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SEVERAL TYPES OF SINGLE-VALUED NEUTROSOPHIC IDEALS AND FILTERS ON A LATTICE

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Abstract. In this paper, we introduce and study the notions of prime, maximal and principal single-valued neutrosophic ideals (resp. filters) on a lattice. Several properties and characterizations of these types of ideals and filters are given, and relationships between them are discussed.

Keywords: Single-valued neutrosophic set, Lattice, Ideal, Filter.

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1. Introduction

The notion of neutrosophic sets (NSs) introduced and developed by Smarandache [17] is a more general platform which extends the concepts of (crisp) fuzzy set, intuitionistic fuzzy set, spherical fuzzy set, and n-hyper spherical fuzzy set. This notion of NS described by three degrees; truth membership function (T), indeterminacy membership function (I) and falsity membership function (F). After that, Wang et al. [20] discussed the notion of single-valued neutrosophic set (SVNS) as a subclass of NS which can independently expresses the truth-membership, the indeterminacy-membership and the falsity-membership degrees and deals with incomplete, indeterminate and inconsistent information. These three components of a SVNS are independent and their values are enclosed in the standard unit interval. Single-valued neutrosophic set theory has useful applications in several branches (see for e.g., [6], [8], [11], [12], [14],[18] and [22]).

Ideals and filters have played a vital role in many mathematical studies and they developed in different algebraic structures (see for e.g., [13], [21], [19]). In particular, various types of them have been extensively investigated in the literature, for instance, Mezzomo et al. [9] provided some types of fuzzy ideals and filters on fuzzy lattice, such as fuzzy principal ideals (filters), proper fuzzy ideals (filters), prime fuzzy ideals (filters) and fuzzy maximal ideals (filters). Zhana and Xu [24] discussed some types of generalized fuzzy filters of BL-algebras. Recently, Alaba and Addis [2] introduced the notions of L-fuzzy

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prime ideals and maximal L-fuzzy ideals of universal algebras. In intuitionistic fuzzy setting and for the same purpose, several authors [4, 10] introduced and studied the notion of intuitionistic fuzzy ideals (resp. filters) on a lattice.

In the neutrosophic setting, the notions of ideals and filters have been done by several authors. For instance, Salama and Smarandache [15] considered the notion of filters via neutrosophic crisp set and investigated several relations between different neutrosophic filters and neutrosophic topologies. Salama and Alagamy [16] introduced the notion of filters on a neutrosophic set as a generalization of the notion of fuzzy filters. Recently, Hamidi et al. [7] studied the notion of single-valued neutrosophic filters on EQ-algebras and its relationship with filters on these kind algebras.

The aim of the present paper is to study some types of single-valued neutrosophic ideals (resp. filters) on a lattice. More precisely, we extend the notion of prime, maximal and principal ideal (resp. filter) to the neutrosophic setting. Furthermore, some interesting characterizations and properties related to these notions in the single-valued neutrosophic setting are discussed.

This paper is structured as follows. In Section 2, we recall basic concepts and properties that will be needed throughout this paper. In Section 3, we recall the notions of single-valued neutrosophics lattice and single-valued neutrosophic ideals (resp. filters), and show some of their properties. In Section 4, we study the notion of prime single-valued neutrosophic ideal (resp. filter). In Section 5, we study the notion of maximal single-valued neutrosophic ideal (resp. filter), and their interesting properties including the relationship between primality and maximality. Section 6 is devoted to the notion of principal single-valued neutrosophic ideal (resp. filter), and their characterization in terms of down-sets (resp. up-sets) generated by a single-valued neutrosophic singleton. Finally, we present some conclusions and we discuss future research in Section 7.

2. Basic concepts

This section contains the basic definitions and properties of single-valued neutrosophic sets and some related notions that will be needed throughout this paper.

Smarandache [17] introduced the notion of a neutrosophic sets as a generalization of the notion of Atanassov's intuitionistic fuzzy sets.

Let X be a nonempty set. A neutrosophic set (NS, for short) A on X is an object of the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle \mid x \in X\}$ characterized by a membership function $\mu_A : X \to]^-0, 1^+[$ and an indeterminacy function $\sigma_A : X \to]^-0, 1^+[$ and a non-membership function $\nu_A : X \to]^-0, 1^+[$ which satisfy the condition:

$$^-0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3^+, \text{ for any } x \in X.$$

For practical use of neutrosophic sets, Wang et al. [20] introduced the notion of single-valued neutrosophic set (SVNS) as a subclass of NSs.

Definition 2.1. [20] *Let X be a nonempty set. A single-valued neutrosophic set (SVNS, for short) A on X is an object of the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle \mid x \in X\}$ characterized by a truth-membership function $\mu_A : X \to [0, 1]$, an indeterminacy-membership function $\sigma_A : X \to [0, 1]$ and a falsity-membership function $\nu_A : X \to [0, 1]$.*

We denote by $SVNS(X)$ the set of all single-valued neutrosophic sets on X.

For any $A,\ B \in SVNS(X)$, several operations are defined (see, e.g., [18, 20, 22, 23]). Here we will present only those which are related to the present paper.

(i) $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$ and $\sigma_A(x) \leq \sigma_B(x)$ and $\nu_A(x) \geq \nu_B(x)$, for any $x \in X$,
(ii) $A = B$ if $\mu_A(x) = \mu_B(x)$ and $\sigma_A(x) = \sigma_B(x)$ and $\nu_A(x) = \nu_B(x)$, for any $x \in X$,

(iii) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\}$,
(iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\}$,
(v) $\overline{A} = \{\langle x, \nu_A(x), \sigma_A(x), \mu_A(x)\rangle \mid x \in X\}$.

In the sequel, we need the following definition of level sets (which is also often called (α, β, γ)-cuts) of a single-valued neutrosophic set.

Definition 2.2. [1] *Let $A \in SVNS(X)$, the level sets of A (or the (α, β, γ)-cuts of A such that $\alpha, \beta, \gamma \in]0, 1]$) are the crisp subsets defined by:*

$$A_{\alpha,\beta,\gamma} = \{x \in X \mid \mu_A(x) \geq \alpha \ , \ \sigma_A(x) \geq \beta \text{ and } \nu_A(x) \leq \gamma\} \, ,$$

for any $\alpha, \beta, \gamma \in]0, 1]$.

