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CODES OVER THE MULTIPLICATIVE HYPERRINGS

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Abstract. Codes over hyperstructures have more codewords than codes over rings(or fields). It implies that they have higher rate than codes over rings (or fields). So, in this paper the codes over multiplicative hyperrings are studied. Linear codes and the cyclic codes over multiplicative hyperrings are constructed.

Keywords: Hyperring, multiplicative hyperring, linear codes, cyclic codes.

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1. Introduction

The fundamental work of algebraic coding theory belongs to Claude Shannon [7]. The paper, pressed in 1948, focused on the problem of how best to encode the information a sender wants to transmit. In this work, he used tools in probability theory. Shannon developed information entropy as a measure for the uncertainty in a message while essentially inventing the field of information theory. The binary Golay code was developed in 1949, [15]. It is an error-correcting code capable of correcting up to three errors in each 24-bit word, and detecting a fourth. Richard Hamming won the Turing Award in 1968 for his work at Bell Labs in numerical methods, automatic coding systems, and error-detecting and error-correcting codes. He invented the concepts known as Hamming codes, Hamming windows, Hamming numbers and Hamming distance.

More recently, the researchers on algebraic coding theory focus on linear codes on fields (especially on the binary fields) because of their many applications in practice. Cyclic codes are important families of linear codes because of their rich algebraic structures and practical applications. Hammons et al. is considered to be a major turning point in coding theory. Because they show an important link between binary (quaternary) linear codes and some well-known binary nonlinear perfect codes in [8]. In later times, most studies focus on the codes on rings [1-5]. However, there are optimal codes on non-chain rings. So, the coding theorists consider construct the codes on different algebraic structures.

The history of popular algebraic hyperstructures that have attracted the interest of many researchers in recent years is based on Marty's study of 1934 [6]. Following this

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work, Krasner's article [9] in 1983 can be regarded as a milestone in this area. In this work, algebraic structures called Krasner hyperfields are defined. Davvaz and Leoreanu-Fotea's book entitled *Hyperring and applications* sheds light on many researchers working on this area [10]. In 2003, Ciampi et al. constructed the hyperring over the set of the polynomials with coefficients in a convenient algebraic structure [12]. In [13], Ameri and Norouzi introduced some notions and they gave some algebraic properties of commutative hyperrings.

The idea of constructing algebraic codes on hyperstructures was first proposed by Davvaz and Musavi [11]. They defined the linear codes and the cyclic codes over a finite Krasner hyperring in the paper. Also, they gave the structure of $l-$quasi-cyclic codes. In 2017, Tsafack et al. studied on codes over hyperfields [14].

This article is about codes over multiplicative hyperrings. The linear codes and the cyclic codes are structured in it. We have a much greater number of code words when we move the codes on the known rings (or fields) to the hyperrings defined by the hyperoperations. Moreover, the length and the alphabet of the code on this new algebraic structure do not change. It is known that the rate of a code (is the amount of non-redundant information per bit in codewords of a code) increases when increasing the number of code words by keeping the length constant.

$$R(C) = \frac{\log |C|}{n \log |A|},$$

where $R(C)$ is the rate of the code C, $|C|$ is the number of elements in C, and $|A|$ is the number of elements in alphabet.

2. Preliminaries

A mapping $\circ : H \times H \longrightarrow \mathcal{P}^*(H)$ is called a *hyperoperation* where $\mathcal{P}^*(H)$ is the set of all the nonempty subsets of H. An algebraic system $(H, \circ)$, where $\circ$ is a hyperoperation defined on H, is called a *hypergroupoid*.
For any two nonempty subsets A and B of H and $x \in H$, the operation is defined

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, A \circ x = A \circ \{x\}.$$

If $a \circ (b \circ c) = (a \circ b) \circ c$ for all $a, b, c \in H$, which means that

$$\bigcup_{u \in b \circ c} a \circ u = \bigcup_{v \in a \circ b} v \circ c,$$

then the hyperoperation $\circ$ is *associative*. A hypergroupoid with the associative hyperoperation is called a *semihypergroup*. A hypergoupoid $(H, \circ)$ is a *quasihypergroup*, whenever $a \circ H = H = H \circ a$ for all $a \in H$. If $(H, \circ)$ is a semihypergroup and a quasihypergroup, then $(H, \circ)$ is called a *hypergroup*. A nonempty subset K of a semihypergroup $(H, \circ)$ is called a subhypergroup if we have $x \circ K = K = K \circ x$ for all $x \in K$.

