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EQUITABLE AND OUTDEGREE EQUITABLE DOMINATION NUMBER OF GRAPHS

THASNEEM T. R.[1*], M. K. MENON[2], §

ABSTRACT. Let $G = (V, E)$ be a simple graph. A subset D of V is said to be a dominating set of G, if each vertex in V is either in D or has a neighbour in D. A subset D of V is said to be an equitable dominating set of G, if for every $v \in V - D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|deg(u) - deg(v)| \leq 1$. The minimum cardinality of an equitable dominating set of G, denoted by $\gamma^e(G)$, is called the equitable domination number of G. The edges from a vertex $u \in D$ to $V - D$ are called the dominating edges of u from D. A dominating set D is called an outdegree equitable dominating set if the difference between the cardinalities of the sets of dominating edges from any two points of D is atmost one. The minimum cardinality of an outdegree equitable dominating set of G, denoted by $\gamma_{oe}(G)$ is called the outdegree equitable domination number of G. In this paper we study equitable domination number and outdegree equitable domination number of some graphs.

Keywords: Equitable domination, Outdegree equitable domination, Equitable isolates.

AMS Subject Classification: 05C, 05C69, 05C99.

1. Introduction

Domination is one of the most relevant topics in the field of Graph Theory. Although the theoretical introduction of domination began around 1960, from literature we can see the references related to the concept of Domination in graphs about 100 years back through the Chess Board Queens problem which was later mentioned by Norwegian mathematician Oystein Ore[8], in his book *Theory of graphs*. The problem is to place the minimum number of queens on an nxn chess board so that every square is dominated by at least one queen. The solution of this problem can be obtained by finding a dominating set in the graph, whose vertices are the squares of the chess board and edges are the possible moves of a queen on the chess board. In 1958, French mathematician Claude Berge[2], introduced

[1] Department of Mathematics, Maharajas College, Ernakulam, Kerala, India, 682011.
e-mail: thasneem@maharajas.ac.in; ORCID: https://orcid.org/0000-0001-5189-8648.
[*] Corresponding author.
[2] Department of Mathematics, St.Pauls College, Kalamassery, Kerala, India, 683503.
e-mail:manju@stpauls.ac.in; ORCID: https://orcid.org/0000-0002-5692-825X.
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the concept of domination as a coefficient of external stability. In 1962, Ore was the first to use the term domination and minimum dominating set. Later a considerable progress in the area of domination can be seen in [1], [4], [5]. The theory of domination in graphs has a wide range of applications which include communication network problems, school bus routing problem and facility location problems. Among these the most often discussed is the communication network which consists of a certain set of sites. The problem is to find the minimum number of sites at which the transmitters are to be placed so that each site in the network has a direct communication link with at least one site with the transmitter. In graphical terms the problem is to find the minimum dominating set.

It is found that people with equal behaviour and thinking often tend to become friends. Likewise in a network, nodes with nearly equal capacity may interact with each other in a better way. In most times a democratic system runs efficiently if it ensures its members equitability in terms of health, wealth, status etc. Sampathkumar in [9], introduced various types of equitability in graph theory. Two adjacent vertices are said to be equitable if the absolute value of the difference of their degrees is atmost one. More detailed works can be referred in [7], [10]. Since 2010, many research works are being carried out in the area of equitable domination in graph theory.

Apart from dominating sets, in addition to finding a neighbour, an equitable dominating set[10], ensures to each vertex outside it, an equitable neighbour. The concept outward equitable dominating set[7] have numerous applications in daily life which includes finding a minimum number of supervisors to be appointed at different stages of production in a manufacturing firm, such that monitoring duties of supervisors are equitable. Likewise the problem of finding a minimum number of locations in a country at which medical camps are to be set up such that facilities in the camps are equitable, can be dealt with finding the outdegree equitable domination number of the graph in which vertices are the locations and roads connecting the locations constitute the edges. Outdegree equitable domination is more important in the situation where the entire graph is to be dominated as well as almost equal powers or responsibilities are to be entrusted with the vertices in the dominating set.