Definition 2.3. [1] *Let A be a single-valued neutrosophic set on a set X. The support of A is the crisp subset on X given by*

$$Supp(A) = \{x \in X \mid \mu_A(x) \neq 0, \ \sigma_A(x) \neq 0 \text{ and } \nu_A(x) \neq 0\}.$$

3. Single-valued neutrosophic lattices, ideals and filters

The notion of single-valued neutrosophic lattice or fuzzy neutrosophic lattice is introduced by Arockiarani and Antony Crispin Sweety [3] as a SVNS on a given crisp lattice stable by its meet and join operations. First, we need to fix the following notations.

Notation 3.1. *In the rest of this paper, L always denotes a lattice $(L, \leqslant, \sqcap, \sqcup)$ and L^d its dual-order lattice $(L, \geqslant, \sqcup, \sqcap)$. To avoid any confusion or misunderstanding in some formulas, we use the notation $(\leqslant, \sqcap, \sqcup)$ to refer the (order, $\min$, $\max$) on the lattice L and $(\leq, \wedge, \vee)$ to refer the (usual order, $\min$, $\max$) on the real interval $[0, 1]$. For more details on lattices, we refer to [5].*

Definition 3.1. [3] *Let L be a lattice and $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle \mid x \in L\}$ be a SVNS on L. Then A is called a single-valued neutrosophic lattice (SVN-lattice, for short) if for any $x, y \in L$, the following conditions are satisfied:*

(i) $\mu_A(x \sqcup y) \geq \mu_A(x) \wedge \mu_A(y)$,
(ii) $\mu_A(x \sqcap y) \geq \mu_A(x) \wedge \mu_A(y)$,
(iii) $\sigma_A(x \sqcup y) \geq \sigma_A(x) \wedge \sigma_A(y)$,
(iv) $\sigma_A(x \sqcap y) \geq \sigma_A(x) \wedge \sigma_A(y)$,
(v) $\nu_A(x \sqcup y) \leq \nu_A(x) \vee \nu_A(y)$,
(vi) $\nu_A(x \sqcap y) \leq \nu_A(x) \vee \nu_A(y)$.

Example 3.1. *Let $L = \{0, a, b, 1\}$ be the lattice given by the following Hasse diagram. The SVNS $A = \{< 0, 0.5, 0.4, 0.1 >, < a, 0.4, 0.3, 0.5 >, < b, 0.4, 0.3, 0.3 >, < 1, 0.7, 0.6, 0.3 >\}$ on L is a SVN-lattice.*

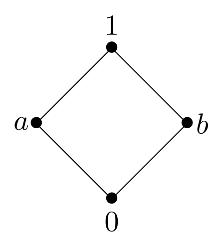

Figure 1. Hasse diagram of a lattice $(L, \leqslant, \sqcap, \sqcup)$ with $L = \{0, a, b, 1\}$.

Definition 3.2. [3] *Let L be a lattice and $I = \{\langle x, \mu_I(x), \sigma_I(x), \nu_I(x)\rangle \mid x \in L\}$ be a SVNS on L. Then I is called a single-valued neutrosophic ideal on L (SVN-ideal, for short) if for all $x, y \in L$, the following conditions are satisfied:*

(i) $\mu_I(x \sqcup y) \geq \mu_I(x) \wedge \mu_I(y)$,
(ii) $\mu_I(x \sqcap y) \geq \mu_I(x) \vee \mu_I(y)$,
(iii) $\sigma_I(x \sqcup y) \geq \sigma_I(x) \wedge \sigma_I(y)$,
(iv) $\sigma_I(x \sqcap y) \geq \sigma_I(x) \vee \sigma_I(y)$,
(v) $\nu_I(x \sqcup y) \leq \nu_I(x) \vee \nu_I(y)$,
(vi) $\nu_I(x \sqcap y) \leq \nu_I(x) \wedge \nu_I(y)$.

Dually, we introduce the notion of a single-valued neutrosophic filter on a lattice.

Definition 3.3. *Let L be a lattice and $F = \{\langle x, \mu_F(x), \sigma_F(x), \nu_F(x)\rangle \mid x \in L\}$ be a SVNS on L. Then F is called a single-valued neutrosophic filter on L (SVN-filter, for short) if for all $x, y \in L$, the following conditions are satisfied:*

(i) $\mu_F(x \sqcup y) \geq \mu_F(x) \vee \mu_F(y)$,
(ii) $\mu_F(x \sqcap y) \geq \mu_F(x) \wedge \mu_F(y)$,
(iii) $\sigma_F(x \sqcup y) \geq \sigma_F(x) \vee \sigma_F(y)$,
(iv) $\sigma_F(x \sqcap y) \geq \sigma_F(x) \wedge \sigma_F(y)$,
(v) $\nu_F(x \sqcup y) \leq \nu_F(x) \wedge \nu_F(y)$,
(vi) $\nu_F(x \sqcap y) \leq \nu_F(x) \vee \nu_F(y)$.

A SVN-ideal or a SVN-filter is said to be proper if it is not equal to the whole lattice L.

Example 3.2. *Let L be the lattice given by the Hasse diagram in Figure 1. Then*

(i) $I = \{< 0, 0.6, 0.5, 0.2 >, < a, 0.5, 0.6, 0.4 >, < b, 0.2, 0.4, 0.3 >, < 1, 0.2, 0.3, 0.4 >\}$ *is a SVN-ideal,*
(ii) $F = \{< 0, 0.2, 0.3, 0.7 >, < a, 0.3, 0.4, 0.7 >, < b, 0.2, 0.3, 0.6 >, < 1, 0.5, 0.4, 0.4 >\}$ *is a SVN-filter.*

The following proposition is immediate.

Proposition 3.1. *Let L be a lattice, L^d be its dual-order lattice and $A \in SVNS(L)$. Then it holds that A is a SVN-ideal on L if and only if A is a SVN-filter on L^d and conversely.*

The following proposition shows that the support of a SVN-ideal (resp. SVN-filter) on a given lattice L is also an ideal (resp. a filter) on L. The proof can be obtained by a direct application of the definition of an ideal (resp. a filter) on a lattice.

Proposition 3.2. *Let L be a lattice and $A \in SVNS(L)$. The following statements hold:*

(i) *if A is a SVN-ideal, then its support $Supp(A)$ is an ideal on L,*
(ii) *if A is a SVN-filter, then its support $Supp(A)$ is a filter on L.*

Proof. We only show (i), as (ii) can be proved by using Proposition 3.1 and (i). Suppose that A is a SVN-ideal on L and we show that $Supp(A)$ is an ideal on L.

