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ON BIPOLAR COMPLEX INTUITIONISTIC FUZZY GRAPHS

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ABSTRACT. In the present article, we devised the novel approach on the bipolar complex intuitionistic fuzzy set, bipolar complex intuitionistic fuzzy graphs and operations namely composition, cartesian product, join and union of the bipolar complex intuitionistic fuzzy graphs are elucidated with certain examples. Finally, the notions of isomorphisms and complement of bipolar complex intuitionistic fuzzy graphs are established and reviewed many of its characteristics.

Keywords: Bipolar complex intuitionistic fuzzy set, bipolar complex intuitionistic fuzzy graph, Isomorphisms of bipolar complex intuitionistic fuzzy graphs, Complement of bipolar complex intuitionistic fuzzy graphs

AMS Subject Classification: 54A40, 03E72, 05C72

1. INTRODUCTION

The main fundamental fuzzy set theoretical concept was introduced, first by Zadeh in 1965[19]. This theory works in uncertainties, ambiguous situations and solves ill-posed problems or problems with incomplete information. After two decades, the intuitionistic fuzzy sets was discovered and generalized by Atanassov [4]. At the same time, Coker [7,8] introduced the notions of an intuitionistic fuzzy points and intuitionistic fuzzy topological spaces with some related concepts. The main idea of intuitionistic fuzzy set theory i.e. the degree of membership and the degree of non membership and its application have been studied in various subjects including image processing, pattern recognition and multi-criteria decision making etc., The complex numbers and fuzzy sets are merged by Buckley [1989] and Nguyen et al [2000]. Later, in the year 2002 the notion of complex fuzzy sets was developed by Ramot et al. [13] whose range of membership function can be extended to the unit circle in the complex plane. The complex intuitionistic fuzzy sets and operations on the complex Atanassov intuitionistic fuzzy set were described by A. M. Alkouri [2,3]. Recently, Naveed Yaqoob et al., initiated the applications of complex intuitionistic fuzzy graphs [18]. The important notion of bipolar fuzzy sets and its membership degree ranges between -1 to 1 was introduced by Zhang in 1994 [20,21]. A general notion of fuzzy graphs and its structure was first presented by A. Rosenfeld [14]. Subsequently, M. Akram[1]

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sketched the idea on bipolar fuzzy graphs in the year 2011 and investigated its properties. Later, the fuzzy hyper graph was elaborated and some of its properties were studied by Mordeson and Nair [11]. After a period of time, Ali Asghar Talebi and Hossein Rashmanlou introduced the complement and isomorphisms on bipolar fuzzy graphs [16]. The Bipolar intuitionistic fuzzy set and its stronger forms were well defined by K. Sankar and D. Ezhilmaran [12,9].

In the current paper, the novel approach on bipolar complex intuitionistic fuzzy set, bipolar complex intuitionistic fuzzy graphs and its operations are devised and elucidated with certain examples. Then the composition, cartesian product, join and union of two BCIF graphs is BCIF graph(respectively). Moreover, we discussed an isomorphism notion of bipolar complex intuitionistic fuzzy graphs and found that the existence of an equivalence (partial ordered) relation, for an isomorphism (weak isomorphism) between any three BCIF graphs. Further we established the complement of bipolar complex intuitionistic fuzzy graphs and then the complement of the sum of two graphs is equivalent to the union of the complement of the graph $G_1\&G_2$ and $\overline{G_1 \bigcup G_2} \cong \overline{G_1} + \overline{G_2}$. Also found that the existence of a strong isomorphism between G_1 and G_2.

2. Preliminaries

Definition 2.1 (10). *Let X be a non empty set. A bipolar fuzzy set B in X is an object having the form $B = \{(x, \mu^P(x), \mu^N(x)) : x \in X\}$, where $\mu^P : X \to [0,1]$, $\mu^N : X \to [-1,0]$, are the mappings such that $0 \le \mu^P + \nu^p \le 1$. $-1 \le \mu^N + \nu^N \le 0$.*

Definition 2.2 (2). *A complex intuitionistic fuzzy set $\mathcal{A}$, defined on a universe of discourse χ is an object of the form $\mathcal{A} = \{(x, \mu_A(x)e^{i\alpha_A(x)},\ \nu_A(x)e^{i\beta_A(x)}) : x \in \chi\}$, where $i = \sqrt{-1}$, $\mu_A(x), \nu_A(x) \in [0,1]$, $\alpha_A(x), \beta_A(x) \in [0, 2\pi]$ and $0 \le \mu_A(x) + \nu_A(x) \le 1$.*

Definition 2.3 (18). *A complex intuitionistic fuzzy graph with an underlying set V is defined to be a pair $G = (A, B)$, where A is a cif-set on V and B is a cif-set on $E \subseteq V \times V$ such that (i)$\mu_B(xy)e^{i\alpha_B(xy)} \le min\{\mu_A(x), \mu_A(y)\}e^{imin\{\alpha_A(x),\alpha_A(y)\}}$ (ii)$\nu_B(xy)e^{i\beta_B(xy)} \le max\{\nu_A(x), \nu_A(y)\}e^{imax\{\beta_A(x),\beta_A(y)\}}$ for all $x, y \in V$.*

3. Bipolar Complex Intuitionistic Fuzzy Graphs

Definition 3.1. *A bipolar complex intuitionistic fuzzy set (in brief BCIF set) $\mathcal{E}$ defined on a universe of discourse χ, which is defined as an object of the form $\mathcal{E} = \{(\xi, \mu_E^+(\xi)e^{i\alpha_E(\xi)},\ \mu_E^-(\xi)e^{i\beta_E(\xi)},\ \nu_E^+(\xi)e^{i\gamma_E(\xi)},\ \nu_E^-(\xi)e^{i\delta_E(\xi)}), : \xi \in \chi\}$, where $i = \sqrt{-1}$, μ_E^+, $\nu_E^+ : X \to [0,1]$, $\mu_E^-, \nu_E^- : X \to [-1,0]$, $\alpha_E, \beta_E, \gamma_E, \delta_E \in [0, 2\pi]$ and $0 \le \mu_E^+ + \nu_E^+ \le 1$. $-1 \le \mu_E^- + \nu_E^- \le 0$.*

Definition 3.2. *A bipolar complex intuitionistic fuzzy graph (in short BCIFG) with an underlying set V is an ordered pair G =(A,B) where $\mathcal{A} = (\mu_A^+e^{i\alpha_A},\ \mu_A^-e^{i\beta_A},\ \nu_A^+e^{i\gamma_A},\ \nu_A^-e^{i\delta_A})$ is a BCIF set on V and $\mathcal{B} = (\mu_B^+e^{i\alpha_B},\ \mu_B^-e^{i\beta_B},\ \nu_B^+e^{i\gamma_B},\ \nu_B^-e^{i\delta_B})$ is a BCIF set on $E \subseteq V \times V$:*

(i) $\mu_B^+(xy)e^{i\alpha_B(xy)} \le min\{\mu_A^+(x), \mu_A^+(y)\}e^{imin\{\alpha_A(x),\alpha_A(y)\}}$
(ii) $\mu_B^-(xy)e^{i\beta_B(xy)} \ge max\{\mu_A^-(x), \mu_A^-(y)\}e^{imax\{\beta_A(x),\beta_A(y)\}}$
(iii) $\nu_B^+(xy)e^{i\gamma_B(xy)} \le max\{\nu_A^+(x), \nu_A^+(y)\}e^{imax\{\gamma_A(x),\gamma_A(y)\}}$
(iv) $\nu_B^-(xy)e^{i\delta_B(xy)} \ge min\{\nu_A^-(x), \nu_A^-(y)\}e^{imin\{\delta_A(x),\delta_A(y)\}}$ $forall x, y \in V$

Here, we take $\mathcal{A}$ as a BCIF vertex set of V and $\mathcal{B}$ as a BCIF edge set of E.

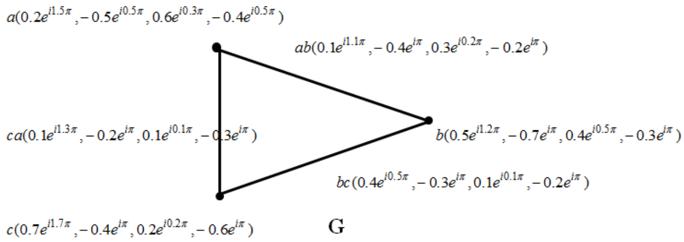

FIGURE 1. Bipolar complex intuitionistic fuzzy graph

Definition 3.3. *Let G=(A,B) be a BCIFG. The order of a BCIF graph G is defined by*

$$O(G) = \Big(\sum_{x\in V} \mu_A^+(x) e^{\sum_{x\in V}\alpha_A}, \sum_{x\in V} \mu_A^-(x) e^{\sum_{x\in V}\beta_A}, \sum_{x\in V} \nu_A^+(x) e^{\sum_{x\in V}\gamma_A}, \sum_{x\in V} \nu_A^-(x) e^{\sum_{x\in V}\delta_A}\Big).$$

Definition 3.4. *Let G=(A,B) be a BCIFG. The degree of a vertex x in BCIF graph G is defined by*

$deg(x) = (deg\mu^+(x), deg\mu^-(x), deg\nu^+(x), deg\nu^-(x))$

where, $deg\mu^+(x) = \sum_{x,y\in E} \mu_B^+(x) e^{\sum_{x,y\in V}\alpha_B}$, $deg\mu^-(x) = \sum_{x,y\in E} \mu_B^-(x) e^{\sum_{x,y\in E}\beta_B}$,

$deg\nu^+(x) \sum_{x,y\in E} \nu_B^+(x) e^{\sum_{x,y\in E}\gamma_B}$, $deg\nu^-(x) \sum_{x,y\in E} \nu_B^-(x) e^{\sum_{x,y\in E}\delta_B}$.

