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CONTROLLABLE SOLITON AND BREATHER INTERACTIONS FOR FIFTH-ORDER VARIABLE COEFFICIENT NONLINEAR SCHRÖDINGER EQUATION IN OPTICAL FIBERS

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Abstract. In this article, we investigate fifth-order variable coefficient nonlinear Schrödinger equation, which govern optical pulse in fiber optics and obtain soliton and breather solutions by applying Darboux transformation. Soliton and breather are responsible for data transmission over the long distance without loss of power. In optical fibers, control soliton and breather interactions have important application in signal processing and transmission. In solutions, With different values of coefficients of dispersion we obtain parabolic, cubic and periodical oscillating soliton. By controlling spectral parameter λ, we obtain different types of interactions like head on, overtaking and parallel on soliton. The effects of coefficients of dispersion and spectral parameter λ on the pattern of breather also presented.

Keywords: Soliton Interactions, Breather Interactions, Optical Soliton, Fiber Optics.

AMS Subject Classification: 35C08, 35P30.

1. Introduction

In different fields, soliton and breather play significant role specially in molecular biology, plasma physics, oceanography and optical fibers [1, 2, 3, 4, 5, 6, 7, 8]. Particularly, when we discuss their control in different media then their significance become more clear. As localized wave structures solitons have been largely investigated in several nonlinear mechanisms like in optics. Optical solitons have fruitful applications in different optical control and photonic signal processing. Fiber optics is one of the such field where, the controllable soliton and breather are responsible for transmission of signals to long distance with out loss of energy. Attenuation, dispersion and other nonlinear effects causes difficulties during propagation of optical signals. A well balance between dispersion and nonlinear effects generate soliton in fiber optics [9, 10, 11, 26, 29]. Nonlinear Schrödinger equation (NLS) is govern the soliton pulse transmission in optical fibers. Different researchers obtained soliton solutions of (NLS) by using different methods like Hirota's bi linear method, Darboux transformation (DT), inverse scattering transformation (IST) and Bäcklund transformation (BT) [12, 13, 14, 15, 16, 17, 18, 19, 24, 25, 27, 28]. The pulse

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in fiber optics gets shorter, when the intensity of optical field become stronger. In this situation we govern the behavior of pulse by using higher-order (NLS). By using (DT), we will investigate fifth-order variable coefficient (NLS) equation proposed in [20] as

$$ir_t + b(t)(r_{xx} + 2r|r|^2) - ia(t)H[r(x,t)] + c(t)P[r(x,t)] - id(t)Q[r(x,t)] = 0 \qquad (1)$$

where

$$\begin{aligned}
H[r(x,t)] &= r_{xxx} + 6|r|^2 r_x, \\
P[r(x,t)] &= r_{xxxx} + 8|r|^2 r_{xx} + 6r|r|^4 + 4r|r_x|^2 + 6r_x^2 r^* + 2r^2 r_{xx}^*, \\
Q[r(x,t)] &= r_{xxxxx} + 10|r|^2 r_{xxx} + 10(r|r|^2)_x + 20r^* r_x r_{xx} + 30|r|^4 r_x.
\end{aligned}$$

where $r(x,t)$ represents the amplitude, the subscripts denoted the corresponding derivatives with respect to spatial coordinate x and temporal coordinate t and asterisk $*$ represent complex conjugate. $a(t)$, $b(t)$, $c(t)$ and $d(t)$ represent third order dispersion, group velocity dispersion, forth order dispersion and fifth order dispersion coefficients respectively. Also $H[r(x,t)]$, $P[r(x,t)]$ and $Q[r(x,t)]$ represent Hirota's operator, Lakshmanan Porsezian Daniel (LPD) operator and quintic operator having orders third, forth and fifth respectively.

The equation (1) reduced to (NLS) [21], (LPD) [22] and Hirota's equation [23] by setting coefficients $a = c = d = 0$, $a = d = 0$ and $c = d = 0$ which are describe in optical fiber applications, ultrashort optical pulse propagation and correction to cubic non linearity in water waves respectively. In this paper, we will apply (DT) to fifth order variable coefficient (NLS) equation to obtain soliton and breather solutions. By controlling different parameters in solutions, we will investigate the interactions between soliton and breather. The division of sections in this article is as follows.

In section 2, we will construct N-soliton solution for equation (1) by applying (DT). With the help of N-soliton solution, we will derive one and two soliton solutions and their interactions in section 3. In section 4, we will obtain breather solution and give their characteristics by controlling parameters. In section 5, we will give conclusions and suggestions that how the controlling parameters in solutions will help to control the optical pulse in fiber optics for stable propagation.