All graphs considered in this paper are finite, simple, connected and undirected. For graph theoretical terminology not defined in this paper, we refer to Clark et al. [6]. Let G be a graph. The degree of a vertex $v \in V(G)$ denoted as $d(v)$, $deg(u)$ or $d_G(v)$ is the number of edges incident at v. Two vertices u and v are adjacent if there exists an edge joining u and v. If u and v are adjacent, we say that u is a neighbour of v and vice versa. For a vertex u, $N(u)$ denotes the set of all neighbours of u. A vertex is called a pendant vertex if its degree is 1. A set $D \subseteq V(G)$ is called a dominating set of G, if each vertex of G is either in D or is adjacent to a vertex in D. The minimum cardinality of a dominating set of G is called the domination number of G, $\gamma(G)$.

The Corona of the graphs G and H denoted as GoH is the graph obtained by taking one copy of G and $|V(G)|$ copies of H, then joining the i^{th} vertex of G to every vertex in the i^{th} copy of H. We use the notation H^v to denote the copy of H attached to the vertex v of G. The domination number of GoH is the number of vertices in G[3].

In this paper, we study the relation between equitable domination number and outdegree equitable domination number with the domination number of a graph. We also investigate the equitable domination number and outdegree equitable domination number of the corona product of graphs.

2. Equitable And Outdegree Equitable Domination Number

Definition 2.1. *Let $G = (V, E)$ be a graph. A subset D of V is said to be an equitable dominating set of G, if for every $v \in V - D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|deg(u) - deg(v)| \leq 1$. The minimum cardinality of an equitable dominating set of G is called equitable domination number of G denoted as $\gamma^e(G)$.*

Definition 2.2. *Consider a vertex u in a dominating set D. The outdegree of u is the number of edges from u to $V - D$. A dominating set D is called the outdegree equitable dominating set if the difference between the outdegrees of any two vertices of D is atmost one. The minimum cardinality of an outdegree equitable dominating set of G is called the outdegree equitable domination number of G denoted as $\gamma_{oe}(G)$.*

Definition 2.3. *A vertex $u \in V$ is an equitable isolate of G, if $|deg(u) - deg(v)| \geq 2$ for all $v \in N(u)$.*

Remark 2.1. *An equitable isolate of a graph cannot be dominated equitably by any other vertex of the graph. Hence, every equitable dominating set of a graph contains the equitable isolates of the graph (if any exists).*

Remark 2.2. *Every equitable (outdegree equitable) dominating set of a graph is a dominating set. Hence, $\gamma(G) \leq \gamma^e(G)$ and $\gamma(G) \leq \gamma_{oe}(G)$ for any graph G.*

Remark 2.3. *If for a graph G, $deg(v) \in \{k, k+1\}$ for all vertices $v \in V(G)$ and $k \geq 1$ then $\gamma^e(G) = \gamma(G)$.*

Remark 2.4. *A universal vertex of a graph is a vertex that is adjacent to all other vertices of the graph. Trivially $\gamma_{oe}(G)$ = 1 if and only if G has a universal vertex.*

Example

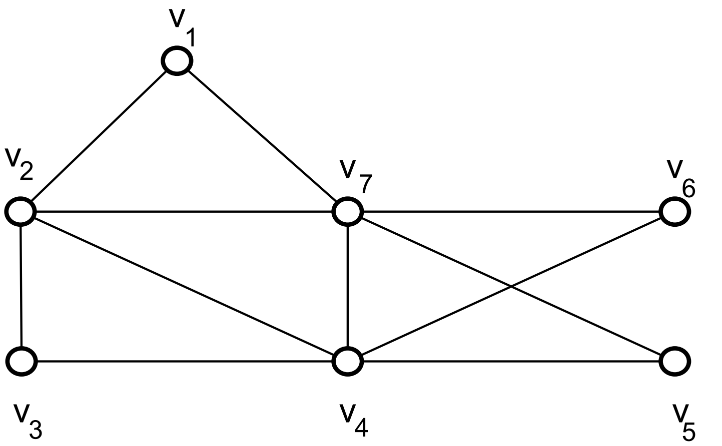

FIGURE 1. Graph G in which the equitable isolates are the vertices v_1, v_3, v_5 and v_6

