SOMEWHAT NEUTROSOFPIC $\delta$-IRRESOLUTE CONTINUOUS MAPPINGS IN NEUTROSOPHIC TOPOLOGICAL SPACES

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ABSTRACT. One of the strongest form in a neutrosophic open sets is a neutrosophic $\delta$-open sets. Based on this open sets, the continuous mapping, open mapping and closed mapping functions are already defined in neutrosophic topological spaces. Also, the extension of these mappings called somewhat neutrosophic $\delta$-continuous (open) mapping functions are defined in previous paper. The concept of somewhat neutrosophic $\delta$-irresolute continuous (open) mapping which is stronger than a somewhat neutrosophic $\delta$-continuous (open) mapping in a neutrosophic topological spaces have been introduced and studied in this paper. Alongside, some interesting properties of those mappings are given in neutrosophic topological spaces.

Keywords: Neutrosophic $\delta$-open set, Somewhat neutrosophic $\delta$-irresolute continuous mapping, Somewhat neutrosophic irresolute $\delta$-open mapping.

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1. INTRODUCTION

In mathematics, concept of fuzzy set between the real standard intervals was first introduced by Zadeh [28] in discipline of logic and set theory. The general topology has been framework with fuzzy set was undertaken by Chang [4] as fuzzy topological space. In 1983, Atanassov [3] initiated intuitionistic fuzzy set which is a combination of membership and non-membership values. Coker [5] created intuitionistic fuzzy set in a topology entitled as intuitionistic fuzzy topological space. In classical topology, the class of somewhat continuous functions was introduced and studied by Karl. R. Gentry and Hughes B. Hoyle [12]. Later, the concept of somewhat in classical topology has been extended to fuzzy topological spaces. Besides giving characterizations of these functions, several interesting properties of these functions are studied. M. N. Mukherjee and S. P. Shina in [15] was introduced and studied the concept of fuzzy irresolute continuous mappings on a fuzzy topological space. The concepts of somewhat fuzzy irresolute continuous mappings [10],

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somewhat fuzzy $\gamma$- irresolute continuous mappings [11] and somewhat fuzzy $\alpha$- irresolute continuous mappings [12] on a fuzzy topological space are successively introduced and studied by Y. B. and others.

The new concepts of neutrosophy and neutrosophic set was introduced Smarandache [21, 22] at the beginning of 20th century which has truth value, neutral value and false value. It has laid the foundation for a whole family of new mathematical theories in both crisp set and fuzzy set. Salama and Alblowi [17, 18] in 2012, originated neutrosophic topology. This gave the way for investigation in terms of neutrosophic topology and its application in decision making problems [13, 20, 23]. In [2], the properties of neutrosophic open sets, neutrosophic closed sets, neutrosophic interior operator and neutrosophic closure operator such as semi open, pre open, $\alpha$ open and their mappings gave the way for applying neutrosophic topology. M.L. Thivagar et al. [19] introduced the notion of $N_n$-open (closed) sets and $N$-neutrosophic topological spaces and R.K Al-Hamido et al. [1] introduced Neutrosophic crisp topology via $N$-Topology. Neutrosophic closed sets as well as Neutrosophic continuous mappings were developed in [14]. R. Dhavaseelan et al. [6, 7, 8, 9] introduced generalized neutrosophic closed sets, neutrosophic generalized contra continuous, neutrosophic generalized $\delta$-contra continuous functions and studied their properties. Recently, Rajesh et al. [16] introduced Neutrosophic pre irresolute functions via $\alpha$ and $\beta$.

S. Saha [24] defined $\delta$-open sets in fuzzy topological spaces. Recently, Vadivel et al. introduced the neutrosophic regular open sets, neutrosophic $\delta$ open sets and neutrosophic $\delta$ continuous functions in [25]. In his paper [26], neutrosophic $\delta$-open mappings and neutrosophic $\delta$-closed mappings are introduced. Also, the concept of somewhat neutrosophic $\delta$-continuous functions and somewhat neutrosophic $\delta$-open sets are introduced and studied in [27].