Example 3.1. *Let us consider a graph $G^* = (V, E)$: $V = \{a,b,c\}$, $E = \{ab,bc,ca\}$. Let A be a BCIF subset of V and Let B be a BCIF subset of $E \subseteq V \times V$, is defined by*

$A=((\frac{0.2e^{i1.5\pi}, -0.5e^{i0.5\pi}, 0.6e^{i0.3\pi}, -0.4e^{i0.5\pi}}{a}), (\frac{0.5e^{i1.2\pi}, -0.7e^{i\pi}, 0.4e^{i0.5\pi}, -0.3e^{i\pi}}{b}),$
$(\frac{0.7e^{i1.7\pi}, -0.4e^{i\pi}, 0.2e^{i0.2\pi}, -0.6e^{i\pi}}{c}))$

$B=((\frac{0.1e^{i1.1\pi}, -0.4e^{i\pi}, 0.3e^{i0.2\pi}, -0.2e^{i0.5\pi}}{ab}), (\frac{0.4e^{i0.5\pi}, -0.3e^{i\pi}, 0.1e^{i0.1\pi}, -0.2e^{i\pi}}{bc}),$
$(\frac{0.1e^{i1.3\pi}, -0.2e^{i\pi}, 0.1e^{i0.1\pi}, -0.3e^{i\pi}}{ca}))$.

(i) Order of BCIFG $O(G) = (1.4e^{i4.4\pi}, -1.6e^{i2.5\pi}, 1.2e^{i1.1\pi}, -1.3e^{i2.5\pi})$

(ii)The degree of each vertex in BCIF graph G is $deg(a) = (deg\mu^+(a), deg\mu^-(a), deg\nu^+(a), deg\nu^-(a))$

$deg(a) = (0.2e^{i2.4\pi}, -0.6e^{i2\pi}, 0.4e^{i0.3\pi}, -0.5e^{i1.5\pi})$

$deg(b) = (0.5e^{i1.6\pi}, -0.7e^{i2\pi}, 0.4e^{i0.3\pi}, -0.4e^{i1.5\pi})$

$deg(c) = (0.5e^{i1.8\pi}, -0.5e^{i2\pi}, 0.2e^{i0.2\pi}, -0.5e^{i2\pi})$

Definition 3.5. *Let A_1, A_2 be two BCIF subsets of V_1 and V_2 and let $B_1$and B_2 be two BCIF subsets of E_1 and E_2 independently. Then the cartesian product $G_1 \times G_2$ of two BCIF graphs is defined to be an ordered pair $G_1 \times G_2 = (A_1 \times A_2, B_1 \times B_2)$:*

1. $\mu^+_{A_1\times A_2}(x_1, x_2)e^{i\alpha_{A_1\times A_2}(x_1,x_2)} = min\{\mu^+_{A_1}(x_1), \mu^+_{A_2}(x_2)\}e^{imin\{\alpha_{A_1}(x_1),\alpha_{A_2}(x_2)\}}$

$\mu^-_{A_1\times A_2}(x_1, x_2)e^{i\beta_{A_1\times A_2}(x_1,x_2)} = max\{\mu^-_{A_1}(x_1), \mu^-_{A_2}(x_2)\}e^{imax\{\beta_{A_1}(x_1),\beta_{A_2}(x_2)\}}$

$\nu^+_{A_1\times A_2}(x_1, x_2)e^{i\gamma_{A_1\times A_2}(x_1,x_2)} = max\{\nu^+_{A_1}(x_1), \nu^+_{A_2}(x_2)\}e^{imax\{\beta_{A_1}(x_1),\beta_{A_2}(x_2)\}}$

$\nu^-_{A_1\times A_2}(x_1, x_2)e^{i\delta_{A_1\times A_2}(x_1,x_2)} = min\{\nu^+_{A_1}(x_1), \nu^+_{A_2}(x_2)\}e^{imin\{\delta_{A_1}(x_1),\delta_{A_2}(x_2)\}}$

$\forall (x_1, x_2) \in V$.

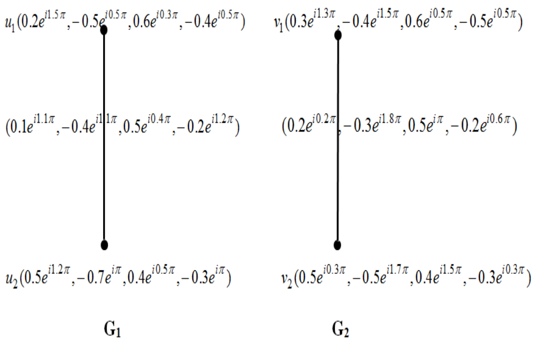

FIGURE 2. Two bipolar complex intuitionistic fuzzy graphs G_1 and G_2

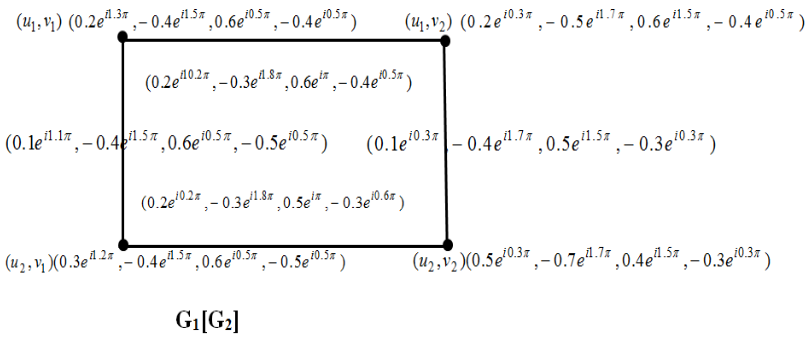

FIGURE 3. The cartesian product $G_1 \times G_2$ of two bipolar complex intuitionistic fuzzy graphs

2. $\mu^+_{B_1\times B_2}((x,x_2)(x,y_2))e^{i\alpha_{B_1\times B_2}((x,x_2)(x,y_2))} = min\{\mu^+_{A_1}(x), \mu^+_{B_2}(x_2y_2)\}e^{imin\{\alpha_{A_1}(x),\alpha_{B_2}(x_2y_2)\}}$
$\mu^-_{B_1\times B_2}((x,x_2)(x,y_2))e^{i\beta_{B_1\times B_2}((x,x_2)(x,y_2))} = max\{\mu^-_{A_1}(x), \mu^-_{B_2}(x_2y_2)\}e^{imax\{\beta_{A_1}(x),\beta_{B_2}(x_2y_2)\}}$
$\nu^+_{B_1\times B_2}((x,x_2)(x,x_2))e^{i\gamma_{B_1\times B_2}((x,x_2)(x,x_2))} = max\{\nu^+_{A_1}(x), \nu^+_{B_2}(x_2y_2)\}e^{imax\{\gamma_{A_1}(x),\gamma_{B_2}(x_2y_2)\}}$
$\nu^-_{B_1\times B_2}((x,x_2)(x,x_2))e^{i\delta_{B_1\times B_2}((x,x_2)(x,x_2))} = min\{\nu^-_{A_1}(x), \nu^-_{B_2}(x_2y_2)\}e^{imin\{\delta_{A_1}(x),\delta_{B_2}(x_2y_2)\}}$
$\forall x \in V_1,\ x_2y_2 \in E_2.$
3.$\mu^+_{B_1\times B_2}((x_1,z)(y_1,z))e^{i\alpha_{B_1\times B_2}((x_1,z)(y_1,z))} = min\{\mu^+_{B_1}(x_1,y_1), \mu^+_{A_2}(z)\}e^{imin\{\alpha_{B_1}(x_1,y_1),\alpha_{A_2}(z)\}}$
$\mu^-_{B_1\times B_2}((x_1,z)(y_1,z))e^{i\beta_{B_1\times B_2}((x_1,z)(y_1,z))} = max\{\mu^-_{B_1}(x_1,y_1), \mu^-_{A_2}(z)\}e^{imax\{\beta_{B_1}(x_1,y_1),\beta_{A_2}(z)\}}$
$\nu^+_{B_1\times B_2}((x_1,z)(y_1,z))e^{i\gamma_{B_1\times B_2}((x_1,z)(y_1,z))} = max\{\nu^+_{B_1}(x_1,y_1), \nu^+_{A_2}(z)\}e^{imax\{\gamma_{B_1}(x_1,y_1),\gamma_{A_2}(z)\}}$
$\nu^-_{B_1\times B_2}((x_1,z)(y_1,z))e^{i\delta_{B_1\times B_2}((x_1,z)(y_1,z))} = min\{\nu^-_{B_1}(x_1,y_1), \mu^-_{A_2}(z)\}e^{imin\{\delta_{B_1}(x_1,y_1),\delta_{A_2}(z)\}}$
$\forall z \in V_2,\ x_1y_1 \in E_1.$

Example 3.2. *Let us consider G_1 and G_2 be the two BCIF graphs then, their cartesian product $G_1 \times G_2$ is BCIF graphs.*