In figure 1, $\{v_2, v_7\}$ is a minimum dominating as well as minimum outdegree equitable dominating set of G, whereas $\{v_1, v_2, v_3, v_5, v_6\}$ is a minimum equitable dominating set of G. Hence $\gamma(G) = 2$, $\gamma^e(G) = 5$ and $\gamma_{oe}(G) = 2$

3. MAIN RESULTS

Proposition 3.1. *For any $k \in N$, there exists a graph G such that $\gamma(G) = \gamma^e(G) = \gamma_{oe}(G) = k$.*

Proof. For $k = 1$ take $G = K_3$, then $\gamma(G) = \gamma^e(G) = \gamma_{oe}(G) = 1$.
Let $k \geq 2$. Take k copies of K_3, where the vertices of the i^{th} copy of K_3 are labelled as v_{i1}, v_{i2}, v_{i3}. Make v_{i3} adjacent to $v_{(i+1)2}$ for all i, where $1 \leq i \leq k-1$ to obtain the resulting graph G, as shown in figure 2.

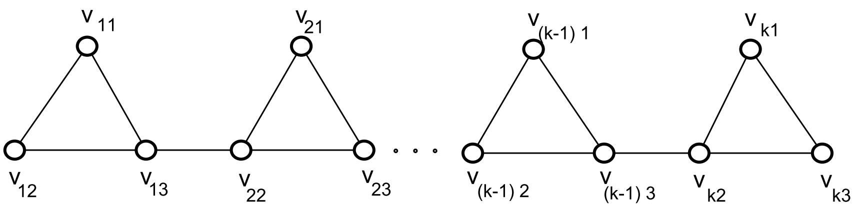

FIGURE 2

Then $\gamma(G) = \gamma^e(G) = \gamma_{oe}(G) = k$, by the set of k vertices $\{v_{11}, v_{21}, ..., v_{k1}\}$. □

Proposition 3.2. *For any graph G of order $n \geq 2$, $1 \leq \gamma_{oe}(G) \leq n - 1$.*

Proof. Consider a graph G with $n \geq 2$ vertices. Obviously, $\gamma_{oe}(G) \geq 1$. Let D be the set consisting of any $n-1$ vertices of G and let v be the vertex of G outside D. Then, clearly the outdegree of vertices in D is either 1 or 0 depending on whether the vertex in D is adjacent to v or not. Thus D is an outdegree equitable dominating set of G. Hence $\gamma_{oe}(G) \leq n-1$. □

Proposition 3.3. *Let a and b be positive integers with $a < b$ and $b - a \geq 4$. Then there exists a connected graph G such that $\gamma(G) = a$ and $\gamma^e(G) = b$.*

Proof. Take a copies of K_3 and connect them as shown below by introducing edges subsequently to obtain the graph H.

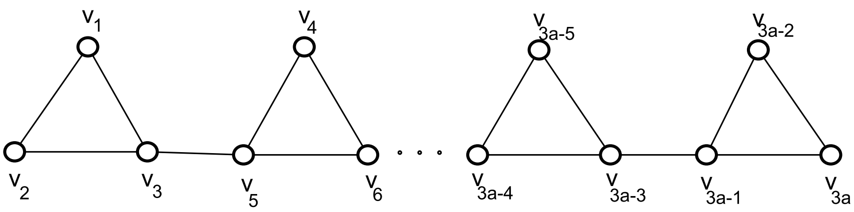

FIGURE 3. Graph H obtained by taking a copies of K_3 and connecting them by edges

Now join $b-a-1$ pendant edges to the vertex v_{3a} to obtain the resulting graph G. Clearly $\gamma(G) = a$. Now we will prove that $\gamma^e(G) = b$.