The main work in this paper is introduced the concepts of somewhat neutrosophic $\delta$- irresolute continuous mappings on a neutrosophic topological spaces in third section. Also, in section four, we discussed about somewhat neutrosophic irresolute $\delta$-open mappings on a neutrosophic topological spaces and further some properties of the mappings are also discussed.

2. Preliminaries

Definition 2.1. [17] Let $Y$ be a non-empty set. A neutrosophic set (briefly, $N_sL$) is an object having the form $L = \{(y, \mu_L(y), \sigma_L(y), \nu_L(y)) : y \in Y\}$ where $\mu_L \rightarrow [0, 1]$ denote the degree of membership function, $\sigma_L \rightarrow [0, 1]$ denote the degree of indeterminacy function and $\nu_L \rightarrow [0, 1]$ denote the degree of non-membership function respectively of each element $y \in Y$ to the set $L$ and $0 \leq \mu_L(y) + \sigma_L(y) + \nu_L(y) \leq 3$ for each $y \in Y$.

Remark 2.1. [17] A $N_sL = \{(y, \mu_L(y), \sigma_L(y), \nu_L(y)) : y \in Y\}$ can be identified to an ordered triple $\langle y, \mu_L(y), \sigma_L(y), \nu_L(y) \rangle$ in $[0, 1]$ on $Y$.

Definition 2.2. [17] Let $Y$ be a non-empty set and the $N_sL$'s $L$ and $M$ in the form $L = \{(y, \mu_L(y), \sigma_L(y), \nu_L(y)) : y \in Y\}$, $M = \{(y, \mu_M(y), \sigma_M(y), \nu_M(y)) : y \in Y\}$, then

(i) $0_N = \langle y, 0, 0, 1 \rangle$ and $1_N = \langle y, 1, 1, 0 \rangle$,
(ii) $L \subseteq M$ iff $\mu_L(y) \leq \mu_M(y), \sigma_L(y) \leq \sigma_M(y) \& \nu_L(y) \geq \nu_M(y) : y \in Y$,
(iii) $L = M$ iff $L \subseteq M$ and $M \subseteq L$,
(iv) $1_N - L = \{(y, \nu_L(y), 1 - \sigma_L(y), \mu_L(y)) : y \in Y\} = L^c$,
(v) $L \cup M = \{(y, \max(\mu_L(y), \mu_M(y)), \max(\sigma_L(y), \sigma_M(y)), \min(\nu_L(y), \nu_M(y))) : y \in Y\}$,
(vi) $L \cap M = \{(y, \min(\mu_L(y), \mu_M(y)), \min(\sigma_L(y), \sigma_M(y)), \max(\nu_L(y), \nu_M(y))) : y \in Y\}$. 

Definition 2.3. [17] A neutrosophic topology (briefly, $N,s,t$) on a non-empty set $Y$ is a family $\Psi_N$ of neutrosophic subsets of $Y$ satisfying

(i) $0_N, 1_N \in \Psi_N$.
(ii) $L_1 \cap L_2 \in \Psi_N$ for any $L_1, L_2 \in \Psi_N$.
(iii) $\bigcup L_x \in \Psi_N$, $\forall x \in X \subseteq \Psi_N$.

Then $(Y, \Psi_N)$ is called a neutrosophic topological space (briefly, $N,s,t$) in $Y$. The $\Psi_N$ elements are called neutrosophic open sets (briefly, $N,s,os$) in $Y$. A $N,s,c$ is called a neutrosophic closed sets (briefly, $N,s,cs$) in $Y$.

