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# DOUBLE FRAMED SOFT FUZZY BI-IDEALS OF GAMMA NEAR-RINGS

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Abstract. The purpose of the article is to study about double framed soft fuzzy bi-ideals of gamma near-ring $\mathcal{R}$ and their some results. We have proved that the uni-int and int-uni of double framed soft fuzzy bi-ideals of gamma near-ring $\mathcal{R}$ are also a double framed soft fuzzy bi-ideal of gamma near-ring $\mathcal{R}$. We investigated some related properties of double framed soft fuzzy bi-ideal of $\mathcal{R}$ using homomorphism and anti-homomorphism. We establish the relationship between bi-ideal and double framed soft fuzzy bi-ideal of $\mathcal{R}$. In this paper we also discuss about double framed soft fuzzy interior ideal of gamma near-ring $\mathcal{R}$ and their some results.

Keywords: Bi-ideal, interior ideal, gamma near-ring, homomorphism, double framed soft fuzzy set.

AMS Subject Classification: Primary 16Y30, 03E72; Secondary 16D25

## 1. Introduction

The fuzzy set was introduced by Zadeh[19] in 1965. It is identified as a better tool for the scientific study of uncertainty, and came as a boost to the researchers working in the field of uncertainty. Many extensions and generalizations of fuzzy set was conceived by a number of researchers and a large number of real-life applications were developed in a variety of areas. In addition to this, parallel development of the classical results of many branches of Mathematics was also carried out in the fuzzy settings. One such abstract area is the branch of fuzzy algebra. It was initiated by Rosenfeld[16], who coined the idea of fuzzy subgroup of a group in 1971 and studied basic properties of this structure. Liu[11] defined fuzzy invariant subgroups and fuzzy ideals and discussed some properties. Soft set theory was introduced by Molodtsov[14]. In some special situation in the study of uncertainty and vague concepts, soft set theory gave proper solution. Pliz[15] introduced the algebraic structure near-rings as a generalization of rings and gave many examples and classification of near-rings. Kim and Kim[10] defined fuzzy ideals of near-rings. Further properties of fuzzy ideals in near-rings was studied by Hong et al.[5]. Jun et al.[6]

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initiated the study on double framed soft sets and presented its applications in BCK/BCI algebras. Also Cho et al.[4] studied the double framed soft near rings and defined some new notions.The monograph by Chinnadurai[1] gives a detailed discussion on fuzzy ideals in algebraic structures. Fuzzy ideals in Gamma near-ring $\mathcal{R}$ was discussed by Jun et al.[7,8] and Satyanarayana[17]. Manikantan[12] studied some properties of fuzzy bi-ideals of near-rings. Meenakumari and Tamizh chelvam[13] have defined fuzzy bi-ideal in Gamma near-ring $\mathcal{R}$ and discussed some results of this structure. Srinivas and Nagaiah[18] have proved some results on $T$-fuzzy ideals of $\Gamma$-near-rings. Chinnadurai and Kadalarasi[2] have defined the direct product of fuzzy ideals in near-rings. Khan et al.[9] studied near-rings in the view of double framed soft fuzzy sets. Chinnadurai and Shakila[3] discussed $T$-Fuzzy bi-ideal of gamma near-ring.
Let $U$ be a universal set and $E$ be a parameter set. A double framed pair $< (F, f); E >$ over $(U, [0, 1])$ is called a double framed soft fuzzy set over $(U, [0, 1])$, where $F$ is a soft set over $U$ and $f$ is a fuzzy set over $[0, 1]$, i.e., $F : E \longrightarrow P(U)$ and $f : E \longrightarrow [0, 1]$. For simplicity, we write $(F, f)$ to denote a double framed soft fuzzy set over $(U, [0, 1])$ with parameter set $E$. In our research work, we initiate double framed soft fuzzy bi-ideals of gamma near-ring $\mathcal{R}$ and establish its properties and study the relationship between bi-ideal and double framed soft fuzzy bi-ideals of gamma near-ring $\mathcal{R}$.

## 2. Double Framed Soft fuzzy bi-ideals of Gamma near-rings

Here we define Double framed Soft Fuzzy Bi-ideal of Gamma near-ring $\mathcal{R}$ and study their basic properties.

**Definition 2.1.** *A double framed soft fuzzy set $(F, f)$ is said to be a double framed soft fuzzy bi-ideal of gamma near-ring over $(U, [0, 1])$, if the following conditions hold for all $u, v, w \in \mathcal{R}$ and $\alpha, \beta \in \Gamma$,*
*(i)* $F(u - v) \subseteq F(u) \cup F(v)$,
*(ii)* $F(u\alpha v\beta w) \subseteq F(u) \cup F(w)$,
*(iii)* $f(u - v) \geq f(u) \cdot f(v)$,
*(iv)* $f(u\alpha v\beta w) \geq f(u) \cdot f(w)$.

**Example 2.1.** *Let $\mathcal{R} = \{0, 1, 2, 3\}$ with binary operation " + " on $\mathcal{R}$, $\Gamma = \{0, 1\}$ and $\mathcal{R} \times \Gamma \times \mathcal{R} \longrightarrow \mathcal{R}$ be a mapping. From the cayley table*

| + | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 |
| 1 | 1 | 0 | 3 | 2 |
| 2 | 2 | 3 | 1 | 0 |
| 3 | 3 | 2 | 0 | 1 |

| 0 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 2 | 0 | 2 | 2 | 2 |
| 3 | 0 | 3 | 3 | 3 |

| 1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |

*Define double framed soft fuzzy set $(F, f)$ over $(U, [0, 1])$ as $F(0) \subset F(1) \subset F(2) = F(3)$ and $f(0) = 0.5, f(1) = 0.6, f(2) = 0.8 = f(3)$, then $(F, f)$ is a doubled framed soft fuzzy bi-ideal of gamma near-ring $\mathcal{R}$ over $(U, [0, 1])$.*

**Definition 2.2.** *The uni-int of two double framed soft fuzzy set $(F, f)$ and $(G, g)$ over $(U, [0, 1])$ is denoted by $(F \cup G, f \wedge g)$ and is defined by*
$(F \cup G)(u) = F(u) \cup G(u)$ *and* $(f \wedge g)(u) = f(u) \wedge g(u)$, *for all* $u \in U$.