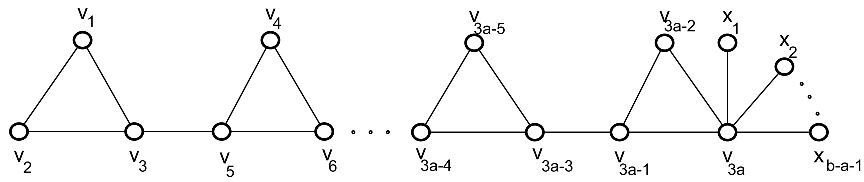

FIGURE 4. Graph G obtained from the graph H by joining $b-a-1$ pendent edges to the vertex v_{3a}

Let $D' = \{v_1, v_4, v_7 ... v_{3a-2}, v_{3a}, x_1, x_2, ... x_{b-a-1}\}$. Then $|D'| = b$. Now for each vertex $v \in V(G) - D'$, there exists a vertex $u \in D'$ such that v is adjacent to u and $|deg(u) - deg(v)| \leq 1$. Thus D' is an equitable dominating set of G. Hence

$$\gamma^e(G) \leq b \tag{1}$$

Let T be an equitable dominating set of G with minimum cardinality. Since the vertices $v_{3a}, x_1, x_2, ... x_{b-a-1}$ are equitable isolates, every equitable dominating set should contain them. Therefore $|T| \geq b - a$. Let $I = \{v_{3a}, x_1, x_2, ... x_{b-a-1}\}$. Then $T - I$ is also an equitable dominating set of $G - I$. Now $G - I$ is the graph obtained as shown in figure 5. Hence

$$\gamma^e(G - I) \leq |T - I| = |T| - (b - a) \tag{2}$$

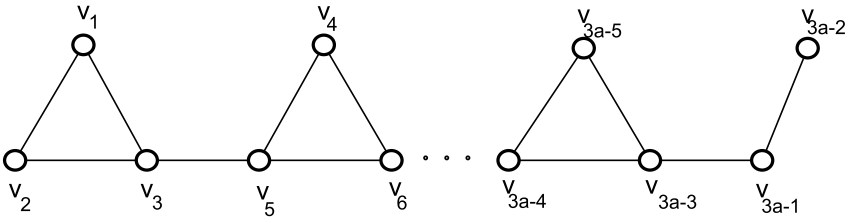

FIGURE 5. Graph $G - I$ obtained by deleting the vertices in I from G

Clearly $\gamma^e(G - I) = a$. Thus from Eq. (2) we get

$$\begin{aligned} a &\leq |T - I| \\ &\leq |T| - (b - a) \\ b &\leq |T| \\ b &\leq \gamma^e(G) \end{aligned}$$

Thus from Eq. (1) we get $\gamma^e(G) = b$. □

Corollary 3.1. *The difference $\gamma^e - \gamma$ can be made arbitrarily large.*

Proposition 3.4. *Let a and b be positive integers with $2 \leq a < b$ and $b - a \geq 2$. Then there exists a connected graph G such that $\gamma(G) = a$ and $\gamma_{oe}(G) = b$.*

Proof. Take a path on a vertices say $P_a = v_1v_2v_3...v_{a-1}v_a$. Now join pendant edges to every vertex of the path except the vertex v_a. Join $b - a + 1$ pendant edges to the vertex v_a to obtain the resulting graph G.

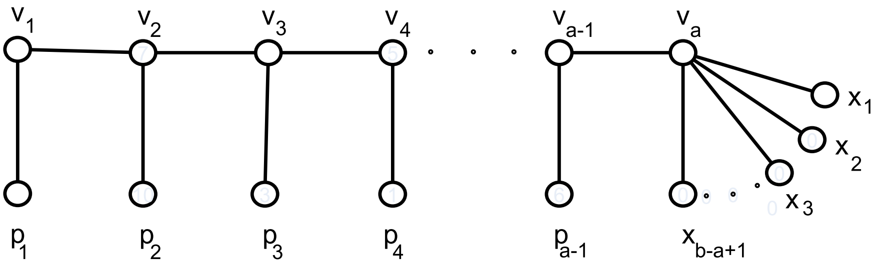

FIGURE 6. Graph G obtained by joining pendant edges to the vertices $v_1, v_2, v_3, ...v_{a-1}$ and $b - a + 1$ pendent edges to the vertex v_a