Definition 2.6. [25] A set $K$ is said to be a neutrosophic $\delta$-open set (briefly, $N,s,os$) if $L = N,s,os$ in $Y$. The complement of a $N,s,os$ is called a neutrosophic regular closed set (briefly, $N,s,rcs$) in $Y$. If $L = N,s,os$ in $Y$, then the neutrosophic closure of $L$ (briefly, $N,s,cl(L)$) and the neutrosophic interior of $L$ (briefly, $N,s,int(L)$) are defined as

$$N,s,int(L) = \bigcup \{I : I \subseteq L \text{ and } I \text{ is a } N,s,os \text{ in } Y\}$$
$$N,s,cl(L) = \bigcap \{J : L \subseteq J \text{ and } J \text{ is a } N,s,cs \text{ in } Y\}.$$
3. Somewhat Neutrosophic $\delta$-Irresolute Continuous Mappings

**Definition 3.1.** A mapping $h : (Y, \tau_Y) \to (Z, \sigma_Z)$ is called neutrosophic $\delta$-irresolute continuous (briefly, $N_s\delta Cts$) if $h^{-1}(\xi) \neq 0_N$ on $(Y, \tau_Y)$ for any $N_s\delta$os $\xi$ on $(Z, \sigma_Z)$.

**Definition 3.2.** A mapping $h : (Y, \tau_Y) \to (Z, \sigma_Z)$ is called somewhat neutrosophic $\delta$-irresolute continuous (briefly, $swN_s\delta Cts$) if there exists a $N_s\delta$os $\xi \neq 0_Y$ on $(Y, \tau_Y)$ such that $\xi \leq h^{-1}(\xi) \neq 0_N$ for any $N_s\delta$os $\zeta$ on $(Z, \sigma_Z)$.

