HARMONIC RECIPROCAL STATUS INDEX AND COINDEX OF GRAPHS

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Abstract. The reciprocal status of a vertex u is defined as the sum of reciprocal of the distances between u and all other vertices of a graph G. In this paper we have defined the harmonic reciprocal status index and coindex of a graph and obtained the bounds for it. Further the harmonic reciprocal status index and coindex of some graphs are obtained.

Keywords: Distance in graph, Reciprocal status of a vertex, Harmonic reciprocal status index.

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1. Introduction

The harmonic index, based on the degrees of the vertices is well studied in the literature [2, 3, 6, 8, 9, 15, 17, 18]. In this paper we study the harmonic index, based on the reciprocal distances in graphs.

Let G be a connected, nontrivial graph on n vertices and m edges. Let $V(G)$ be the vertex set and $E(G)$ be the edge set of G. The edge joining the vertices u and v is denoted by uv. The *degree* of a vertex u is the number of edges incident to it and is denoted by $d(u)$. If all the vertices of G have same degree equal to r, then G is called a *regular graph* of degree r. The *distance* between the vertices u and v, denoted by $d(u, v)$, is the length of the shortest path joining u and v in G. The *eccentricity* of a vertex u in a graph G is defined as $e(u) = \max\{d(u, v) \mid v \in V(G)\}$. The maximum distance between any pair of vertices in G is called the *diameter* of G and is denoted by $diam(G)$ [1].

The *status* [5] of a vertex u is defined as the sum of its distances from every other vertex of G and is denoted by $\sigma(u)$. That is,

$$\sigma(u) = \sum_{v \in V(G)} d(u, v).$$

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In [14], the first and second *status connectivity indices* of a connected graph G are defined respectively as

$$S_1(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v)] \quad \text{and} \quad S_2(G) = \sum_{uv \in E(G)} \sigma(u)\sigma(v).$$

The *reciprocal status* of a vertex u is defined as the sum of reciprocal of its distances from every other vertex of G and is denoted by $rs(u)$. That is,

$$rs(u) = \sum_{v \in V(G),\, u \neq v} \frac{1}{d(u,v)}.$$

The *Harary index* $HI(G)$ of a connected graph G is defined as the sum of reciprocal of the distances between all pairs of vertices of G [7]. That is,

$$HI(G) = \sum_{\{u,v\} \subseteq V(G),\, u \neq v} \frac{1}{d(u,v)} = \frac{1}{2} \sum_{u \in V(G)} rs(u).$$

For more about Harary index one can refer [10, 16].

The *first reciprocal status connectivity index* $RS_1(G)$ and *second reciprocal status connectivity index* $RS_2(G)$ of a connected graph G are defined respectively as [12, 13]

$$RS_1(G) = \sum_{uv \in E(G)} [rs(u) + rs(v)] \quad \text{and} \quad RS_2(G) = \sum_{uv \in E(G)} rs(u)rs(v).$$

The *harmonic index* of a graph G is defined as [4]

$$H(G) = \sum_{uv \in E(G)} \frac{2}{d(u) + d(v)}.$$

Recent results on the harmonic index can be found in [2, 3, 6, 8, 9, 15, 17, 18].

The *Harmonic status index* of a graph G is defined as [11]

$$HS(G) = \sum_{uv \in E(G)} \frac{2}{\sigma(u) + \sigma(v)}.$$

Motivated by the harmonic index and harmonic status index of a graph, we introduce and study here the harmonic reciprocal status index and harmonic reciprocal status coindex of connected graphs.

The *harmonic reciprocal status index* of a connected graph G is defined as

$$HRS(G) = \sum_{uv \in E(G)} \frac{2}{rs(u) + rs(v)}$$

and *harmonic reciprocal status coindex* of a connected graph G is defined as

$$\overline{HRS}(G) = \sum_{uv \notin E(G)} \frac{2}{rs(u) + rs(v)}.$$

For a graph given in Fig. 1, $HRS(G) = \frac{913}{420} \approx 2.1738$ and $\overline{HRS}(G) = \frac{8}{13} \approx 0.6153$.

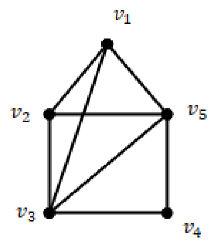

Figure 1

2. Harmonic Reciprocal Status Index

First we give bounds for the harmonic reciprocal status index.

