = \sum_{i=1}^{n-1} \frac{1}{F_D(v_i,v_{i+1})}.$$

where $l_T(P), l_I(P)$ and $l_F(P)$ are called the T-length, I-length and F-length of path P, respectively. The distance between two vertices α and β in V, which is denoted by $\delta(\alpha,\beta)$, is defined by

$$\delta(\alpha,\beta) = (\delta_T(\alpha,\beta), \delta_I(\alpha,\beta), \delta_F(\alpha,\beta)),$$

where

$$\delta_T(\alpha,\beta) = \min(l_T(P)),\ \delta_I(\alpha,\beta) = \min(l_I(P)),\ \delta_F(\alpha,\beta) = \min(l_F(P)),$$

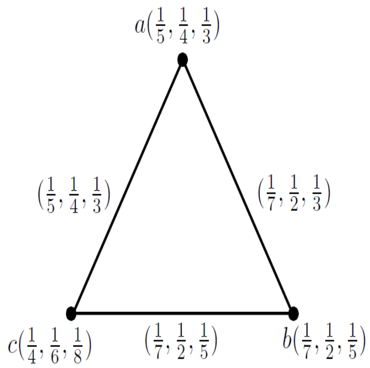

FIGURE 1. SVNG

where $\delta_T(\alpha,\beta)$, $\delta_T(\alpha,\beta)$ and $\delta_T(\alpha,\beta)$ are called the T-distance, I-distance and F-distance of any path $\alpha-\beta$, respectively. The eccentricity of $v_i \in V$, which is denoted by is $e(v_i)$, is defined by $e(v_i) = (e_T(v_i), e_I(v_i), e_F(v_i))$, where

$$e_T(v_i) = \max\{\delta_T(v_i,v_j) : v_j \in V, v_i \neq v_j\}$$

$$e_I(v_i) = \min\{\delta_I(v_i,v_j) : v_j \in V, v_i \neq v_j\}$$

$$e_F(v_i) = \min\{\delta_F(v_i,v_j) : v_j \in V, v_i \neq v_j\}$$

where $e_T(v_i), e_I(v_i)$ and $e_F(v_i)$ are called the T-eccentricity, I-eccentricity and F-eccentricity of vertex v_i, respectively. The radius of G, which is denoted by $r(G)$, is defined by $r(G) = (r_T(G), r_I(G), r_F(G))$, where

$$r_T(G) = \min\{e_T(v_i) : v_i \in V\}$$

$$r_I(G) = \min\{e_I(v_i) : v_i \in V\}$$

$$r_F(G) = \min\{e_F(v_i) : v_i \in V\}$$

where $r_T(G), r_I(G)$ and $r_F(G)$ are called the T-radius, I-radius and F-radius of graph G, respectively. The diameter of G, which is denoted by $d(G)$, is defined by $d(G) = (d_T(G), d_I(G), d_F(G))$, where

$$d_T(G) = \max\{e_T(v_i) : v_i \in V\}$$

$$d_I(G) = \max\{e_I(v_i) : v_i \in V\}$$

$$d_F(G) = \max\{e_F(v_i) : v_i \in V\}$$

where $d_T(G), d_I(G)$ and $d_F(G)$ are called the T-diameter, I-diameter and F-diameter of graph G, respectively.

Definition 3.3. *An antipodal single valued neutrosophic graph (ASVNG) $A(G) = (Q, R)$ of a SVNG $G = (A, B)$ is the SVNG, such that*
(**a**) *$Q = A$ on V.* (**b**) *If $\delta(p,q) = d(G)$, then*
(**i**) *If p and q are adjacent in G, then $R = B$ on E.*
(**ii**) *If p and q are not adjacent in G, then*

$$T_R(p,q) = \min(T_A(p), T_A(q))$$

$$I_R(p,q) = \max(I_A(p), I_A(q))$$

$$F_R(p,q) = \max(F_A(p), F_A(q))$$

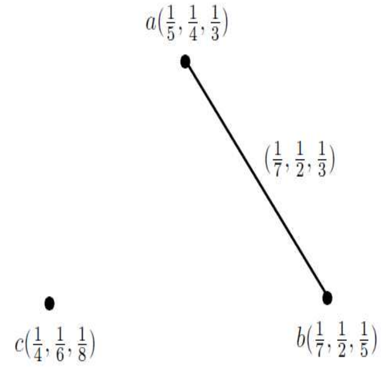

FIGURE 2. Antipodal SVNG

Example 3.1. *Consider the SVNG $G = (A, B)$ of $G^* = (V, E)$, which is shown in Figure 1. Then by routine calculations, $\delta(a, b) = (7, 2, 3)$, $\delta(a, c) = (5, 4, 3)$, $\delta(b, c) = (7, 2, 5)$, $e(a) = (7, 2, 3)$, $e(b) = (7, 2, 3)$, $e(c) = (7, 2, 3)$, $d(G) = (7, 2, 3) = \delta(a, b)$. Hence an ASVNG $A(G) = (Q, R)$, which is shown in Figure* 2.

Definition 3.4. *An eccentric SVNG $G_e = (P, Q)$ of a SVNG $G = (A, B)$ is the SVNG, such that*
(**a**) *$P = A$ on V.* (**b**) *If*

$$\delta_T(p, q) = \min(e_T(p), e_T(q))$$
$$\delta_I(p, q) = \max(e_I(p), e_I(q))$$
$$\delta_F(p, q) = \max(e_F(p), e_F(q))$$

then
(**i**) *If p and q are neighbors in G, then $Q = B$ on E.*
(**ii**) *If p and q are not neighbors in G, then*

$$T_Q(p, q) = \min(T_A(p), T_A(q))$$
$$I_Q(p, q) = \max(I_A(p), I_A(q))$$
$$F_Q(p, q) = \max(F_A(p), F_A(q))$$

(**c**) *else $Q = O = (0, 0, 0)$.*

Example 3.2. *Consider the SVNG $G = (A, B)$ of $G^* = (V, E)$, which is given in Example 3.1. Then by calculations, $\delta(a, b) = (7, 2, 3)$, $\delta(a, c) = (5, 4, 3)$, $\delta(b, c) = (7, 2, 5)$, $e(a) = (7, 2, 3)$, $e(b) = (7, 2, 3)$, $e(c) = (7, 2, 3)$, $d(G) = (7, 2, 3) = \delta(a, b)$ here, $\delta_T(a, b) = 7 = \min(e_T(a), e_T(b))$, $\delta_I(a, b) = 2 = \max(e_I(a), e_I(b))$, $\delta_F(a, b) = 3 = \max(e_F(a), e_F(b))$, $\delta_F(a, c) = 3 = \max(e_F(a), e_F(c))$, $\delta_T(b, c) = 7 = \min(e_T(b), e_T(c))$, $\delta_I(b, c) = 2 = \max(e_I(b), e_I(c))$. The ESVNG is shown in Figure* 3.

Proposition 3.1. *The ASVNG of the SVNG is the generalization of antipodal fuzzy graph of fuzzy graph and antipodal intuitionistic fuzzy graph of intuitionistic fuzzy graph.*

Proposition 3.2. *The ESVNG of SVNG is the generalization of eccentric fuzzy graph of fuzzy graph and eccentric intuitionistic fuzzy graph of intuitionistic fuzzy graph.*

Proposition 3.3. *$A(G)$ is always a single valued neutrosophic subgraph of G_e. Further $A(G)$ and G_e are same, whenever $G = (A, B)$ be a complete SVNG.*

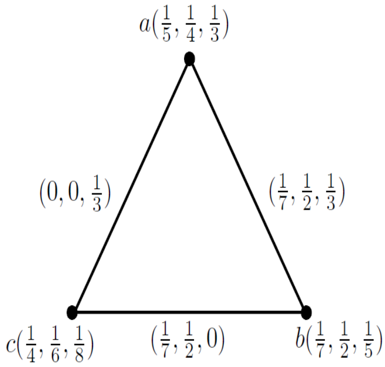

FIGURE 3. Eccentric SVNG

Definition 3.5. *The connected SVNG $G=(A,B)$ is distance regular SVNG, whenever*

$$\delta(x,y)=k=(k_1,k_2,k_3)$$

$\forall\ x,y\in V$.