**Remark 3.1.** The implications contained in the following diagram are true and the reverse implications need not be true.

```
\begin{center}
\begin{tikzpicture}[node distance=2cm, auto]
  \node (A) {$N_s\delta Cts$};
  \node (B) [below of=A] {$swN_s\delta Cts$};
  \node (C) [below of=B] {$N_s\delta Cts$};
  \node (D) [below of=C] {$swN_s\delta Cts$};
  \draw[->] (A) -- (B);
  \draw[->] (B) -- (C);
  \draw[->] (C) -- (D);
\end{tikzpicture}
\end{center}
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**Figure 1.** $swN_s\delta Cts$ function in $Nts$.

**Example 3.1.** Let $X = \{l, m, n\} \& Y = \{x, y, z\}$ and define $N_s$'s $X_1, X_2$ and $X_3$ in $X$ and $Y_1, Y_2$ and $Y_3$ in $Y$ by

- $X_1 = (X, (\mu_x \mu_y \mu_z, (\sigma_x, \sigma_y, \sigma_z), (\nu_x, \nu_y, \nu_z)))$,
- $X_2 = (X, (\mu_x \mu_y \mu_z, (\sigma_x, \sigma_y, \sigma_z), (\nu_x, \nu_y, \nu_z)))$,
- $X_3 = (X, (\mu_x \mu_y \mu_z, (\sigma_x, \sigma_y, \sigma_z), (\nu_x, \nu_y, \nu_z)))$,
- $Y_1 = (Y, (\mu_x \mu_y \mu_z, (\sigma_x, \sigma_y, \sigma_z), (\nu_x, \nu_y, \nu_z)))$,
- $Y_2 = (Y, (\mu_x \mu_y \mu_z, (\sigma_x, \sigma_y, \sigma_z), (\nu_x, \nu_y, \nu_z)))$,
- $Y_3 = (Y, (\mu_x \mu_y \mu_z, (\sigma_x, \sigma_y, \sigma_z), (\nu_x, \nu_y, \nu_z)))$.

Then we have $\tau_N = \{0_N, X_1, X_2, 1_N\}$ and $\sigma_N = \{0_N, Y_1, Y_2, Y_3, 1_N\}$. Let $h : (X, \tau_N) \to (Y, \sigma_N)$ be defined by

- $h(l) = y, h(m) = y, h(n) = y$.

Then we have $h^{-1}(Y_1) = X_1$, $X_1 \leq h^{-1}(Y_2) = X_2$ and $X_1 \leq h^{-1}(Y_3) = X_3$. Since $X_1$ is a $N_s\delta$os on $(X, \tau_N)$, $h$ is $swN_s\delta Cts$. But $h^{-1}(Y_3) = X_3$ is not $N_s\delta$os on $(X, \tau_N)$. Thus $h$ is not a $N_s\delta Cts$ mapping.
**Example 3.2.** Let $X = \{l, m, n\} \& Y = \{x, y, z\}$ and define $N_s$'s $X_1$ in $X$ and $Y_1$ in $Y$

\[
X_1 = (X, (\mu_l \mu_m \mu_n, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.2}, \frac{\mu_n}{0.2})), (\sigma_l \sigma_m \sigma_n, (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5})), (\nu_l \nu_m \nu_n, (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.8}, \frac{\nu_n}{0.8})),
Y_1 = (Y, (\mu_l \mu_m \mu_n, (\frac{\mu_x}{0.4}, \frac{\mu_y}{0.4}, \frac{\mu_z}{0.4})), (\sigma_x \sigma_y \sigma_z, (\frac{\sigma_x}{0.5}, \frac{\sigma_y}{0.5}, \frac{\sigma_z}{0.5})), (\nu_x \nu_y \nu_z, (\frac{\nu_x}{0.6}, \frac{\nu_y}{1.0}, \frac{\nu_z}{0.6})).
\]

Then we have $\tau_N = \{0, X_1, X_1^c, 1_N\}$ and $\sigma_N = \{0, Y_1, Y_1^c, 1_N\}$. Let $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ be defined by

\[
h(l) = y, h(m) = y, h(n) = y.
\]

Then we have $h^{-1}(Y_1) = 0_N$ and $h^{-1}(Y_1^c) = 1_N$, $h$ is $N_s\delta Cts$. But for a $N_s\delta os$ $Y_1 = 0_N$ on $Y$, $h^{-1}(Y_1) = 0_N$. Thus $h$ is not a $swN_s\delta irr Cts$ mapping.