**Definition 2.3.** *The int-uni of two double framed soft fuzzy set $(F, f)$ and $(G, g)$ over $(U, [0, 1])$ is denoted by $(F \cap G, f \vee g)$ and is defined by*
$(F \cap G)(u) = F(u) \cap G(u)$ *and* $(f \vee g)(u) = f(u) \vee g(u)$, *for all* $u \in U$.

**Definition 2.4.** *Let $(F_1, f_1)$ and $(F_2, f_2)$ be two double framed soft fuzzy set over $(U, [0, 1])$, then the direct product is defined by*
$(F_1 \times F_2)(u, v) = F_1(u, v) \cup F_2(u, v)$ *and* $(f_1 \times f_2)(u, v) = f_1(u, v) \cdot f_2(u, v)$, *for all* $u, v \in U$.

**Proposition 2.1.** *Let $(F, f)$ be a double framed soft fuzzy bi-ideal of gamma near-ring over $(U, [0, 1])$. Then the following results hold,*
*(i)* $F(0) \subseteq F(u)$ *and* $f(0) \geq f(u)^2$ *for all* $u \in \mathcal{R}$,
*(ii)* $F(u) = F(-u)$ *and* $f(u) = f(-u)$
*(iii) If* $F(u - v) = F(0)$ *and* $f(u - v) = f(0)$, *then* $F(u) = F(v)$ *and* $f(u) \geq f(v)^3$.

*Proof.* (i) $F(0) = F(u - u) \subseteq F(u) \cup F(u) = F(u)$, which implies that
$F(0) \subseteq F(u)$.
$f(0) = f(u - u) \geq f(u) \cdot f(u) = f(u)^2$ for all $u \in \mathcal{R}$, which implies that
$f(0) \geq f(u)^2$.
(ii) $F(u) = F(0 - (-u)) \subseteq F(0) \cup F(-u) \subseteq F(-u)$, which implies that
$F(u) \subseteq F(-u)$ ...... (1)
$F(-u) = F(0 - u) \subseteq F(0) \cup F(u) \subseteq F(u)$, which implies that
$F(-u) \subseteq F(u)$. ...... (2)
From (1) and (2), $F(u) = F(-u)$.
Now,$f(u) = f(0 - (-u)) \geq f(0) \cdot f(-u) \geq f(u)^2 \cdot f(-u) \geq f(-u)$ and $f(-u) = f(0 - u) \geq f(0) \cdot f(u) \geq f(u)2 \cdot f(u) \geq f(u)$, then $f(u) = f(-u)$
(iii) $F(u) = F(u - v + v) = F((u - v) + v) \subseteq F(u - v) \cup F(v) = F(0) \cup F(v) = F(v)$, which implies that
$F(u) \subseteq F(v)$ ...... (3)
$F(v) = F(v - u + u) = F((v - u) + u) \subseteq F(v - u) \cup F(u) = F(0) \cup F(u) = F(u)$, which implies that
$F(v) \subseteq F(u)$ ...... (4)
From (3) and (4), $F(u) = F(v)$.
$f(u) = f(u - v + v) = f((u - v) + v) \geq f(u - v) \cdot f(v) = f(0) \cdot f(v) \geq f(v)^3$, which implies that $f(u) \geq f(v)^3$. □

**Theorem 2.1.** *The uni-int of two double framed soft fuzzy bi-ideals of gamma near-ring $(F, f)$ and $(G, g)$ over $(U, [0, 1])$ is a double framed soft fuzzy bi-ideal of gamma near-ring over $(U, [0, 1])$.*

*Proof.* Let $u, v \in \mathcal{R}$, then
(i) $(F \cup G)(u - v) = F(u - v) \cup G(u - v)$
$\subseteq (F(u) \cup F(v)) \cup (G(u) \cup G(v))$
$= (F(u) \cup G(u)) \cup (F(v) \cup G(v))$
$= (F \cup G)(u) \cup (F \cup G)(v)$.
(ii) $(F \cup G)(u\alpha v\beta w) = F(u\alpha v\beta w) \cup G(u\alpha v\beta w)$
$\subseteq (F(u) \cup F(w)) \cup (G(u) \cup G(w))$
$= (F(u) \cup G(u)) \cup (F(w) \cup G(w))$
$= (F \cup G)(u) \cup (F \cup G)(w)$.
(iii) $(f \wedge g)(u - v) = f(u - v) \wedge g(u - v)$
$\geq (f(u) \cdot f(v)) \wedge (g(u) \cdot g(v))$
$= (f(u) \wedge g(u)) \cdot (f(v) \wedge g(v))$
$= (f \wedge g)(u) \cdot (f \wedge g)(v)$.

(iv) $(f \wedge g)(u\alpha v\beta w) = f(u\alpha v\beta w) \wedge g(u\alpha v\beta w)$
$\geq (f(u) \cdot f(w)) \wedge (g(u) \cdot g(w))$
$= (f(u) \wedge g(u)) \cdot (f(w) \wedge g(w))$
$= (f \wedge g)(u) \cdot (f \wedge g)(w).$

Hence $< F \cup G, f \wedge g, \mathcal{R} >$ is a double framed soft fuzzy bi-ideal of gamma near-ring over $(U, [0,1])$. □

**Theorem 2.2.** *The uni-int of n- double framed soft fuzzy bi-ideals of gamma near-ring* $(F_1, f_1), (F_2, f_2), (F_3, f_3), ......, (F_n, f_n)$ *over* $(U, [0,1])$ *is a double framed soft fuzzy bi-ideal of gamma near-ring over* $(U, [0,1])$.

*Proof.* We can extend the Theorem 2.1. to $n$-double framed soft fuzzy bi-ideals of gamma near-ring over $(U, [0,1])$. □

**Theorem 2.3.** *The int-uni of two double framed soft fuzzy bi-ideals of gamma near-ring* $(F, f)$ *and* $(G, g)$ *over* $(U, [0,1])$ *is a double framed soft fuzzy bi-ideal of gamma near-ring over* $(U, [0,1])$.