Proposition 3.4. *If $G=(A,B)$, is distance regular SVNG, then G is single valued neutrosophic spanning subgraph of $A(G)$, such that $A(G)$ is same as G_e.*

Theorem 3.1. *If $G=(A,B)$ be a complete SVNG, then G and $A(G)$ are isomorphic.*

Proof. Since A is constant function, that is $A(x)=c=(c_1,c_2,c_3)$ where c_1, c_2 and c_3 are constants, hence we get $\delta(p,q)=d=(d_1,d_2,d_3)$ for all $p,q\in V$, therefore eccentricity of G $e(p)=d=(d_1,d_2,d_3)$ for all $p\in V$. Hence $d(G)=d=(d_1,d_2,d_3)=\delta(p,q)$ for all $p\in V$. Thus adjacency between every two vertices in $A(G)$ such that (i) $Q=A$ on V. (ii) Since p and q are neighbors in G, hence $R=B$ on E. Therefore G is isomorphic to $A(G)$. □

Theorem 3.2. *Let $G=(A,B)$ be a connected SVNG, then ASVNG $A(G)$ is subgraph of G.*

Proof. Since by the definition of ASVNG, $A(G)$ and G have same vertex set, such that (i) $Q=A$ on V. (ii) If $\delta(p,q)=d(G)$, then (a) If p and q are adjacent in G, then $R=B$ on E. (b) If p and q are not adjacent in G, then $T_R(p,q)=\min(T_A(p),T_A(q)),I_R(p,q)=\max(I_A(p),I_A(q))$ and $F_R(p,q)=\max(F_A(p),F_A(q))$. □

Theorem 3.3. *If $G_1=(A_1,B_1)$ and $G_2=(A_2,B_2)$ are isomorphic, then so $A(G_1)$ and $A(G_2)$.*

Proof. By hypothesis there is an isomorphism f between them, which preserves the weights of edges, so the length and distance will be preserved. Hence if vertex α has maximum T-eccentricity, minimum I-eccentricity and minimum F-eccentricity in G_1, then $f(\alpha)$ has maximum T-eccentricity, minimum I-eccentricity and minimum F-eccentricity in G_2, so G_1 and G_2 will have same diameter. If distance between α and β is $k=(k_1,k_2,k_3)$ in G_1, then $f(\alpha)$ and $f(\beta)$ will also have their distance as $k=(k_1,k_2,k_3)$ in G_2, f is a bijective function between $A(G_1)$ and $A(G_2)$ with $Q_1(\alpha)=A_1(\alpha)=A_2(\alpha)=Q_2(\alpha)$ for all $\alpha\in V_1$ and (i) If α and β are adjacent in E_1, then $R_1=B_1$. (ii) If α and β are not adjacent in E_1, then $T_{R_1}(\alpha,\beta)=\min(T_{A_1}(\alpha),T_{A_1}(\beta))$, $I_{R_1}(\alpha,\beta)=\max(I_{A_1}(\alpha),I_{A_1}(\beta))$ and $F_{R_1}(\alpha,\beta)=\max(F_{A_1}(\alpha),F_{A_1}(\beta))$ as $f:G_1\to G_2$ is an isomorphism, then α and β are adjacent in E_1, then $R_1(\alpha,\beta)=B_2(f(\alpha),f(\beta))$, if α and β are not adjacent in E_1, then $T_{R_1}(\alpha,\beta)=\min(f(\alpha),f(\beta))$, $I_{R_1}(\alpha,\beta)=\max(f(\alpha),f(\beta))$ and $F_{R_1}(\alpha,\beta)=\max(f(\alpha),f(\beta))$, thus we conclude that $R_1(\alpha,\beta) = R_2(f(\alpha), f(\beta))$, so the same isomorphism f is an isomorphism between $A(G_1)$ and $A(G_2)$. □

Theorem 3.4. *If $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be two connected SVNGs, If G_1 and G_2 are co-weak isomorphic, then $A(G_1)$ is homomorphic to $A(G_2)$.*

Proof. Since G_1 and G_2 are co-weak isomorphic SVNGs, then there exists a bijection $f : G_1 \to G_2$ satisfying the conditions $T_{A_1}(\alpha) \le T_{A_2}(f(\alpha)), I_{A_1}(\alpha) \ge I_{A_2}(f(\alpha)), F_{A_1}(\alpha) \ge F_{A_2}(f(\alpha))$ for all $\alpha \in V_1$ and $B_1(\alpha,\beta) = B_2(f(\alpha), f(\beta))$ for all $(\alpha,\beta) \in E_1$, so the distance and diameters will preserved. Let $d(G_1) = d(G_2) = k = (k_1, k_2, k_3)$ if $u, v \in V_1$ are at a distance k in G_1, then they are made as neighbors in $A(G_1)$, so $f(u), f(v) \in V_2$ are at a distance k in G_2, then they are made as neighbors in $A(G_2)$. If u and v are neighbors in G_1, then $R_1(u, v) = B_1(u, v) = B_2(f(u), f(v)) = R_2(f(u), f(v))$. If u and v are not neighbors in G_1, then $T_{R_1}(u, v) = \min(T_{A_1}(u), T_{A_1}(v)) \le \min(T_{A_2}(f(u)), T_{A_2}(f(v))) = T_{R_2}(f(u), f(v))$ similarly $I_{R_1}(u, v) \ge I_{R_2}(f(u), f(v))$ and $F_{R_1}(u, v) \ge F_{R_2}(f(u), f(v))$. Hence $A(G_1)$ is homomorphic to $A(G_2)$. □

Theorem 3.5. *If $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be two complete SVNGs, then if G_1 is co-weak isomorphic to G_2, then $A(G_1)$ is co-weak isomorphic to $A(G_2)$.*

Proof. Straight forward as Theorem 3.4 is proved. □

Definition 3.6. *Let $G = (A, B)$ be a SVNG, a vertex $v_i \in V$ is said to be a central vertex if $r(G) = e(v_i)$. The set of all central vertices of G, is denoted by $C(G)$. The connected SVNG $G = (A, B)$ is said to be self centered single valued neutrosophic graph (SCSVNG), if $r(G) = e(v_i)$ $\forall v_i \in V$.*

Example 3.3. *Consider the SVNG $G = (A, B)$ of $G^* = (V, E)$, which is given in example 3.1. Then by calculations, $\delta(a, b) = (7, 2, 3)$, $\delta(a, c) = (5, 4, 3)$, $\delta(b, c) = (7, 2, 5)$, $e(a) = (7, 2, 3)$, $e(b) = (7, 2, 3)$, $e(c) = (7, 2, 3)$, $r(G) = (7, 2, 3) = e(a) = e(b) = e(c)$. Therefore G is a self centered SVNG.*

Definition 3.7. *A path cover of a SVNG $G = (A, B)$ is the set Q of paths so that every vertex of G is incident to some path of Q.*

Definition 3.8. *An edge cover of a SVNG $G = (A, B)$ is the set E of edges such that every vertex of G is incident to some edge of E.*

Theorem 3.6. *Every complete SVNG $G = (A, B)$ is a self centered SVNG and*

$$r(G) = \left(\frac{1}{T_{Ai}}, \frac{1}{I_{Ai}}, \frac{1}{F_{Ai}}\right)$$

where T_{Ai} is minimal, I_{Ai} and F_{Ai} are maximal.

Proof. Let $v_i \in V$ such that T_{Ai} is least truth membership of vertex value in G.
Case(i) $\forall$ $v_i - v_j$ paths P of length n in G for all $v_j \in V$.
for $n = 1$ trivially holds, if $n > 1$, the T-strength of one edge T_{Ai} and therefore T-length of a $v_i - v_j$ path will exceed $\frac{1}{T_{Ai}}$, thus T-length of path $P = l_T(P) > \frac{1}{T_{Ai}}$, hence $\delta_T(v_i, v_j) = \min(l_T(P)) = \frac{1}{T_{Ai}}$ for all $v_j \in V$.
Case(ii) Let $v_k \neq v_i \in V$, consider all $v_k - v_j$ paths Q of length n in G for all $v_j \in V$.
Subcase(i) Whenever $n = 1$, then $T_B(v_k, v_j) = \min(T_{Ak}, T_{Aj}) \ge T_{Ai}$ since T_{Ai} is minimal, hence T-length of $Q = l_T(Q) = \frac{1}{T_B(v_k,v_j)} \le \frac{1}{T_{Ai}}$.
Subcase(ii) Whenever $n = 2$, then $l_T(Q) = \frac{1}{T_B(v_k,v_{k+1})} + \frac{1}{T_B(v_{k+1},v_j)} \le \frac{2}{T_{Ai}}$ since T_{Ai} is minimal.

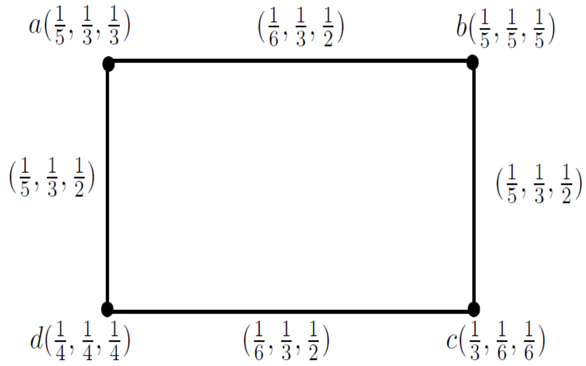

FIGURE 4. SVNG

Subcase(iii) Whenever $n > 2$, then $l_T(Q) \leq \frac{n}{T_{Ai}}$ since T_{Ai} is minimal, hence $\delta_T(v_k, v_j) = \min(l_T(Q)) \leq \frac{1}{T_{Ai}}$ for all $v_k, v_j \in V$. Thus we have $e_T(v_i) = \min(\delta_T(v_i, v_j)) = \frac{1}{T_{Ai}}$ for all $v_i \in V$. Next $r_T(G) = \min(e_T(v_i)) = \frac{1}{T_{Ai}}$, hence $r_T(G) = \frac{1}{T_{Ai}}$ where $T_A(v_i)$ is minimal. Similarly others can be proved. Hence G is self centered SVNG. $\square$

Remark 3.2. *In general converse part does not hold of Theorem 3.6.*

Example 3.4. *Consider a SVNG* $G = (A, B)$ *of* $G^* = (V, E)$, *which is shown in Figure* 4. *Then by calculations,* $\delta(\alpha, \beta) = (6, 3, 2)$, $\delta(\alpha, \delta) = (5, 3, 2)$, $\delta(\beta, \gamma) = (5, 3, 2)$, $\delta(\gamma, \delta) = (6, 3, 2)$, $\delta(\alpha, \gamma) = (11, 6, 4)$, $\delta(\beta, \delta) = (11, 6, 4)$, $e(\alpha) = (11, 3, 2)$, $e(\beta) = (11, 3, 2)$, $e(\gamma) = (11, 3, 2)$, $e(\delta) = (11, 3, 2)$. *Here* $r(G) = e(G) = (11, 3, 2)$. *Thus* G *is self centered SVNG, but* G *is not complete SVNG.*

Remark 3.3. *A SVNG* $G = (A, B)$ *is self centered SVNG if and only if* $d(G) = r(G)$.