(a) Let $x \in Supp(A)$ and $y \leqslant x$, then it holds that

$$\mu_A(x) \neq 0,\ \sigma_A(x) \neq 0 \text{ and } \nu_A(x) \neq 0\,.$$

Since $y \leqslant x$, then it holds that $x \sqcap y = y$. This implies that $\mu_A(y) = \mu_I(x \sqcap y) \geq \mu_I(x) \vee \mu_I(y)$. Hence, $\mu_A(y) \geq \mu_I(x) \neq 0$. In similar way, we obtain that $\sigma_A(x) \neq 0$ and $\nu_A(x) \neq 0$. Thus, $y \in Supp(A)$.

(b) Let $x, y \in Supp(A)$, we need to show that $x \sqcup y \in Supp(A)$. Since A is a SVN-ideal, then it follows from Definition 3.2 (i) that $\mu_A(x \sqcup y) \geq \mu_I(x) \wedge \mu_I(y) \neq 0$. Similarly, we show that $\sigma_A(x \sqcup y) \neq 0$ and $\nu_A(x \sqcup y) \neq 0$. Thus, $x \sqcup y \in Supp(A)$.

Therefore, $Supp(A)$ is an ideal on L. □

The following result provides a characterization of a SVN-ideal (resp. SVN-filter) on a given lattice in terms of its level sets.

Proposition 3.3. *Let L be a lattice and $A \in SVNS(L)$. The following statements hold:*
(i) *A is a SVN-ideal if and only if its level sets are ideals on L,*
(ii) *A is a SVN-filter if and only if its level sets are filters on L.*

Proof. We only show (i), as (ii) can be proved by using Proposition 3.1 and (i). Let A be a SVN-ideal on L and $A_{\alpha,\beta,\gamma}$ such that $\alpha, \beta, \gamma \in]0, 1]$ their level sets.
(a) Let $x \in A_{\alpha,\beta,\gamma}$ and $y \leqslant x$. From Definition 3.2 of a SVN-ideal, it follows that $\mu_A(y) \geq \mu_A(x)$, $\sigma_A(y) \geq \sigma_A(x)$ and $\nu_A(y) \leq \nu_A(x)$. Now, the fact that $\mu_A(x) \geq \alpha$, $\sigma_A(x) \geq \beta$ and $\nu_A(x) \leq \gamma$ imply that $\mu_A(y) \geq \alpha$, $\sigma_A(y) \geq \beta$ and $\nu_A(y) \leq \gamma$. Hence, $y \in A_{\alpha,\beta,\gamma}$.
(b) Let $x, y \in A_{\alpha,\beta,\gamma}$, then it holds that ($\mu_A(x) \geq \alpha$, $\sigma_A(x) \geq \beta$ and $\nu_A(x) \leq \gamma$) and ($\mu_A(y) \geq \alpha$, $\sigma_A(y) \geq \beta$ and $\nu_A(y) \leq \gamma$). From Definition 3.2 of a SVN-ideal, it follows that $\mu_A(x \sqcup y) \geq \mu_A(x) \wedge \mu_A(y) \geq \alpha$, $\sigma_A(x \sqcup y) \geq \sigma_A(x) \wedge \sigma_A(y) \geq \beta$ and $\nu_A(x \sqcup y) = \nu_A(x) \vee \nu_A(y) \leq \gamma$. Hence, $x \sqcup y \in A_{\alpha,\beta,\gamma}$.

Thus, $A_{\alpha,\beta,\gamma}$ is an ideal on L, for any $\alpha, \beta, \gamma \in]0, 1]$.

Conversely, suppose that all level sets of A are ideals on L and we show that A is a SVN-ideal on L. Let $x, y \in L$ and assume that $\alpha = \mu_A(x) \wedge \mu_A(y)$, $\beta = \sigma_A(x) \wedge \sigma_A(y)$ and $\gamma = \nu_A(x) \vee \nu_A(y)$. Since $A_{\alpha,\beta,\gamma}$ is an ideal on L, then it holds that $x \sqcup y \in A_{\alpha,\beta,\gamma}$, for any $\alpha, \beta, \gamma \in]0, 1]$. This implies that $\mu_A(x \sqcup y) \geq \alpha$, $\sigma_A(x \sqcup y) \geq \beta$ and $\nu_A(x \sqcup y) \leq \gamma$. Hence, $\mu_A(x \sqcup y) \geq \mu_A(x) \wedge \mu_A(y)$, $\sigma_A(x \sqcup y) \geq \sigma_A(x) \wedge \sigma_A(y)$ and $\nu_A(x \sqcup y) \leq \nu_A(x) \vee \nu_A(y)$. The other three conditions (ii), (iv) and (vi) of Definition 3.2 can be proved similarly. Thus, A is a SVN-ideal on L. □

4. Prime SVN-ideals and filters on a lattice

In this section, we introduce and characterize the notion of prime SVN-ideals (resp. SVN-filters) on a given lattice.

Definition 4.1. *A SVN-ideal I on a lattice L is called a prime SVN-ideal if, for any $x, y \in L$, the following conditions hold:*
(i) $\mu_I(x \sqcap y) \leq \mu_I(x) \vee \mu_I(y)$,
(ii) $\sigma_I(x \sqcap y) \leq \sigma_I(x) \vee \sigma_I(y)$,
(iii) $\nu_I(x \sqcap y) \geq \nu_I(x) \wedge \nu_I(y)$.

Definition 4.2. *A SVN-filter F on a lattice L is called a prime SVN-filter if, for any $x, y \in L$, the following conditions hold:*
(i) $\mu_F(x \sqcup y) \leq \mu_F(x) \vee \mu_F(y)$,
(ii) $\sigma_F(x \sqcup y) \leq \sigma_F(x) \vee \sigma_F(y)$,
(iii) $\nu_F(x \sqcup y) \geq \nu_F(x) \wedge \nu_F(y)$.

The following result provides a characterization of prime SVN-ideal on a given lattice.