*Proof.* Let $u, v \in \mathcal{R}$, then

(i) $(F \cap G)(u - v) = F(u - v) \cap G(u - v)$
$\subseteq (F(u) \cup F(v)) \cap (G(u) \cup G(v))$
$= (F(u) \cap G(u)) \cup (F(v) \cap G(v))$
$= (F \cap G)(u) \cup (F \cap G)(v).$

(ii) $(F \cap G)(u\alpha v\beta w) = F(u\alpha v\beta w) \cap G(u\alpha v\beta w)$
$\subseteq (F(u) \cup F(w)) \cap (G(u) \cup G(w))$
$= (F(u) \cap G(u)) \cup (F(w) \cap G(w))$
$= (F \cap G)(u) \cup (F \cap G)(w).$

(iii) $(f \vee g)(u - v) = f(u - v) \vee g(u - v)$
$\geq (f(u) \cdot f(v)) \vee (g(u) \cdot g(v))$
$= (f(u) \vee g(u)) \cdot (f(v) \vee g(v))$
$= (f \vee g)(u) \cdot (f \vee g)(v).$

(iv) $(f \vee g)(u\alpha v\beta w) = f(u\alpha v\beta w) \vee g(u\alpha v\beta w)$
$\geq (f(u) \cdot f(w)) \vee (g(u) \cdot g(w))$
$= (f(u) \vee g(u)) \cdot (f(w) \vee g(w))$
$= (f \vee g)(u) \cdot (f \vee g)(w).$

Hence $< F \cap G, f \vee g, \mathcal{R} >$ is a double framed soft fuzzy bi-ideal of gamma near-ring over $(U, [0,1])$. □

**Theorem 2.4.** *The int-uni of n-double framed soft fuzzy bi-ideals of gamma near-ring* $(F_1, f_1), (F_2, f_2), (F_3, f_3), ......, (F_n, f_n)$ *over* $(U, [0,1])$ *is a double framed soft fuzzy bi-ideal of gamma near-ring over* $(U, [0,1])$.

*Proof.* We can extend the Theorem 2.3. to $n$-double framed soft fuzzy bi-ideals of gamma near-ring over $(U, [0,1])$. □

**Theorem 2.5.** *If* $(F_1, f_1)$ *and* $(F_2, f_2)$ *be two double framed soft fuzzy bi-ideals of gamma near-rings* $R_1$ *and* $R_2$ *respectively over* $(U, [0,1])$. *Then* $(F_1 \times F_2, f_1 \times f_2)$ *is also a double framed soft fuzzy bi-ideal of gamma near-ring* $R_1 \times R_2$ *over* $(U, [0,1])$.

*Proof.* Let $(u_1, u_2), (v_1, v_2), (w_1, w_2) \in R_1 \times R_2$ and $\alpha, \beta \in \Gamma$. Now take $F = F_1 \times F_2$ and $f = f_1 \times f_2$, we have,

(i) $F((u_1, u_2) - (v_1, v_2)) = F((u_1 - v_1), (u_2 - v_2))$
$= (F_1 \times F_2)((u_1 - v_1), (u_2 - v_2))$
$= F_1(u_1 - v_1) \cup F_2(u_2 - v_2)$
$\subseteq (F_1(u_1) \cup F_1(v_1)) \cup (F_2(u_2) \cup F_2(v_2))$
$= (F_1 \times F_2)(u_1, u_2) \cup (F_1 \times F_2)(v_1, v_2)$
$= F(u_1, u_2) \cup F(v_1, v_2)$.

(ii) $F((u_1, u_2)\alpha(v_1, v_2)\beta(w_1, w_2)) = F((u_1v_1w_1)\alpha\beta, (u_2v_2w_2)\alpha\beta)$
$=(F_1 \times F_2)((u_1v_1w_1)\alpha\beta, (u_2v_2w_2)\alpha\beta)$
$= F_1(u_1\alpha v_1\beta w_1) \cup F_2(u_2\alpha v_2\beta w_2)$
$\subseteq (F_1(u_1) \cup F_1(w_1)) \cup (F_2(u_2) \cup F_2(w_2))$
$= (F_1 \times F_2)(u_1, u_2) \cup (F_1 \times F_2)(w_1, w_2)$
$= F(u_1, u_2) \cup F(w_1, w_2)$.

(iii) $f((u_1, u_2) - (v_1, v_2)) = f((u_1 - v_1), (u_2 - v_2))$
$= (f_1 \times f_2)((u_1 - v_1), (u_2 - v_2))$
$= f_1(u_1 - v_1) \cdot f_2(u_2 - v_2)$
$\geq (f_1(u_1) \cdot f_1(v_1)) \cdot (f_2(u_2) \cdot f_2(v_2))$
$= (f_1 \times f_2)(u_1, u_2) \cdot (f_1 \times f_2)(v_1, v_2)$
$= f(u_1, u_2) \cdot f(v_1, v_2)$.

(iv) $f((u_1, u_2)\alpha(v_1, v_2)\beta(w_1, w_2)) = f((u_1v_1w_1)\alpha\beta, (u_2v_2w_2)\alpha\beta)$
$=(f_1 \times f_2)((u_1v_1w_1)\alpha\beta, (u_2v_2w_2)\alpha\beta)$
$= f_1(u_1\alpha v_1\beta w_1) \cdot f_2(u_2\alpha v_2\beta w_2)$
$\geq (f_1(u_1) \cdot f_1(w_1)) \cdot (f_2(u_2) \cdot f_2(w_2))$
$= (f_1 \times f_2)(u_1, u_2) \cdot (f_1 \times f_2)(w_1, w_2)$
$= f(u_1, u_2) \cdot f(w_1, w_2)$.