Theorem 2.1. *Let G be a connected graph with n vertices and let $diam(G) = D$. Then*

$$\sum_{uv\in E(G)} \frac{2}{n-1+\frac{1}{2}\left[d(u)+d(v)\right]} \leq HRS(G) \leq \sum_{uv\in E(G)} \frac{2}{\frac{2}{D}(n-1)+\left(1-\frac{1}{D}\right)\left[d(u)+d(v)\right]}.$$

Equality on both sides holds if and only if $diam(G) \leq 2$.

Proof. Upper bound: For any vertex u of G, there are $d(u)$ vertices which are at distance 1 from u and the remaining $n-1-d(u)$ vertices are at distance at most D. Therefore for any vertex $u \in V(G)$,

$$rs(u) \geq d(u) + \frac{1}{D}(n-1-d(u)) = \frac{1}{D}(n-1) + d(u)\left(1-\frac{1}{D}\right).$$

Therefore

$$\begin{aligned} HRS(G) &= \sum_{uv\in E(G)} \frac{2}{rs(u)+rs(v)} \\ &\leq \sum_{uv\in E(G)} \frac{2}{\frac{2}{D}(n-1)+\left(1-\frac{1}{D}\right)\left(d(u)+d(v)\right)}. \end{aligned}$$

Lower bound: For any vertex u of G, there are $d(u)$ vertices which are at distance 1 from u and the remaining $n-1-d(u)$ vertices are at distance at least 2. Therefore for any vertex $u \in V(G)$,

$$rs(u) \leq d(u) + \frac{1}{2}(n-1-d(u)) = \frac{1}{2}\left[d(u)+n-1\right].$$

Therefore

$$\begin{aligned} HRS(G) &= \sum_{uv\in E(G)} \frac{2}{rs(u)+rs(v)} \\ &\geq \sum_{uv\in E(G)} \frac{2}{(n-1)+\frac{1}{2}\left[d(u)+d(v)\right]}. \end{aligned}$$

For equality: If the diameter of G is 1 or 2 then the equality holds.

Conversely, let

$$HRS(G) = \sum_{uv\in E(G)} \frac{2}{(n-1)+\frac{1}{2}\left[d(u)+d(v)\right]}.$$

Suppose, $diam(G) \geq 3$, then there exists at least one pair of vertices, say u_1 and u_2 such that $d(u_1, u_2) \geq 3$.

Therefore

$$rs(u_1) \leq d(u_1) + \frac{1}{3} + \frac{1}{2}(n - 2 - d(u_1)) = \frac{n}{2} - \frac{2}{3} + \frac{d(u_1)}{2}.$$

Similarly $rs(u_2) \leq \frac{n}{2} - \frac{2}{3} + \frac{d(u_2)}{2}$ and for all other vertices u of G, $rs(u) \leq \frac{n}{2} - \frac{1}{2} + \frac{d(u)}{2}$.

Partition the edge set of G into three sets E_1, E_2 and E_3, such that

$$E_1 = \left\{ u_1 v \mid rs(u_1) \leq \frac{n}{2} - \frac{2}{3} + \frac{d(u_1)}{2} \text{ and } rs(v) \leq \frac{n}{2} - \frac{1}{2} + \frac{d(v)}{2} \right\},$$

$$E_2 = \left\{ u_2 v \mid rs(u_2) \leq \frac{n}{2} - \frac{2}{3} + \frac{d(u_2)}{2} \text{ and } rs(v) \leq \frac{n}{2} - \frac{1}{2} + \frac{d(v)}{2} \right\}$$

and

$$E_3 = \left\{ uv \mid rs(u) \leq \frac{n}{2} - \frac{1}{2} + \frac{d(u)}{2} \text{ and } rs(v) \leq \frac{n}{2} - \frac{1}{2} + \frac{d(v)}{2} \right\}.$$