Theorem 3.7. *Let* $G = (A, B)$ *be a connected SVNG with path covers* P_1, P_2 *and* P_3 *of* G, *respectively. Then* G *is self centered SVNG if and only if*

$$\delta_T(v_i, v_j) = d_T(G),\ \forall(v_i, v_j) \in P_1$$
$$\delta_I(v_i, v_j) = d_I(G),\ \forall(v_i, v_j) \in P_2$$
$$\delta_F(v_i, v_j) = d_F(G),\ \forall(v_i, v_j) \in P_3$$

Proof. Assume that $G = (A, B)$ be self centered SVNG. Suppose that conditions are false, that is

$$\delta_T(v_i, v_j) = d_T(G),\ \exists(v_i, v_j) \in P_1$$
$$\delta_I(v_i, v_j) = d_I(G),\ \exists(v_i, v_j) \in P_2$$
$$\delta_F(v_i, v_j) = d_F(G),\ \exists(v_i, v_j) \in P_3$$

then by above remark, the above inequality becomes

$$\delta_T(v_i, v_j) \neq r_T(G),\ \exists(v_i, v_j) \in P_1$$
$$\delta_I(v_i, v_j) \neq r_I(G),\ \exists(v_i, v_j) \in P_2$$
$$\delta_F(v_i, v_j) \neq r_F(G),\ \exists(v_i, v_j) \in P_3$$

Thus we conclude that, for some $v_i \in V$

$$e_T(v_i) \neq r_T(G),\ e_I(v_i) \neq r_I(G),\ e_F(v_i) \neq r_F(G)$$

which shows that G is not self centered SVNG, which contradict the assumption. Thus

$$\delta_T(v_i, v_j) = d_T(G),\ \forall(v_i, v_j) \in P_1$$
$$\delta_I(v_i, v_j) = d_I(G),\ \forall(v_i, v_j) \in P_2$$
$$\delta_F(v_i, v_j) = d_F(G),\ \forall(v_i, v_j) \in P_3$$

Next assume that

$$\delta_T(v_i, v_j) = d_T(G),\ \forall(v_i, v_j) \in P_1$$
$$\delta_I(v_i, v_j) = d_I(G),\ \forall(v_i, v_j) \in P_2$$
$$\delta_F(v_i, v_j) = d_F(G),\ \forall(v_i, v_j) \in P_3$$

then by our hypothesis, we have

$$\delta_T(v_i, v_j) = e_T(v_i),\ \forall(v_i, v_j) \in P_1$$
$$\delta_I(v_i, v_j) = e_I(v_i),\ \forall(v_i, v_j) \in P_2$$
$$\delta_F(v_i, v_j) = e_F(v_i),\ \forall(v_i, v_j) \in P_3$$

this implies that, $v_i \in V$

$$e_T(v_i) = r_T(G),\ e_I(v_i) = r_I(G),\ e_F(v_i) = r_F(G)$$

hence $e(G) = r(G)$, this shows that G is SCSVNG. □

Theorem 3.8. *If $G = (A, B)$ be a connected SVNG, with edge covers L_1,L_2 and L_3 of G, G is self centered SVNG if and only if*

$$\delta(v_i, v_j) = d_T(G) \text{ for all } (v_i, v_j) \in L_1,$$
$$\delta(v_i, v_j) = d_I(G) \text{ for all } (v_i, v_j) \in L_2,$$
$$\delta(v_i, v_j) = d_F(G) \text{ for all } (v_i, v_j) \in L_3.$$

Proof. Similarly as Theorem 3.7 proved. □

Theorem 3.9. *Let $H = (A^{'}, B^{'})$ be connected self centered SVNG, then there exists a connected SVNG $G = (A, B)$ for which $< C(G) >$ is isomorphic with H and $d_T(G) = 2r_T(G), d_I(G) = 2r_I(G),\ d_F(G) = 2r_F(G)$.*

Proof. Let $H = (A^{'}, B^{'})$ be a connected self centered SVNG. Let $d_T(H) = l, d_I(H) = m$, and $d_F(H) = n$. For two vertices $v_i, v_j \in V$ with $T_A(v_i) = T_A(v_j) = \frac{1}{l}, I_A(v_i) = I_A(v_j) = \frac{1}{2m}, F_A(v_i) = F_A(v_j) = \frac{1}{2n}$. Also all the vertices of H are neighbors to both v_i and v_j with $T_B(v_i, v_k) = T_B(v_j, v_k) = \frac{1}{l}, I_B(v_i, v_k) = I_B(v_j, v_k) = \frac{1}{2m}, F_B(v_i, v_k) = F_B(v_j, v_k) = \frac{1}{2n}$ for all $v_k \in V^{'}_{.}$. Next put $T_A = T_{A^{'}},\ I_A = I_{A^{'}}$ and $F_A = F_{A^{'}}$ for all vertices in H and $T_B = T_{B^{'}}, I_B = I_{B^{'}}$ and $F_B = F_{B^{'}}\ \forall\ (\alpha, \beta) \in E$.
If possible $T_A(v_i) > T_A(v_k)$ for at least one vertex $v_k \in V^{'}_{,}$, then $\frac{1}{l} > T_A(v_k)$ that is $l < \frac{1}{T_A(v_k)} \leq \frac{1}{T_B(v_k,v_l)}$, this holds for all $v_l \in V^{'}$ because H is SVNG, thus $\frac{1}{T_B(v_k,v_l)} > l$ for all $v_k \in V^{'}$ which contradict to fact $d_T(H) = l$, therefore $T_A(v_i) \leq T_A(v_k)$ for all $v_k \in V^{'}$ and $T_B(v_i, v_k) \leq \min(T_{Ai}, T_{Ak}) = \frac{1}{l}$, similarly $T_B(v_j, v_k) \leq \min(T_{Aj}, T_{Ak}) = \frac{1}{l}$ for all $v_k \in V^{'}_{,}$ note that $I_A(v_i) \geq I_A(v_k)$ and $I_A(v_j) \geq I_A(v_k)$ for all $v_k \in V^{'}$ since $d_I(H) = m$, therefore $I_B(v_i, v_k) \geq \max(I_{Ai}, I_{Ak}) = \frac{1}{2m}$, similarly $I_B(v_j, v_k) \geq \max(I_{Aj}, I_{Ak}) = \frac{1}{2m}$ for all $v_k \in V^{'}_{,}$, similarly $F_A(v_i) \geq F_A(v_k)$ and $F_A(v_j) \geq F_A(v_k)$ for all $v_k \in V^{'}_{,}$ since $d_F(H) = n$, therefore $F_B(v_i\ , v_k) \leq \max(F_{Ai}, F_{Ak}) = \frac{1}{2n}$, similarly $F_B(v_j, v_k) \geq \max(F_{Aj}, F_{Ak}) = \frac{1}{2n}$ for all $v_k \in V^{'}$ hence G is SVNG.
Next $e_T(v_k) = l$ for all $v_k \in V^{'}$ and $e_T(v_i) = e_T(v_j) = \frac{1}{T_B(v_i,v_k)} + \frac{1}{T_B(v_i,v_k)} = 2l$, $r_T(G) = l$, $d_T(G) = 2l$. Next $e_I(v_k) = m$ for all $v_k \in V^{'}$ and $e_I(v_i) = e_I(v_j) = \frac{1}{I_B(v_l,v_k)} = 2m$, $r_I(G) =$

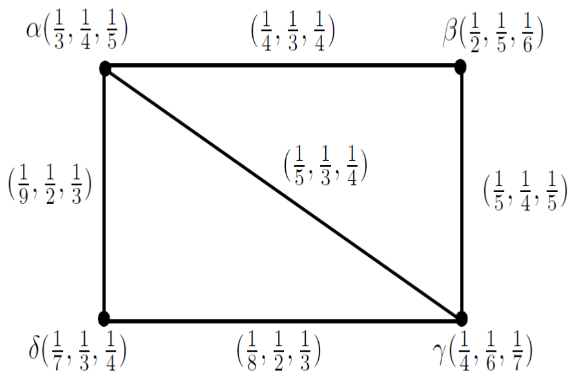

FIGURE 5. SVNG

m, $d_I(G) = 2m$. Similarly $e_F(v_k) = n$ for all $v_k \in V'$ and $e_F(v_i) = e_F(v_j) = \frac{1}{F_B(v_l,v_k)} = 2n$, $r_F(G) = n$, $d_F(G) = 2n$. □

Definition 3.9. *Let $G = (A, B)$ be a connected SVNG, the status of a vertex α, which is denoted by $S(\alpha)$, is defined by $S(\alpha) = (S_T(\alpha), S_I(\alpha), S_F(\alpha))$, where*

$$S_T(\alpha) = \sum_{\beta \in V} \delta_T(\alpha, \beta),\ S_I(\alpha) = \sum_{\beta \in V} \delta_I(\alpha, \beta),\ S_F(\alpha) = \sum_{\beta \in V} \delta_F(\alpha, \beta)$$

where $S_T(\alpha), S_I(\alpha)$ and $S_F(\alpha)$ are called T-status, I-status and F-status of the vertex α, respectively. The connected SVNG G is called self-median if all vertices have same status.