Clearly $\gamma(G) = a$. Now we will prove that $\gamma_{oe}(G) = b$.
Let $D = \{v_1, v_2, ...v_{a-1}, x_1, x_2, ...x_{b-a+1}\}$. Then $|D| = b$. Clearly D is a dominating set of G. Now the outdegree of each vertex in D is less than or equal to 2. Note that the outdegree of v_{a-1} is 2 and the outdegree of all other vertices in D is 1. Hence D is an outdegree equitable dominating set of G. Thus

$$\gamma_{oe}(G) \leq b \tag{3}$$

If possible let T be an outdegree equitable dominating set of G with $|T| < b$. Since the vertices $p_1, p_2, p_3, ...p_{a-1}$ are pendant vertices and none of these vertices have a common neighbour, T should either contain $p_1, p_2, p_3, ...p_{a-1}$ or their neighbours. In both cases T should contain at least $a-1$ vertices. The number of vertices in T excluding the above mentioned vertices is $< b-(a-1) = b-a+1$ and these vertices dominate the remaining $b-a+2$ vertices $v_a, x_1, x_2, ...x_{b-a+1}$ of G. Among these vertices, $x_1, x_2, ...x_{b-a+1}$ are pendant vertices, all having a common neighbour v_a. Inorder for T to be a dominating set of G, v_a should be a member of T. But this then violates the outdegree equitability condition of T, which is a contradiction. Thus there exists no outdegree equitable dominating set of G with cardinality less than b. Thus from Eq. (3), $\gamma_{oe}(G) = b$. □

Corollary 3.2. *The difference $\gamma_{oe} - \gamma$ can be made arbitrarily large.*

3.1. **Corona of Graphs.**

Theorem 3.1. *Let G and H be two graphs and let m be the order of G. Then $\gamma^e(GoH) \leq \gamma^e(G) + m\gamma^e(H)$.*

Proof. Suppose $v_1, v_2, ...v_m$ be the vertices of G. Let D^{v_i} be a minimum equitable dominating set of H^{v_i}, the copy of H attached to the vertex v_i and let D' be a minimum equitable dominating set of G. Let $D = D' \cup D^{v_1} \cup D^{v_2} \cup D^{v_3} ... \cup D^{v_m}$. Then clearly D is an equitable dominating set of GoH . Hence

$$\gamma^e(GoH) \leq |D| = \gamma^e(G) + m\gamma^e(H).$$

□

Corollary 3.3. *Let G be a graph with m vertices . Then $\gamma^e(GoK_n) \leq m + \gamma^e(G)$.*

Corollary 3.4. *Let G be a graph with m vertices. Then $\gamma^e(GoP_n) \leq \gamma^e(G) + m\lceil \frac{n}{3} \rceil$ and equality holds if $n \geq 4$.*

Proof. From [10], $\gamma^e(P_n) = \lceil \frac{n}{3} \rceil$.
From Theorem 3.1

$$\gamma^e(GoP_n) \leq \gamma^e(G) + m\lceil \frac{n}{3} \rceil.$$

Suppose $n \geq 4$. If possible let

$$\gamma^e(GoP_n) < \gamma^e(G) + m\lceil \frac{n}{3} \rceil. \tag{4}$$

Let $v_1, v_2, ...v_m$ be the vertices of G. Let $u \in V({P_n}^{v_i})$, the copy of P_n attached to the vertex v_i. Then $d_{GoP_n}(u)$ is either 2 or 3. Hence

$$|d_{GoP_n}(v_i) - d_{GoP_n}(u)| \geq 2.$$

which implies that v_i does not equitabily dominate ${P_n}^{v_i}$. As $\gamma^e[P_n] = \lceil \frac{n}{3} \rceil$, to dominate m copies of P_n we require $m\lceil \frac{n}{3} \rceil$ vertices and also none of these vertices equitably dominate any vertex of G. But then Eq. (4) implies G is equitably dominated by a set of vertices whose cardinality is less than $\gamma^e(G)$ which is a contradiction. Hence for $n \geq 4$,