Theorem 4.1. *Let L be a lattice and $I \in SVNS(L)$. Then it holds that I is a prime SVN-ideal on L if and only if the following conditions hold:*
(i) $\mu_I(x \sqcup y) = \mu_I(x) \wedge \mu_I(y)$, *for any* $x, y \in L$,
(ii) $\mu_I(x \sqcap y) = \mu_I(x) \vee \mu_I(y)$, *for any* $x, y \in L$,
(iii) $\sigma_I(x \sqcup y) = \sigma_I(x) \wedge \sigma_I(y)$, *for any* $x, y \in L$,
(iv) $\sigma_I(x \sqcap y) = \sigma_I(x) \vee \sigma_I(y)$, *for any* $x, y \in L$,

(v) $\nu_I(x \sqcup y) = \nu_I(x) \vee \nu_I(y)$, *for any* $x, y \in L$,
(vi) $\nu_I(x \sqcap y) = \nu_I(x) \wedge \nu_I(y)$, *for any* $x, y \in L$.

Proof. Suppose that I is a prime SVN-ideal on L. We only give the proof of the condition (i), as the others can be proved analogously. By hypothesis we have that $\mu_I(x \sqcup y) \geq \mu_I(x) \wedge \mu_I(y)$. Further, for any $x, y \in L$ it holds from Definition 3.2 (ii) that $\mu_I(x) = \mu_I(x \sqcap (x \sqcup y)) \geq \mu_I(x \sqcup y)$ and $\mu_I(y) = \mu_I(y \sqcap (x \sqcup y)) \geq \mu_I(x \sqcup y)$. Hence, $\mu_I(x) \wedge \mu_I(y) \geq \mu_I(x \sqcup y)$. Thus, $\mu_I(x \sqcup y) = \mu_I(x) \wedge \mu_I(y)$.

Conversely, suppose that μ_I, σ_I and ν_I satisfying the above six conditions. Then it is obvious to see that I is a prime SVN-ideal on L. □

Similarly, combining Theorem 4.1 and Proposition 3.1 leads to the following characterization of prime SVN-filters.

Theorem 4.2. *Let* L *be a lattice and* $F \in SVNS(L)$. *Then it holds that* F *is a prime SVN-filter on* L *if and only if the following conditions hold:*

(i) $\mu_F(x \sqcup y) = \mu_F(x) \vee \mu_F(y)$, *for any* $x, y \in L$,
(ii) $\mu_F(x \sqcap y) = \mu_F(x) \wedge \mu_F(y)$, *for any* $x, y \in L$,
(iii) $\sigma_F(x \sqcup y) = \sigma_F(x) \vee \sigma_F(y)$, *for any* $x, y \in L$,
(iv) $\sigma_F(x \sqcap y) = \sigma_F(x) \wedge \sigma_F(y)$, *for any* $x, y \in L$,
(v) $\nu_F(x \sqcup y) = \nu_F(x) \wedge \nu_F(y)$, *for any* $x, y \in L$,
(vi) $\nu_F(x \sqcap y) = \nu_F(x) \vee \nu_F(y)$, *for any* $x, y \in L$.

The following proposition shows that the support of a prime SVN-ideal (resp. prime SVN-filter) on a lattice is a prime ideal (resp. prime filter) on that lattice.

Proposition 4.1. *Let* L *be a lattice and* $A \in SVNS(L)$. *Then it holds that*

(i) *if* A *is a prime SVN-ideal, then its support* $Supp(A)$ *is a prime ideal on* L,
(ii) *if* A *is a prime SVN-filter, then its support* $Supp(A)$ *is a prime filter on* L.

Proof. (i) Suppose that A is a prime SVN-ideal on a lattice L. From Proposition 3.2, it holds that $Supp(A)$ is an ideal on L. Next, we prove that $Supp(A)$ is prime. Let $x, y \in L$ such that $x \sqcap y \in Supp(A)$. It then holds that $\mu_A(x \sqcap y) \neq 0$, $\sigma_A(x \sqcap y) \neq 0$ and $\nu_A(x \sqcap y) \neq 0$. Then the fact that A is a prime SVN-ideal on L implies that $\mu_A(x) \vee \mu_A(y) = \mu_A(x \sqcap y) \neq 0$, $\sigma_A(x) \vee \sigma_A(y) = \sigma_A(x \sqcap y) \neq 0$ and $\nu_A(x) \wedge \nu_A(y) = \nu_A(x \sqcap y) \neq 0$. This implies that either $(\mu_A(x) \neq 0,\ \sigma_A(x) \neq 0$ and $\nu_A(x) \neq 0)$ or $(\mu_A(y) \neq 0,\ \sigma_A(y) \neq 0$ and $\nu_A(y) \neq 0)$. Hence, either $x \in Supp(A)$ or $y \in Supp(A)$. Therefore, $Supp(A)$ is a prime ideal on L.
(ii) Follows in the same way by using Proposition 3.1 and (i).

□

In the same direction, we get the following theorem which provides a characterization of prime SVN-ideals (resp. prime SVN-filters) in terms of their level sets.

Theorem 4.3. *Let* L *be a lattice and* $A \in SVNS(L)$. *Then it holds that*

(i) A *is a prime SVN-ideal if and only if its level sets are prime ideals,*
(ii) A *is a prime SVN-filter if and only if its level sets are prime filters.*