Definition 3.10. *minimum and maximum status of connected SVNG the G is denoted and defined by respectively*

$$m(S(G)) = (\min(S_T(G)), \min(S_I(G)), \min(S_F(G)))$$

$$M(S(G)) = (\max(S_T(G)), \max(S_I(G)), \max(S_F(G)))$$

Definition 3.11. *The total status of of connected SVNG the G is given by*

$$t(S(G)) = (t(S_T(G)), t(S_I(G)), t(S_F(G)))$$

where

$$t(S_T(G)) = \sum_{\alpha \in V} S_T(\alpha),\ t(S_I(G)) = \sum_{\alpha \in V} S_I(\alpha),\ t(S_F(G)) = \sum_{\alpha \in V} S_F(\alpha)$$

Example 3.5. *Consider the SVNG $G = (A, B)$ of $G^* = (V, E)$, which is shown in Figure 5. Then by calculations, $S(\alpha) = (5, 3, 4)$, $S(\beta) = (18, 9, 12)$, $S(\gamma) = (13, 6, 8)$, $S(\delta) = (21, 7, 10)$. Thus G is not self median SVNG.*

Remark 3.4. *Let $G = (C, D)$ be a connected SVNG of $G^* = (V, E)$, which is an even cycle, then G is self-median SVNG, if alternative edges have same truth, indeterminacy and falsity membership values.*

Example 3.6. *Consider the SVNG $G = (A, B)$ of $G^* = (V, E)$ which is given in Example 3.4. Then by routine calculations, we get $\delta(\alpha, \beta) = (6, 3, 2)$, $\delta(\alpha, \delta) = (5, 3, 2)$, $\delta(\beta, \gamma) = (5, 3, 2)$, $\delta(\gamma, \delta) = (6, 3, 2)$, $\delta(\alpha, \gamma) = (11, 6, 4)$, $\delta(\beta, \delta) = (11, 6, 4)$, $S(\alpha) = (22, 12, 8)$, $S(\beta) = (22, 12, 8)$, $S(\gamma) = (22, 12, 8)$, $S(\delta) = (22, 12, 8)$, $t(S(G)) = S(\alpha) = S(\beta) = S(\gamma) = S(\delta)$. Thus G is self median SVNG.*

4. The Classes of $m-$PSVNGs

In this section, we discuss the $m-$PSVNGs and special classes of $m-$PSVNGs such as, antipodal, eccentric, self centered and self median $m-$PSVNGs. Let G denotes $m-$PSVNG and $G^* = (V, E)$ denotes underlying crisp graph. In the whole article the results and definitions hold $\forall r = 1, 2, 3, \cdots, m$.

Definition 4.1. *Let X be a space of points (objects) with generic elements in X denoted by x. A $m-$Polar single valued neutrosophic set A ($m-$PSVNS A) is characterized by $m-$Polar truth-membership function $T_A(x) : X \longrightarrow [0, 1]^m$ $m-$Polar indeterminacy-membership function $I_A(x) : X \longrightarrow [0, 1]^m$ and $m-$Polar falsity-membership function $F_A(x) : X \longrightarrow [0, 1]^m$. The $m-$PSVNS is the generalization of $m-$Polar fuzzy set and $m-$Polar intuitionistic fuzzy set. Note that a $[0, 1]^m$-set is an L-set. An L-set on the set X is a synonym of a mapping $A : X \longrightarrow L$, where L is a lattice. So, $[0, 1]^m$ is considered to be a partial order set with the point-wise order $\leq$, where m is an arbitrary ordinal number, $\leq$ is defined by $x \leq y \Leftrightarrow p_r(x) \leq p_r(y)$ for each $r \in m$ and $p_r : [0, 1]^m \longrightarrow [0, 1]$ is the $r-th$ projection mapping ($r \in m$), when $L = [0, 1]$, an L-set on X will be called a fuzzy set on X.*

Definition 4.2. *An $m-$Polar single valued neutrosophic graph is a pair $G = (A, B)$, where $A : V \longrightarrow [0, 1]^m$ is an $m-$Polar single valued neutrosophic set in V such that,*

$$0 \leq p_r \circ T_A(x) + p_r \circ I_A(x) + p_r \circ F_A(x) \leq 3,$$

$\forall x \in V$, $\forall r = 1, 2, 3, \cdots, m$ and $B : V \times V \longrightarrow [0, 1]^m$ is an $m-$Polar single valued neutrosophic relation on V, such that

$$p_r \circ T_B(x, y) \leq \inf(p_r \circ T_A(x), p_r \circ T_A(y)),$$
$$p_r \circ I_B(x, y) \geq \sup(p_r \circ I_A(x), p_r \circ I_A(y)),$$
$$p_r \circ F_B(x, y) \geq \sup(p_r \circ F_A(x), p_r \circ F_A(y)),$$

$\forall x, y \in V$, whenever

$$0 \leq p_r \circ T_B(x, y) + p_r \circ I_B(x, y) + p_r \circ F_B(x, y) \leq 3,$$

$\forall (x, y) \in E \subseteq V \times V$ and $\forall r = 1, 2, 3, \cdots, m$. Note that $p_r \circ B(x, y) = 0$, $\forall (x, y) \in V \times V - E$, $\forall r = 1, 2, 3, \cdots, m$. Also A is called the $m-$Polar SVN vertex set of G and B is called the $m-$Polar SVN edge set of G, respectively. An $m-$Polar SVN relation B on V is called symmetric if $p_r \circ B(x, y) = p_r \circ B(y, x)$ $\forall x, y \in V$.
The $m-$PSVNG is the generalization of $m-$Polar fuzzy graph and $m-$Polar intuitionistic fuzzy graph. The graph G is said to be a complete (strong) $m-$Polar SVNG, if

$$p_r \circ T_B(x, y) = \inf(p_r \circ T_A(x), p_r \circ T_A(y)),$$
$$p_r \circ I_B(x, y) = \sup(p_r \circ I_A(x), p_r \circ I_A(y)),$$
$$p_r \circ F_B(x, y) = \sup(p_r \circ F_A(x), p_r \circ F_A(y)),$$

$\forall x, y \in V ((x, y) \in E)$ and $\forall r = 1, 2, 3, \cdots, m$. The order of G, which is denoted by $O(G)$, is defined by

$$O(G) = ((p_1 \circ O_T(G), p_1 \circ O_I(G), p_1 \circ O_F(G)), \cdots, (p_m \circ O_T(G), p_m \circ O_I(G), p_m \circ O_F(G)))$$

where

$$p_r \circ O_T(G) = \sum_{x \in V} p_r \circ T_A(x),\ \ p_r \circ O_I(G) = \sum_{x \in V} p_r \circ I_A(x),\ \ p_r \circ O_F(G) = \sum_{x \in V} p_r \circ F_A(x)$$

$\forall r = 1, 2, 3, \cdots, m$. The size of G, which is denoted by $S(G)$, is defined by

$$S(G) = ((p_1 \circ S_T(G), p_1 \circ S_I(G), p_1 \circ S_F(G)), \cdots, (p_m \circ S_T(G), p_m \circ S_I(G), p_m \circ S_F(G)))$$

where

$$p_r \circ S_T(G) = \sum_{\substack{(x,y)\in E\\ x\neq y}} p_r \circ T_B(x,y),$$
$$p_r \circ S_I(G) = \sum_{\substack{(x,y)\in E\\ x\neq y}} p_r \circ I_B(x,y),$$
$$p_r \circ S_F(G) = \sum_{\substack{(x,y)\in E\\ x\neq y}} p_r \circ F_B(x,y),$$

$\forall r = 1, 2, 3, \cdots, m$. *The degree of vertex x, which is denoted by $d_G(x)$, is defined by*

$$d_G(x) = ((p_1 \circ d_T(x), p_1 \circ d_I(x), p_1 \circ d_F(x)), \cdots, (p_m \circ d_T(x), p_m \circ d_I(x), p_m \circ d_F(x)))$$

where

$$p_r \circ d_T(x) = \sum_{\substack{(x,y)\in E\\ x\neq y}} p_r \circ T_B(x,y),$$
$$p_r \circ d_I(x) = \sum_{\substack{(x,y)\in E\\ x\neq y}} p_r \circ I_B(x,y),$$
$$p_r \circ d_F(x) = \sum_{\substack{(x,y)\in E\\ x\neq y}} p_r \circ F_B(x,y)$$

$\forall r = 1, 2, 3, \cdots, m$.