**Example 3.3.** In Example 3.1, for a $N_s\delta os$ on $Y$, $h^{-1}(Y_1) = X_1, X_1 \leq h^{-1}(Y_2) = X_2$ and $X_1 \leq h^{-1}(Y_3) = X_3$. Since $X_3$ is a $N_s\delta os$ on $X$, $h$ is $swN_s\delta Cts$. But $h^{-1}(Y_3) = X_3$ is not a $N_s\delta os$ on $X$. Thus $h$ is not a $N_s\delta Cts$ mapping.

**Theorem 3.1.** Let $h : (Y, \tau_N) \rightarrow (Z, \sigma_N)$ be a mapping. Then the statements

(i) $h$ is $swN_s\delta irr Cts$.

(ii) If $\xi$ is a $N_s\delta os$ of $Z \ni h^{-1}(\xi) \neq 1_Y$ then there exists a $N_s\delta os$ $\xi \neq 1_Y$ of $Y \ni h^{-1}(\xi) \leq \xi$.

(iii) If $\xi$ is a $N_s\delta D$ set on $Y$, then $h(\xi)$ is a $N_s\delta D$ set on $Z$.

are equivalent.

**Proof.** (i) $\Rightarrow$ (ii): Let $\xi$ be a $N_s\delta os$ on $Z \ni h^{-1}(\xi) \neq 1_Y$. Then $\xi^c$ is a $N_s\delta os$ on $Z$ and $h^{-1}(\xi^c) = h^{-1}(\xi)^c \neq 0_Y$. Since $h$ is $swN_s\delta irr Cts$, $\exists$ $N_s\delta os$ $\lambda \neq 0_Y$ on $Y \ni \lambda \leq h^{-1}(\xi)^c$. Let $\xi = X^c$. Then $\xi \neq 1_Y$ is $N_s\delta C$ $\ni h^{-1}(\xi) = 1 - h^{-1}(\xi)^c \leq 1 - \xi^c = \xi$.

(ii) $\Rightarrow$ (iii): Let $\xi$ be a $N_s\delta D$ set on $Y$ and suppose $h(\xi)$ is not $N_s\delta D$ set on $Z$. Then $\exists$ $N_s\delta os$ $\xi$ on $Z \ni h(\xi) < \xi < 1$. Since $\xi < 1$ and $h^{-1}(\xi) \neq 1_Y$, there exists a $N_s\delta os$ $\eta \neq 1_Y$ $\ni \exists$

\[
\xi \leq h^{-1}(h(\xi)) < h^{-1}(\xi) \leq \eta.
\]

$\not\exists$ $\xi$ is a $N_s\delta D$ set on $Y$. Thus $h(\xi)$ is a $N_s\delta D$ set on $Z$.

(iii) $\Rightarrow$ (i): Let $\xi \neq 0_Z$ be a $N_s\delta os$ on $Z$ and $h^{-1}(\xi) \neq 0_Y$. Suppose there exists no $N_s\delta os$ $\xi \neq 0_Y$ on $Y \ni \xi \leq h^{-1}(\xi)$. Then $h^{-1}(\xi^c)$ is a $N_s\delta s$ on $Y \ni \exists$ is no $N_s\delta os$ $\eta$ on $Y$ with $h^{-1}(\xi^c) < \eta < 1$. In fact, if $\exists$ $N_s\delta os$ $\delta^c \ni \delta^c < h^{-1}(\xi)$, then it is a contradiction. Thus $h^{-1}(\xi)^c$ is a $N_s\delta D$ set on $Y$. So $h(\xi^c) \neq 0_Y$ on $Y \ni \lambda \leq h^{-1}(\xi)^c$. Let $h((h^{-1})(\xi)^c) = h((h^{-1}(\xi)^c)^c) \neq 0_Y$. This contradicts to the fact that $h((h^{-1}(\xi)^c)^c)$ is $N_s\delta D$ on $Z$. Hence $\exists$ $N_s\delta os$ $\xi \neq 0_Y$ on $Y \ni \lambda \leq h^{-1}(\xi)$. Consequently, $h$ is $swN_s\delta irr Cts$.

**Theorem 3.2.** Let $Y_1$ be product related to $Y_2$ and let $Z_1$ be product related to $Z_2$. Then the product $h_1 \times h_2 : Y_1 \times Y_2 \rightarrow Z_1 \times Z_2$ of $swN_s\delta irr Cts$ mappings $h_1 : Y_1 \rightarrow Z_1$ and $h_2 : Y_2 \rightarrow Z_2$ is also $swN_s\delta irr Cts$.

**Proof.** Let $\lambda = \bigvee_{i,j} (\xi_i \wedge \xi_j)$ be a $N_s\delta os$ on $Z_1 \times Z_2$ where $\xi_i \neq 0_{Z_1}$ and $\xi_j \neq 0_{Z_2}$ are $N_s\delta os$'s on $Z_1$ and $Z_2$ respectively. Then

\[