Proof. (i) From Proposition 3.3, A is an SVN-ideal on L if and only if $A_{\alpha,\beta,\gamma}$ are an ideals on L, for any $\alpha, \beta, \gamma \in]0, 1]$. It remains to show the primality conditions. Suppose that A is a prime SVN-ideal on L, and let $x, y \in L$ such that $x \sqcap y \in A_{\alpha,\beta,\gamma}$. Then from Theorem 4.1 it follows that $(\mu_A(x \sqcap y) = \mu_A(x) \vee \mu_A(y) \geq \alpha$, $\sigma_A(x \sqcap y) = \sigma_A(x) \vee \sigma_A(y) \geq \beta$ and $\nu_A(x \sqcap y) = \nu_A(x) \wedge \nu_A(y) \leq \gamma)$. These imply that either ($\mu_A(x) \geq \alpha$, $\sigma_A(x) \geq \beta$ and $\nu_A(x) \leq \gamma$) or ($\mu_A(y) \geq \alpha$, $\sigma_A(y) \geq \beta$ and $\nu_A(y) \leq \gamma$). Hence, either $x \in A_{\alpha,\beta,\gamma}$ or $y \in A_{\alpha,\beta,\gamma}$. Thus, $A_{\alpha,\beta,\gamma}$ are a prime ideals, for any $\alpha, \beta, \gamma \in]0,1]$. Conversely, suppose that $A_{\alpha,\beta,\gamma}$ are a prime ideals for any $\alpha, \beta, \gamma \in]0,1]$ and A is not a prime SVN-ideal on L. Then it holds that there exist $x, y \in L$ such that $\mu_A(x \sqcap y) > \mu_A(x) \vee \mu_A(y)$, $\sigma_A(x \sqcap y) > \sigma_A(x) \vee \sigma_A(y)$ and $\nu_A(x \sqcap y) < \nu_A(x) \wedge \nu_A(y)$. These imply that ($\mu_A(x \sqcap y) > \mu_A(x)$ and $\mu_A(x \sqcap y) > \mu_A(y)$), ($\sigma_A(x \sqcap y) > \sigma_A(x)$ and $\sigma_A(x \sqcap y) > \sigma_A(y)$) and ($\nu_A(x \sqcap y) < \nu_A(x)$ and $\nu_A(x \sqcap y) < \nu_A(y)$). If we put $\mu_A(x \sqcap y) = \alpha$, $\sigma_A(x \sqcap y) = \beta$ and $\nu_A(x \sqcap y) = \gamma$, it follows that ($\mu_A(x) < \alpha$, $\sigma_A(x) < \beta$ and $\nu_A(x) > \gamma$) and ($\mu_A(y) < \alpha$, $\sigma_A(y) < \beta$ and $\nu_A(y) > \gamma$). Hence, $x \sqcap y \in A_{\alpha,\beta,\gamma}$ and $x, y \notin A_{\alpha,\beta,\gamma}$. That is a contradiction with the fact that $A_{\alpha,\beta,\gamma}$ are a prime ideals on L for any $\alpha, \beta, \gamma \in]0,1]$. Hence, A is a prime SVN-ideal.

(ii) Follows from Proposition 3.1 and (i).

□

5. Maximal SVN-ideals and filters on a lattice

In this section, we introduce and study various properties and characterizations of maximal SVN-ideals (resp. SVN-filters) on a lattice.

Definition 5.1. *Let L be a lattice and let $I \in SVNS(L)$. We say that I is a maximal SVN-ideal if there is no proper SVN-ideal on L containing I.*

The following proposition shows that the support of a maximal SVN-ideal (resp. filter) on a given lattice coincides with that lattice.

Proposition 5.1. *Let L be a lattice and $A \in SVNS(L)$. Then it holds that*

(i) *if A is a maximal SVN-ideal, then its support $Supp(A)$ coincides with L,*
(ii) *if A is a maximal SVN-filter, then its support $Supp(A)$ coincides with L.*

Proof. (i) Suppose that A is a maximal SVN-ideal on a lattice L, then it holds from Proposition 3.2 that $Supp(A)$ is an ideal on L. Now, the facts that any ideal is a SVN-ideal on L and $A \subseteq Supp(A)$ imply that $Supp(A) = L$.

(ii) Follows from Proposition 3.1 and (i).

□

The following proposition shows that every proper SVN-ideal (resp. SVN-filter) is contained in a maximal SVN-ideal (resp. maximal SVN-filter). It is a natural generalization of the crisp case.

Proposition 5.2. *Let L be a lattice and $A \in SVNS(L)$. Then it holds that*

(i) *if A is a proper SVN-ideal, then A is contained in a maximal SVN-ideal,*
(ii) *if A is a proper SVN-filter, then A is contained in a maximal SVN-filter.*

Proof. The proof is straightforward and similar to that used for the crisp case. □

In the following theorem, we show the relationship between maximal SVN-ideal (resp. SVN-filter) and prime SVN-ideal (resp. SVN-filter) on a given distributive lattice.

Theorem 5.1. *Let L be a distributive lattice and $A \in SVNS(L)$. Then it holds that*

(i) *if A is a maximal SVN-ideal, then A is a prime SVN-ideal,*
(ii) *if A is a maximal SVN-filter, then A is a prime SVN-filter.*

Proof. (i) Suppose that A is a maximal SVN-ideal on a lattice L and it isn't prime. Then it holds that there exist $a, b \in L$ such that $\mu_A(a \sqcap b) > \mu_A(a) \vee \mu_A(b)$, $\sigma_A(a \sqcap b) > \sigma_A(a) \vee \sigma_A(b)$ and $\nu_A(a \sqcap b) < \nu_A(a) \wedge \nu_A(b)$. We define an SVNS on L as follows:

$$B = \{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle \mid x \in L\},$$

where

$$\mu_B(x) = \bigvee_{\{u \in L \ \mid \ x \leqslant u \sqcup a\}} \mu_A(u)\,,$$

$$\sigma_B(x) = \bigvee_{\{u \in L \ \mid \ x \leqslant u \sqcup a\}} \sigma_A(u)$$

and

$$\nu_B(x) = \bigwedge_{\{u \in L \ \mid \ x \leqslant u \sqcup a\}} \nu_A(u)\,.$$

It is clear that $A \subset B$. Now, we show that B is a SVN-ideal on L. Let $x, y \in L$, then $\mu_B(x \sqcup y) = \bigvee_{\{u \in L \ \mid \ x \sqcup y \leqslant u \sqcup a\}} \mu_A(u) = (\bigvee_{\{u \in L \ \mid \ x \leqslant u \sqcup a\}} \mu_A(u)) \wedge (\bigvee_{\{u \in L \ \mid \ y \leqslant u \sqcup a\}} \mu_A(u))\,.$
Hence,

$$\mu_B(x \sqcup y) = \mu_B(x) \wedge \mu_B(y)\,.$$

The other conditions (ii)-(vi) of Definition 3.2 can be proved similarly. Thus, A is a SVN-ideal on L. Hence, B is a SVN-ideal on L. The fact that $A \subset B$ implies that A isn't a maximal SVN-ideal, a contradiction. Thus, A is a prime SVN-ideal on L.

(ii) Follows from Proposition 3.1 and (i).