Definition 2.1. [10] *A commutative hypergroup $(H, \circ)$ is canonical if the followings are hold:*

- *There exists $e \in H$, such that $e \circ x = \{x\}$, for every $x \in H$;*
- *For all $x \in H$ there exits a unique $x^{-1} \in H$, such that $e \in x \circ x^{-1}$;*
- *$x \in y \circ z$ implies $y \in x \circ z^{-1}$.*

Definition 2.2. [10] *An algebraic structure* $(R, +, \cdot)$ *is said to be:*

(A) *General hyperring, if*
(a_1) $(R, +)$ *is a hypergroup;*
(a_2) $(R, \cdot)$ *is a semihypergroup;*
(a_3) $\cdot$ *is distributive with respect to* $+$*;*
(B) *Krasner hyperring, if*
(b_1) $(R, +)$ *is a canonical hypergroup;*
(b_2) $(R, \cdot)$ *is a semigroup having zero element,i.e. for all* $x \in R$ $x \cdot 0 = 0 \cdot x = 0$*;*
(b_3) $x \cdot (y + z) = x \cdot y + x \cdot z$ *and* $(y + z) \cdot x = y \cdot x + z \cdot x$*, for all* $x, y, z \in R$*;*
(C) *Multiplicative hyperring, if*
(c_1) $(R, +)$ *is a commutative group;*
(c_2) $(R, \cdot)$ *is a semihypergroup;*
(c_3) *For all* $x, y, z \in R$, $x \cdot (y + z) \subseteq x \cdot y + x \cdot z$ *and* $(y + z) \cdot x \subseteq y \cdot x + z \cdot x$*;*
(c_4) *For all* $x, y \in R$, $x \cdot (-y) = (-x) \cdot y = -(x \cdot y)$.

Example 2.1. $\mathbb{Z}_4$ *is a commutative multiplicative hyperring with unit element that* $\overline{0}$ *is a zero element with the operations as follows:*

$\oplus$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$
$\overline{0}$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$
$\overline{1}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{0}$
$\overline{2}$	$\overline{2}$	$\overline{3}$	$\overline{0}$	$\overline{1}$
$\overline{3}$	$\overline{3}$	$\overline{0}$	$\overline{1}$	$\overline{2}$

$*$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$
$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$
$\overline{1}$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$
$\overline{2}$	$\overline{0}$	$\overline{2}$	$\mathbb{Z}_4$	$\overline{2}$
$\overline{3}$	$\overline{0}$	$\overline{3}$	$\overline{2}$	$\mathbb{Z}_4$

Definition 2.3. [10] *A non empty subset* I *of a multiplicative hyperring* R *is a left (right) hyperideal if the followings are hold:*

- *for every* $a, b \in I$ *implies that* $a - b \in I$
- *for every* $a \in I$, $r \in R$ *implies that* $r \cdot a \subseteq I$ *(or* $a \cdot r \subseteq I$*).*

In 2003, Ciampi and Rota gave the structure of the set of polynomials which are over the multiplicative hyperring. However, they got a lot of properties of it [12]. Now, let remember some necessary identities for us from that article.

Definition 2.4. [12] *Let* $(R, +, \circ)$ *be a commutative multiplicative hyperring such that, for all* $a \in R$, $a \circ 0 = \{0\}$ *and let* x *be any element out of* R*. Then a polynomial in* x *over* A *is in the form as;* $f(x) = f_0x^0 + f_1x^1 + ... = \sum f_kx^k$*, where* $k \in \mathbb{N}$ *and* $f_k \in R$*.*
Denote by $R[x]$ *the set of all polynomials in* x *over* R *and let the operations over* $R[x]$ *be ;*

$$\sum f_kx^k + \sum g_kx^k = \sum (f_k + g_k)x^k$$
$$f(x) * g(x) = \{\sum a_ix^i, i = 0, ..., n + m | a_i \in \sum f_sg_t, s + t = i\},$$

where $f(x) = \sum f_ix^i, i = 0, 1, ..., n$ *and* $g(x) = \sum g_ix^i, i = 0, 1, ..., m$.

Theorem 2.1. [12] *The hyperstructure* $(R[x], +, *)$ *is a commutative multiplicative hyperring.*

3. Linear Codes over Multiplicative Hyperrings

From now, the operations and some properties of multiplicative hyperrings have recalled. In this section, we define a code and a linear code over a multiplicative hyperring.

Defining the codes over a multiplicative hyperring means that the alphabet will be a finite multiplicative hyperring. Throughout this paper, without loss of generality, the left multiplication will be used as a second operation. Every result can be obtained for right one.