$$\gamma^e(GoP_n) = \gamma^e(G) + m\lceil \frac{n}{3} \rceil.$$

□

Theorem 3.2. *Let G and H be two graphs, then $\gamma_{oe}(GoH) = |V(G)|$.*

Proof. Let $|V(G)| = m$. Consider the graph GoH. Now $V(G)$ is a dominating set of GoH and each vertex in $V(G)$ is having the same outdegree which is equal to the number of vertices in H. Therefore $V(G)$ is an outdegree equitable dominating set of GoH. Thus

$$\gamma_{oe}(GoH) \leq m \tag{5}$$

But we know that

$$\gamma(GoH) = m \text{ and } \gamma(GoH) \leq \gamma_{oe}(GoH) \tag{6}$$

Thus from Eq. (5) and Eq. (6) we get $\gamma_{oe}(GoH)$ =m. □

Theorem 3.3. *Let G be a graph with m vertices out of which p vertices are pendant vertices. If all the pendant vertices are equitable isolates of G, then $\gamma^e(GoK_n) = \gamma^e(G) + m - p$.*

Proof. Let $v_1, v_2, ...v_m$ be the vertices of G. Let u_i be any vertex of ${K_n}^{v_i}$, the copy of K_n attached to the vertex v_i where $i = 1, 2, ...m$. Let D' be an equitable dominating set of G with minimum cardinality. Hence $|D'| = \gamma^e(G)$. Let $D = D' \cup \{u_i : v_i$ is not a pendant vertex of $G\}$. Then

$$|D| = \gamma^e(G) + m - p. \tag{7}$$

Now we claim that D is an equitable dominating set of GoK_n. Let $x \in V - D$ where $V = V(GoK_n)$. Then either $x \in V(G)$ or $x \in {K_n}^{v_i}$, for some i. If $x \in V(G)$, then there exists $x' \in D'$ such that $xx' \in E(G)$ and $|d_G(x) - d_G(x')| \leq 1$. In GoK_n, $d(x)$ and $d(x')$ are both increased by n. Hence $|d_{GoK_n}(x) - d_{GoK_n}(x')| \leq 1$ Now let $x \in {K_n}^{v_i}$, for some i. If that v_i is a pendant vertex then $xv_i \in E(GoK_n)$ and $|d_{GoK_n}(x) - d_{GoK_n}(v_i)| = |n - (n+1)| = 1$. Suppose v_i is not a pendant vertex of G, then there exists $u_i \in D$ such that $xu_i \in E(GoK_n)$ and $|d_{GoK_n}(x) - d_{GoK_n}(u_i)| = |n - n| = 0$. Thus in all cases, there exists some $v \in D$ such that $xv \in E(GoK_n)$ and $|d_{GoK_n}(x) - d_{GoK_n}(v)| \leq 1$ which implies that D is an equitable dominating set of GoK_n and from Eq. (7) we get

$$\gamma^e(GoK_n) \leq |D| = \gamma^e(G) + m - p. \tag{8}$$

If possible let

$$\gamma^e(GoK_n) = t < \gamma^e(G) + m - p. \tag{9}$$

We know that $m-p$ vertices are required to equitably dominate the copies of K_n attached to the $m-p$ vertices of G with degree ≥ 2. Now the vertices which remain to be equitably dominated are the vertices of G and the vertices in the copies of K_n attached to the pendant vertices of G. These vertices are dominated by the $t - (m - p)$ vertices in the equitable dominating set of GoK_n with minimum cardinality. From Eq. (9), $t - (m - p) < \gamma^e(G)$. That is G is equitably dominated by a set of vertices whose cardinality is less than $\gamma^e(G)$ which is a contradiction. Hence $\gamma^e(GoK_n) = \gamma^e(G) + m - p$. □

Theorem 3.4. *Let G and H be two graphs. Then $\gamma^e(GoH) = 1$ if and only if the following conditions hold.*

(a) G is identical to K_1 .