The total degree of vertex x is denoted and defined by

$$td_G(x) = ((p_1 \circ td_T(x), p_1 \circ td_I(x), p_1 \circ td_F(x)), \cdots, (p_m \circ td_T(x), p_m \circ td_I(x), p_m \circ td_F(x)))$$

where,

$$p_i \circ td_T(x) = \sum_{\substack{(x,y)\in E\\ x\neq y}} p_i \circ T_B(x,y) + p_i \circ T_A(x)$$
$$p_i \circ td_I(x) = \sum_{\substack{(x,y)\in E\\ x\neq y}} p_i \circ I_B(x,y) + p_i \circ I_A(x)$$
$$p_i \circ td_F(x) = \sum_{\substack{(x,y)\in E\\ x\neq y}} p_i \circ F_B(x,y) + p_i \circ F_A(x)$$

$\forall i = 1, 2, 3, \cdots, m$. *The degree of edge $e = xy$ is denoted and defined by*

$$d_G(x,y) = (\cdots, (p_i \circ d_T(x,y), p_i \circ d_I(x,y), p_i \circ d_F(x,y)), \cdots)$$

where

$$p_i \circ d_T(x,y) = \sum_{\substack{(x,w)\in E\\ x\neq w}} p_i \circ T_B(x,w) + \sum_{\substack{(w,y)\in E\\ w\neq y}} p_i \circ T_B(w,y)$$
$$p_i \circ d_I(x,y) = \sum_{\substack{(x,w)\in E\\ x\neq w}} p_i \circ I_B(x,w) + \sum_{\substack{(w,y)\in E\\ w\neq y}} p_i \circ I_B(w,y)$$
$$p_i \circ d_F(x,y) = \sum_{\substack{(x,w)\in E\\ x\neq w}} p_i \circ F_B(x,w) + \sum_{\substack{(w,y)\in E\\ w\neq y}} p_i \circ F_B(w,y)$$

$\forall i = 1, 2, 3, \cdots, m$. The total degree of edge $e = xy$ is denoted and defined by

$$td_G(x, y) = (\cdots, (p_i \circ td_T(x, y), p_i \circ td_I(x, y), p_i \circ td_F(x, y)), \cdots)$$

where

$$p_i \circ td_T(x, y) = \sum_{\substack{(x,w)\in E \\ x\neq w}} p_i \circ T_B(x, w) + \sum_{\substack{(w,y)\in E \\ w\neq y}} p_i \circ T_B(w, y) + p_i \circ T_B(x, y)$$

$$p_i \circ td_I(x, y) = \sum_{\substack{(x,w)\in E \\ x\neq w}} p_i \circ I_B(x, w) + \sum_{\substack{(w,y)\in E \\ w\neq y}} p_i \circ I_B(w, y) + p_i \circ I_B(x, y)$$

$$p_i \circ td_F(x, y) = \sum_{\substack{(x,w)\in E \\ x\neq w}} p_i \circ F_B(x, w) + \sum_{\substack{(w,y)\in E \\ w\neq y}} p_i \circ F_B(w, y) + p_i \circ F_B(x, y)$$

$\forall i = 1, 2, 3, \cdots, m$.

Definition 4.3. *A strong (complete) $m-$Polar single valued neutrosophic graph is a pair $G = (A, B)$, where $A : V \longrightarrow [0, 1]^m$ is an $m-$Polar single valued neutrosophic set in V and $B : V \times V \longrightarrow [0, 1]^m$ is an $m-$Polar single valued neutrosophic relation on V, such that*

$$p_r \circ T_B(x, y) = \inf(p_r \circ T_A(x), p_r \circ T_A(y))$$
$$p_r \circ I_B(x, y) = \sup(p_r \circ I_A(x), p_r \circ I_A(y))$$
$$p_r \circ F_B(x, y) = \sup(p_r \circ F_A(x), p_r \circ F_A(y))$$

$\forall (x, y) \in V$ $(\forall x, y \in V)$ and $\forall r = 1, 2, 3, \cdots, m$.

Definition 4.4. *The Partial $m-$PSVN-subgraph of $m-$PSVNG $G = (A, B)$ on a crisp graph $G^* = (V, E)$ is a $m-$PSVNG $H = (A^{'}, B^{'})$, such that*
(1) *$A^{'} \subseteq A$, i.e $\forall r = 1, 2, 3, \cdots, m$ and $\forall x \in V$*

$$p_r \circ T_{A'}(x) \leq p_r \circ T_A(x),\ p_r \circ I_{A'}(x) \geq p_r \circ I_A(x),\ p_r \circ F_{A'}(x) \geq p_r \circ F_A(x)$$

(2) *$B^{'} \subseteq B$, i.e $\forall r = 1, 2, 3, \cdots, m$ and $\forall xy \in E$*

$$p_r \circ T_{B'}(x, y) \leq p_r \circ T_B(x, y),\ p_r \circ I_{B'}(x, y) \geq p_r \circ I_B(x, y),\ p_r \circ F_{B'}(x, y) \geq p_r \circ F_B(x, y)$$

Definition 4.5. *The $m-$Polar SVN-subgraph of $m-$PSVNG $G = (A, B)$, on a crisp graph $G^* = (V, E)$ is a $m-$PSVNG $H = (A^{'}, B^{'})$, on a crisp graph $H^* = (V^{'}, E^{'})$, such that*
(1) *$A^{'} = A$, i.e $\forall x \in V^{'} \subseteq V$, with*

$$p_r \circ T_{A'}(x) = p_r \circ T_A(x),\ p_r \circ I_{A'}(x) = p_r \circ I_A(x),\ p_r \circ F_{A'}(x) = p_r \circ F_A(x)$$

$\forall r = 1, 2, 3, \cdots, m$.
(2) *$B^{'} = B$, i.e $\forall xy \in E^{'} \subseteq E$, with*

$$p_r \circ T_{B'}(x, y) = p_r \circ T_B(x, y),\ p_r \circ I_{B'}(x, y) = p_r \circ I_B(x, y),\ p_r \circ F_{B'}(x, y) = p_r \circ F_B(x, y)$$

$\forall r = 1, 2, 3, \cdots, m$.

Definition 4.6. *A $m-$PSVN path P in a $m-$PSVNG $G = (A, B)$, is a sequence of distinct vertices $v_0, v_1, \cdots, v_n$, such that*

$$p_r \circ T_B(v_{j-1}, v_j) > 0,\ p_r \circ I_B(v_{j-1}, v_j) > 0,\ p_r \circ F_B(v_{j-1}, v_j) > 0$$

for $0 \leq j \leq n$ and $\forall r = 1, 2, 3, \cdots, m$. Here $n \geq 1$ is called length of path P. A single node or a vertex v may also be considered as a path. In this case path is of length

$((0,0,0),\cdots,(0,0,0))$. *The consecutive pairs* (v_{j-1},v_j) *are called edges of path. We call* P *a cycle if* $v_0=v_n$ *and* $n\geq 3$. *An* $m-PSVNG$ $G=(A,B)$, *is said to be connected if every pair of vertices has at least one* $m-PSVN$ *path between them, otherwise it is disconnected.*

Definition 4.7. *Let* $G_1=(C_1,D_1)$ *and* $G_2=(C_2,D_2)$ *be two* $m-PSVNGs$ *of* $G_1^*=(V_1,E_1)$ *and* $G_2^*=(V_2,E_2)$, *respectively. The homomorphism* $\chi:V_1\to V_2$ *is a mapping from* V_1 *into* V_2 *satisfying following conditions:*

$$p_r\circ T_{C_1}(\xi)\leq p_r\circ T_{C_2}(\chi(\xi)),\ p_r\circ I_{C_1}(\xi)\geq p_r\circ I_{C_2}(\chi(\xi)),\ p_r\circ F_{C_1}(\xi)\geq p_r\circ F_{C_2}(\chi(\xi))$$

$\forall\xi\in V_1$ *and* $\forall r=1,2,3,\cdots,m$.

$$\begin{aligned}p_r\circ T_{D_1}(\xi,\eta)&\leq p_r\circ T_{D_2}(\chi(\xi),\chi(\eta))\\ p_r\circ I_{D_1}(\xi,\eta)&\geq p_r\circ I_{D_2}(\chi(\xi),\chi(\eta))\\ p_r\circ F_{D_1}(\xi,\eta)&\geq p_r\circ F_{D_2}(\chi(\xi),\chi(\eta))\end{aligned}$$

$\forall(\xi,\eta)\in E_1$ *and* $\forall r=1,2,3,\cdots,m$. *The weak isomorphism* $\upsilon:V_1\to V_2$ *is an bijective homomorphism from* V_1 *into* V_2 *satisfying following conditions:*

$$p_r\circ T_{C_1}(\xi)=p_r\circ T_{C_2}(\upsilon(\xi)),\ p_r\circ I_{C_1}(\xi)=p_r\circ I_{C_2}(\upsilon(\xi)),\ p_r\circ F_{C_1}(\xi)=p_r\circ F_{C_2}(\upsilon(\xi))$$

$\forall\xi\in V_1$ *and* $\forall r=1,2,3,\cdots,m$. *The co-weak isomorphism* $\kappa:V_1\to V_2$ *is an bijective homomorphism from* V_1 *into* V_2 *satisfying following conditions:*

$$\begin{aligned}p_r\circ T_{D_1}(\xi,\eta)&=p_r\circ T_{D_2}(\chi(\xi),\chi(\eta))\\ p_r\circ I_{D_1}(\xi,\eta)&=p_r\circ I_{D_2}(\chi(\xi),\chi(\eta))\\ p_r\circ F_{D_1}(\xi,\eta)&=p_r\circ F_{D_2}(\chi(\xi),\chi(\eta))\end{aligned}$$

$\forall(\xi,\eta)\in E_1$ *and* $\forall r=1,2,3,\cdots,m$. *An isomorphism* $\psi:V_1\to V_2$ *is a bijective homomorphism from* V_1 *into* V_2 *satisfying following conditions:*

$$p_r\circ T_{C_1}(\xi)=p_r\circ T_{C_2}(\psi(\xi)),\ p_r\circ I_{C_1}(\xi)=p_r\circ I_{C_2}(\psi(\xi)),\ p_r\circ F_{C_1}(\xi)=p_r\circ F_{C_2}(\psi(\xi))$$

$\forall\xi\in V_1$ *and* $\forall r=1,2,3,\cdots,m$.

$$\begin{aligned}p_r\circ T_{D_1}(\xi,\eta)&=p_r\circ T_{D_2}(\chi(\xi),\chi(\eta))\\ p_r\circ I_{D_1}(\xi,\eta)&=p_r\circ I_{D_2}(\chi(\xi),\chi(\eta))\\ p_r\circ F_{D_1}(\xi,\eta)&=p_r\circ F_{D_2}(\chi(\xi),\chi(\eta))\end{aligned}$$

$\forall(\xi,\eta)\in E_1$ *and* $\forall r=1,2,3,\cdots,m$.