(h_1 \times h_2)^{-1}(\lambda) = \bigvee_{i,j} (h_1^{-1}(\xi_i) \times h_2^{-1}(\xi_j)).
\]

since $h_1$ is $swN_s\delta irr Cts$, $\exists$ $N_s\delta os$ $\delta_i \neq 0_{Y_1} \ni \delta_i \leq h_1^{-1}(\xi_i) \neq 0_{Y_1}$. And, since $h_2$ is $swN_s\delta irr Cts$, $\exists$ $N_s\delta os$ $\eta_j \neq 0_{Y_2}$ such that $\eta_j \leq h_2^{-1}(\xi_j) \neq 0_{Y_2}$. Now $\delta_i \times \eta_j \leq h_1^{-1}(\xi_i) \times
Remark 4.1. The implications contained in the following diagram are true and the reverse is bijective.

Proof. (i) If $\delta \times \eta \neq 0_{Y_1} \times 0_{Y_2}$ is a $N_\delta \delta$ on $Y_1 \times Y_2$. Thus
\[
\bigvee_{i,j} (\delta_i \times \eta_j) \leq \bigvee_{i,j} (h_1^{-1}(\xi_i) \times h_2^{-1}(\xi_i))
\]
\[
= (h_1 \times h_2)^{-1} \left( \bigvee_{i,j} (\xi_i \times \xi_j) \right)
\]
\[
= (h_1 \times h_2)^{-1} (\lambda) \neq 0_{Y_1 \times Y_2}
\]

So, $h_1 \times h_2$ is $swN_\delta \delta$.

Theorem 3.3. Let $h : (Y, \tau_N) \rightarrow (Z, \sigma_N)$ be a mapping. If the graph $g_\delta : Y \rightarrow Y \times Z$ of $h$ is a $swN_\delta \delta$ mapping, then $h$ is also $swN_\delta \delta$.

Proof. Let $\varsigma$ be a $N_\delta \delta$ on $Z$. Then $h^{-1}(\varsigma) = 1 \land h^{-1}(\varsigma) = g_\delta^{-1}(1 \times \varsigma)$. Since $g_\delta$ is $swN_\delta \delta$ and $(1 \times \varsigma)$ is a $N_\delta \delta$ on $Y \times Z$, $\exists \ N_\delta \delta \xi \neq 0_Y$ on $Y \ni
\[
\xi \leq g_\delta^{-1}(1 \times \varsigma) = h^{-1}(\varsigma) \neq 0_Y
\]

Thus, $h$ is $swN_\delta \delta$.

4. SOMEWHAT NEUTROSOPHIC IRRESOLUTE $\delta$-OPEN MAPPINGS

Definition 4.1. A mapping $h : (Y, \tau_N) \rightarrow (Z, \sigma_N)$ is called neutrosophic irresolute $\delta$-open (briefly, $N_\delta \delta \delta O$) if $h(\xi)$ is a $N_\delta \delta$ on $Z$ for any $N_\delta \delta$ on $Y$.

Definition 4.2. A fuzzy mapping $h : (Y, \tau_N) \rightarrow (Z, \sigma_N)$ is called somewhat neutrosophic irresolute $\delta$-open (briefly, $swN_\delta \delta \delta O$) if there exists a $N_\delta \delta$ on $Z$ such that $\varsigma \leq h(\xi) \neq 0_Z$ for any $N_\delta \delta \xi \neq 0_Y$.

Theorem 4.1. Let $h : (Y, \tau_N) \rightarrow (Z, \sigma_N)$ be a bijection. Then the statements

(i) $h$ is $swN_\delta \delta \delta O$.
(ii) If $\xi$ is a $N_\delta \delta \delta$ on $Y$ such that $h(\xi) \neq 1_Z$, then there exists a $N_\delta \delta \delta \varsigma \neq 1_Z$ on $Z$ such that $h(\xi) < \varsigma$

are equivalent.

Proof. (i) $\Rightarrow$ (ii): Let $\xi$ be a $N_\delta \delta \delta$ on $Y$ such that $h(\xi) \neq 1_Z$. Since $h$ is bijective and $\xi^c$ is a $N_\delta \delta \delta$ on $Y$, $h(\xi^c) = (h(\xi))^c \neq 0_Z$. And, since $h$ is $swN_\delta \delta \delta O$, $\exists \ N_\delta \delta \delta \delta \neq 0_Z$ on $Z \ni \delta < h(\xi^c) = (h(\xi))^c$. Consequently, $h(\xi) < \delta^c = \varsigma \neq 1_Z$ and $\varsigma$ is a $N_\delta \delta \delta$ on $Z$.