□

Remark 5.1. *The converse of the above implications is not necessarily hold. Indeed, let us consider the distributive lattice* $L = \{0, a, b, c, 1\}$ *represented by the following Hasse diagram*

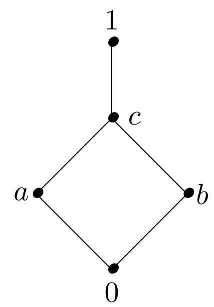

FIGURE 2. Hasse diagram of $(L, \leqslant, \sqcap, \sqcup)$ with $L = \{0, a, b, c, 1\}$.

Let $I = \{< 0, 1, 0.6, 0 >, < a, 1, 0.6, 0 >, < b, 0.5, 0.3, 0.2 >, < c, 0.5, 0.3, 0.2 >, < 1, 0.5, 0.3, 0.2 >\}$ *be a SVNS on* L*. One can easily verifies that* I *is a prime SVN-ideal on* L*, but it isn't maximal.*

6. Principal SVN-ideals and filters on a lattice

In this section, we introduce the notion of principal SVN-ideal (resp. SVN-filter) on a lattice and we characterize these notions in terms of a down-set and an up-set generated by single-valued neutrosophic singletons.

6.1. **Single-valued neutrosophic down-sets and up-sets.** This subsection is devoted to study the notion of single-valued neutrosophic down-set (resp. single-valued neutrosophic up-set) on a lattice analogously to the crisp down-set (resp. up-set), and then we show their interesting properties.

6.1.1. *Definitions.* Analogously to a crisp down-set and up-set on a lattice L, we introduce the notions of a single-valued neutrosophic down-set and a single-valued neutrosophic up-set. Also, for a given SVNS S on L, we define two associated SVNSs denoted by $\Downarrow S$ and $\Uparrow S$ analogous to those for crisp sets.

Definition 6.1. *Let L be a lattice and $S \in SVNS(L)$.*

(i) *S is called a single-valued neutrosophic down-set (SVN-down-set, for short) if $\mu_S(x) \geq \mu_S(y)$, $\sigma_S(x) \geq \sigma_S(y)$ and $\nu_S(x) \leq \nu_S(y)$, for all $x \leqslant y$.*
(ii) *Dually, S is called a single-valued neutrosophic up-set (SVN-up-set, for short) if $\mu_S(x) \leq \mu_S(y)$, $\sigma_S(x) \leq \sigma_S(y)$ and $\nu_S(x) \geq \nu_S(y)$, for all $x \leqslant y$.*

Definition 6.2. *Let $S \in SVNS(L)$ we denote by:*

(i) *$\Downarrow S$ the SVNS associated to S defined as*

$$\mu_{\Downarrow S}(x) = \sup_{y \in \uparrow x} \mu_S(y),$$
$$\sigma_{\Downarrow S}(x) = \sup_{y \in \uparrow x} \sigma_S(y),$$
$$\nu_{\Downarrow S}(x) = \inf_{y \in \uparrow x} \nu_S(y).$$

(ii) *$\Uparrow S$ the SVNS associated to S defined as*

$$\mu_{\Uparrow S}(x) = \sup_{y \in \downarrow x} \mu_S(y),$$
$$\sigma_{\Uparrow S}(x) = \sup_{y \in \downarrow x} \sigma_S(y),$$
$$\nu_{\Uparrow S}(x) = \inf_{y \in \downarrow x} \nu_S(y).$$

Remark 6.1. *For any crisp set S on a given lattice L, it holds that*

(i) $\Downarrow S = \downarrow S$;
(ii) $\Uparrow S = \uparrow S$.

For a given lattice L and $S \in SVNS(L)$, it is clear that

(i) $\mu_{\Downarrow S}$ and $\sigma_{\Downarrow S}$ are an antitone mapping and $\nu_{\Downarrow S}$ is a monotone mapping;
(ii) $\mu_{\Uparrow S}$ and $\sigma_{\Uparrow S}$ are a monotone mapping and $\nu_{\Uparrow S}$ is an antitone mapping.

6.1.2. *Properties of SVN-down sets and SVN-up sets.* In this subsection, we present some interesting properties of SVN-down-sets and up-sets on a lattice. We start with the easier one.

Proposition 6.1. *Let L be a lattice, L^d be its dual-order lattice and $S \in SVNS(L)$. The following statements hold:*

(i) *S is a SVN-down-set on L if and only if S is a SVN-up-set on L^d,*
(ii) *S is a SVN-up-set on L if and only if S is a SVN-down-set on L^d,*
(iii) *$\Downarrow S$ on L coincides with $\Uparrow S$ on L^d,*
(iv) *$\Uparrow S$ on L coincides with $\Downarrow S$ on L^d.*

Proposition 6.2. *Let L be a lattice and $S \in SVNS(L)$. It holds that*

(i) *$\Downarrow S$ is the smallest SVN-down-set containing S;*

(ii) $\Uparrow S$ *is the smallest SVN-up-set containing* S.

Proof. (i) At first we show that $S \subseteq \Downarrow S$. Let $a \in L$ and $b \in \uparrow a$, then it holds that $\mu_S(a) \leq \sup_{b\in\uparrow a}\mu_S(b) = \mu_{\Downarrow S}(a)$. Similarly, $\sigma_S(a) \leq \sup_{b\in\uparrow a}\sigma_S(b) = \sigma_{\Downarrow S}(a)$ and $\nu_S(a) \geq \inf_{b\in\uparrow a}\nu_S(b) = \nu_{\Downarrow S}(a)$. Hence, $S \subseteq \Downarrow S$. Now, we show that $\Downarrow S$ is a SVN-down-set. Let $x, y \in L$ such that $x \leqslant y$. We have $\mu_{\Downarrow S}(x) = \sup_{t\in\uparrow x}\mu_S(t) \geq \sup_{t\in\uparrow y}\mu_S(t)$. Hence, $\mu_{\Downarrow S}(x) \geq \mu_{\Downarrow S}(y)$. In a similar way, we obtain that $\sigma_{\Downarrow S}(x) \geq \sigma_{\Downarrow S}(y)$ and $\nu_{\Downarrow S}(x) \leq \mu_{\Downarrow S}(y)$. Thus, $\Downarrow S$ is a SVN-down-set. Furthermore, let R be a SVN-down-set containing S. This implies that $\mu_S(x) \leq \mu_R(x)$, $\sigma_S(x) \leq \sigma_R(x)$ and $\nu_S(x) \geq \nu_R(x)$, for any $x \in L$. Hence, $\sup_{t\in\uparrow x}\mu_S(t) \leq \sup_{t\in\uparrow x}\mu_R(t)$ and $\sup_{t\in\uparrow x}\sigma_S(t) \leq \sup_{t\in\uparrow x}\sigma_R(t)$ and $\inf_{t\in\uparrow x}\nu_S(t) \geq \inf_{t\in\uparrow x}\nu_R(t)$. Thus, $\Downarrow S \subseteq \Downarrow R$. Finally, we conclude that $\Downarrow S$ is the smallest SVN-down-set containing S.