Definition 3.1. *Let the code alphabet be a finite multiplicative hyperring* $(R, +, \cdot)$ *and the number of elements of* R *be* $|R| = r$. *A commutative hypergroup* G *with the map*

$$\cdot : R \times G \longrightarrow G$$

is called a left hypermodule over R, *the following conditions are satisfied;*

$$\begin{array}{ll} (i) & r(g_1 + g_2) = rg_1 + rg_2, \\ (ii) & (r+s)g_1 = rg_1 + sg_1, \\ (iii) & (rs)g_1 = r(sg_1). \end{array}$$

for all $r, s \in R$ *and* $g_1, g_2 \in G$;

For example, $(\mathbb{Z}_4, \oplus, *)$ is a commutative multiplicative hyperring. So, $\mathbb{Z}_4^n$ is a hypermodule over $\mathbb{Z}_4$.

Definition 3.2. *An arbitrary code* S *is a subset of* R^n *which is a left hypermodule of finite multiplicative hyperring* R.

Definition 3.3. *A linear code* C *of length* n *over* R *is a left* $R-$*subhypermodule of* R^n. *Namely, for every* $c_1, c_2 \in C$ *and* $a_1, a_2 \in R$, *we have* $a_1c_1 + a_2c_2 \subseteq C$.

Example 3.1. $(\mathbb{Z}_4, \oplus, *)$ *is a commutative multiplicative hyperring with 4 elements.* $C = \{\bar{0}\bar{0}, \bar{1}\bar{2}, \bar{2}\bar{0}, \bar{2}\bar{1}, \bar{2}\bar{2}, \bar{2}\bar{3}, \bar{3}\bar{2}\}$ *is a linear code over* $\mathbb{Z}_4^2$ *with 7 codewords.*

Definition 3.4. *Let* R *be a multiplicative hyperring and* $x = (x_1, x_2, ..., x_n)$ *and* $y = (y_1, y_2, ..., y_n)$ *be* $n-$ *tuples in* R^n. *Then, the inner product vectors* x *and* y *are defined as* $x \cdot y^T = \bigcup_{i=1}^{n} x_i y_i$.

Definition 3.5. *Let* R *be a finite multiplicative hyperring and* C *be a linear code over* R, *namely* C *is a left hypermodule of* R^n. *Then, the left dual code of* C *is defined by* $C^{\perp} = \{y \in R^n | \{0\} \subseteq x \cdot y^T, \forall x \in C\}$.

Proposition 3.1. *Let* R *be a finite multiplicative hyperring,* C *be a linear code over* R *with* n *length and* $C^{\perp}$ *be the dual code of* C. *Then,* $C^{\perp}$ *is a linear code over* R *with same length.*

Proof. Let $x = (x_1, x_2, ..., x_n), y = (y_1, y_2, ..., y_n) \in C^{\perp}$ and $a, b \in R$. Then, for all $c = (c_1, c_2, ..., c_n) \in C$

$$\{0\} \subseteq c \cdot x^T = \{c_1x_1\} \cup \{c_2x_2\} \cup ... \cup \{c_nx_n\}$$
$$\{0\} \subseteq c \cdot y^T = \{c_1y_1\} \cup \{c_2y_2\} \cup ... \cup \{c_ny_n\}.$$

So, we get $\{0\} \subseteq c \cdot (ax + by)^T = cax^T \cup cby^T$. Here,

$$\begin{array}{rcl} c \cdot (ax+by)^T & = & \{(c_1, c_2, ..., c_n) \cdot (t_1, t_2, ..., t_n)^T | \{t_i\} \subseteq ax_i \cup by_i, 1 \le i \le n\} \\ & \subseteq & c_1(ax_1 + by_1) \cup ... \cup c_n(ax_n + by_n) \\ & = & \{c_1ax_1\} \cup \{c_1by_1\} \cup ... \cup \{c_nax_n\} \cup \{c_nby_n\} \\ & = & a(\{c_1x_1\} \cup ... \cup \{c_nx_n\}) + b(\{c_1y_1\} \cup ... \cup \{c_ny_n\}) \\ & = & a(c \cdot x^T) \cup b(c \cdot y^T). \end{array}$$

$\{0\} \subseteq a(c \cdot x^T) \cup b(c \cdot y^T)$. Hence, $\{0\} \subseteq c(ax + by)^T$ and $ax + by \in C^T$. So, C^T is a linear code. □

Example 3.2. *Let C be the linear code in Example 3.1 over $\mathbb{Z}_4$. Hence, the dual code of C is $C^{\perp} = \{\bar{0}\bar{0}, \bar{2}\bar{1}, \bar{0}\bar{2}, \bar{1}\bar{2}, \bar{2}\bar{2}, \bar{3}\bar{2}, \bar{2}\bar{3}\}$ with 2 length and 7 codewords.*