Definition 3.6. *Let G_1 and G_2 be the two BCIF graphs. Then the degree of a vertex $(x_1, x_2 \in V_1 \times V_2)$ of $G_1 \times G_2$ is defined by for any $(x_1, x_2 \in V_1 \times V_2)$*
$deg_{G_1\times G_2}(x_1,x_2) = (deg\mu^+_{G_1\times G_2}(x_1,x_2), deg\mu^-_{G_1\times G_2}(x_1,x_2), deg\nu^+_{G_1\times G_2}(x_1,x_2), deg\nu^-_{G_1\times G_2}(x_1,x_2))$
where,

$$deg\mu^+_{G_1\times G_2}(x_1,x_2) = \sum_{x_1=y_1,x_2y_2\in E_2} min\{\mu^+_{A_1}(x_1), \mu^+_{B_2}(x_2,y_2)\}e^{\sum_{x_1=y_1,x_2y_2\in E_2} min\{\alpha_{A_1}(x_1),\alpha_{B_2}(x_2,y_2)\}}$$

$$+\sum_{x_2=y_2,x_1y_1\in E_1} min\{\mu^+_{B_1}(x_1,y_1),\mu^+_{A_2}(y_2)\}e^{\sum_{x_2=y_2,x_1y_1\in E_1} min\{\alpha^+_{B_1}(x_1,y_1),\alpha^+_{A_2}(y_2)\}}$$

$$deg\mu^-_{G_1\times G_2}(x_1,x_2)=\sum_{x_1=y_1,x_2y_2\in E_2} max\{\mu^-_{A_1}(x_1),\mu^-_{B_2}(x_2,y_2)\}e^{\sum_{x_1=y_1,x_2y_2\in E_2} max\{\beta_{A_1}(x_1),\beta_{B_2}(x_2,y_2)\}}$$

$$+\sum_{x_2=y_2,x_1y_1\in E_1} max\{\mu^-_{B_1}(x_1,y_1),\mu^-_{A_2}(y_2)\}e^{\sum_{x_2=y_2,x_1y_1\in E_1} max\{\beta_{B_1}(x_1,y_1),\beta^+_{A_2}(y_2)\}}$$

$$deg\nu^+_{G_1\times G_2}(x_1,x_2)=\sum_{x_1=y_1,x_2y_2\in E_2} max\{\nu^+_{A_1}(x_1),\nu^+_{B_2}(x_2,y_2)\}e^{\sum_{x_1=y_1,x_2y_2\in E_2} max\{\gamma_{A_1}(x_1),\gamma_{B_2}(x_2,y_2)\}}$$

$$+\sum_{x_2=y_2,x_1y_1\in E_1} max\{\nu^+_{B_1}(x_1,y_1),\nu^+_{A_2}(y_2)\}e^{\sum_{x_2=y_2,x_1y_1\in E_1} max\{\gamma_{B_1}(x_1,y_1),\gamma^+_{A_2}\}}$$

$$deg\nu^-_{G_1\times G_2}(x_1,x_2)=\sum_{x_1=y_1,x_2y_2\in E_2} min\{\nu^-_{A_1}(x_1),\nu^-_{B_2}(x_2,y_2)\}e^{\sum_{x_1=y_1,x_2y_2\in E_2} min\{\delta_{A_1}(x_1),\delta_{B_2}(x_2,y_2)\}}$$

$$+\sum_{x_2=y_2,x_1y_1\in E_1} min\{\nu^-_{B_1}(x_1,y_1),\ \nu^-_{A_2}(y_2)\}e^{\sum_{x_2=y_2,x_1y_1\in E_1} max\{\delta_{B_1}(x_1,y_1),\delta^-_{A_2}(y_2)\}}.$$

Proposition 3.1. *Let G_1 and G_2 be the two BCIF graphs, then the product $G_1\times G_2$ is also a BCIFG.*

Definition 3.7. *Let $\mathcal{A}_1=(\mu^+_{A_1},\mu^-_{A_1},\nu^+_{A_1},\nu^-_{A_1})$ and $\mathcal{A}_2=(\mu^+_{A_2},\mu^-_{A_2},\nu^+_{A_2},\nu^-_{A_2})$ be BCIF subsets of V_1 and V_2 and let $\mathcal{B}_1=(\mu^+_{B_1},\mu^-_{B_1},\nu^+_{B_1},\nu^-_{B_1})$ and $\mathcal{B}_2=(\mu^+_{B_2},\mu^-_{B_2},\nu^+_{B_2},\nu^-_{B_2})$ respectively be BCIF subsets of E_1 and E_2. Then the composition $G_1\circ G_2$ of two BCIF-graphs is a pair $G_1[G_2]=(\mathcal{A}_1\circ\mathcal{A}_2,\mathcal{B}_1\circ\mathcal{B}_2)$ as follows.*
(i) $\mu^+_{A_1\circ A_2}(x_1,x_2)e^{i\alpha_{A_1\circ A_2}(x_1,x_2)}=min\{\mu^+_{A_1}(x_1),\mu^+_{A_2}(x_2)\}e^{imin\{\alpha_{A_1}(x_1),\alpha_{A_2}(x_2)\}}$
$\mu^-_{A_1\circ A_2}(x_1,x_2)e^{i\beta_{A_1\circ A_2}(x_1,x_2)}=max\{\mu^-_{A_1}(x_1),\mu^-_{A_2}(x_2)\}e^{imax\{\beta_{A_1}(x_1),\beta_{A_2}(x_2)\}}$
$\nu^+_{A_1\circ A_2}(x_1,x_2)e^{i\gamma_{A_1\circ A_2}(x_1,x_2)}=max\{\nu^+_{A_1}(x_1),\nu^+_{A_2}(x_2)\}e^{imax\{\beta_{A_1}(x_1),\beta_{A_2}(x_2)\}}$
$\nu^-_{A_1\circ A_2}(x_1,x_2)e^{i\delta_{A_1\circ A_2}(x_1,x_2)}=min\{\nu^+_{A_1}(x_1),\nu^+_{A_2}(x_2)\}e^{imin\{\delta_{A_1}(x_1),\delta_{A_2}(x_2)\}}$
$\forall(x_1,x_2)\in V$.
(ii)$\mu^+_{B_1\circ B_2}((x,x_2)(x,y_2))e^{i\alpha_{B_1\circ B_2}((x,x_2)(x,y_2))}=min\{\mu^+_{A_1}(x),\mu^+_{B_2}(x_2y_2)\}e^{imin\{\alpha_{A_1}(x),\alpha_{B_2}(x_2y_2)\}}$
$\mu^-_{B_1\circ B_2}((x,x_2)(x,y_2))e^{i\beta_{B_1\circ B_2}((x,x_2)(x,y_2))}=max\{\mu^-_{A_1}(x),\mu^-_{B_2}(x_2y_2)\}e^{imax\{\beta_{A_1}(x),\beta_{B_2}(x_2y_2)\}}$
$\nu^+_{B_1\circ B_2}((x,x_2)(x,x_2))e^{i\gamma_{B_1\circ B_2}((x,x_2)(x,x_2))}=max\{\nu^+_{A_1}(x),\nu^+_{B_2}(x_2y_2)\}e^{imax\{\gamma_{A_1}(x),\gamma_{B_2}(x_2y_2)\}}$
$\nu^-_{B_1\circ B_2}((x,x_2)(x,x_2))e^{i\delta_{B_1\circ B_2}((x,x_2)(x,x_2))}=min\{\nu^-_{A_1}(x),\nu^-_{B_2}(x_2y_2)\}e^{imin\{\delta_{A_1}(x),\delta_{B_2}(x_2y_2)\}}$
$\forall x\in V_1\ x_2y_2\in E_2$.
(iii)$\mu^+_{B_1\circ B_2}((x_1,z)(y_1,z))e^{i\alpha_{B_1\circ B_2}((x_1,z)(y_1,z))}=min\{\mu^+_{B_1}(x_1,y_1),\mu^+_{A_2}(z)\}e^{imin\{\alpha_{B_1}(x_1,y_1),\alpha_{A_2}(z)\}}$
$\mu^-_{B_1\circ B_2}((x_1,z)(y_1,z))e^{i\beta_{B_1\circ B_2}((x_1,z)(y_1,z))}=max\{\mu^-_{B_1}(x_1,y_1),\mu^-_{A_2}(z)\}e^{imax\{\beta_{B_1}(x_1,y_1),\beta_{A_2}(z)\}}$
$\nu^+_{B_1\circ B_2}((x_1,z)(y_1,z))e^{i\gamma_{B_1\circ B_2}((x_1,z)(y_1,z))}=max\{\nu^+_{B_1}(x_1,y_1),\nu^+_{A_2}(z)\}e^{imax\{\gamma_{B_1}(x_1,y_1),\gamma_{A_2}(z)\}}$
$\nu^-_{B_1\circ B_2}((x_1,z)(y_1,z))e^{i\delta_{B_1\circ B_2}((x_1,z)(y_1,z))}=min\{\nu^-_{B_1}(x_1,y_1),\mu^-_{A_2}(z)\}e^{imin\{\delta_{B_1}(x_1,y_1),\delta_{A_2}(z)\}}$
$\forall z\in V_2\ x_1y_1\in E_1$.