(b) either $\gamma^e(H) = 1$ or H is an n regular graph, for some even n with $|V(H)| = n+2$.

Proof. (a) Suppose $\gamma^e(GoH) = 1$. To prove that G is identical to K_1 , it is enough to prove that G has only one vertex. If possible, let $|V(G)| = m \geq 2$. Since $\gamma(GoH) \leq \gamma^e(GoH)$, we get

$$2 \leq m \leq \gamma^e(GoH)$$

which is a contradiction to the assumption $\gamma^e(GoH) = 1$. Hence G has only one vertex.

(b) From part (a), G is identical to K_1 and by assumption $\gamma^e(K_1oH) = 1$. Let $\{u\}$ be an equitable dominating set of K_1oH (with minimum cardinality). Then either $u \in V(K_1)$ or $u \in V(H)$. In both cases

$$d_{K_1oH}(u) = |V(H)| = t(say).$$

Since $\{u\}$ is an equitable dominating set of K_1oH, degree of other vertices in K_1oH is either t or $t-1$.
i.e

$$either \quad d_{K_1oH}(v) = t \quad or \quad d_{K_1oH}(v) = t-1, \quad for \;\; all \;\; v \in V(K_1oH)$$

$$either \quad d_H(v) = t-1 \quad or \quad d_H(v) = t-2, \quad for \;\; all \;\; v \in V(H)$$

Case 1.
Suppose $d_H(v) = t-1$, for all $v \in V(H)$. Then each vertex of H equitably dominates H which implies $\gamma^e(H)$=1.

Case 2.
Suppose G is an irregular graph with either $d_H(v) = t-1$ or $d_H(v) = t-2$, for $v \in V(H)$. Here also there exists a vertex say $v' \in H$ with $d(v') = t-1$ and $|d(v') - d(v)| \leq 1$ for all $v \in V(H)$. Therefore $\{v'\}$ forms an equitable dominating set of H and hence $\gamma^e(H) = 1$.

Case 3.
Suppose $d_H(v) = t-2$, for all $v \in V(H)$. Then t cannot be odd, because if t is odd, then the sum of degree of vertices in H is equal to $t(t-2)$, an odd number which is a contradiction to the fact that sum of degree of vertices in H is equal to two times the number of edges in H, which is an even number. Therefore t is even. Let $n = t-2$. Then n is also even and we get H is an n regular graph with $|V(H)| = t = n+2$.

Conversely suppose conditions (a) and (b) hold. Suppose in (b) H satifies the condition $\gamma^e(H) = 1$. Then an equitable dominating set of H with minimum cardinality itself is an equitable dominating set of GoH with minimum cardinality. Thus $\gamma^e(GoH) = 1$. Now suppose in (b) H satifies the condition that H is an n regular graph, for some even n with $|V(H)| = n+2$. Then the vertex of K_1 equitably domintes all vertices of K_1oH. Therefore $\gamma^e(K_1oH) = 1$. □

4. Conclusions

In this paper we found that there are graphs for which the domination number, equitable domination number and outdegree equitable domination numbers coincide. Although $\gamma(G) \leq \gamma^e(G)$ and $\gamma(G) \leq \gamma_{oe}(G)$, from corollary 3.1 and 3.2 we conclude that the difference $\gamma^e - \gamma$ as well as $\gamma_{oe} - \gamma$ can be made arbitrarily large. We also obtained sharp upperbounds for equitable domination number of corona product of graphs and exact value for outdegree equitable domination number of corona product of graphs.

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Thasneem T.R graduated from Maharajas College, Ernakulam with the first position in Mahatma Gandhi University, Kerala, India. Currently, she is working as an Assistant Professor and doing research in the Department of Mathematics, Maharajas College, Ernakulam under the supervision of Dr. Manju K. Menon in the field of Graph Theory.

Dr. Manju K. Menon graduated from Union Christian College, Kerala, India. She completed her Ph.D in Graph Theory from Cochin University of Science and Technology, India in 2010. Currently, she is working as an Assistant Professor in the Department of Mathematics, St. Paul's College, Kalamassery, Kerala, India, where she started her career in 2005.