Proposition 3.2. *Let R be a finite multiplicative hyperring, C_1 and C_2 be linear codes over R. So, $C_1 + C_2 = \{x + y \in R^n | every\, x \in C_1, y \in C_2\}$ and $C_1 \cap C_2 = \{x \in R^n | x \in C_1\, and\, x \in C_2\}$ are linear codes over R.*

Proof. Let C_1 and C_2 be the linear codes over R, where R is a finite multiplicative hyperring. So, they are left $R-$subhypermodules of R^n. It is clear that, $C_1 + C_2$ and $C_1 \cap C_2$ are left $R-$subhypermodules of R^n. Consequently, $C_1 + C_2$ and $C_1 \cap C_2$ are linear codes over R. □

Definition 3.6. *Let R be a multiplicative hyperring. Then, the Hamming distance between $x, y \in R^n$ is defined as;*

$$\begin{aligned} d_H : R^n \times R^n &\rightarrow \mathbb{N} \\ (x, y) &\rightarrow d_H(x, y) = |\{i \in \mathbb{N} | x_i \neq y_i\}|. \end{aligned}$$

Example 3.3. *The distance between $(\bar{1}, \bar{2}), (\bar{2}, \bar{0})$ which are the codewords of the linear code in Example 3.1 is 2.*

Definition 3.7. *Let R be a multiplicative hyperring and C be a linear code over R. Then, the minimum distance of C is $d = d_{min}(C) = \min\{d_H(x, y)\}$, for every $x, y \in C$.*

Example 3.4. *For the linear code C in Example 3.1, $d_{min}(C) = 1$ and $d_{min}(C^{\perp}) = 1$.*

Definition 3.8. *Let R be a multiplicative hyperring and C be a linear code R^n.*

- *A generator matrix for C is a matrix G whose rows form a basis for C.*
- *A parity-check matrix H for C is a generator matrix for the dual code $C^{\perp}$.*
- *The number of linearly independent rows of G is called the dimension of C.*

Example 3.5. *A generator matrix G for the linear code C in Example 3.1 is $G = [\bar{1}\bar{2}]$. Also, a parity-check matrix H is $H = [\bar{2}\bar{1}]$ and the dimension of C is 1.*

Remark 3.1. *The generator matrix G generates a code has 4 elements over ring $\mathbb{Z}_4$ and the rate of the code is $1/2 = 0,5$. But, with hypermultiplication, G generates the code with 7 codewords over multiplicative hyperring $\mathbb{Z}_4$ and the rate of C is nearly $\frac{\log 7}{2 \log 4} = 0,7$.*

4. Cyclic Codes over Multiplicative Hyperrings

Let construct the cyclic codes over a finite multiplicative hyperring and give an illustrative example.

Definition 4.1. *Let c be a vector of length n over R. Then, the cyclic shift $T(c)$ is defined as;*

$$T(c_1, c_2, ..., c_n) = (c_n, c_1, ..., c_{n-1}).$$

Definition 4.2. *A linear code C of length n over a finite multiplicative hyperring R is said to be cyclic if $T(c) \in C$ whenever $c \in C$, i.e. $T(C) = C$.*

Example 4.1. *Let $\mathbb{Z}_4$ as a multiplicative hyperring with hyperoperations in Example 2.1 and assume that the generator matrix of C be $G = \begin{bmatrix} \bar{1} & \bar{2} \\ \bar{2} & \bar{1} \end{bmatrix}$ over $\mathbb{Z}_4$. Hence, C is a cyclic code with 2 length, 2 dimensional and 2 minimum distance.*

Proposition 4.1. *If C_1 and C_2 are cyclic codes of length n over a finite multiplicative hyperring R, then $C_1 + C_2$ is cyclic code.*

Proof. Assume that $t = (t_0, t_1, ..., t_{n-1}) \in C_1 + C_2 = \{a | a = c + d, c \in C_1, d \in C_2\}$. So, there exist $c = (c_0, c_1, ..., c_{n-1}) \in C_1$ and $d = (d_0, d_1, ..., d_{n-1}) \in C_2$ such that $t = c + d$. Hence, we have to show that $(t_0, t_1, ..., t_{n-1}) = T(t) \in C_1 + C_2$. Since C_1 and C_2 are cyclic, $c = (c_{n-1}, c_0, ..., c_{n-2}) \in C_1$ and $(d_{n-1}, d_0, ..., d_{n-2}) \in C_2$. Therefore, $(c_{n-1}, c_0, ..., c_{n-2}) + (d_{n-1}, d_0, ..., d_{n-2}) \in C_1, d \in C_2$, i.e.,
$\{(s_{n-1}, s_0, ..., s_{n-2}) | s_i = c_i + d_i, c_i \in C_1, d_i \in C_2, 1 \leq i \leq n-1\}$ in $C_1 + C_2$ such that $(t_{n-1}, t_0, ..., t_{n-2}) \in C_1 + C_2$. Thus, $C_1 + C_2$ is cyclic. □