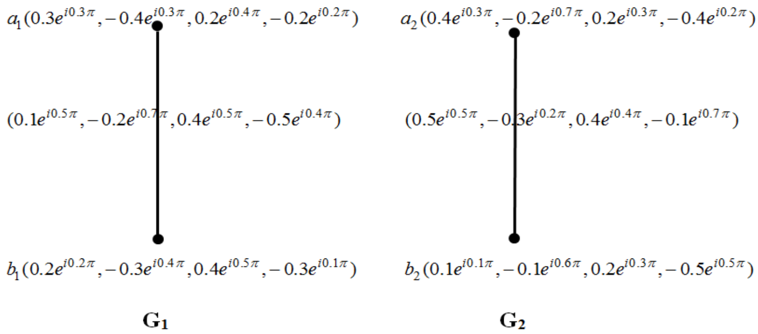

FIGURE 4. G_1 and G_2- bipolar complex intuitionistic fuzzy graphs

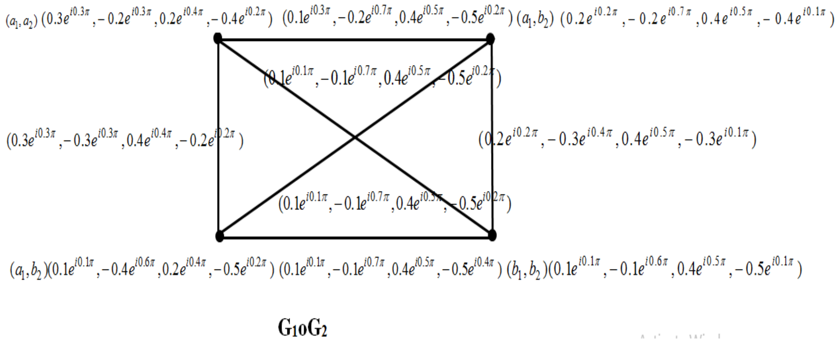

FIGURE 5. The composition of two bipolar complex intuitionistic fuzzy graphs

(iv) $\mu^+_{B_1 \circ B_2}((x_1, x_2)(y_1, y_2))e^{i\alpha_{B_1 \circ B_2}((x_1,x_2)(y_1,y_2))}$
$= min\{\mu^+_{A_2}(x_2), \mu_{A_2}(y_2), \mu^+_{B_1}(x_1, y_1)\}e^{imin\{\alpha_{A_2}(x_2), \alpha_{A_2}(y_1), \alpha_{B_1}(x_1,y_1)\}}$
$\mu^-_{B_1 \circ B_2}((x_1, x_2)(y_1, y_2))e^{i\beta_{B_1 \circ B_2}((x_1,x_2)(y_1,y_2))}$
$= max\{\mu^-_{A_2}(x_2), \mu_{A_2}(y_2), \mu^-_{B_1}(x_1, y_1)\}e^{imax\{\beta_{A_2}(x_2), \beta_{A_2}(y_1), \beta_{B_1}(x_1,y_1)\}}$
$\nu^+_{B_1 \circ B_2}((x_1, x_2)(y_1, y_2))e^{i\gamma_{B_1 \circ B_2}((x_1,x_2)(y_1,y_2))}$
$= max\{\nu^+_{A_2}(x_2), \nu_{A_2}(y_2), \nu^+_{B_1}(x_1, y_1)\}e^{imax\{\gamma_{A_2}(x_2), \gamma_{A_2}(y_1), \gamma_{B_1}(x_1,y_1)\}}$
$\nu^-_{B_1 \circ B_2}((x_1, x_2)(y_1, y_2))e^{i\delta_{B_1 \circ B_2}((x_1,x_2)(y_1,y_2))}$
$= min\{\nu^-_{A_2}(x_2), \nu_{A_2}(y_2), \nu^-_{B_1}(x_1, y_1)\}e^{imin\{\delta_{A_2}(x_2), \delta_{A_2}(y_1), \delta_{B_1}(x_1,y_1)\}}$, *for all* $x_2, y_2 \in V_2$, *and* $x_1 y_1 \in E_2$.

Example 3.3. *Let us consider the two BCIF graphs* G_1 *and* G_2 *then, their composition* $G_1 \circ G_2$ *is a BCIFG.*

Definition 3.8. *Let* G_1 *and* G_2 *be the two BCIF graphs. Then the degree of a vertex* $(x_1, x_2 \in V_1 \circ V_2)$ *of* $G_1 \circ G_2$ *is defined by for any* $(x_1, x_2 \in V_1 \circ V_2)$
$deg_{G_1 \circ G_2}(x_1, x_2) = (deg\mu^+_{G_1 \circ G_2}(x_1, x_2), deg\mu^-_{G_1 \circ G_2}(x_1, x_2), deg\nu^+_{G_1 \circ G_2}(x_1, x_2), deg\nu^-_{G_1 \circ G_2}(x_1, x_2))$
where,

$$deg\mu^+_{G_1 \circ G_2}(x_1, x_2) = \sum_{x_1 = y_1, x_2 y_2 \in E_2} min\{\mu^+_{A_1}(x_1), \mu^+_{B_2}(x_2, y_2)\}e^{\sum_{x_1 = y_1, x_2 y_2 \in E_2} min\{\alpha_{A_1}(x_1), \alpha_{B_2}(x_2, y_2)\}}$$

$$+ \sum_{x_2=y_2, x_1y_1 \in E_1} min\{\mu^+_{B_1}(x_1,y_1),\ \mu^+_{A_2}(y_2)\} e^{\sum_{x_2=y_2,x_1y_1\in E_1} min\{\alpha_{B_1}(x_1,y_1), \alpha^+_{A_2}(y_2)\}}$$
$$+ \sum_{x_2\neq y_2, x_1y_1 \in E_1} min\{\mu^+_{A_2}(x_2), \mu^+_{A_2}(y_2), \mu^+_{B_1}(x_1,y_1)\}$$

$$deg\mu^-_{G_1\circ G_2}(x_1,x_2) = \sum_{x_1=y_1, x_2y_2\in E_2} max\{\mu^-_{A_1}(x_1), \mu^-_{B_2}(x_2,y_2)\} e^{\sum_{x_1=y_1,x_2y_2\in E_2} max\{\beta_{A_1}(x_1), \beta_{B_2}(x_2,y_2)\}}$$
$$+ \sum_{x_2=y_2, x_1y_1 \in E_1} max\{\mu^-_{B_1}(x_1,y_1), \mu^-_{A_2}(y_2)\} e^{\sum_{x_2=y_2,x_1y_1\in E_1} max\{\beta_{B_1}(x_1,y_1), \beta^+_{A_2}(y_2)\}}$$
$$+ \sum_{x_2\neq y_2, x_1y_1 \in E_1} max\{\mu^-_{A_2}(x_2), \mu^-_{A_2}(y_2), \mu^-_{B_1}(x_1,y_1)\}$$

$$deg\nu^+_{G_1\circ G_2}(x_1,x_2) = \sum_{x_1=y_1, x_2y_2\in E_2} max\{\nu^+_{A_1}(x_1), \nu^+_{B_2}(x_2,y_2)\} e^{\sum_{x_1=y_1,x_2y_2\in E_2} max\{\gamma_{A_1}(x_1), \gamma_{B_2}(x_2,y_2)\}}$$
$$+ \sum_{x_2=y_2, x_1y_1 \in E_1} max\{\nu^+_{B_1}(x_1,y_1), \nu^+_{A_2}(y_2)\} e^{\sum_{x_2=y_2,x_1y_1\in E_1} max\{\gamma_{B_1}(x_1,y_1), \gamma^+_{A_2}(y_2)\}}$$
$$+ \sum_{x_2\neq y_2, x_1y_1 \in E_1} max\{\nu^+_{A_2}(x_2), \nu^+_{A_2}(y_2), \nu^+_{B_1}(x_1,y_1)\}$$

$$deg\nu^-_{G_1\circ G_2}(x_1,x_2) = \sum_{x_1=y_1, x_2y_2\in E_2} min\{\nu^-_{A_1}(x_1),\ \nu^-_{B_2}(x_2,y_2)\} e^{\sum_{x_1=y_1,x_2y_2\in E_2} min\{\delta_{A_1}(x_1), \delta_{B_2}(x_2,y_2)\}}$$
$$+ \sum_{x_2=y_2, x_1y_1 \in E_1} min\{\nu^-_{B_1}(x_1,y_1),\ \nu^-_{A_2}(y_2)\} e^{\sum_{x_2=y_2,x_1y_1\in E_1} max\{\delta_{B_1}(x_1,y_1), \delta^-_{A_2}(y_2)\}}$$
$$+ \sum_{x_2\neq y_2, x_1y_1 \in E_1} min\{\nu^-_{A_2}(x_2), \nu^-_{A_2}(y_2), \nu^-_{B_1}(x_1,y_1)\}.$$

Proposition 3.2. *For any two BCIF graphs G_1 and G_2 then $G_1 \circ G_2$ is also a BCIFG.*

Definition 3.9. *Let $A_1 = (\mu^+_{A_1}, \mu^-_{A_1}, \nu^+_{A_1}, \nu^-_{A_1})$ and $A_2 = (\mu^+_{A_2}, \mu^-_{A_2}, \nu^+_{A_2}, \nu^-_{A_2})$ be two bipolar complex intuitionistic fuzzy subsets of V_1 and V_2 and let $B_1 = (\mu^+_{B_1}, \mu^-_{B_1}, \nu^+_{B_1}, \nu^-_{B_1})$ and $B_2 = (\mu^+_{B_2}, \mu^-_{B_2}, \nu^+_{B_2}, \nu^-_{B_2})$ respectively be two bipolar complex intuitionistic fuzzy subsets of E_1 and E_2. Then their Union of two BCIF graphs G_1 and G_2 are defined as follows $G_1 \cup G_2 = (A_1 \cup A_2, B_1 \cup B_2)$:*
1. $\mu^+_{A_1\cup A_2}(x)e^{i\alpha_{A_1\cup A_2}(x)} = \mu^+_{A_1}(x)e^{i\alpha_{A_1}(x)}$
$\mu^-_{A_1\cup A_2}(x)e^{i\beta_{A_1\cup A_2}(x)} = \mu^-_{A_1}(x)e^{i\beta_{A_1}(x)}$
$\nu^+_{A_1\cup A_2}(x)e^{i\gamma_{A_1\cup A_2}(x)} = \nu^+_{A_1}(x)e^{i\gamma_{A_1}(x)}$
$\nu^-_{A_1\cup A_2}(x)e^{i\delta_{A_1\cup A_2}(x)} = \nu^-_{A_1}(x)e^{i\delta_{A_1}(x)}$ *for* $x \in V_1$ *and* $x \notin V_2$.
2. $\mu^+_{A_1\cup A_2}(x)e^{i\alpha_{A_1\cup A_2}(x)} = \mu^+_{A_2}(x)e^{i\alpha_{A_2}(x)}$
$\mu^-_{A_1\cup A_2}(x)e^{i\beta_{A_1\cup A_2}(x)} = \mu^-_{A_2}(x)e^{i\beta_{A_2}(x)}$
$\nu^+_{A_1\cup A_2}(x)e^{i\gamma_{A_1\cup A_2}(x)} = \nu^+_{A_2}(x)e^{i\gamma_{A_2}(x)}$