(ii) Follows from Proposition 6.1 and (i).

□

From Proposition 6.2, we obtain the following corollary. It shows a characterization of SVN-down-sets and SVN-up-sets.

Corollary 6.1. *Let* L *be a lattice and* $S \in SVNS(L)$. *The following equivalences hold:*

(i) S *is a SVN-down-set if and only if* $S = \Downarrow S$,
(ii) S *is a SVN-up-set if and only if* $S = \Uparrow S$.

The following propositions list some properties of SVN-down and SVN-up sets.

Proposition 6.3. *Let* L *be a lattice and* $R, S \in SVNS(L)$. *The following statements hold:*

(i) *If* $S \subseteq R$, *then* $\Downarrow S \subseteq \Downarrow R$,
(ii) $\Downarrow (\Downarrow S) = \Downarrow S$,
(iii) $\Downarrow (S \cup R) = \Downarrow S \cup \Downarrow R$,
(iv) $\Downarrow (S \cap R) \subseteq \Downarrow S \cap \Downarrow R$.

Proof. (i) Since $R \subseteq \Downarrow R$, it holds that $S \subseteq \Downarrow R$. From Proposition 6.2, it trivially holds that $\Downarrow S \subseteq \Downarrow R$.

(ii) Follows from Proposition 6.2 and Corollary 6.1.

(iii) On the one hand, we easily verify from (i) that $\Downarrow S \cup \Downarrow R \subseteq \Downarrow (S \cup R)$. On the other hand, since $\Downarrow S \cup \Downarrow R$ is a SVN-down-set and $S \cup R \subseteq \Downarrow S \cup \Downarrow R$, it follows from Proposition 6.2 that $\Downarrow (S \cup R) \subseteq \Downarrow S \cup \Downarrow R$. Thus, $\Downarrow (S \cup R) = \Downarrow S \cup \Downarrow R$.

(iv) Follows from Proposition 6.2 and (i).

□

Remark 6.2. *In the same direction, a dual version of Proposition 6.3 can also obtained for SVN-up-sets. Its proof follows from Propositions 6.1 and 6.3.*

In the following result, we show that any SVN-ideal (resp. SVN-filter) on a lattice L is a SVN-down-set (resp. SVN-up-set) on L.

Theorem 6.1. *Let* L *be a lattice and* $S \in SVNS(L)$. *The following implications hold:*

(i) *If* S *is a SVN-ideal, then* S *is a SVN-down-set.*
(ii) *If* S *is a SVN-filter, then* S *is a SVN-up-set.*

Proof. (i) Let $x, y \in L$ such that $x \leqslant y$. Since S is a SVN-ideal, it follows that $\mu_S(x) = \mu_S(x \sqcap y) \geq \mu_S(x) \vee \mu_S(y)$. Hence, $\mu_S(x) \geq \mu_S(y)$. Similarly, we obtain that $\sigma_S(x) \geq \sigma_S(y)$ and $\nu_S(x) \leq \nu_S(y)$. Thus, S is a SVN-down-set.

(ii) Follows from Proposition 3.1, (i) and Proposition 6.1.

□

Combining Theorem 6.1 and Corollary 6.1 leads to the following corollary.

Corollary 6.2. *Let L be a lattice and $S \in SVNS(L)$. The following implications hold:*

(i) *If S is a SVN-ideal, then $\Downarrow S = S$.*
(ii) *If S is a SVN-filter, then $\Uparrow S = S$.*

Remark 6.3. *The converse of the implications in the above Theorem 6.1 and Corollary 6.2 does not necessarily hold. Indeed, consider L the lattice given by the Hasse diagram in Figure 1 and $S \in SVNS(L)$ given by $S = \{< 0, 0.7, 0.6, 0.1 >, < a, 0.4, 0.3, 0.2 >, < b, 0.3, 0.2, 0.1 >, < c, 0.2, 0.1, 0.1 >, < 1, 0.1, 0, 0.3 >\}$. We easily verify that*

x	0	a	b	c	1
$\mu_{\Downarrow S}(x)$	0.7	0.4	0.3	0.2	0.1
$\sigma_{\Downarrow S}(x)$	0.6	0.3	0.2	0.1	0
$\nu_{\Downarrow S}(x)$	0.1	0.2	0.1	0.1	0.3

Then $\Downarrow S = \{< 0, 0.7, 0.6, 0.1 >, < a, 0.4, 0.3, 0.2 >, < b, 0.3, 0.2, 0.1 >, < c, 0.2, 0.1, 0.1 >, < 1, 0.1, 0, 0.3 >\}$. Hence, $\Downarrow S = S$, i.e., S is an SVN-down-set. But, $\mu_S(1) = \mu_S(a \sqcup b) \ngeq \min\{0.4, 0.3\}$, which implies that S is not a SVN-ideal on L .

6.2. **Principal SVN-ideals and filters on a lattice.** In this section, we introduce the notion of principal SVN-ideal (resp. SVN-filter) on a lattice. First, we generalize the notion of crisp singleton to the single-valued neutrosophic setting.

Definition 6.3. *Let L be a lattice. For any $x \in L$, a single-valued neutrosophic singleton (SVN- singleton, for short) $\tilde{x}$ is a SVNS on L given by $\tilde{x} = \{\langle t, \mu_{\tilde{x}}(t), \sigma_{\tilde{x}}(t), \nu_{\tilde{x}}(t)\rangle \mid t \in L\}$, where*

$$\mu_{\tilde{x}}(t) = \begin{cases} 1 \ , \ if \ x = t \\ f(t) \ , \ otherwise \end{cases} ,$$

$$\sigma_{\tilde{x}}(t) = \begin{cases} 1 \ , \ if \ x = t \\ g(t) \ , \ otherwise \end{cases}$$

and

$$\nu_{\tilde{x}}(t) = \begin{cases} 0 \ , \ if \ x = t \\ h(t) \ , \ otherwise \end{cases} ,$$

such that f and g are mappings from L into $[0, 1[$, and h is a mapping from L into $]0, 1]$.