Proposition 4.2. *If C_1 and C_2 are cyclic codes of length n over a finite multiplicative hyperring R, then $C_1 \cap C_2$ is cyclic code.*

Proof. Let we show that $C_1 \cap C_2$ is a cyclic code, when C_1 and C_2 are cyclic. So, take $t = (t_0, t_1, ..., t_{n-1}) \in C_1 \cap C_2$. Since C_1 and C_2 are cyclic codes, then $(t_{n-1}, t_0, ..., t_{n-2}) \in C_1$ and C_2. Consequently, $(t_{n-1}, t_0, ..., t_{n-2}) \in C_1 \cap C_2$ and $C_1 \cap C_2$ is cyclic. □

Let C be a cyclic code of length n over a multiplicative hyperring R, then C is a (left) ideal of $\frac{R[x]}{\langle x^n - 1 \rangle}$. C is called splitting if it is a direct summand of $\frac{R[x]}{\langle x^n - 1 \rangle}$. Note that C does not have to be complemented left ideal of $\frac{R[x]}{\langle x^n - 1 \rangle}$. It is obvious that for R being a Krasner hyperfield all definitions given coincide with [11]. Only the notion of a splitting code is a specialization to a proper subclass of linear codes over rings.

Theorem 4.1. *Let R be a finite multiplicative hyperring, and let $gh = x^n - 1$ for some $g, h \in R[x]$. The followings are satisfied:*

- *g and h commute i.e., $hg = x^n - 1$,*
- *$(R[x]h)$ is a free left module,*
- *$(R[x]g)$ is a direct summand of $R[x]$.*

Proof. Let the constant of $g(x)$ and $h(x)$ be g_0 and h_0, respectively. We have $g_0 h_0 = -1$ because $hg = gh = x^n - 1$. It implies that g_0 and h_0 are units of R, since R is finite. From this, we get that $fh = 0$ implies that $f = 0$ for all $f \in R[x]$. This leads to the $R[x]-$isomorphy and hence to the $R-$isomorphy of $R[x]$ and $R[x]h$ which proves this module to be free. Computing;

$$\begin{aligned} &(hg - (x^n - 1))h = hgh - (x^n - 1)h = 0 \\ &\Rightarrow hg - (x^n - 1) = 0 \\ &\Rightarrow hg = x^n - 1. \end{aligned}$$

Let us finally consider the $R-$ linear epimorphism

$$R[x] \to \frac{(R[x]h)}{\langle x^n - 1 \rangle}.$$

The kernel of the epimorphism above is $(R[x]g)$. $\frac{(R[x]h)}{\langle x^n - 1 \rangle}$ to be a projective $R-$ module, because $(R[x](x^n - 1))$ is a direct summand of the free module $(R[x]h)$. This shows that $(R[x]g)$ is a direct summand of $R[x]$. □

Corollary 4.1. *For a finite multiplicative hyperring R every divisors of $x^n - 1$ in $R[x]$ generates a cyclic splitting code of length n.*

Proof. Let g be a divisor of $x^n - 1$ in $R[x]$. Then $R[x]g$ to be a direct summand of $R[x]$ which contains the submodule $R[x](x^n - 1)$. Hence, we obtain $\frac{R[x]g}{\langle x^n-1\rangle}$ to be a direct summand in $\frac{R[x]}{\langle x^n-1\rangle}$ which proves our claim. □

Corollary 4.2. *For a cyclic (left) code of length n over a finite multiplicative hyperring, the followings are equivalent:*

- *C is splitting cyclic code,*
- *There exists a divisor g of $x^n - 1$ in $R[x]$ such that $C = \frac{(R[x]g)}{\langle x^n-1\rangle}$.*

5. Conclusion

In this paper, we introduce the codes over a multiplicative hyperring. Because, compared to the known codes defined on the fields and rings, the codes on the hyperrings have more codewords with same length. So, the rate of codes increases. Firstly, we define the linear codes over a multiplicative hyperring. Secondly, we give the structure of the cyclic codes over a finite multiplicative hyperring. Finally, we show that every divisor of $x^n - 1$ in $R[x]$ corresponds a cyclic code over R, as usual.

In future work, the MDS and perfect code can be searched by examining known boundaries for linear codes over a hyperstructure.

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