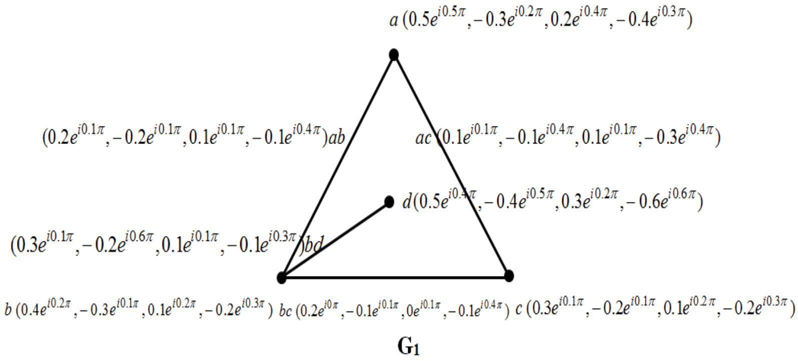

FIGURE 6. G_1 bipolar complex intuitionistic fuzzy graph

$\nu^-_{A_1\cup A_2}(x)e^{i\delta_{A_1\cup A_2}(x)} = \nu^-_{A_2}(x)e^{i\delta_{A_2}(x)}$ *for* $x \notin V_1$ *and* $x \in V_2$.
3. $\mu^+_{A_1\cup A_2}(x)e^{i\alpha_{A_1\cup A_2}(x)} = max\{\mu^+_{A_1}(x), \mu^+_{A_2}(x)\}e^{imax\{\alpha_{A_1}(x),\alpha_{A_2}(x)\}}$
$\mu^-_{A_1\cup A_2}(x)e^{i\beta_{A_1\cup A_2}(x)} = min\{\mu^-_{A_1}(x), \mu^-_{A_2}(x)\}e^{imin\{\beta_{A_1}(x),\beta_{A_2}(x)\}}$
$\nu^+_{A_1\cup A_2}(x)e^{i\gamma_{A_1\cup A_2}(x)} = max\{\nu^+_{A_1}(x), \nu^+_{A_2}(x)\}e^{imax\{\gamma_{A_1}(x),\gamma_{A_2}(x)\}}$
$\nu^-_{A_1\cup A_2}(x)e^{i\delta_{A_1\cup A_2}(x)} = min\{\nu^-_{A_1}(x), \nu^-_{A_2}(x)\}e^{imin\{\delta_{A_1}(x),\delta_{A_2}(x)\}}$ *for* $x \in V_1 \cap V_2$
4. $\mu^+_{B_1\cup B_2}(xy)e^{i\alpha_{B_1\cup B_2}(xy)} = \mu^+_{B_1}(xy)e^{i\alpha_{B_1}(xy)}$
$\mu^-_{B_1\cup B_2}(xy)e^{i\beta_{B_1\cup B_2}(xy)} = \mu^-_{B_1}(xy)e^{i\beta_{B_1}(xy)}$
$\nu^+_{B_1\cup B_2}(xy)e^{i\gamma_{B_1\cup B_2}(xy)} = \nu^+_{B_1}(xy)e^{i\gamma_{B_1}(xy)}$
$\nu^-_{B_1\cup B_2}(xy)e^{i\delta_{B_1\cup B_2}(xy)} = \nu^-_{B_1}(xy)e^{i\delta_{B_1}(xy)}$*for* $xy \in E_1$ *and* $xy \notin E_2$.
5. $\mu^+_{B_1\cup B_2}(xy)e^{i\alpha_{B_1\cup B_2}(xy)} = \mu^+_{B_2}(xy)e^{i\alpha_{B_2}(xy)}$
$\mu^-_{B_1\cup B_2}(xy)e^{i\beta_{B_1\cup B_2}(xy)} = \mu^-_{B_2}(xy)e^{i\beta_{B_2}(xy)}$
$\nu^+_{B_1\cup B_2}(xy)e^{i\gamma_{B_1\cup B_2}(xy)} = \nu^+_{B_2}(xy)e^{i\gamma_{B_2}(xy)}$
$\nu^-_{B_1\cup B_2}(xy)e^{i\delta_{B_1\cup B_2}(xy)} = \nu^-_{B_2}(xy)e^{i\delta_{B_2}(xy)}$*for* $xy \in E_2$ *and* $xy \notin E_1$.
6. $\mu^+_{B_1\cup B_2}(xy)e^{i\alpha_{B_1\cup B_2}(xy)} = max\{\mu^+_{B_1}(xy), \mu^+_{B_2}(xy)\}e^{imax\{\alpha_{B_1}(xy),\alpha_{B_2}(xy)\}}$
$\mu^-_{B_1\cup B_2}(xy)e^{i\beta_{B_1\cup B_2}(xy)} = min\{\mu^-_{B_1}(xy), \mu^-_{B_2}(xy)\}e^{imin\{\beta_{B_1}(xy),\beta_{B_2}(xy)\}}$
$\nu^+_{B_1\cup B_2}(xy)e^{i\gamma_{B_1\cup B_2}(xy)} = max\{\nu^+_{B_1}(xy), \nu^+_{B_2}(xy)\}e^{imax\{\gamma_{B_1}(xy),\gamma_{B_2}(xy)\}}$
$\nu^-_{B_1\cup B_2}(xy)e^{i\delta_{B_1\cup B_2}(xy)} = min\{\nu^-_{B_1}(xy), \nu^-_{B_2}(xy)\}e^{imin\{\delta_{B_1}(xy),\delta_{B_2}(xy)\}}$ *for* $xy \in E_1 \cap E_2$.

Example 3.4. *Let us consider* G_1 *and* G_2 *be the two BCIF graphs then, their union* $G_1 \cup G_2$ *is BCIFG.*

Proposition 3.3. *Let* G_1 *and* G_2 *be the two bipolar complex intuitionistic fuzzy graphs. Then* $G_1 \cup G_2$ *is also a bipolar complex intuitionistic fuzzy graph.*

Definition 3.10. *Let* $A_1 = (\mu^+_{A_1}, \mu^-_{A_1}, \nu^+_{A_1}, \nu^-_{A_1})$ *and* $A_2 = (\mu^+_{A_2}, \mu^-_{A_2}, \nu^+_{A_2}, \nu^-_{A_2})$ *be two BCIF subsets of* V_1 *and* V_2 *and let* $B_1 = (\mu^+_{B_1}, \mu^-_{B_1}, \nu^+_{B_1}, \nu^-_{B_1})$ *and* $B_2 = (\mu^+_{B_2}, \mu^-_{B_2}, \nu^+_{B_2}, \nu^-_{B_2})$ *correspondingly be two BCIF subsets of* E_1 *and* E_2*. Then the join of two BCIF graphs is defined by* $G_1 + G_2 = (A_1 + A_2, B_1 + B_2)$ *as*
1. $\mu^+_{A_1+A_2}(x)e^{i\alpha_{A_1+A_2}(x)} = \mu^+_{A_1\cup A_2}(x)e^{i\alpha_{A_1}(x)}$
$\mu^-_{A_1+A_2}(x)e^{i\beta_{A_1+A_2}(x)} = \mu^-_{A_1\cup A_2}(x)e^{i\beta_{A_1}(x)}$
$\nu^+_{A_1+A_2}(x)e^{i\gamma_{A_1+A_2}(x)} = \nu^+_{A_1\cup A_2}(x)e^{i\gamma_{A_1}(x)}$
$\nu^-_{A_1+A_2}(x)e^{i\delta_{A_1+A_2}(x)} = \nu^-_{A_1\cup A_2}(x)e^{i\delta_{A_1}(x)}$*for* $x \in V_1$ *and* $x \notin V_2$,*if* $x \in V_1 \cup V_2$,
2. $\mu^+_{B_1+B_2}(xy)e^{i\alpha_{B_1+B_2}(xy)} = \mu^+_{B_1\cup B_2}(xy)e^{i\alpha_{B_1\cup B_2}(xy)}$

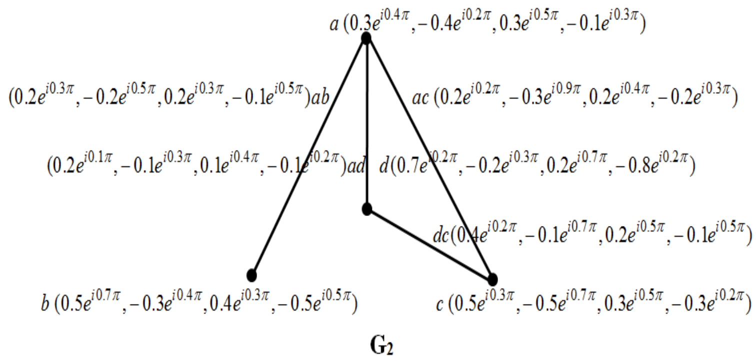

FIGURE 7. bipolar complex intuitionistic fuzzy graph G_2

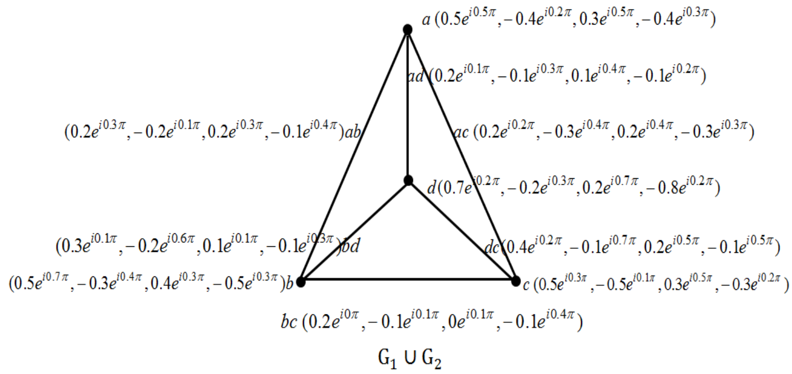

FIGURE 8. The union of two bipolar complex intuitionistic fuzzy graphs $G_1 \bigcup G_2$

$\mu^-_{B_1+B_2}(xy)e^{i\beta_{B_1+B_2}(xy)} = \mu^-_{B_1\cup B_2}(xy)e^{i\beta_{B_1\cup B_2}(xy)}$
$\nu^+_{B_1+B_2}(xy)e^{i\gamma_{B_1+B_2}(xy)} = \nu^+_{B_1\cup B_2}(xy)e^{i\gamma_{B_1\cup B_2}(xy)}$
$\nu^-_{B_1+B_2}(xy)e^{i\delta_{B_1+B_2}(xy)} = \nu^-_{B_1\cup B_2}(xy)e^{i\delta_{B_1\cup B_2}(xy)}$ *if* $xy \in E_1 \cap E_2$,
3. $\mu^+_{B_1+B_2}(xy)e^{i\alpha_{B_1+B_2}(xy)} = min\{\mu_{A_1^+}(x), \mu^+_{A_2}(y)\}e^{imin\{\alpha_{A_1}(x),\alpha_{A_2}(y)\}}$
$\mu^-_{B_1+B_2}(xy)e^{i\beta_{B_1+B_2}(xy)} = max\{\mu_{A_1^-}(x), \mu^-_{A_2}(y)\}e^{imax\{\beta_{A_1}(x),\beta_{A_2}(y)\}}$
$\nu^+_{B_1+B_2}(xy)e^{i\gamma_{B_1+B_2}(xy)} = min\{\nu_{A_1^+}(x), \nu^+_{A_2}(y)\}e^{imin\{\gamma_{A_1}(x),\gamma_{A_2}(y)\}}$
$\nu^-_{B_1+B_2}(xy)e^{i\delta_{B_1+B_2}(xy)} = max\{\nu_{A_1^-}(x), \nu^-_{A_2}(y)\}e^{imax\{\delta_{A_1}(x),\delta_{A_2}(y)\}}$ *if* $xy \in E'$, *where E' is the collection of edges joining the point of* V_1 *and* V_2, *for* $xy \in E_2$ *and* $xy \notin E_1$.