It is east to check that $|E_1| = d(u_1)$, $|E_2| = d(u_2)$ and $|E_3| = m - d(u_1) - d(u_2)$. Therefore

$$\begin{aligned}
HRS(G) &= \sum_{uv \in E(G)} \frac{2}{rs(u) + rs(v)} \\
&= \sum_{u_1 v \in E_1} \frac{2}{rs(u_1) + rs(v)} + \sum_{u_2 v \in E_2} \frac{2}{rs(u_2) + rs(v)} + \sum_{uv \in E_3} \frac{2}{rs(u) + rs(v)} \\
&\geq \sum_{u_1 v \in E_1} \frac{2}{\left[n - \frac{7}{6} + \frac{1}{2}(d(u_1) + d(v))\right]} + \sum_{u_2 v \in E_2} \frac{2}{\left[n - \frac{7}{6} + \frac{1}{2}(d(u_2) + d(v))\right]} \\
&\quad + \sum_{uv \in E_3} \frac{2}{\left[n - 1 + \frac{1}{2}(d(u) + d(v))\right]} \\
&> \sum_{uv \in E(G)} \frac{2}{n - 1 + \frac{1}{2}\left[d(u) + d(v)\right]},
\end{aligned}$$

which is a contradiction. Hence $diam(G) \leq 2$. □

Corollary 2.1. *Let G be a connected graph with n vertices, m edges and $diam(G) = D$. Let δ and Δ be the minimum and maximum degree of the vertices of G respectively. Then*

$$\frac{2m}{n - 1 + \Delta} \leq HRS(G) \leq \frac{m}{\frac{n-1}{D} + \left(1 - \frac{1}{D}\right)\delta}.$$

Proof. For any vertex u of G, $\delta \leq d(u) \leq \Delta$. Therefore substituting $d(u) + d(v) \geq 2\delta$ in the upper bound and $d(u) + d(v) \leq 2\Delta$ in the lower bound of Theorem 2.1, we get the results. □

Corollary 2.2. *Let G be a connected regular graph of degree r on n vertices and m edges and let $diam(G) = D$. Then*

$$\frac{2m}{n - 1 + r} \leq HRS(G) \leq \frac{m}{\frac{n-1}{D} + \left(1 - \frac{1}{D}\right) r}.$$

Equality on both side holds if and only if $diam(G) \leq 2$.

Proof. For any vertex u of G, $d(u) = r$. Therefore the results follows by the Theorem 2.1. □

Now we compute the harmonic reciprocal status index of some specific graphs.

Proposition 2.1. *For a complete graph K_n on n vertices, $HRS(K_n) = \frac{n}{2}$.*

Proof. For any vertex u of K_n, $rs(u) = n-1$. Therefore by the definition of harmonic reciprocal status index, $HRS(K_n) = \frac{n}{2}$. □

Proposition 2.2. *For a complete bipartite graph $K_{p,q}$, $HRS(K_{p,q}) = \frac{4pq}{3(p+q)-2}$.*

Proof. The vertex set $V(K_{p,q})$ can be partitioned into two independent sets V_1 and V_2 such that for every edge uv of $K_{p,q}$, the vertex $u \in V_1$ and $v \in V_2$. Therefore $d(u) = q$ and $d(v) = p$, where $|V_1| = p$ and $|V_2| = q$. The graph $K_{p,q}$ has $n = p+q$ vertices and $m = pq$ edges. Also $diam(K_{p,q}) \leq 2$. Threfore by the equality part of Theorem 2.1,

$$HRS(K_{p,q}) = \sum_{uv \in E(K_{p,q})} \frac{2}{p+q-1+\frac{1}{2}[p+q]} = \frac{4pq}{3(p+q)-2}.$$

□

Proposition 2.3. *For a path P_n on n vertices,*

$$HRS(P_n) = \left[\frac{4}{\frac{n}{n-1} + 2\sum_{i=1}^{n-2}\frac{1}{i}}\right] + \sum_{i=2}^{n-2}\left[\frac{2}{\frac{n}{i(n-i)} + 2\left[\sum_{j=1}^{i-1}\frac{1}{j} + \sum_{j=1}^{n-i-1}\frac{1}{j}\right]}\right].$$

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of P_n, where v_i is adjacent to v_{i+1}, $i = 1, 2, \ldots, n-1$. Therefore for $i = 1, 2, \ldots, n$,

$$\begin{aligned} rs(v_1) &= \sum_{i=1}^{n-1}\frac{1}{i}, \\ rs(v_i) &= \sum_{j=1}^{i-1}\frac{1}{j} + \sum_{j=1}^{n-i}\frac{1}{j}, \quad \text{for } 2 \leq i \leq n-1 \\ \text{and} \quad rs(v_n) &= \sum_{i=1}^{n-1}\frac{1}{i}. \end{aligned}$$