Hence $(F_1 \times F_2, f_1 \times f_2)$ is a double framed soft fuzzy bi-ideal of gamma near-ring $R_1 \times R_2$ over $(U, [0, 1])$. □

**Theorem 2.6.** *If $(F_1, f_1), (F_2, f_2), (F_3, f_3), ......, (F_n, f_n)$ be n-double framed soft fuzzy bi-ideals of gamma near-rings $R_1, R_2, R_3, ......, R_n$ respectively over $(U, [0, 1])$. Then $(F_1 \times F_2 \times ...... \times F_n, f_1 \times f_2 \times f_3, ...... \times f_n)$ is also a double framed soft fuzzy bi-ideal of gamma near-ring $R_1 \times R_2 \times R_3, ...... \times R_n$ over $(U, [0, 1])$.*

*Proof.* We can extend the Theorem 2.5. to $n$-double framed soft fuzzy bi-ideals of gamma near-ring over $(U, [0, 1])$. □

**Theorem 2.7.** *If $(F, f)$ be a double framed soft fuzzy bi-ideal of gamma near-ring $\mathcal{R}$ over $(U, [0, 1])$ if and only if the following conditions hold for any $u, v, w \in \mathcal{R}$*
*(i) $F(u + v) \subseteq F(u) \cup F(v)$ and $F(u\alpha v\beta w) \subseteq F(u) \cup F(w)$*
*(ii) $f(u + v) \geq f(u) \cdot f(v)$ and $f(u\alpha v\beta w) \geq f(u) \cdot f(w)$*
*(iii) $F(u) = F(-u)$ and $f(u) = f(-u)$.*

*Proof.* Let $(F, f)$ be a double framed soft fuzzy bi-ideal of gamma near-ring $R$ over $(U, [0, 1])$. Now we have,

(i) $F(u + v) = F(u - (-v))$
$\subseteq F(u) \cup F(-v)$,

$= F(u) \cup F(v)$, since from Proposition 2.1. (ii), we get $F(-v) = F(v)$.

And $F(u\alpha v\beta w) \subseteq F(u) \cup F(w)$

(ii) $f(u+v) = f(u-(-v))$
$\geq f(u) \cdot f(-v)$,
$= f(u) \cdot f(v)$, since from Proposition 2.1. (ii), we get $f(-v) = f(v)$.

And $f(u\alpha v\beta w) \geq f(u) \cdot f(w)$

(iii) It comes obviously.

The converse of the theorem is obvious. □

**Theorem 2.8.** *Let $(F, f)$ be a double framed soft fuzzy bi-ideal of gamma near-ring $\mathcal{R}$ over $(U, [0,1])$. Then $F(0) = F(u)$ if and only if $F(u+v) = F(v+u) = F(v)$ for a fixed $u \in \mathcal{R}$ and for all $v \in \mathcal{R}$.*

*Proof.* Let $F(0) = F(u)$. Since by Proposition 2.1. (i), we have $F(0) \subseteq F(u)$, for all $u \in \mathcal{R}$.

Now, $F(u) = F(0) \subseteq F(v)$, for all $v \in \mathcal{R}$.

$F(u+v) \subseteq F(u) \cup F(v) = F(v)$ and $F(v+u) \subseteq F(v) \cup F(u) = F(v)$.

Thus $F(u+v) = F(v+u) \subseteq F(v)$.

Also, $F(v) = F(u-u+v)$
$\subseteq F(u) \cup F(u+v)$
$= F(0) \cup F(u+v)$
$= F(u+v)$.

And, $F(v) = F(v+u-u)$
$\subseteq F(v+u) \cup F(u)$
$= F(v+u) \cup F(0)$
$= F(v+u)$.

Thus $F(v) \subseteq F(u+v) = F(v+u)$.

Hence $F(u+v) = F(v+u) = F(v)$ for a fixed $u \in \mathcal{R}$ and for all $v \in \mathcal{R}$.

The converse part of the theorem is obvious. □

**Theorem 2.9.** *Let $A$ be non-empty set of a gamma near-ring $\mathcal{R}$. If $A$ is a bi-ideal of $\mathcal{R}$, then $(F, f)$ be a double framed soft fuzzy bi-ideal of gamma near-ring $\mathcal{R}$.*

*Proof.* Let $A$ is a bi-ideal of a gamma near-ring $\mathcal{R}$, we define $F : \mathcal{R} \longrightarrow P(U)$ and $f : \mathcal{R} \longrightarrow [0,1]$ by

$$F(u) = \begin{cases} 1, & \text{if } u \in A \\ 0, & \text{if } u \notin A \end{cases} \quad \textit{and } f(u) = \begin{cases} 1, & \text{if } u \in A \\ 0, & \text{if } u \notin A \end{cases}$$

respectively. For $u, v \in A$, $u - v \in A$.

(i) Let $u, v \in \mathcal{R}$.

case(a): If $u, v \in A$, then $F(u) = 1$ and $F(v) = 1$.
Thus $F(u-v) = 1 \subseteq F(u) \cup F(v)$.

case(b): $u \in A$ and $v \notin A$, then $F(u) = 1$ and $F(v) = 0$.
Thus $F(u-v) = 0 \subseteq F(u) \cup F(v)$.

case(c): If $u \notin A$ and $v \in A$, then $F(u) = 0$ and $F(v) = 1$.
Thus $F(u-v) = 0 \subseteq F(u) \cup F(v)$.

case(d): If $u \notin A$ and $v \notin A$, then $F(u) = 0$ and $F(v) = 0$.
Thus $F(u-v) = 0 \subseteq F(u) \cup F(v)$.

(ii) Let $u, v, w \in \mathcal{R}$.

case(a): If $u \in A$ and $w \in A$, then $F(u) = 1$ and $F(w) = 1$.
Thus $F(u\alpha v\beta w) = 1 \subseteq F(u) \cup F(w)$.
case(b): If $u \in A$ and $w \notin A$, then $F(u) = 1$ and $F(w) = 0$.
Thus $F(u\alpha v\beta w) = 0 \subseteq F(u) \cup F(w)$.
case(c): If $u \notin A$ and $w \in A$, then $F(u) = 0$ and $F(w) = 1$.
Thus $F(u\alpha v\beta w) = 0 \subseteq F(u) \cup F(w)$.
case(d): If $u \notin A$ and $w \notin A$, then $F(u) = 0$ and $F(w) = 0$.
Thus $F(u\alpha v\beta w) = 0 \subseteq F(u) \cup F(w)$.

(iii) Let $u, v \in \mathcal{R}$.
case(a): If $u, v \in A$, then $f(u) = 1$ and $f(v) = 1$.
Thus $f(u - v) = 1 \geq f(u) \cdot f(v)$.
case(b): $u \in A$ and $v \notin A$, then $f(u) = 1$ and $f(v) = 0$.
Thus $f(u - v) = 0 \geq f(u) \cdot f(v)$.
case(c): If $u \notin A$ and $v \in A$, then $f(u) = 0$ and $f(v) = 1$.
Thus $f(u - v) = 0 \geq f(u) \cdot f(v)$.
case(d): If $u \notin A$ and $v \notin A$, then $f(u) = 0$ and $f(v) = 0$.
Thus $f(u - v) = 0 \geq f(u) \cdot f(v)$.