Definition 6.4. *Let L be a lattice, then*

(i) *the principal SVN-ideal generated by a SVN-singleton $\tilde{x}$ is the smallest SVN-ideal contains $\tilde{x}$,*
(ii) *the principal SVN-filter generated by a SVN-singleton $\tilde{x}$ is the smallest SVN-filter contains $\tilde{x}$.*

The following theorem shows that the SVN-down-set (resp. the SVN-up-set) generated by a SVN-singleton on a lattice L is a SVN-ideal (resp. is a SVN-filter) on L.

Theorem 6.2. *Let L be a lattice and x be an element on L. Then it holds that*

(i) *$\Downarrow \tilde{x}$ is a SVN-ideal on L,*
(ii) *$\Uparrow \tilde{x}$ is a SVN-filter on L.*

Proof. (i) Let $x, y \in L$, we will show that $\mu_{\Downarrow\tilde{x}}(x \sqcup y) = \mu_{\Downarrow\tilde{x}}(x) \wedge \mu_{\Downarrow\tilde{x}}(y)$, $\sigma_{\Downarrow\tilde{x}}(x \sqcup y) = \sigma_{\Downarrow\tilde{x}}(x) \wedge \sigma_{\Downarrow\tilde{x}}(y)$ and $\nu_{\Downarrow\tilde{x}}(x \sqcup y) = \nu_{\Downarrow\tilde{x}}(x) \vee \nu_{\Downarrow\tilde{x}}(y)$. Let $a, b \in L$. On the one hand, by Proposition 6.2, $\Downarrow \tilde{x}$ is a SVN-down-set, which implies that $\mu_{\Downarrow\tilde{x}}(a) \geq \mu_{\Downarrow\tilde{x}}(a \sqcup b)$ and $\mu_{\Downarrow\tilde{x}}(b) \geq \mu_{\Downarrow\tilde{x}}(a \sqcup b)$. Hence, $\mu_{\Downarrow\tilde{x}}(a) \wedge \mu_{\Downarrow\tilde{x}}(a) \geq \mu_{\Downarrow\tilde{x}}(a \sqcup b)$. On the other hand, since $\mu_{\tilde{x}}$ is a monotone mapping, it holds that $\mu_{\tilde{x}}(a) \leq \mu_{\tilde{x}}(a \sqcup b)$ and $\mu_{\tilde{x}}(b) \leq \mu_{\tilde{x}}(a \sqcup b)$. This implies that $\sup\limits_{a \leqslant t} \mu_{\tilde{x}}(t) \leq \sup\limits_{a \sqcup b \leqslant t} \mu_{\tilde{x}}(t)$ and $\sup\limits_{b \leqslant t} \mu_{\tilde{x}}(t) \leq \sup\limits_{a \sqcup b \leqslant t} \mu_{\tilde{x}}(t)$. Hence, $\sup\limits_{a \leqslant t} \mu_{\tilde{x}}(t) \wedge \sup\limits_{b \leqslant t} \mu_{\tilde{x}}(t) \leq \sup\limits_{a \sqcup b \leqslant t} \mu_{\tilde{x}}(t)$. Thus, $\mu_{\Downarrow\tilde{x}}(a) \wedge \mu_{\Downarrow\tilde{x}}(b) \leq \mu_{\Downarrow\tilde{x}}(a \sqcup b)$. Therefore, $\mu_{\Downarrow\tilde{x}}(a \sqcup b) = \mu_{\Downarrow\tilde{x}}(a) \wedge \mu_{\Downarrow\tilde{x}}(b)$, for all $a, b \in L$. In analogous way, we easily prove that $\sigma_{\Downarrow\tilde{x}}(a \sqcup b) = \sigma_{\Downarrow\tilde{x}}(a) \wedge \sigma_{\Downarrow\tilde{x}}(b)$ and $\nu_{\Downarrow\tilde{x}}(a \sqcup b) = \nu_{\Downarrow\tilde{x}}(a) \vee \nu_{\Downarrow\tilde{x}}(b)$. Finally, we conclude that $\Downarrow \tilde{x}$ is a SVN-ideal.

(ii) Follows dually by using Proposition 6.1, (i) and Proposition 3.1.

□

In the following result, we show a characterization of a principal SVN-ideal (resp. SVN-filter) in terms of a down-set (resp. up-set) generated by an SVN-singleton.

Theorem 6.3. *Let L be a lattice and I (resp. F) be a SVN-ideal (resp. SVN-filter) on L. Then it holds that*

(i) *I is a principal SVN-ideal on L if and only if there exists $x \in L$ such that $I = \Downarrow \tilde{x}$,*
(ii) *F is a principal SVN-filter on L if and only if there exists $x \in L$ such that $F = \Uparrow \tilde{x}$.*

Proof. We only prove (i), as (ii) can be proved analogously by using Proposition 6.1 and Proposition 3.1. Suppose that I is a principal SVN-ideal on L. Then there exists a SVN-singleton $\tilde{x}$ such that I is the smallest SVN-ideal contains $\tilde{x}$. Since $\tilde{x} \subseteq I$, it follows from Proposition 6.3 that $\Downarrow \tilde{x} \subseteq \Downarrow I = I$. On the other hand, Theorem 6.2 guarantees that $\Downarrow \tilde{x}$ is an ideal. Then the fact that I is the smallest ideal contains $\tilde{x}$ implies that $I \subseteq \Downarrow \tilde{x}$. Thus, $I = \Downarrow \tilde{x}$. Conversely, $I = \Downarrow \tilde{x}$ is a SVN-ideal contains $\tilde{x}$. Now, suppose that J is an other SVN-ideal contains $\tilde{x}$. From Proposition 6.3, it holds that $\Downarrow \tilde{x} \subseteq \Downarrow J = J$. Hence, $I = \Downarrow \tilde{x}$ is the smallest SVN-ideal contains $\tilde{x}$. Thus, I is a principal SVN-ideal. □

7. Conclusion

In this work, we have introduced the notion of prime, maximal and principal single-valued neutrosophic ideals (resp. filters) on a lattice. Moreover, we have investigated their various properties and characterizations. Future work will be focused on the characterizations of these types on a single-valued neutrosophic ordered lattice.

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