Therefore,

$$\begin{aligned} HRS(P_n) &= \sum_{uv \in E(P_n)} \frac{2}{rs(u)+rs(v)} \\ &= \left[\frac{2}{rs(v_1)+rs(v_2)}\right] + \sum_{i=2}^{n-2}\left[\frac{2}{rs(v_i)+rs(v_{i+1})}\right] + \left[\frac{2}{rs(v_{n-1})+rs(v_n)}\right] \\ &= \left[\frac{2}{\sum_{i=1}^{n-1}\frac{1}{i} + 1 + \sum_{j=1}^{n-2}\frac{1}{j}}\right] + \sum_{i=2}^{n-2}\left[\frac{2}{\sum_{j=1}^{i-1}\frac{1}{j} + \sum_{j=1}^{n-i}\frac{1}{j} + \sum_{j=1}^{i}\frac{1}{j} + \sum_{j=1}^{n-i-1}\frac{1}{j}}\right] \\ &\quad + \left[\frac{2}{\sum_{j=1}^{n-2}\frac{1}{j} + 1 + \sum_{i=1}^{n-1}\frac{1}{i}}\right] \\ &= \left[\frac{4}{\frac{n}{n-1} + 2\sum_{i=1}^{n-2}\frac{1}{i}}\right] + \sum_{i=2}^{n-2}\left[\frac{2}{\frac{n}{i(n-i)} + 2\left[\sum_{j=1}^{i-1}\frac{1}{j} + \sum_{j=1}^{n-i-1}\frac{1}{j}\right]}\right]. \end{aligned}$$

□

Proposition 2.4. *For a cycle C_n on $n \geq 3$ vertices,*

$$HRS(C_n) = \begin{cases} \frac{n}{\frac{2}{n}+2\sum_{i=1}^{(n-2)/2}\frac{1}{i}}, & \text{if } n \text{ is even} \\ \frac{n}{2\sum_{i=1}^{(n-1)/2}\frac{1}{i}}, & \text{if } n \text{ is odd.} \end{cases}$$

Proof. Case (i): If n is even number then for any vertex u of C_n,

$$rs(u) = 2\left[1 + \frac{1}{2} + \cdots + \frac{1}{\frac{n-2}{2}}\right] + \frac{1}{\frac{n}{2}} = \frac{2}{n} + 2\sum_{i=1}^{(n-2)/2}\frac{1}{i}.$$

Therefore,

$$\begin{aligned} HRS(C_n) &= \sum_{uv \in E(C_n)} \frac{2}{rs(u)+rs(v)} \\ &= \sum_{uv \in E(C_n)} \left[\frac{2}{\frac{2}{n}+2\sum_{i=1}^{(n-2)/2}\frac{1}{i}+\frac{2}{n}+2\sum_{i=1}^{(n-2)/2}\frac{1}{i}}\right] \\ &= \frac{n}{\frac{2}{n}+2\sum_{i=1}^{(n-2)/2}\frac{1}{i}}. \end{aligned}$$

Case (ii): If n is odd then for any vertex u of C_n,

$$rs(u) = 2\left[1 + \frac{1}{2} + \cdots + \frac{1}{\frac{n-1}{2}}\right] = 2\sum_{i=1}^{(n-1)/2}\frac{1}{i}.$$

Therefore

$$\begin{aligned} HRS(C_n) &= \sum_{uv \in E(C_n)} \frac{2}{rs(u)+rs(v)} \\ &= \sum_{uv \in E(C_n)} \frac{2}{2\sum_{i=1}^{(n-1)/2}\frac{1}{i}+2\sum_{i=1}^{(n-1)/2}\frac{1}{i}} \\ &= \frac{n}{2\sum_{i=1}^{(n-1)/2}\frac{1}{i}}. \end{aligned}$$

□

A *wheel* W_{k+1} is a graph obtained from the cycle C_k, $k \geq 3$, by adding a new vertex and making it adjacent to all the vertices of C_k. The degree of a cental vertex of W_{k+1} is k and the degree of all other vertices is 3.