Remark 4.1. *One can see the following.*

(1) *The weak isomorphism between two* $m-PSVNGs$ *preserves the orders.*
(2) *The weak isomorphism between* $m-PSVNGs$ *is a partial order relation.*
(3) *The co-weak isomorphism between two* $m-PSVNGs$ *preserves the sizes.*
(4) *The co-weak isomorphism between* $m-PSVNGs$ *is a partial order relation.*
(5) *The isomorphism between two* $m-PSVNGs$ *is an equivalence relation.*
(6) *The isomorphism between two* $m-PSVNGs$ *preserves the orders and sizes.*
(7) *The isomorphism between two* $m-PSVNGs$ *preserves the degrees of their vertices's.*

Definition 4.8. *Let* G *be a* $m-PSVNG$ *of* G^*, *the* $m-PSVN$*-Length of path* $Q:v_1,v_2,\ldots,v_n$, *which is denoted by* $l(Q)$, *is defined by*

$$l(Q)=(p_r\circ l_T(Q),p_r\circ l_I(Q),p_r\circ l_F)$$

where

$$p_r\circ l_T(Q)=\sum_{i=1}^{n-1}\frac{1}{p_r\circ T_B(v_i,v_{i+1})}$$

$$p_r \circ l_I(Q) = \sum_{i=1}^{n-1} \frac{1}{p_r \circ I_B(v_i, v_{i+1})}$$

$$p_r \circ l_F(Q) = \sum_{i=1}^{n-1} \frac{1}{p_r \circ F_B(v_i, v_{i+1})}$$

The $p_r \circ l_T(Q), p_r \circ l_I(Q)$ and $p_r \circ l_F(Q)$ are called the $m-$PSVN-T-Length, $m-$PSVN-I-Length and $m-$PSVN-F-Length of path Q, respectively. The $m-$PSVN-Distance between two vertices α and β in V, which is denoted by $\delta(\alpha, \beta)$, is defined by

$$\delta(\alpha, \beta) = (p_r \circ \delta_T(\alpha, \beta), p_r \circ \delta_I(\alpha, \beta), p_r \circ \delta_F(\alpha, \beta))$$

where

$$p_r \circ \delta_T(\alpha, \beta) = \inf(l_T(Q)), \ p_r \circ \delta_I(\alpha, \beta) = \inf(l_I(Q)), \ p_r \circ \delta_F(\alpha, \beta) = \inf(l_F(Q)).$$

where $p_r \circ \delta_T(\alpha, \beta), p_r \circ \delta_I(\alpha, \beta)$ and $p_r \circ \delta_F(\alpha, \beta)$ are called the $m-$PSVN-T-Distance, $m-$PSVN-I-Distance and $m-$PSVN-F-Distance, respectively of any path $\alpha-\beta$. The $m-$PSVN-Eccentricity of $v_i \in V$, which is denoted by $e(v_i)$, is defined by

$$e(v_i) = (p_r \circ e_T(v_i), p_r \circ e_I(v_i), p_r \circ e_F(v_i))$$

where

$$p_r \circ e_T(v_i) = \sup\{p_r \circ \delta_T(v_i, v_j) : v_j \in V, v_i \neq v_j\}$$

$$p_r \circ e_I(v_i) = \inf\{p_r \circ \delta_T(v_i, v_j) : v_j \in V, v_i \neq v_j\}$$

$$p_r \circ e_F(v_i) = \inf\{p_r \circ \delta_T(v_i, v_j) : v_j \in V, v_i \neq v_j\}$$

where $p_r \circ e_T(v_i), p_r \circ e_I(v_i)$ and $p_r \circ e_F(v_i)$ are called the $m-$PSVN-T-Eccentricity, $m-$PSVN-I-Eccentricity and $m-$PSVN-F-Eccentricity of vertex v_i, respectively. The $m-$PSVN-Radius of G, which is denoted by $r(G)$, is defined by

$$r(G) = (p_r \circ r_T(G), p_r \circ r_I(G), p_r \circ r_F(G))$$

where

$$p_r \circ r_T(G) = \inf\{p_r \circ e_T(v_i) : v_i \in V\}$$

$$p_r \circ r_I(G) = \inf\{p_r \circ e_I(v_i) : v_i \in V\}$$

$$p_r \circ r_F(G) = \inf\{p_r \circ e_F(v_i) : v_i \in V\}$$

where $p_r \circ r_T(G), p_r \circ r_I(G)$ and $p_r \circ r_F(G)$ are called the $m-$PSVN-T-Radius, $m-$PSVN-I-Radius and $m-$PSVN-F-Radius, respectively. The $m-$PSVN-Diameter of G, which is denoted by $d(G)$, is defined by

$$d(G) = (p_r \circ d_T(G), p_r \circ d_I(G), p_r \circ d_F(G))$$

where

$$p_r \circ d_T(G) = \sup\{p_r \circ e_T(v_i) : v_i \in V\}$$

$$p_r \circ d_I(G) = \sup\{p_r \circ e_I(v_i) : v_i \in V\}$$

$$p_r \circ d_F(G) = \sup\{p_r \circ e_F(v_i) : v_i \in V\}$$

where $p_r \circ d_T(G), p_r \circ d_I(G)$ and $p_r \circ d_F(G)$ are called the $m-$PSVN-T-Diameter, $m-$PSVN-I-Diameter and $m-$PSVN-F-Diameter, respectively.

Definition 4.9. *An $m-$Polar antipodal single valued neutrosophic graph (mPASVNG) $A(G) = (Q, R)$ of a $m-$PSVNG $G = (A, B)$ is the $m-$PSVNG in which*
(**a**) *$Q = A$ on V.* (**b**) *If $p_r \circ \delta(p, q) = p_r \circ d(G)$, then*
(**i**) *If $(p, q) \in E$, then $R = B$ on E.*
(**ii**) *If $(p, q) \notin E$, then*

$$p_r \circ T_R(p, q) = \inf(p_r \circ T_A(p), p_r \circ T_A(q))$$

$$p_r \circ I_R(p, q) = \sup(p_r \circ I_A(p), p_r \circ I_A(q))$$

$$p_r \circ F_R(p, q) = \sup(p_r \circ F_A(p), p_r \circ F_A(q))$$

Example 4.1. *Consider the crisp graph $G^* = (V, E)$ of 3-PSVNG $G = (A, B)$, the 3-PSVNSs A and B of $V = \{\xi, \eta, \zeta\}$ and $E = \{(\xi, \eta), (\eta, \zeta)(\zeta, \xi)\}$ are defined in Table. 1.*

$p_1 \circ A$	$p_1 \circ T_A$	$p_1 \circ I_A$	$p_1 \circ F_A$	$p_1 \circ B$	$p_1 \circ T_B$	$p_1 \circ I_B$	$p_1 \circ F_B$
ξ	1/5	1/4	1/3	(ξ, η)	1/7	1/2	1/3
η	1/7	1/2	1/5	(η, ζ)	1/7	1/2	1/5
ζ	1/4	1/6	1/8	(ζ, ξ)	1/5	1/4	1/3
$p_2 \circ A$	$p_2 \circ T_A$	$p_2 \circ I_A$	$p_2 \circ F_A$	$p_2 \circ B$	$p_2 \circ T_B$	$p_2 \circ I_B$	$p_2 \circ F_B$
ξ	1/4	1/6	1/2	(ξ, η)	1/4	1/5	1/2
η	1/3	1/5	1/3	(η, ζ)	1/3	1/5	1/3
ζ	1/2	1/8	1/7	(ζ, ξ)	1/4	1/6	1/2
$p_3 \circ A$	$p_3 \circ T_A$	$p_3 \circ I_A$	$p_3 \circ F_A$	$p_3 \circ B$	$p_3 \circ T_B$	$p_3 \circ I_B$	$p_3 \circ F_B$
ξ	1/3	1/2	1/6	(ξ, η)	1/8	1/2	1/6
η	1/8	1/3	1/7	(η, ζ)	1/8	1/3	1/3
ζ	1/6	1/5	1/3	(ζ, ξ)	1/6	1/2	1/3

TABLE 1. 3-PSVNSs of 3-PSVNG

By calculations 3-PSVNSs of 3-PASVNG, which are defined in Table. 2.