(iv) Let $u, v, w \in \mathcal{R}$.
case(a): If $u \in A$ and $w \in A$, then $f(u) = 1$ and $f(w) = 1$.
Thus $f(u\alpha v\beta w) = 1 \geq f(u) \cdot f(w)$.
case(b): If $u \in A$ and $w \notin A$, then $f(u) = 1$ and $f(w) = 0$.
Thus $f(u\alpha v\beta w) = 0 \geq f(u) \cdot f(w)$.
case(c): If $u \notin A$ and $w \in A$, then $f(u) = 0$ and $f(w) = 1$.
Thus $f(u\alpha v\beta w) = 0 \geq f(u) \cdot f(w)$.
case(d): If $u \notin A$ and $w \notin A$, then $f(u) = 0$ and $f(w) = 0$.
Thus $f(u\alpha v\beta w) = 0 \geq f(u) \cdot f(w)$.

$(F, f)$ is a double framed soft fuzzy bi-ideal of gamma near-ring $\mathcal{R}$. □

## 3. Homomorphism of double framed soft fuzzy bi-ideal of gamma near-ring

In this section, we define gamma near-ring homomorphism of fuzzy bi-ideal and discuss the properties of double framed soft fuzzy bi-ideal of gamma near-ring using homomorphism.

**Definition 3.1.** *A gamma near-ring homomorphism of fuzzy bi-ideal is a mapping $\phi$ from a gamma near-ring $R_1$ into a gamma near-ring $R_2$, that is $\phi : R_1 \longrightarrow R_2$ such that*
*(i) $\phi(u - v) = \phi(u) - \phi(v)$, for all $u, v \in R_1$.*
*(ii) $\phi(u\alpha v\beta w) = \phi(u)\alpha\phi(v)\beta\phi(w)$, for all $u, v, w \in R_1$ and $\alpha, \beta \in \Gamma$.*

**Theorem 3.1.** *Let $\phi : R_1 \longrightarrow R_2$ be a gamma near-ring homomorphism and $((F, f), R_1)$ be a double framed soft fuzzy bi-ideal of gamma near-ring $R_1$ over $(U, [0, 1])$. Then the image $((\phi(F), \phi(f)), R_2)$ is also a double framed soft fuzzy bi-ideal of gamma near-ring of $R_2$ over $(U, [0, 1])$.*

*Proof.* Let $u, v, w \in R_1$ . Since $\phi$ is a gamma near-ring homomorphism and $(F, f)$ is a double framed soft fuzzy bi-ideal gamma near-ring. We have,

(i) $\phi(F)(u - v) \subseteq \phi(F)(u) \cup \phi(F)(v)$
(ii) $\phi(F)(u\alpha v\beta w) \subseteq \phi(F)(u) \cup \phi(F)(w)$
(iii) $\phi(f)(u - v) \geq \phi(f)(u) \cdot \phi(f)(v)$

(iv) $\phi(f)(u\alpha v\beta w) \geq \phi(f)(u) \cdot \phi(f)(w)$.

Hence the image $(\phi(F), \phi(f), R_2)$ is a double framed soft fuzzy bi-ideal of gamma near-ring of $R_2$ over $(U, [0,1])$. □

**Theorem 3.2.** *Let $\phi : R_1 \longrightarrow R_2$ be a gamma near-ring homomorphism and $((G, g), R_2)$ be a double framed soft fuzzy bi-ideal of gamma near-ring $R_2$ over $(U, [0,1])$. Then the pre-image $((\phi^{-1}(G), \phi^{-1}(g)), R_1)$ is also a double framed soft fuzzy bi-ideal of gamma near-ring of $R_1$ over $(U, [0,1])$.*

*Proof.* Let $u, v, w \in R_1$ . Since $\phi$ is a gamma near-ring homomorphism and $(G, g)$ is a double framed soft fuzzy bi-ideal gamma near-ring. We have,

(i)
$$\begin{aligned}\phi^{-1}(G)(u - v) &= G(\phi(u - v)) \\ &= G(\phi(u) - \phi(v)) \\ &\subseteq G(\phi(u)) \cup G(\phi(v)) \\ &= \phi^{-1}(G)(u) \cup \phi^{-1}(G)(v).\end{aligned}$$

(ii)
$$\begin{aligned}\phi^{-1}(G)(u\alpha v\beta w) &= G(\phi(u\alpha v\beta w)) \\ &= G(\phi(u)\alpha\phi(v)\beta\phi(w) \\ &\subseteq G(\phi(u)) \cup G(\phi(w)) \\ &= \phi^{-1}(G)(u) \cup \phi^{-1}(G)(w).\end{aligned}$$

(iii)
$$\begin{aligned}\phi^{-1}(g)(u - v) &= g(\phi(u - v)) \\ &= g(\phi(u) - \phi(v)) \\ &\geq g(\phi(u)) \cdot g(\phi(v)) \\ &= \phi^{-1}(g)(u) \cdot \phi^{-1}(g)(v).\end{aligned}$$

(iv)
$$\begin{aligned}\phi^{-1}(g)(u\alpha v\beta w) &= g(\phi(u\alpha v\beta w)) \\ &= g(\phi(u)\alpha\phi(v)\beta\phi(w) \\ &\geq g(\phi(u)) \cdot g(\phi(w)) \\ &= \phi^{-1}(g)(u) \cdot \phi^{-1}(g)(w).\end{aligned}$$

Hence the pre-image $(\phi^{-1}(G), \phi^{-1}(g), R_1)$ is a double framed soft fuzzy bi-ideal of gamma near-ring of $R_1$ over $(U, [0,1])$. □

## 4. Anti-Homomorphism of double framed soft fuzzy bi-ideal of gamma near-ring

In this section, we define gamma near-ring anti-homomorphism of fuzzy bi-ideal and discuss the properties of double framed soft fuzzy bi-ideal of gamma near-ring using anti-homomorphism.