Example 3.5. *Let us consider* G_1 *and* G_2 *be the two BCIF graphs. Then, their join* $G_1 + G_2$ *is BCIFG.*

Proposition 3.4. *Let* G_1 *and* G_2 *be the two BCIF graphs, then the join* $G_1 + G_2$ *is also a BCIFG.*

Proposition 3.5. *Let* $G_1 = (A_1, B_1)$ *and* $G_2 = (A_2, B_2)$ *be the two BCIF graphs of the graphs* G_1^* *and* G_2^* *and let* $V_1 \cap V_2 = \phi$. *Then, union* $G_1 \cup G_2 = (A_1 \cup A_2, B_1 \cup B_2)$ *respectively is a BCIFG of* G^* *if and only if* G_1 *and* G_2 *are BCIF-graphs* G_1^* *and* G_2^*.

Proposition 3.6. *Let* $G_1 = (A_1, B_1)$ *and* $G_2 = (A_2, B_2)$ *be the BCIF graphs of the graphs* G_1^* *and* G_2^* *and let* $V_1 \cap V_2 = \phi$. *Then, join* $G_1 + G_2 = (A_1 + A_2, B_1 + B_2)$ *is a BCIFG of* G^* *if and only if* G_1 *and* G_2 *are BCIF-graphs* G_1^* *and* G_2^* *individually.*

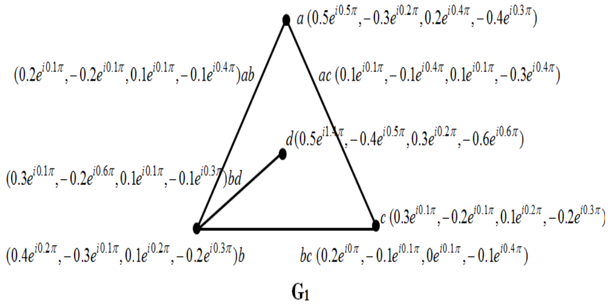

FIGURE 9. The bipolar complex intuitionistic fuzzy graph G_1

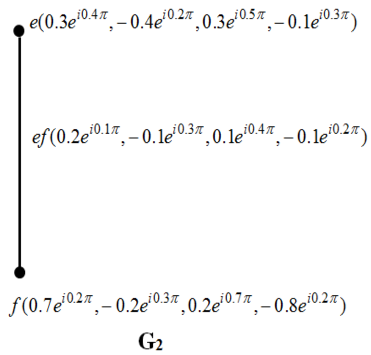

FIGURE 10. The bipolar complex intuitionistic fuzzy graph G_2

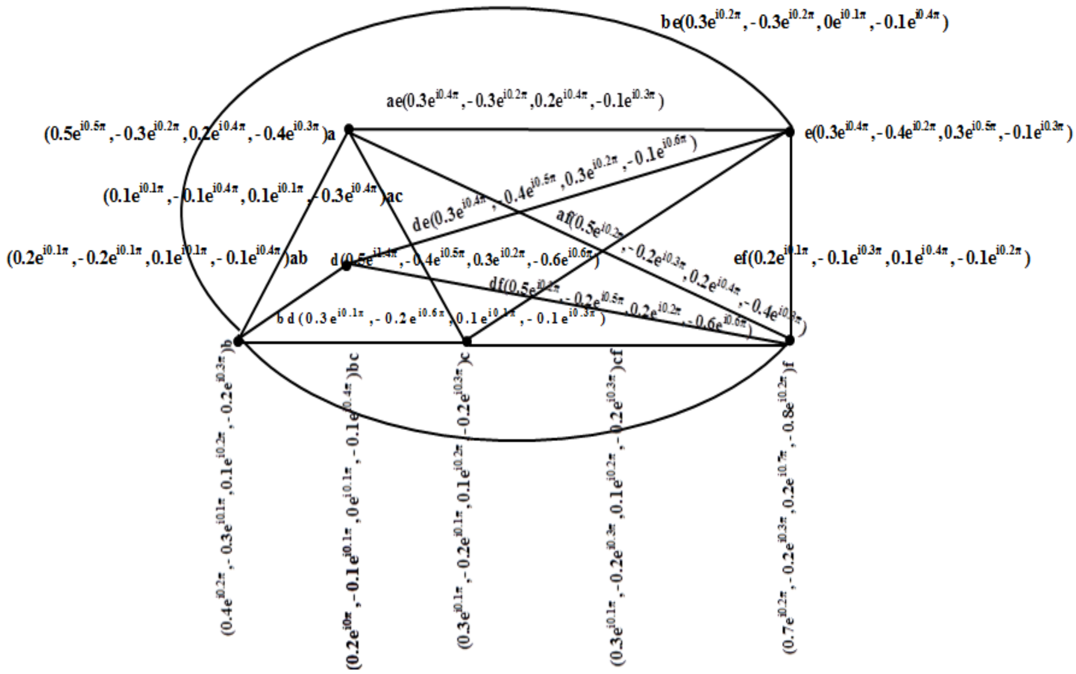

FIGURE 11. The join of two bipolar complex intuitionistic fuzzy graphs

4. Isomorphisms of BCIF-Graphs

Definition 4.1. *Let $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be two BCIF-graphs. A homomorphism $h : G_1 \to G_2$ is a function $h : V_1 \to V_2$ which satisfies the following conditions:*
(i)$\mu^+_{A_1}(x_1)e^{i\alpha_{A_1}(x_1)} \leq \mu^+_{A_2}(h(x_1))e^{i\alpha_{A_2}(h(x_1))}$
$\mu^-_{A_1}(x_1)e^{i\beta_{A_1}(x_1)} \geq \mu^-_{A_2}(h(x_1))e^{i\beta_{A_2}(h(x_1))}$
$\nu^+_{A_1}(x_1)e^{i\gamma_{A_1}(x_1)} \leq \nu^+_{A_2}(h)(x_1))e^{i\gamma_{A_2}(h(x_1))}$
$\nu^-_{A_1}(x_1)e^{i\delta_{A_1}(x_1)} \geq \nu^-_{A_2}(h(x_1))e^{i\delta_{A_2}(h(x_1))}$ $\forall x_1, y_1 \in V_1$.
(ii)$\mu^+_{B_1}(x_1y_1)e^{i\alpha_{B_1}(x_1y_1)} \leq \mu^+_{B_2}(h(x_1)h(y_1))e^{i\alpha_{B_2}(h(x_1)h(y_1))}$
$\mu^-_{B_1}(x_1y_1)e^{i\beta_{B_1}(x_1y_1)} \geq \mu^-_{B_2}(h(x_1)h(y_1))e^{i\beta_{B_2}(h(x_1)h(y_1))}$
$\nu^+_{B_1}(x_1y_1)e^{i\gamma_{B_1}(x_1y_1)} \leq \nu^+_{B_2}(h(x_1)h(y_1))e^{i\gamma_{B_2}(h(x_1)h(y_1))}$
$\nu^-_{B_1}(x_1y_1)e^{i\delta_{B_1}(x_1y_1)} \geq \nu^-_{B_2}(h(x_1)h(y_1))e^{i\delta_{B_2}(h(x_1)h(y_1))}$ $\forall x_1y_1 \in E_1$.

Definition 4.2. *Let $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be two BCIF-graphs. A function, $h : G_1 \to G_2$, a map $h : V_1 \to V_2$ which is one-one, onto homomorphism is said to be a weak isomorphism if (i)*$\mu^+_{A_1}(x_1)e^{i\alpha_{A_1}(x_1)} = \mu^+_{A_2}(h(x_1))e^{i\alpha_{A_2}(h(x_1))}$
$\mu^-_{A_1}(x_1)e^{i\beta_{A_1}(x_1)} = \mu^-_{A_2}(h(x_1))e^{i\beta_{A_2}(h(x_1))}$
$\nu^+_{A_1}(x_1)e^{i\gamma_{A_1}(x_1)} = \nu^+_{A_2}(h)(x_1))e^{i\gamma_{A_2}(h(x_1))}$
$\nu^-_{A_1}(x_1)e^{i\delta_{A_1}(x_1)} = \nu^-_{A_2}(h(x_1))e^{i\delta_{A_2}(h(x_1))}$ $\forall x_1 \in V_1$. *Hence, the weights of the vertices are preserved by a weak isomorphism but not necessarily by the weights of the edges.*

Definition 4.3. *Let $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be two BCIF-graphs. Then a co-weak isomomorphism $h : G_1 \to G_2$ is a bijective function $h : V_1 \to V_2$: which meets the following properties: (i) h is homomorphism*
(ii)$\mu^+_{B_1}(x_1y_1)e^{i\alpha_{B_1}(x_1y_1)} = \mu^+_{B_2}(h(x_1)h(y_1))e^{i\alpha_{B_2}(h(x_1)h(y_1))}$
$\mu^-_{B_1}(x_1y_1)e^{i\beta_{B_1}(x_1y_1)} = \mu^-_{B_2}(h(x_1)h(y_1))e^{i\beta_{B_2}(h(x_1)h(y_1))}$
$\nu^+_{B_1}(x_1y_1)e^{i\gamma_{B_1}(x_1y_1)} = \nu^+_{B_2}(h(x_1)h(y_1))e^{i\gamma_{B_2}(h(x_1)h(y_1))}$
$\nu^-_{B_1}(x_1y_1)e^{i\delta_{B_1}(x_1y_1)} = \nu^-_{B_2}(h(x_1)h(y_1))e^{i\delta_{B_2}(h(x_1)h(y_1))}$ $\forall x_1y_1 \in E_1$. *Hence, for a co-weak isomorphism the weights of the edges are preserved and not necessarily by the weights of the vertices.*