$p_1 \circ Q$	$p_1 \circ T_Q$	$p_1 \circ I_A$	$p_1 \circ F_Q$	$p_1 \circ R$	$p_1 \circ T_R$	$p_1 \circ I_R$	$p_1 \circ F_R$
ξ	1/5	1/4	1/3	(ξ, η)	1/7	1/2	1/3
η	1/7	1/2	1/5	(η, ζ)	0	0	0
ζ	1/4	1/6	1/8	(ζ, ξ)	0	0	0
$p_2 \circ Q$	$p_2 \circ T_Q$	$p_2 \circ I_A$	$p_2 \circ F_Q$	$p_2 \circ R$	$p_2 \circ T_R$	$p_2 \circ I_R$	$p_2 \circ F_R$
ξ	1/4	1/6	1/2	(ξ, η)	1/4	1/5	1/2
η	1/3	1/5	1/3	(η, ζ)	0	0	0
ζ	1/2	1/8	1/7	(ζ, ξ)	0	0	0
$p_3 \circ Q$	$p_3 \circ T_Q$	$p_3 \circ I_A$	$p_3 \circ F_Q$	$p_3 \circ R$	$p_3 \circ T_R$	$p_3 \circ I_R$	$p_3 \circ F_R$
ξ	1/3	1/2	1/6	(ξ, η)	0	0	0
η	1/8	1/3	1/7	(η, ζ)	0	0	0
ζ	1/6	1/5	1/3	(ζ, ξ)	0	0	0

TABLE 2. 3-PSVNSs of 3-PASVNG

Definition 4.10. *An eccentric $m-$PSVNG $G_e = (Q, R)$ of a $m-$PSVNG $G = (A, B)$, which is the $m-$PSVNG, is defined by*
(**a**) *$Q = A$ on V.* (**b**) *If*

$$p_r \circ \delta_T(\alpha, \beta) = \inf(p_r \circ e_T(\alpha), p_r \circ e_T(\beta))$$

$$p_r \circ \delta_I(\alpha, \beta) = \sup(p_r \circ e_I(\alpha), p_r \circ e_I(\beta))$$

$$p_r \circ \delta_F(\alpha, \beta) = \sup(p_r \circ e_F(\alpha), p_r \circ e_F(\beta))$$

then
(**i**) *If* $(\alpha, \beta) \in E$, *then* $R = B$ *on* E.
(**ii**) *If* $(\alpha, \beta) \notin E$, *then*

$$p_r \circ T_Q(\alpha, \beta) = \inf(p_r \circ T_A(\alpha), p_r \circ T_A(\beta))$$

$$p_r \circ I_Q(\alpha, \beta) = \sup(p_r \circ I_A(\alpha), p_r \circ I_A(\beta))$$

$$p_r \circ F_Q(\alpha, \beta) = \sup(p_r \circ F_A(\alpha), p_r \circ F_A(\beta))$$

(**c**) *Otherwise* $R = O = (0, \cdots, 0)$.

Example 4.2. *Consider the 3-PSVNG* $G = (A, B)$ *of* $G^* = (V, E)$, *which is given in Example. 4.1. By calculations 3-PSVNSs of eccentrc 3-PSVNG are given in Table. 3.*

$p_1 \circ Q$	$p_1 \circ T_Q$	$p_1 \circ I_A$	$p_1 \circ F_Q$	$p_1 \circ R$	$p_1 \circ T_R$	$p_1 \circ I_R$	$p_1 \circ F_R$
ξ	1/5	1/4	1/3	(ξ, η)	1/7	1/2	1/3
η	1/7	1/2	1/5	(η, ζ)	1/7	1/2	0
ζ	1/4	1/6	1/8	(ζ, ξ)	0	0	1/3
$p_2 \circ Q$	$p_2 \circ T_Q$	$p_2 \circ I_A$	$p_2 \circ F_Q$	$p_2 \circ R$	$p_2 \circ T_R$	$p_2 \circ I_R$	$p_2 \circ F_R$
ξ	1/4	1/6	1/2	(ξ, η)	0	0	0
η	1/3	1/5	1/3	(η, ζ)	0	0	1/3
ζ	1/2	1/8	1/7	(ζ, ξ)	0	0	0
$p_3 \circ Q$	$p_3 \circ T_Q$	$p_3 \circ I_A$	$p_3 \circ F_Q$	$p_3 \circ R$	$p_3 \circ T_R$	$p_3 \circ I_R$	$p_3 \circ F_R$
ξ	1/3	1/2	1/6	(ξ, η)	1/8	1/2	0
η	1/8	1/3	1/7	(η, ζ)	1/8	0	1/3
ζ	1/6	1/5	1/3	(ζ, ξ)	0	1/2	1/3

TABLE 3. 3-PSVNSs of eccentric 3-PSVNG

Proposition 4.1. *The* $m-$*PASVNG of the* $m-$*PSVNG is the generalization of* $m-$*Polar antipodal bipolar fuzzy graph and* $m-$*Polar antipodal intuitionistic bipolar fuzzy graph.*

Proposition 4.2. *The eccentric* $m-$*PSVNG is the generalization of* $m-$*Polar eccentric bipolar fuzzy graph and eccentric* $m-$*Polar intuitionistic bipolar fuzzy graph.*

Proposition 4.3. *The* $A(G)$ *is always a* $m-$*PSVN subgraph of* G_e. *For a complete* $m-$*PSVNG* $G = (C, D)$, $A(G)$ *is same as* G_e *and they are* $m-$*PSVN subgraphs of* G.

Definition 4.11. *The connected* $m-$*PSVNG* $G = (X, Y)$ *is distance regular* $m-$*PSVNG, whenever*

$$p_r \circ \delta(x, y) = k_r = (k_{1r}, k_{2r}, k_{3r})$$

$\forall\ x, y \in V$.

Theorem 4.1. *For the complete* $m-$*PSVNG* $G = (A, B)$ *where* A *be constant* $m-$*PSVNS then* G *and* $A(G)$ *are isomorphic.*

Proof. Since A is constant function, that is $A(x) = c_r = (c_{1r}, c_{2r}, c_{3r})$ where c_{1r}, c_{2r}, and c_{3r} are constants, hence $p_r \circ \delta(p, q) = d_r = (d_{1r}, d_{2r}, d_{3r})$ $\forall p, q \in V$, therefore eccentricity $p_r \circ e(\alpha) = d_r = (d_{1r}, d_{2r}, d_{3r})$ $\forall \alpha \in V$. Hence $p_r \circ d(G) = d_r = (d_{1r}, d_{2r}, d_{3r}) = \delta(\alpha, \beta)$ $\forall \alpha \in V$. Thus adjacency between every two vertices in $A(G)$ such that (i) $Q = A$ on V. (ii) Since α and β are neighbors in G, hence $R = B$ on E. Therefore G is isomorphic to $A(G)$. $\square$

Theorem 4.2. *If $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ are isomorphic $m-PSVNGs$, then so $A(G_1)$ and $A(G_2)$.*

Proof. By hypothesis there is an isomorphism τ between them preserves the weights of edges. Hence if vertex α has maximum T-eccentricity, minimum I-eccentricity, minimum F-eccentricity in G_1, then $\tau(\alpha)$ has maximum T-eccentricity, minimum I-eccentricity and minimum F-eccentricity in G_2, so G_1 and G_2 will have same diameter. If distance between α and β is $k_r = (k_{1r}, k_{2r}, k_{3r})$ in G_1, then $p_r \circ \tau(\alpha)$ and $p_r \circ \tau(\beta)$ will also have their distance as k_r in G_2, since τ is a bijective function between $A(G_1)$ and $A(G_2)$ with $p_r \circ Q_1(\alpha) = p_r \circ A_1(\alpha) = p_r \circ A_2(\alpha) = p_r \circ Q_2(\alpha)$ $\forall \alpha \in V_1$ and (i) If $(\alpha, \beta) \in E_1$, then $p_r \circ R_1 = p_r \circ B_1$. (ii) If $(\alpha, \beta) \notin E_1$, then

$$p_r \circ T_{R_1}(\alpha, \beta) = \inf(p_r \circ T_{A_1}(\alpha), p_r \circ T_{A_1}(\beta))$$

$$p_r \circ I_{R_1}(\alpha, \beta) = \sup(p_r \circ I_{A_1}(\alpha), p_r \circ I_{A_1}(\beta))$$

$$p_r \circ F_{R_1}(\alpha, \beta) = \sup(p_r \circ F_{A_1}(\alpha), p_r \circ F_{A_1}(\beta))$$

Since $\tau : G_1 \to G_2$ is an isomorphism, so if $(\alpha, \beta) \in E_1$ this implies $p_r \circ R_1(\alpha, \beta) = p_r \circ R_2(\tau(\alpha), \tau(\beta))$, if $(\alpha, \beta) \notin E_1$, then

$$p_r \circ T_{R_1}(\alpha, \beta) = \inf(p_r \circ \tau(\alpha), p_r \circ \tau(\beta))$$

$$p_r \circ I_{R_1}(\alpha, \beta) = \sup(p_r \circ \tau(\alpha), p_r \circ \tau(\beta))$$

$$p_r \circ F_{R_1}(\alpha, \beta) = \sup(p_r \circ \tau(\alpha), p_r \circ \tau(\beta))$$

Therefore we conclude that $p_r \circ R_1(\alpha, \beta) = p_r \circ R_2(\tau(\alpha), \tau(\beta))$. □

Theorem 4.3. *Let $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be two connected $m-PSVNGs$, If G_1 and G_2 are co-weak isomorphic, then $A(G_1)$ is homomorphic to $A(G_2)$.*

Theorem 4.4. *Let $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be two complete $m-PSVNGs$, if G_1 is co-weak isomorphic to G_2, then $A(G_1)$ is co-weak isomorphic to $A(G_2)$.*

Definition 4.12. *A vertex $v_i \in V$ is said to be a central vertex if $p_r \circ r(G) = p_r \circ e(v_i)$. The set of all central vertices of a $m-PSVNG$ G is $C(G)$, G is said to be self centered $m-PSVNG$ whenever $p_r \circ r(G) = p_r \circ e(v_i)$ $\forall v_i \in V$.*

Example 4.3. *The 3-PSVNG $G = (A, B)$ of G^*, which is given in Example 3.1 is a self centered 3-PSVNG.*

Remark 4.2. *Every complete $m-PSVNG$ $G = (A, B)$ is a self centered $m-PSVNG$.*

Remark 4.3. *In general converse part does not hold of Remark 4.2.*

Example 4.4. *Consider a crisp graph $G^* = (V, E)$, of 3-PSVNG $G = (A, B)$, the 3-PSVNSs A and B of V and E, which are defined in Table. 4. Here G is self centered 3-PSVNG, but G is not complete 3-PSVNG.*