**Definition 4.1.** *A gamma near-ring anti-homomorphism of fuzzy bi-ideal is a mapping $\phi$ from a gamma near-ring $R_1$ into a gamma near-ring $R_2$, that is $\phi : R_1 \longrightarrow R_2$ such that*

*(i) $\phi(u - v) = \phi(v) - \phi(u)$, for all $u, v \in R_1$.*

*(ii) $\phi(u\alpha v\beta w) = \phi(w)\alpha\phi(v)\beta\phi(u)$, for all $u, v, w \in R_1$ and $\alpha, \beta \in \Gamma$.*

**Theorem 4.1.** *Let $\phi : R_1 \longrightarrow R_2$ be a gamma near-ring anti-homomorphism and $((F, f), R_1)$ be a double framed soft fuzzy bi-ideal of gamma near-ring $R_1$ over $(U, [0,1])$. Then the image $((\phi(F), \phi(f)), R_2)$ is also a double framed soft fuzzy bi-ideal of gamma near-ring of $R_2$ over $(U, [0,1])$.*

*Proof.* Let $u, v, w \in R_1$ . Since $\phi$ is a gamma near-ring anti-homomorphism and $(F, f)$ is a double framed soft fuzzy bi-ideal gamma near-ring. We have,

(i) $\phi(F)(u - v) \subseteq \phi(F)(u) \cup \phi(F)(v)$
(ii) $\phi(F)(u\alpha v\beta w) \subseteq \phi(F)(u) \cup \phi(F)(w)$
(iii) $\phi(f)(u - v) \geq \phi(f)(u) \cdot \phi(f)(v)$
(iv) $\phi(f)(u\alpha v\beta w) \geq \phi(f)(u) \cdot \phi(f)(w)$.

Hence the image $(\phi(F), \phi(f), R_2)$ is a double framed soft fuzzy bi-ideal of gamma near-ring of $R_2$ over $(U, [0, 1])$. □

**Theorem 4.2.** *Let $\phi : R_1 \longrightarrow R_2$ be a gamma near-ring anti-homomorphism and $((G, g), R_2)$ be a double framed soft fuzzy bi-ideal of gamma near-ring $R_2$ over $(U, [0, 1])$. Then the pre-image $((\phi^{-1}(G), \phi^{-1}(g)), R_1)$ is also a double framed soft fuzzy bi-ideal of gamma near-ring of $R_1$ over $(U, [0, 1])$.*

*Proof.* Let $u, v, w \in R_1$ . Since $\phi$ is a gamma near-ring anti-homomorphism and $(G, g)$ is a double framed soft fuzzy bi-ideal gamma near-ring. We have,

(i)
$$\begin{aligned}\phi^{-1}(G)(u - v) &= G(\phi(u - v)) \\ &= G(\phi(v) - \phi(u)) \\ &\subseteq G(\phi(v)) \cup G(\phi(u)) \\ &= \phi^{-1}(G)(v) \cup \phi^{-1}(G)(u). \\ &= \phi^{-1}(G)(u) \cup \phi^{-1}(G)(v).\end{aligned}$$

(ii)
$$\begin{aligned}\phi^{-1}(G)(u\alpha v\beta w) &= G(\phi(u\alpha v\beta w)) \\ &= G(\phi(w)\alpha\phi(v)\beta\phi(u) \\ &\subseteq G(\phi(w)) \cup G(\phi(u)) \\ &= \phi^{-1}(G)(w) \cup \phi^{-1}(G)(u). \\ &= \phi^{-1}(G)(u) \cup \phi^{-1}(G)(w).\end{aligned}$$

(iii)
$$\begin{aligned}\phi^{-1}(g)(u - v) &= g(\phi(u - v)) \\ &= g(\phi(v) - \phi(u)) \\ &\geq g(\phi(v)) \cdot g(\phi(u)) \\ &= \phi^{-1}(g)(v) \cdot \phi^{-1}(g)(u). \\ &= \phi^{-1}(g)(u) \cdot \phi^{-1}(g)(v).\end{aligned}$$

(iv)
$$\begin{aligned}\phi^{-1}(g)(u\alpha v\beta w) &= g(\phi(u\alpha v\beta w)) \\ &= g(\phi(w)\alpha\phi(v)\beta\phi(u) \\ &\geq g(\phi(w)) \cdot g(\phi(u)) \\ &= \phi^{-1}(g)(w) \cdot \phi^{-1}(g)(u). \\ &= \phi^{-1}(g)(u) \cdot \phi^{-1}(g)(w).\end{aligned}$$

Hence the pre-image $(\phi^{-1}(G), \phi^{-1}(g), R_1)$ is a double framed soft fuzzy bi-ideal of gamma near-ring of $R_1$ over $(U, [0, 1])$. □

## 5. Double Framed Soft fuzzy interior ideal of Gamma near-ring

Here we define Double framed Soft Fuzzy interior ideal of Gamma near-ring $\mathcal{R}$ and study their basic properties.

**Definition 5.1.** *A double framed soft fuzzy set $(F, f)$ is said to be a double framed soft fuzzy interior ideal of gamma near-ring over $(U, [0, 1])$, if the following conditions hold for all $u, v, w \in \mathcal{R}$ and $\alpha, \beta \in \Gamma$,*
*(i)* $F(u - v) \subseteq F(u) \cup F(v)$,
*(ii)* $F(u\alpha v\beta w) \subseteq F(v)$,
*(iii)* $f(u - v) \geq f(u) \cdot f(v)$,
*(iv)* $f(u\alpha v\beta w) \geq f(v)$.