Definition 4.4. *Let $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be two BCIF-graphs. A function, $h : G_1 \to G_2$, a map $h : V_1 \to V_2$ which is one-one, onto is called an isomorphism if*
(i) $\mu^+_{A_1}(x_1)e^{i\alpha_{A_1}(x_1)} = \mu^+_{A_2}(h(x_1)e^{i\alpha_{A_2}(h(x_1))}$
$\mu^-_{A_1}(x_1)e^{i\beta_{A_1}(x_1)} = \mu^-_{A_2}(h(x_1))e^{i\beta_{A_2}(h(x_1))}$
$\nu^+_{A_1}(x_1)e^{i\gamma_{A_1}(x_1)} = \nu^+_{A_2}(h)(x_1))e^{i\gamma_{A_2}(h(x_1))}$
$\nu^-_{A_1}(x_1)e^{i\delta_{A_1}(x_1)} = \nu^-_{A_2}(h(x_1)e^{i\delta_{A_2}(h(x_1))}$ $\forall x_1 \in V_1$.
(ii)$\mu^+_{B_1}(x_1y_1)e^{i\alpha_{B_1}(x_1y_1)} = \mu^+_{B_2}(h(x_1)h(y_1))e^{i\alpha_{B_2}(h(x_1)h(y_1))}$
$\mu^-_{B_1}(x_1y_1)e^{i\beta_{B_1}(x_1y_1)} = \mu^-_{B_2}(h(x_1)h(y_1))e^{i\beta_{B_2}(h(x_1)h(y_1))}$
$\nu^+_{B_1}(x_1y_1)e^{i\gamma_{B_1}(x_1y_1)} = \nu^+_{B_2}(h(x_1)h(y_1))e^{i\gamma_{B_2}(h(x_1)h(y_1))}$
$\nu^-_{B_1}(x_1y_1)e^{i\delta_{B_1}(x_1y_1)} = \nu^-_{B_2}(h(x_1)h(y_1))e^{i\delta_{B_2}(h(x_1)h(y_1))}$ $\forall x_1y_1 \in E_1$.

Definition 4.5. *A BCIF set $A = (\mu^+_A e^{i\alpha_A}, \mu^-_A e^{i\beta_A}, \nu^+_A e^{i\gamma_A}, \nu^-_A e^{i\delta_A})$ in a semigroup S is called a BCIF sub semigroup of S if it satisfies:*
(i) $\mu^+_A(xy)e^{i\alpha_A(xy)} \geq min\{\mu^+_A(x), \mu^+_A(y)\}e^{imin\{\alpha_A(x),\alpha_A(y)\}}$,
(ii)$\mu^-_A(xy)e^{i\beta_A(xy)} \leq max\{\mu^-_A(x), \mu^-_A(y)\}e^{imax\{\beta_A(x),\beta_A(y)\}}$,
(iii) $\nu^+_A(xy)e^{i\gamma_A(xy)} \geq min\{\nu^+_A(x), \nu^+_A(y)\}e^{imin\{\gamma_A(x),\gamma_A(y)\}}$,

(iv) $\nu^-_A(xy)e^{i\delta_A(xy)} \leq max\{\nu^-_A(x), \nu^-_A(y)\}e^{imax\{\delta_A(x),\delta_A(y)\}}$ $\forall x, y \in S$.
A BCIF set $A_1 = (\mu^+_{A_1}, \mu^-_{A_1}, \nu^+_{A_1}, \nu^-_{A_1})$ *in a group* G *is called a BCIF subgroup of a group* G *if it satisfies:*
(i) $\mu^+_A(x^{-1})e^{i\alpha_A(x^{-1})} = \mu^+_A(x)e^{i\alpha_A(x)}$,
(ii) $\mu^-_A(x^{-1})e^{i\beta_A(x^{-1})} = \mu^-_A(x)e^{i\beta_A(x)}$,
(iii)$\nu^+_A(x^{-1})e^{i\gamma_A(x^{-1})} = \nu^+_A(x)e^{i\gamma_A(x)}$,
(iv) $\nu^-_A(x^{-1})e^{i\delta_A(x^{-1})} = \nu^-_A(x)e^{i\delta_A(x)}$ $\forall x \in G$.

Proposition 4.1. *Let* $G = (A, B)$ *be a BCIFG and let* $Aut(G)$ *be the collection of automorphisms of* G. *Then the ordered pair* $(Aut(G), \circ)$ *forms a group.*

Proposition 4.2. *Let* $G = (A, B)$ *be a bipolar complex intuitionistic fuzzy graph and let* $Aut(G)$ *be the set of all automorphisms of* G. *Let* $j_1 = (\mu^+_{j_1}, \mu^-_{j_1}, \nu^+_{j_1}, \nu^-_{j_1})$ *be a bipolar complex intuitionistic fuzzy set in* $Aut(G)$ *defined by*
$\mu^+_{j_1}(\varsigma) = Sup\{\mu^+_B(\varsigma(x, \varsigma(y))) : (x, y) \in V \times V\}$
$\mu^-_{j_1}(\varsigma) = Inf\{\mu^-_B(\varsigma(x, \varsigma(y))) : (x, y) \in V \times V\}$
$\nu^+_{j_1}(\varsigma) = Inf\{\nu^+_B(\varsigma(x, \varsigma(y))) : (x, y) \in V \times V\}$
$\nu^-_{j_1}(\varsigma) = Sup\{\nu^-_B(\varsigma(x, \varsigma(y))) : (x, y) \in V \times V\}$ $\forall \varsigma \in Aut(G)$. *Thus,* $j_1 = (\mu^+_{j_1}, \mu^-_{j_1}, \nu^+_{j_1}, \nu^-_{j_1})$ *is a bipolar complex intuitionistic group on* $Aut(G)$.

Proposition 4.3. *Every BCIF group has an embedding into the BCIF group of the group of automorphisms of some BCIFG.*

Proposition 4.4. *Let* G_1, G_2 *and* G_3 *be the three BCIF graphs. Hence there exists an equivalence relation, for an isomorphism between these BCIF graphs.*

Proposition 4.5. *Let* G_1, G_2 *and* G_3 *be the BCIF graphs. Then there* $\exists$ *a partial ordered relation, for the weak isomorphism between these BCIF graphs.*

5. Complement of BCIF-Graphs

Definition 5.1. *A BCIF graph* $G = (A, B)$ *of a graph* $G^* = (V, E)$ *is said to be self weak complement if* G *is weak isomorphism with its complement* $\overline{G}$ *(i.e)* $\exists$ *a 1-1 homomorphism* f *from* G *to* $\overline{G}$: $\forall x, y \in V$.
$\mu^+_A(x)e^{i\alpha_A(x)} = \overline{\mu^+_A}(f(x))e^{i\alpha_{\overline{A}}(f(x))}$
$\mu^-_A(x)e^{i\beta_A(x)} = \overline{\mu^-_A}(f(x))e^{i\beta_{\overline{A}}(f(x))}$
$\nu^+_A(x)e^{i\gamma_A(x)} = \overline{\nu^+_A}(f(x))e^{i\gamma_{\overline{A}}(f(x))}$
$\nu^-_A(x)e^{i\delta_A(x)} = \overline{\nu^-_A}(f(x))e^{i\delta_{\overline{A}}(f(x))}$ *and*
$\mu^+_A(xy)e^{i\alpha_A(xy)} \leq \overline{\mu^+_A}(f(x)f(y))e^{i\alpha_{\overline{A}}(f(x)f(y))}$
$\mu^-_A(xy)e^{i\beta_A(xy)} \geq \overline{\mu^-_A}(f(x)f(y))e^{i\beta_{\overline{A}}(f(x)f(y))}$
$\nu^+_A(xy)e^{i\gamma_A(xy)} \leq \overline{\mu^+_A}(f(x)f(y))e^{i\gamma_{\overline{A}}(f(x)f(y))}$
$\nu^-_A(xy)e^{i\delta_A(xy)} \geq \overline{\mu^-_A}(f(x)f(y))e^{i\delta_{\overline{A}}(f(x)f(y))}$

Definition 5.2. *The complement of BCIFG* $G = (A, B)$ *of a graph* $G^* = (V, E)$ *is a BCIFG* $\overline{G} = (\overline{A}, \overline{B})$ *of* $\overline{G^*} = (V, V^2)$ *where* $A = (\mu^+_A e^{i\alpha_A}, \mu^-_A e^{i\beta_A}, \nu^+_A e^{i\gamma_A}, \nu^-_A e^{i\delta_A})$,
$B = (\mu^+_B e^{i\alpha_B}, \mu^-_B e^{i\beta_B}, \nu^+_B e^{i\gamma_B}, \nu^-_B e^{i\delta_B})$ *is defined as*
$\overline{\mu^+_B}(xy)e^{i\alpha_{\overline{B}(xy)}} = min\{\mu^+_A(x)e^{i\alpha_A(x)}, \mu^+_A(y)e^{i\alpha_A(y)}\} - \mu^+_A(xy)e^{i\alpha_A(xy)}$,
$\overline{\mu^-_B}(xy)e^{i\beta_{\overline{B}(xy)}} = max\{\mu^-_A(x)e^{i\beta_A(x)}, \mu^-_A(y)e^{i\beta_A(y)}\} - \mu^+_A(xy)e^{i\beta_A(xy)}$,
$\overline{\nu^+_B}(xy)e^{i\gamma_{\overline{B}(xy)}} = max\{\nu^+_A(x)e^{i\gamma_A(x)}, \nu^+_A(y)e^{i\gamma_A(y)}\} - \mu^+_A(xy)e^{i\gamma_A(xy)}$,
$\overline{\nu^-_B}(xy)e^{i\delta_{\overline{B}(xy)}} = min\{\nu^-_A(x)e^{i\delta_A(x)}, \nu^-_A(y)e^{i\delta_A(y)}\} - \mu^+_A(xy)e^{i\delta_A(xy)}$.