(ii) $\Rightarrow$ (i): Let $\xi$ be a $N_\delta \delta \delta$ on $Y$ such that $h(\xi) \neq 0_Z$. Then $\xi^c$ is a $N_\delta \delta \delta$ on $Y$ and $h(\xi^c) \neq 1_Z$. Thus there exists a $N_\delta \delta \delta \varsigma \neq 1_Z$ on $Z$ such that $h(\varsigma^c) < \varsigma$. Since $h$ is bijective, $h(\varsigma^c) = (h(\varsigma))^c < \varsigma$. So $\varsigma^c < h(\varsigma)$ and $\varsigma^c \neq 0_Y$ is a $N_\delta \delta \delta$ on $Z$. Hence, $h$ is $swN_\delta \delta \delta O$.

Remark 4.1. The implications contained in the following diagram are true and the reverse implications need not be true.
Then we have

\[ \tau \] be defined by

\[ Y = \{ x, y, z \} \]

and define \( N_s \)'s \( X_1 \) and \( X_2 \) in \( X \) and \( Y_1 \) and \( Y_2 \) in \( Y \) by

\[
X_1 = (X, (\mu_i, \mu_m, \mu_n), (\sigma_i, \sigma_m, \sigma_n), (\nu_i, \nu_m, \nu_n), (0.3, 0.3, 0.3, 0.5, 0.5, 0.5, 0.7, 0.7, 0.7))
\]

\[
X_2 = (X, (\mu_i, \mu_m, \mu_n), (\sigma_i, \sigma_m, \sigma_n), (\nu_i, \nu_m, \nu_n), (0.7, 0.7, 0.7, 0.5, 0.5, 0.5, 0.3, 0.3, 0.3))
\]

\[
Y_1 = (Y, (\mu_i, \mu_m, \mu_n), (\sigma_i, \sigma_m, \sigma_n), (\nu_i, \nu_m, \nu_n), (0.6, 0.3, 0.0, 0.5, 0.5, 0.5, 1.0, 0.7, 1.0))
\]

\[
Y_2 = (Y, (\mu_i, \mu_m, \mu_n), (\sigma_i, \sigma_m, \sigma_n), (\nu_i, \nu_m, \nu_n), (0.3, 0.7, 0.0, 0.5, 0.5, 0.5, 1.0, 0.3, 1.0))
\]

Then we have \( \tau_N = \{ 0_N, X_1, X_2, 1_N \} \) and \( \sigma_N = \{ 0_N, Y_1, Y_2, 1_N \} \). Let \( h : (X, \tau_N) \to (Y, \sigma_N) \) be defined by

\[ h(l) = y, h(m) = y, h(n) = y. \]

Then we have \( Y_1 \leq h(X_1) = Y_1 \) and \( Y_2 \leq h(X_2) = Y_2 \). Since \( Y_1 \) is a \( N_s \) on \( Y, \sigma_N \), \( h \) is \( swN_s \delta O \). But for a \( N_s \) \( X_2 \) on \( X \), \( h(X_2) = Y_2 \) is not \( N_s \) \( \delta O \) on \( Y, \sigma_N \). Thus \( h \) is not a \( swN_s \delta O \) mapping.

**Example 4.2.** Let \( X = \{ l, m, n \} \) and \( Y = \{ x, y, z \} \) and define \( N_s \)'s \( X_1 \) and \( X_2 \) in \( X \) and \( Y_1 \) in \( Y \) by

\[
X_1 = (X, (\mu_i, \mu_m, \mu_n), (\sigma_i, \sigma_m, \sigma_n), (\nu_i, \nu_m, \nu_n), (0.3, 0.0, 0.3, 0.5, 0.5, 0.5, 0.7, 1.0, 0.7))
\]

\[
X_2 = (X, (\mu_i, \mu_m, \mu_n), (\sigma_i, \sigma_m, \sigma_n), (\nu_i, \nu_m, \nu_n), (0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.3, 0.6, 0.6))
\]

\[
Y_1 = (Y, (\mu_i, \mu_m, \mu_n), (\sigma_i, \sigma_m, \sigma_n), (\nu_i, \nu_m, \nu_n), (0.5, 0.5, 0.5, 0.5, 0.5, 0.5))
\]

Then we have \( \tau_N = \{ 0_N, X_1, X_2, 1_N \} \) and \( \sigma_N = \{ 0_N, Y_1, 1_N \} \). Let \( h : (X, \tau_N) \to (Y, \sigma_N) \) be defined by