Proposition 2.5. *For a wheel W_{k+1}, $k \geq 3$,*

$$HRS(W_{k+1}) = \frac{2k(5k+9)}{3k^2+12k+9}.$$

Proof. Partition the edge set $E(W_{k+1})$ into two sets E_1 and E_2, such that $E_1 = \{uv \mid d(u) = k \text{ and } d(v) = 3\}$ and $E_2 = \{uv \mid d(u) = 3 \text{ and } d(v) = 3\}$. It is easy to check that $|E_1| = |E_2| = k$. Also $diam(W_{k+1}) = 2$. Therefore by the equality part of Theorem 2.1,

$$
\begin{aligned}
HRS(W_{k+1}) &= \sum_{uv\in E(W_{n+1})} \frac{2}{k+\frac{1}{2}\left[d(u)+d(v)\right]} \\
&= \sum_{uv\in E_1} \frac{2}{k+\frac{1}{2}\left[d(u)+d(v)\right]} + \sum_{uv\in E_2} \frac{2}{k+\frac{1}{2}\left[d(u)+d(v)\right]} \\
&= \sum_{uv\in E_1} \frac{2}{k+\frac{1}{2}(k+3)} + \sum_{uv\in E_2} \frac{2}{k+\frac{1}{2}(3+3)} \\
&= \frac{2k}{k+\frac{1}{2}(k+3)} + \frac{2k}{k+3} \\
&= \frac{2k(5k+9)}{3k^2+12k+9}.
\end{aligned}
$$

□

A *windmill graph* $F_k, k \geq 2$, is a graph that can be constructed by coalescence k copies of the cycle C_3 of length 3 with a common vertex. It has $2k+1$ vertices and $3k$ edges. The degree of a coalescence vertex of F_k is $2k$ and the degree of all other vertices is 2.

Proposition 2.6. *For a windmill graph F_k, $k \geq 2$,*

$$HRS(F_k) = \frac{k(7k+5)}{3k^2+4k+1}.$$

Proof. Partition the edge set $E(F_k)$ into two sets E_1 and E_2, such that $E_1 = \{uv \mid d(u) = 2k \text{ and } d(v) = 2\}$ and $E_2 = \{uv \mid d(u) = 2 \text{ and } d(v) = 2\}$. It is easy to check that $|E_1| = 2k$ ans $|E_2| = k$. Also $diam(F_k) = 2$. Therefore by the equality part of Theorem 2.1,

$$
\begin{aligned}
HRS(F_k) &= \sum_{uv\in E(F_k)} \frac{2}{2k+\frac{1}{2}\left[d(u)+d(v)\right]} \\
&= \sum_{uv\in E_1} \frac{2}{2k+\frac{1}{2}\left[d(u)+d(v)\right]} + \sum_{uv\in E_2} \frac{2}{2k+\frac{1}{2}\left[d(u)+d(v)\right]} \\
&= \sum_{uv\in E_1} \frac{2}{2k+\frac{1}{2}\left[2k+2\right]} + \sum_{uv\in E_2} \frac{2}{2k+\frac{1}{2}\left[2+2\right]} \\
&= \frac{4k}{3k+1} + \frac{2k}{2k+2} \\
&= \frac{k(7k+5)}{3k^2+4k+1}.
\end{aligned}
$$

□

3. Harmonic Reciprocal Status Coindex of Graphs

Theorem 3.1. *Let G be a connected graph on n vertices and let $diam(G) = D$. Then*

$$\sum_{uv\notin E(G)} \frac{2}{n-1+\frac{1}{2}\left[d(u)+d(v)\right]} \leq \overline{HRS}(G) \leq \sum_{uv\notin E(G)} \frac{2}{\frac{2}{D}(n-1)+\left(1-\frac{1}{D}\right)\left[d(u)+d(v)\right]}.$$

Equality on both sides holds if and only if $diam(G) \leq 2$.

Proof. Proof is analogous to that of Theorem 2.1. □

Corollary 3.1. *Let G be a connected graph with n vertices, m edges and $diam(G) = D$. Let δ and Δ be the minimum and maximum degree of the vertices of G respectively. Then*

$$\frac{n(n-1)-2m}{n-1+\Delta} \leq \overline{HRS}(G) \leq \frac{n(n-1)-2m}{2\left[\frac{n-1}{D}+\left(1-\frac{1}{D}\right)\delta\right]}.$$

Proof. For any vertex $u \in V(G)$, $\delta \leq d(u) \leq \Delta$. Therefore $2\delta \leq d(u)+d(v) \leq 2\Delta$. The graph G has $\frac{n(n-1)}{2} - m$ pair of non adjacent vertices. Substituting $d(u)+d(v) \geq 2\delta$ in the upper bound and $d(u)+d(v) \leq 2\Delta$ in the lower bound of Theorem 3.1 we get the results. □

Corollary 3.2. *Let G be a connected r-regular graph on n vertices and let $diam(G) = D$. Then*

$$\frac{n(n-1)-nr}{n-1+r} \leq \overline{HRS}(G) \leq \frac{n(n-1)-nr}{2\left[\frac{n-1}{D}+\left(1-\frac{1}{D}\right)r\right]}.$$

Equality on both sides holds if and only if $diam(G) \leq 2$.