**Example 5.1.** *Let $\mathcal{R} = \{0, 1, 2, 3\}$ with binary operation " $+$ " on $\mathcal{R}$, $\Gamma = \{0, 1\}$ and $\mathcal{R} \times \Gamma \times \mathcal{R} \longrightarrow \mathcal{R}$ be a mapping. From the cayley table*

| + | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 |
| 1 | 1 | 0 | 3 | 2 |
| 2 | 2 | 3 | 1 | 0 |
| 3 | 3 | 2 | 0 | 1 |

| 0 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 2 | 0 | 2 | 2 | 2 |
| 3 | 0 | 3 | 3 | 3 |

| 1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |

*Define double framed soft fuzzy set $(F, f)$ over $(U, [0, 1])$ as $F(0) \subset F(1) \subset F(2) = F(3)$ and $f(0) = 0.5, f(1) = 0.2, f(2) = 0.1 = f(3)$, then $(F, f)$ is a doubled framed soft fuzzy interior ideal of gamma near-ring $\mathcal{R}$ over $(U, [0, 1])$.*

**Theorem 5.1.** *The uni-int of two double framed soft fuzzy interior ideals of gamma near-ring $(F, f)$ and $(G, g)$ over $(U, [0, 1])$ is a double framed soft fuzzy interior ideal of gamma near-ring over $(U, [0, 1])$.*

*Proof.* Let $u, v \in \mathcal{R}$, then

(i) $(F \cup G)(u - v) = F(u - v) \cup G(u - v)$
$\subseteq (F(u) \cup F(v)) \cup (G(u) \cup G(v))$
$= (F(u) \cup G(u)) \cup (F(v) \cup G(v))$
$= (F \cup G)(u) \cup (F \cup G)(v)$.

(ii) $(F \cup G)(u\alpha v\beta w) = F(u\alpha v\beta w) \cup G(u\alpha v\beta w)$
$\subseteq F(v) \cup G(v)$
$= (F \cup G)(v)$.

(iii) $(f \wedge g)(u - v) = f(u - v) \wedge g(u - v)$
$\geq (f(u) \cdot f(v)) \wedge (g(u) \cdot g(v))$
$= (f(u) \wedge g(u)) \cdot (f(v) \wedge g(v))$
$= (f \wedge g)(u) \cdot (f \wedge g)(v)$.

(iv) $(f \wedge g)(u\alpha v\beta w) = f(u\alpha v\beta w) \wedge g(u\alpha v\beta w)$
$\geq f(v) \wedge g(v)$
$= (f \wedge g)(v)$.

Hence $< F \cup G, f \wedge g, \mathcal{R} >$ is a double framed soft fuzzy interior ideal of gamma near-ring over $(U, [0, 1])$. □

**Theorem 5.2.** *The uni-int of $n$- double framed soft fuzzy interior ideals of gamma near-ring $(F_1, f_1), (F_2, f_2), (F_3, f_3), ......, (F_n, f_n)$ over $(U, [0, 1])$ is a double framed soft fuzzy interior ideal of gamma near-ring over $(U, [0, 1])$.*

*Proof.* We can extend the Theorem 5.1. to $n$-double framed soft fuzzy interior ideals of gamma near-ring over $(U, [0, 1])$. □

**Theorem 5.3.** *The int-uni of two double framed soft fuzzy interior ideals of gamma near-ring $(F, f)$ and $(G, g)$ over $(U, [0, 1])$ is a double framed soft fuzzy interior ideal of gamma* *near-ring over* $(U,[0,1])$.

*Proof.* Let $u, v \in \mathcal{R}$, then

(i) $(F \cap G)(u-v) = F(u-v) \cap G(u-v)$
$\subseteq (F(u) \cup F(v)) \cap (G(u) \cup G(v))$
$= (F(u) \cap G(u)) \cup (F(v) \cap G(v))$
$= (F \cap G)(u) \cup (F \cap G)(v)$.

(ii) $(F \cap G)(u\alpha v\beta w) = F(u\alpha v\beta w) \cap G(u\alpha v\beta w)$
$\subseteq F(v) \cap G(v)$
$= (F \cap G)(v)$.

(iii) $(f \vee g)(u-v) = f(u-v) \vee g(u-v)$
$\geq (f(u) \cdot f(v)) \vee (g(u) \cdot g(v))$
$= (f(u) \vee g(u)) \cdot (f(v) \vee g(v))$
$= (f \vee g)(u) \cdot (f \vee g)(v)$.

(iv) $(f \vee g)(u\alpha v\beta w) = f(u\alpha v\beta w) \vee g(u\alpha v\beta w)$
$\geq f(v) \vee g(v)$
$= (f \vee g)(v)$.

Hence $< F \cap G, f \vee g, \mathcal{R} >$ is a double framed soft fuzzy interior ideal of gamma near-ring over $(U,[0,1])$. □

**Theorem 5.4.** *The int-uni of n-double framed soft fuzzy interior ideals of gamma near-ring* $(F_1, f_1), (F_2, f_2), (F_3, f_3), ......, (F_n, f_n)$ *over* $(U,[0,1])$ *is a double framed soft fuzzy interior-ideal of gamma near-ring over* $(U,[0,1])$.

*Proof.* We can extend the Theorem 5.3. to $n$-double framed soft fuzzy interior ideals of gamma near-ring over $(U,[0,1])$. □

**Theorem 5.5.** *If* $(F_1, f_1)$ *and* $(F_2, f_2)$ *be two double framed soft fuzzy interior ideals of gamma near-rings* $R_1$ *and* $R_2$ *respectively over* $(U,[0,1])$. *Then* $(F_1 \times F_2, f_1 \times f_2)$ *is also a double framed soft fuzzy interior ideal of gamma near-ring* $R_1 \times R_2$ *over* $(U,[0,1])$.

*Proof.* Let $(u_1, u_2), (v_1, v_2), (w_1, w_2) \in R_1 \times R_2$ and $\alpha, \beta \in \Gamma$. Now take $F = (F_1 \times F_2$ and $f = f_1 \times f_2$, we have,

(i) $F((u_1, u_2) - (v_1, v_2)) = F((u_1 - v_1), (u_2 - v_2))$
$= (F_1 \times F_2)((u_1 - v_1), (u_2 - v_2))$
$\subseteq F_1(u_1 - v_1) \cup F_2(u_2 - v_2)$
$\subseteq (F_1(u_1) \cup F_1(v_1)) \cup (F_2(u_2) \cup F_2(v_2))$
$= (F_1 \times F_2)(u_1, u_2) \cup (F_1 \times F_2)(v_1, v_2)$
$= F(u_1, u_2) \cup F(v_1, v_2)$.

(ii) $F((u_1, u_2)\alpha(v_1, v_2)\beta(w_1, w_2)) = F((u_1 v_1 w_1)\alpha\beta, (u_2 v_2 w_2)\alpha\beta)$
$=(F_1 \times F_2)((u_1 v_1 w_1)\alpha\beta, (u_2 v_2 w_2)\alpha\beta)$
$\subseteq F_1(u_1 \alpha v_1 \beta w_1) \cup F_2(u_2 \alpha v_2 \beta w_2)$
$\subseteq F_1(v_1) \cup F_2(v_2)$
$= (F_1 \times F_2)(v_1, v_2)$
$= F(v_1, v_2)$.