\[ h(l) = y, h(m) = y, h(n) = y. \]

Since \( h(X_1) = 0_N, h(X_2) = 1_N \) and \( h(X_2) = Y_1 \) are \( N_s \) \( \delta os \)'s on \( (Y, \sigma_N) \), \( h \) is \( N_s \) \( \delta o \). But for a \( N_s \) \( X_1 \), \( h(X_1) = 0_N \). Thus \( h \) is not a \( swN_s \delta O \) mapping.

**Example 4.3.** In Example 4.1, we have \( Y_1 \leq h(X_1) = Y_1 \) and \( Y_2 \leq h(X_2) = Y_2 \). Since \( Y_1 \) is a \( N_s \) \( \delta os \) on \( X \), \( h \) is \( swN_s \delta O \) mapping. But \( h(X_2) = Y_2 \) is not \( N_s \) \( \delta os \) on \( (Y, \sigma_N) \). Thus \( h \) is not a \( N_s \) \( \delta O \) mapping.

**Theorem 4.2.** Let \( h : (Y, \tau_N) \to (Z, \sigma_N) \) be a surjection. Then the statements
(i) $h$ is $swN_s$irr$\delta O$.

(ii) If $\varsigma$ is a $N_s\delta D$ set on $Z$, then $h^{-1}(\varsigma)$ is a $N_s\delta D$ set on $Y$ are equivalent.

Proof. (i) $\Rightarrow$ (ii): Let $\varsigma$ be a $N_s\delta D$ set on $Z$. Suppose $h^{-1}(\varsigma)$ is not $N_s\delta D$ on $Y$. Then $\exists N_s\delta c\varsigma$ on $Y$ such that $h^{-1}(\varsigma) < \varsigma < 1$. Since $h$ is $swN_s$irr$\delta O$ and $\xi^c$ is a $N_s\delta o\varsigma$ on $Y$, $\exists N_s\delta o\varsigma \delta \neq 0 Z$ on $Z \ni \delta \leq h(N_sint\xi^c) \leq h(\xi^c)$. Since $h$ is surjective, $\delta \leq h(\xi^c) < h(h^{-1}(\varsigma)) = \varsigma^c$. Thus $\exists N_s\delta c\varsigma^c$ on $Z$ such that $\varsigma < \varsigma^c < 1$. This is a contradiction. So $h^{-1}(\varsigma)$ is $N_s\delta D$ on $Y$.

(ii) $\Rightarrow$ (i): Let $\xi$ be a $N_s\delta o\varsigma$ on $Y$ and $h(\xi) \neq 0 Z$. Suppose $\exists$ no $N_s\delta O \varsigma \neq 0 Z$ on $Z \ni \varsigma \leq h(\xi)$. Then $(h(\xi))^c$ is a $N_s\varsigma$ on $Z \ni \exists$ no $N_s\delta c\varsigma$ on $Z$ with $(h(\xi))^c < \delta < 1$. This means that $(h(\xi))^c$ is $N_s\delta D$ on $Z$. Thus $h^{-1}((h(\xi))^c) = N_s\delta D$ on $Y$. But $h^{-1}((h(\xi))^c) = (h^{-1}(h(\xi)))^c \leq \xi^c < 1$. This contradicts to the fact that $h^{-1}(h(\xi))^c$ is $N_s\delta D$ on $Y$. So, hence $\exists N_s\delta o\varsigma \neq 0 Z$ on $Z \ni \varsigma \leq h(\xi)$. Hence, $h$ is $swN_s$irr$\delta O$.

□

5. Conclusions

We have studied the concepts of somewhat neutrosophic $\delta$-irresolute continuous mappings in $N_s$ts is introduced and studied in this paper. Also, we discuss about a somewhat neutrosophic irresolute $\delta$-open mappings in a $N_s$ts. In future, neutrosophic $\delta$-irresolute mappings in a $N_s$ts can be extended. Also, this can be carried out to be neutrosophic $\delta S$ mapping, neutrosophic $\delta P$ mapping, neutrosophic $\delta \beta$ mapping such as continuous and irresolute in a neutrosophic topological spaces.

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