$p_1 \circ A$	$p_1 \circ T_A$	$p_1 \circ I_A$	$p_1 \circ F_A$	$p_1 \circ B$	$p_1 \circ T_B$	$p_1 \circ I_B$	$p_1 \circ F_B$
α	1/5	1/3	1/3	$(\alpha,\beta))$	1/6	1/3	1/2
β	1/5	1/5	1/5	(β,γ)	1/5	1/3	1/2
γ	1/3	1/6	1/6	(γ,ξ)	1/6	1/3	1/2
ξ	1/4	1/4	1/4	(ξ,α)	1/5	1/3	1/2
$p_2 \circ A$	$p_2 \circ T_A$	$p_2 \circ I_A$	$p_2 \circ F_A$	$p_2 \circ B$	$p_2 \circ T_B$	$p_2 \circ I_B$	$p_2 \circ F_B$
α	1/4	1/5	1/6	(α,β)	1/4	1/4	1/6
β	1/3	1/4	1/7	(β,γ)	1/3	1/3	1/3
γ	1/2	1/3	1/3	(γ,ξ)	1/6	1/2	1/2
ξ	1/6	1/2	1/2	(ξ,α)	1/6	1/2	1/2
$p_3 \circ A$	$p_3 \circ T_A$	$p_3 \circ I_A$	$p_3 \circ F_A$	$p_3 \circ B$	$p_3 \circ T_B$	$p_3 \circ I_B$	$p_3 \circ F_B$
α	1/2	1/7	1/8	(α,β)	1/6	1/2	1/7
β	1/6	1/2	1/7	(β,γ)	1/6	1/2	1/4
γ	1/3	1/9	1/4	(γ,ξ)	1/5	1/7	1/3
ξ	1/5	1/7	1/3	(ξ,α)	1/5	1/7	1/3

TABLE 4. 3-PSVNSs of self centered 3-PSVNG

Remark 4.4. *The $m-PSVNG$ $G = (A, B)$ is self centered $m-PSVNG$ if and only if $p_r \circ d(G) = p_r \circ r(G)$.*

Theorem 4.5. *Let $H = (A^{'}, B^{'})$ be self centered $m-PSVNG$, then there exists a $m-PSVNG$ $G = (A, B)$ for which $< C(G) >$ and H are isomorphic. Further $2(p_r \circ r(G)) = p_r \circ d(G)$.*

Proof. Let $p_r \circ d_T(H) = l_r, p_r \circ d_I(H) = m_r$, and $p_r \circ d_F(H) = n_r$ next take two vertices $v_i, v_j \in V$ with $p_r \circ T_A(v_i) = p_r \circ T_A(v_j) = \frac{1}{l_r}, p_r \circ I_A(v_i) = p_r \circ I_A(v_j) = \frac{1}{2m_r}, p_r \circ F_A(v_i) = p_r \circ F_A(v_j) = \frac{1}{2n_r}$ and all the vertices of H are neighbors to both v_i and v_j with $p_r \circ T_B(v_i, v_k) = p_r \circ T_B(v_j, v_k) = \frac{1}{l_r}, p_r \circ I_B(v_i, v_k) = p_r \circ I_B(v_j, v_k) = \frac{1}{2m_r}, p_r \circ F_B(v_i, v_k) = p_r \circ F_B(v_j, v_k) = \frac{1}{2n_r}$ $\forall v_k \in V^{'}$. Next put $p_r \circ T_A = p_r \circ T_{A'}$, $p_r \circ I_A = p_r \circ I_{A'}$ and $p_r \circ F_A = p_r \circ F_{A'}$ for all vertices's in H and $p_r \circ T_B = p_r \circ T_{B'}, p_r \circ I_B = p_r \circ I_{B'}$ and $p_r \circ F_B = p_r \circ F_{B'}$ $\forall\ \alpha\beta \in E(H)$. If possible $p_r \circ T_A(v_i) > p_r \circ T_A(v_k)$ for at least one vertex $v_k \in V,^{'}$, then $\frac{1}{l_r} > p_r \circ T_A(v_k)$ that is $l_r < \frac{1}{p_r \circ T_A(v_k)} \leq \frac{1}{p_r \circ T_B(v_k, v_l)}$ this holds $\forall v_l \in V^{'}$ because H is $m-$PSVNG, thus $\frac{1}{p_r \circ T_B(v_k, v_l)} > l_r$ $\forall v_k \in V^{'}$ which contradict to fact $p_r \circ d_T(H) = l_r$, therefore $p_r \circ T_A(v_i) \leq T_A(v_k)$ $\forall v_k \in V^{'}$ and $p_r \circ T_B(v_i, v_k) \leq \inf(p_r \circ T_{Ai}, p_r \circ T_{Ak}) = \frac{1}{l_r}$, similarly $p_r \circ T_B(v_j, v_k) \leq \inf(p_r \circ T_{Aj}, p_r \circ T_{Ak}) = \frac{1}{l_r}$ $\forall v_k \in V,^{'}$ note that $p_r \circ I_A(v_i) \geq p_r \circ I_A(v_k)$ and $p_r \circ I_A(v_j) \geq p_r \circ I_A(v_k)$ $\forall v_k \in V^{'}$ since $p_r \circ d_I(H) = m_r$, therefore $p_r \circ I_B(v_i, v_k) \geq \sup(p_r \circ I_{Ai}, p_r \circ I_{Ak}) = \frac{1}{2m_r}$, similarly $p_r \circ I_B(v_j, v_k) \geq \sup(p_r \circ I_{Aj}, p_r \circ I_{Ak}) = \frac{1}{2m_r}$ $\forall v_k \in V,^{'}$ similarly $p_r \circ F_A(v_i) \geq p_r \circ F_A(v_k)$ and $p_r \circ F_A(v_j) \geq p_r \circ F_A(v_k)$ $\forall v_k \in V,^{'}$ since $p_r \circ d_F(H) = n_r$, therefore $p_r \circ F_B(v_i, v_k) \leq \sup(p_r \circ F_{Ai}, p_r \circ F_{Ak}) = \frac{1}{2n_r}$, similarly $p_r \circ F_B(v_j, v_k) \geq \sup(p_r \circ F_{Aj}, p_r \circ F_{Ak}) = \frac{1}{2n_r}$ $\forall v_k \in V^{'}$. Hence G is $m-$PSVNG. Also $p_r \circ e_T(v_k) = l_r$ $\forall v_k \in V^{'}$ and $p_r \circ e_T(v_i) = p_r \circ e_T(v_j) = \frac{1}{p_r \circ T_B(v_i, v_k)} + \frac{1}{p_r \circ T_B(v_i, v_k)} = 2l_r$, $p_r \circ r_T(G) = l_r, p_r \circ d_T(G) = 2l_r$. Next $p_r \circ e_I(v_k) = m_r$ $\forall v_k \in V^{'}$ and $p_r \circ e_I(v_i) = p_r \circ e_I(v_j) = \frac{1}{p_r \circ I_B(v_l, v_k)} = 2m_r, p_r \circ r_I(G) = m_r, p_r \circ d_I(G) = 2m_r$. Similarly $p_r \circ e_F(v_k) = n_r$ $\forall v_k \in V^{'}$ and $p_r \circ e_F(v_i) = p_r \circ e_F(v_j) = \frac{1}{p_r \circ F_B(v_l, v_k)} = 2n_r$, $p_r \circ r_F(G) = n_r$, $p_r \circ d_F(G) = 2n_r$. □

Definition 4.13. *The status of vertex ξ, which is denoted by $S(\xi)$, is defined by*

$$S(\xi) = (p_r \circ S_T(\xi), p_r \circ S_I(\xi), p_r \circ S_F(\xi))$$

where

$$p_r \circ S_T(\xi) = \sum_{\eta \in V} p_r \circ \delta_T(\xi, \eta),\ p_r \circ S_I(\xi) = \sum_{\eta \in V} p_r \circ \delta_I(\xi, \eta),\ p_r \circ S_F(\xi) = \sum_{\eta \in V} p_r \circ \delta_F(\xi, \eta)$$

where $p_r \circ S_T(\xi), p_r \circ S_I(\xi)$ and $p_r \circ S_F(\xi)$ are called $m-$PSVN-T-status, $m-$PSVN-I-status and $m-$PSVN-F-status of the vertex ξ, respectively. The connected $m-$PSVNG is called self median $m-$PSVNG, if every vertex has the same status.

Remark 4.5. *Let $G = (C, D)$ be a connected $m-$PSVNG of G^*, which is an even cycle, then G is self median $m-$PSVNG, if alternative edges have same truth, indeterminacy and falsity membership values.*

Example 4.5. *The 3-PSVNG which is given in Example 4.4 is also self median 3-PSVNG.*

5. Conclusion

In this paper, we discussed the special classes of SVNGs, antipodal SVNGs, eccentric SVNGs, self centered SVNGs and self-median SVNGs of the given SVNGs. We also investigated isomorphism properties on antipodal SVNGs. Next, we generalize into the $m-$Polar single valued neutrosophic graph which is the generalization of $m-$Polar fuzzy, $m-$Polar intuitionistic fuzzy, $m-$Polar bipolar fuzzy, $m-$Polar bipolar intuitionistic fuzzy graphs. The $m-$PSVNGs gives more flexibility than BSVNGs. The $m-$PSVNGs have many applications in path problems, networks and computer science. The concept of $m-$Polar antipodal SVNG, eccentric $m-$PSVNG, self centered $m-$PSVNG and self median $m-$PSVNG of the given $m-$PSVNG introduced here. The weak isomorphism, co weak isomorphism and isomorphism properties of antipodal $m-$PSVNG, eccentric $m-$PSVNG and self centered $m-$PSVNG discussed in this article.

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