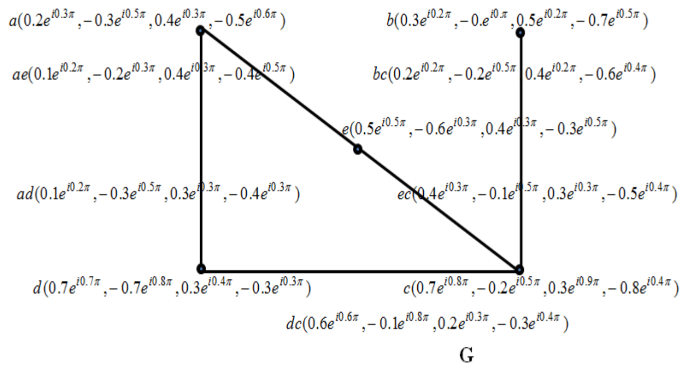

FIGURE 12. Bipolar complex intuitionistic fuzzy graph of G

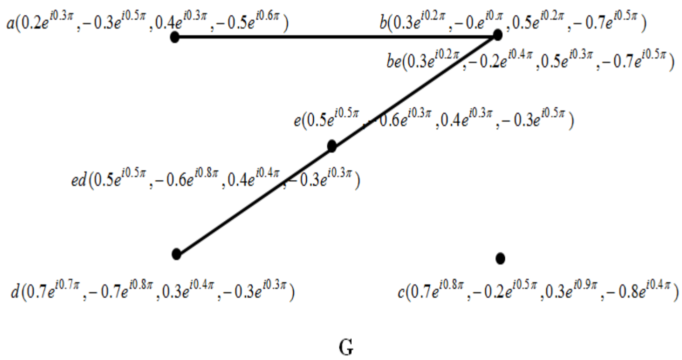

FIGURE 13. Complement of BCIF-graph of G is $\overline{G}$

Definition 5.3. *The complement of BCIFG $G = \overline{(A, B)}$ of a graph $G^* = (V, E)$ is a BCIFG $\overline{G} = (\overline{A}, \overline{B})$ on $\overline{G^*}$, where $\overline{A} = (\overline{\mu_A}^+ e^{i\alpha_{\overline{A}}}, \overline{\mu_A^-} e^{i\beta_{\overline{A}}}, \overline{\nu_A^+} e^{i\gamma_{\overline{A}}}, \overline{\nu_A^-} e^{i\delta_{\overline{A}}})$,*
$\overline{B} = (\overline{\mu_B}^+ e^{i\alpha_{\overline{B}}}, \overline{\mu_B^-} e^{i\beta_{\overline{B}}}, \overline{\nu_B^+} e^{i\gamma_{\overline{B}}}, \overline{\nu_B^-} e^{i\delta_{\overline{B}}})$ are defined by (i) $\overline{V} = V$
(ii)$\overline{\mu_A^+}(x) e^{i\alpha_{\overline{A}}}(x) = \mu_A^+(x) e^{i\alpha_A(x)}$,
$\overline{\mu_A^-}(x) e^{i\beta_{\overline{A}}}(x) = \mu_A^+(x) e^{i\beta_A(x)}$,
$\overline{\nu_A^+}(x) e^{i\gamma_{\overline{A}}}(x) = \nu_A^+(x) e^{i\gamma_A(x)}$,
$\overline{\nu_A^-}(x) e^{i\delta_{\overline{A}}}(x) = \nu_A^-(x) e^{i\delta_A(x)}$ for all $x \in V$

$$(iii)\ \overline{\mu_B^+}(xy) e^{i\alpha_{\overline{B}}}(xy) = \begin{cases} 0 & if \quad \mu_B^+(xy) e^{i\alpha_B(xy)} \neq 0 \\ min\{\mu_A^+(x), \mu_A^+(y)\} e^{imin\alpha_A(x), \alpha_A(y)} & if \quad \mu_B^+(xy) e^{i\alpha_B(xy)} = 0, \end{cases}$$

$$\overline{\mu_B^-}(xy) e^{i\beta_{\overline{B}}}(xy) = \begin{cases} 0 & if \quad \mu_B^+(xy) e^{i\beta_B(xy)} \neq 0 \\ max\{\mu_A^-(x), \mu_A^+(y)\} e^{imax\beta_A(x), \beta_A(y)} & if \quad \mu_B^+(xy) e^{i\beta_B(xy)} = 0, \end{cases}$$

$$\overline{\nu_B^+}(xy) e^{i\gamma_{\overline{B}}}(xy) = \begin{cases} 0 & if \quad \nu_B^+(xy) e^{i\gamma_B(xy)} \neq 0 \\ max\{\nu_A^+(x), \nu_A^+(y)\} e^{imax\gamma_A(x), \gamma_A(y)} & if \quad \nu_B^+(xy) e^{i\gamma_B(xy)} = 0, \end{cases}$$

$$\overline{\nu_B^-}(xy) e^{i\delta_{\overline{B}}}(xy) = \begin{cases} 0 & if \quad \nu_B^-(xy) e^{i\delta_B(xy)} \neq 0 \\ min\{\nu_A^-(x), \nu_A^-(y)\} e^{imin\delta_A(x), \delta_A(y)} & if \quad \nu_B^-(xy) e^{i\delta_B(xy)} = 0, \end{cases}$$

Definition 5.4. *A BCIFG G is said to be self complementary if $\overline{G} \approx G$.*

Example 5.1. *Let us assume a BCIFG G as in Figure 12. Then, the complement $\overline{G}$ of G as in Figure 13.*

Proposition 5.1. *Let $G = (A, B)$ be a self complementary BCIFG. Then,*
$\sum_{x \neq y} \mu_B^+(xy) e^{i\alpha_B}(xy) = \sum_{x \neq y} min\{\mu_A^+(x), \mu_A^+(y)\} e^{imin\{\alpha_A(x), \alpha_A(y)\}}$,
$\sum_{x \neq y} \mu_B^-(xy) e^{i\beta_B}(xy) = \sum_{x \neq y} max\{\mu_A^-(x), \mu_A^-(y)\} e^{imax\{\beta_A(x), \beta_A(y)\}}$,
$\sum_{x \neq y} \nu_B^+(xy) e^{i\gamma_B}(xy) = \sum_{x \neq y} max\{\nu_A^+(x), \nu_A^+(y)\} e^{imax\{\gamma_A(x), \gamma_A(y)\}}$,
$\sum_{x \neq y} \nu_B^-(xy) e^{i\delta_B}(xy) = \sum_{x \neq y} min\{\nu_A^-(x), \nu_A^-(y)\} e^{imin\{\delta_A(x), \delta_A(y)\}}$.

Proposition 5.2. *Let $G_1 = (A_1, B_2)$ and $G_2 = (A_2, B_2)$ be two BCIF graphs such that $V_1 \bigcap V_2 = \phi$. Then the complement of the sum of two graphs is equivalent to the union of the complement of the graph G_1 and G_2. (i.e) $\overline{G_1 + G_2} \cong \overline{G_1} \bigcup \overline{G_2}$.*

Proposition 5.3. *Let $G_1 = (A_1, B_2)$ and $G_2 = (A_2, B_2)$ be two BCIF graphs such that $V_1 \bigcap V_2 = \phi$. Then the complement of the sum of two graphs is equivalent to the union of the complement of the graph G_1 and G_2.(i.e) $\overline{G_1 \bigcup G_2} \cong \overline{G_1} + \overline{G_2}$.*

Proposition 5.4. *Let $G = (A, B)$ be a BCIF-graph. If,*
$\mu_B^+(xy) e^{i\alpha_B}(xy) = min\{\mu_A^+(x), \mu_A^+(y)\} e^{imin\{\alpha_A(x), \alpha_A(y)\}}$,
$\mu_B^-(xy) e^{i\beta_B}(xy) = max\{\mu_A^-(x), \mu_A^-(y)\} e^{imax\{\beta_A(x), \beta_A(y)\}}$,
$\nu_B^+(xy) e^{i\gamma_B}(xy) = max\{\nu_A^+(x), \nu_A^+(y)\} e^{imax\{\gamma_A(x), \gamma_A(y)\}}$,
$\nu_B^-(xy) e^{i\delta_B}(xy) = min\{\nu_A^-(x), \nu_A^-(y)\} e^{imin\{\delta_A(x), \delta_A(y)\}}$ *for all $x, y \in V$, then G is self complementary.*

Proposition 5.5. *Let G_1 and G_2 be BCIF-graphs. If there exists a strong isomorphism between G_1 and G_2, then there exists a strong isomorphism between $\overline{G_1}$ and $\overline{G_2}$.*

Proposition 5.6. *Let us consider G_1 and G_2 be BCIF-graphs. Then, $G_1 \cong G_2$ if and only if $\overline{G_1} \cong \overline{G_2}$.*

Proposition 5.7. *Let G_1 and G_2 be BCIF-graphs. If there $\exists$ is a co-strong isomorphism between $G_1 \cong G_2$, then there is a homomorphism between $\overline{G_1} \cong \overline{G_2}$.*

6. Conclusions

The combinatorial problems are solved by using a dynamic tool i.e. Graph Theory, in numerous areas such as algebra, topology, chemistry, geometry, computer science and operation research. It has massive applications in traffic signal, cellular networks, GPS and decision making etc. Here, we developed a notion of BCIF-sets and BCIF-graphs and then, we defined and established the composition, cartesian product, join and union of BCIF-graphs and the degree of vertices of the graphs $G_1[G_2]$ and $G_1 \times G_2$. In addition, we also introduced complement and isomorphisms on BCIF-graphs and some of its special features are derived.

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