(iii) $f((u_1, u_2) - (v_1, v_2)) = f((u_1 - v_1), (u_2 - v_2))$
$= (f_1 \times f_2)((u_1 - v_1), (u_2 - v_2))$
$\geq f_1(u_1 - v_1) \cdot f_2(u_2 - v_2)$

$$\geq (f_1(u_1) \cdot f_1(v_1)) \cdot (f_2(u_2) \cdot f_2(v_2))$$
$$= (f_1 \times f_2)(u_1, u_2) \cdot (f_1 \times f_2)(v_1, v_2)$$
$$= f(u_1, u_2) \cdot f(v_1, v_2).$$

(iv) $f((u_1, u_2)\alpha(v_1, v_2)\beta(w_1, w_2)) = f((u_1v_1w_1)\alpha\beta, (u_2v_2w_2)\alpha\beta)$
$$=(f_1 \times f_2)((u_1v_1w_1)\alpha\beta, (u_2v_2w_2)\alpha\beta)$$
$$\geq f_1(u_1\alpha v_1\beta w_1) \cdot f_2(u_2\alpha v_2\beta w_2)$$
$$\geq f_1(v_1) \cdot f_2(v_2)$$
$$= (f_1 \times f_2)(v_1, v_2)$$
$$= f(v_1, v_2).$$

Hence $(F_1 \times F_2, f_1 \times f_2)$ is a double framed soft fuzzy interior ideal of gamma near-ring $R_1 \times R_2$ over $(U, [0, 1])$. □

**Theorem 5.6.** *If $(F_1, f_1), (F_2, f_2), (F_3, f_3), ......, (F_n, f_n)$ be n-double framed soft fuzzy interior ideals of gamma near-rings $R_1, R_2, R_3, ......, R_n$ respectively over $(U, [0, 1])$. Then $(F_1 \times F_2 \times ...... \times F_n, f_1 \times f_2 \times f_3, ...... \times f_n)$ is also a double framed soft fuzzy interior ideal of gamma near-ring $R_1 \times R_2 \times R_3, ...... \times R_n$ over $(U, [0, 1])$.*

*Proof.* We can extend the Theorem 5.5. to $n$-double framed soft fuzzy interior ideals of gamma near-ring over $(U, [0, 1])$. □

**Theorem 5.7.** *Let $A$ be non-empty set of a gamma near-ring $\mathcal{R}$. If $A$ is an interior ideal of $\mathcal{R}$, then $(F, f)$ be a double framed soft fuzzy interior ideal of gamma near-ring $\mathcal{R}$.*

*Proof.* Let $A$ is an interior ideal of a gamma near-ring $\mathcal{R}$, we define $F : \mathcal{R} \longrightarrow P(U)$ and $f : \mathcal{R} \longrightarrow [0, 1]$ by

$$F(u) = \begin{cases} 1, & \text{if } u \in A \\ 0, & \text{if } u \notin A \end{cases} \quad \textit{and } f(u) = \begin{cases} 1, & \text{if } u \in A \\ 0, & \text{if } u \notin A \end{cases}$$

respectively. For $u, v \in A$, $u - v \in A$.

(i) Let $u, v \in \mathcal{R}$.

case(a): If $u, v \in A$, then $F(u) = 1$ and $F(v) = 1$.
Thus $F(u - v) = 1 \subseteq F(u) \cup F(v)$.

case(b): $u \in A$ and $v \notin A$, then $F(u) = 1$ and $F(v) = 0$.
Thus $F(u - v) = 0 \subseteq F(u) \cup F(v)$.

case(c): If $u \notin A$ and $v \in A$, then $F(u) = 0$ and $F(v) = 1$.
Thus $F(u - v) = 0 \subseteq F(u) \cup F(v)$.

case(d): If $u \notin A$ and $v \notin A$, then $F(u) = 0$ and $F(v) = 0$.
Thus $F(u - v) = 0 \subseteq F(u) \cup F(v)$.

(ii) Let $u, v, w \in \mathcal{R}$.

case(a): If $v \in A$, then $F(v) = 1$.
Thus $F(u\alpha v\beta w) = 1 \subseteq F(v)$.

case(b): If $v \notin A$, then $F(v) = 0$.
Thus $F(u\alpha v\beta w) = 0 \subseteq F(v)$.

(iii) Let $u, v \in \mathcal{R}$.

case(a): If $u, v \in A$, then $f(u) = 1$ and $f(v) = 1$.
Thus $f(u - v) = 1 \geq f(u) \cdot f(v)$.

case(b): $u \in A$ and $v \notin A$, then $f(u) = 1$ and $f(v) = 0$.
Thus $f(u - v) = 0 \geq f(u) \cdot f(v)$.

case(c): If $u \notin A$ and $v \in A$, then $f(u) = 0$ and $f(v) = 1$.

Thus $f(u - v) = 0 \geq f(u) \cdot f(v)$.

case(d): If $u \notin A$ and $v \notin A$, then $f(u) = 0$ and $f(v) = 0$.

Thus $f(u - v) = 0 \geq f(u) \cdot f(v)$.

(iv) Let $u, v, w \in \mathcal{R}$.

case(a): If $v \in A$, then $f(v) = 1$.

Thus $f(u\alpha v\beta w) = 1 \geq f(v)$.

case(b): If $v \notin A$, then $f(v) = 0$.

Thus $f(u\alpha v\beta w) = 0 \geq f(v)$.

$(F, f)$ is a double framed soft fuzzy interior ideal of gamma near-ring $\mathcal{R}$. □

## 6. Conclusions

We obtained the uni-int and int-uni of two double framed soft fuzzy bi-ideals (resp. interior ideals) of gamma near-ring $(F, f)$ and $(G, g)$ over $(U, [0, 1])$ are also a double framed soft fuzzy bi-ideal (resp. interior ideal) of gamma near-ring over $(U, [0, 1])$. In future we will discuss the double framed soft fuzzy sets in some other algebraic structures.

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