Proof. Substituting $d(u) = r$ for all $u \in V(G)$ in Theorem 3.1, we get the results. □

Proposition 3.1. *For a complete graph K_n, $\overline{HRS}(K_n) = 0$.*

Proposition 3.2. *For a complete bipartite graph $K_{p,q}$,*

$$\overline{HRS}(K_{p,q}) = \frac{p(p-1)}{2q+p-1} + \frac{q(q-1)}{2p+q-1}.$$

Proof. Let V_1 and V_2 be the partite sets of $V(K_{p,q})$, where $|V_1| = p$ and $|V_2| = q$ such that for every edge of $K_{p,q}$ has one end in V_1 and other end in V_2. If $u \in V_1$ then $rs(u) = q + \frac{1}{2}(p-1)$ and if $u \in V_2$ then $rs(u) = p + \frac{1}{2}(q-1)$. Therefore for $u, v \in V_1$, $rs(u) + rs(v) = 2q + (p-1)$ and for $u, v \in V_2$, $rs(u) + rs(v) = 2p + (q-1)$. Therefore,

$$\begin{aligned}
\overline{HRS}(K_{p,q}) &= \sum_{uv \notin E(K_{p,q})} \frac{2}{rs(u)+rs(v)} \\
&= \sum_{\{u,v\} \subseteq V_1} \frac{2}{rs(u)+rs(v)} + \sum_{\{u,v\} \subseteq V_2} \frac{2}{rs(u)+rs(v)} \\
&= \frac{p(p-1)}{2q+p-1} + \frac{q(q-1)}{2p+q-1}.
\end{aligned}$$

□

Proposition 3.3. *For a cycle C_n on $n \geq 3$ vertices,*

$$\overline{HRS}(C_n) = \begin{cases} \frac{n^2-3n}{\frac{4}{n}+4\sum_{i=1}^{(n-2)/2}\frac{1}{i}}, & \text{if } n \text{ is even} \\ \frac{n^3-3n}{4\sum_{i=1}^{(n-1)/2}\frac{1}{i}}, & \text{if } n \text{ is odd.} \end{cases}$$

Proof. There are $\frac{n(n-1)}{2} - n$ pairs of non-adjacent vertices in C_n. As seen in Proposition 2.4, we have for a vertex u of C_n,

$$rs(u) = \begin{cases} \frac{2}{n} + 2\sum_{i=1}^{(n-2)/2}\frac{1}{i}, & \text{if } n \text{ is even} \\ 2\sum_{i=1}^{(n-1)/2}\frac{1}{i}, & \text{if } n \text{ is odd.} \end{cases}$$

Therefore by the definition of harmonic reciprocal status coindex, we get the results. □

Proposition 3.4. *For a wheel W_{k+1}, $k \geq 3$,*

$$\overline{HRS}(W_{k+1}) = \frac{k(k-3)}{k+3}.$$

Proof. The non adjacent pairs of vertices of the wheel W_{k+1} has degree 3 and there are $\frac{(k+1)k}{2} - 2k$ pairs of non adjacent vertices in W_{k+1}. Also $diam(W_{k+1}) = 2$. Therefore by the equality part of Theorem 3.1, we get the result. □

Proposition 3.5. *For a windmill graph F_k, $k \geq 2$,*

$$\overline{HRS}(F_k) = \frac{2k(k-1)}{k+1}.$$

Proof. The non adjacent pairs of vertices of the windmill graph F_k has degree 2 and there are $\frac{2k(2k+1)}{2} - 3k$ such pairs in F_k. Also $diam(F_k) = 2$. Therefore by the equality part of Theorem 3.1, we get the result. □

4. Conclusion

We have introduced harmonic reciprocal status index and coindex of connected graphs and obtained bounds for these indices. Also these indices of certain standard graphs have been obtained.

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