2. Lax pair and N-soliton solution

The Lax pair for the system in equation (1) is given as

$$\begin{aligned}
&\Psi_x = L\Psi, \qquad L = i\begin{pmatrix} \lambda & r^*(x,t) \\ r(x,t) & -\lambda \end{pmatrix}, \\
&\Psi_t = M\Psi\,, \;\; M = \sum_{j=0}^{5} \lambda^j M_j, \;\; M_j = \begin{pmatrix} A_j & B_j^* \\ B_j & -A_j \end{pmatrix}.
\end{aligned} \qquad (2)$$

where

$$A_0 = -\frac{1}{2}|r|^2 - 3c(t)|r|^4 - ia(t)\Upsilon_1 - c(t)\Upsilon_2 - id(t)(r^*_{xxx}r - r^*_{xx}r_x + r_{xx}r^*_x - r_{xxx}r^*)$$
$$- 6id(t)(r^*_t r - r_t r^*)|r|^2,$$
$$A_1 = 2a(t)|r|^2 + 6d(t)|r|^4 - 2ic(t)\Upsilon_1 + 2d(t)\Upsilon_2,\ A_2 = 2b(t) + 4c(t)|r|^2 + 4id(t)\Upsilon_1,$$
$$A_3 = -4a(t) - 8d(t)|r|^2,\ A_4 = -8c(t),\ A_5 = 16d(t),$$
$$B_0 = 2a(t)|r|^2 r + 6d(t)|r|^4 r + i\frac{1}{2}r_x + 6ic(t)|r|^2 r_x + a(t)r_{xx} + 2d(t)r^*_{xx}r^2 + 4d(t)|r_x|^2 r$$
$$+ 6d(t)(r_x)^2 r^* + 8d(t)r_{xx}|r|^2 + ic(t)r_{xxx} + d(t)r_{xxxx},$$
$$B_1 = r + 4c(t)|r|^2 r - 2ia(t)r_x - 12id(t)|r|^2 r_x + 2c(t)r_{xx} - 2id(t)r_{xxx},$$
$$B_2 = -4a(t)r - 8d(t)|r|^2 r - 4ic(t)r_x - 4d(t)r_{xx},\ B_3 = -8c(t)r + 8id(t)r_x,\ B_4 = 16d(t)r,$$
$$B_5 = 0, \Upsilon_1 = r^*_x r - r_x r^*,\ \Upsilon_2 = r^*_{xx}r - |r_x|^2 + r_{xx}r^*.$$

In equation (2), $\Psi = (\Psi_1, \Psi_2)^T$ represent the vector function and Ψ_1 and Ψ_2 are the function of x and t. The T represent the transpose of the matrix. λ denoted the spectral parameter which is independent of x and t. The compatibility condition $L_x - M_t + LM - ML = 0$ give the system in equation (1).

By applying gauge transformation $\Psi[1] = D[1]\Psi$ the Lax pair in equation (2) transforms the matrices into $\Psi[1]_x = L[1]\Psi[1]$ and $\Psi[1]_t = M[1]\Psi[1]$ having same form as L and M except that r, r^* are replaced by $r[1]$, $r[1]^*$. The matrix $D[1]$ can be derived as

$$D[1] = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} \psi_{1,1} & \psi^*_{2,1} \\ \psi_{2,1} & -\psi^*_{1,1} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda^*_1 \end{bmatrix} \begin{bmatrix} \psi_{1,1} & \psi^*_{2,1} \\ \psi_{2,1} & -\psi^*_{1,1} \end{bmatrix}^{-1} \tag{3}$$

where $(\psi_{1,1}, \psi_{2,1})^T$ and $\left(\psi^*_{2,1}, -\psi^*_{1,1}\right)^T$ are the solutions of equation (2) with $\lambda = \lambda_1$ and $\lambda = \lambda^*_1$. $\psi_{1,1}$ and $\psi_{2,1}$ are complex functions of x and t.

Then the DT for the system in equation (1) can be written as

$$r[1] = r[0] + \frac{(\lambda^*_1 - \lambda_1)\,\psi^*_{1,1}\psi_{2,1}}{\psi_{1,1}\psi^*_{1,1} + \psi_{2,1}\psi^*_{2,1}} \tag{4}$$

where $r[0]$ represent the seed solution for system in equation (1).

By setting the eigenfunctions $(\psi_{1,1}, \psi_{2,1})^T, (\psi_{1,2}, \psi_{2,2})^T, ..., (\psi_{1,N}, \psi_{2,N})^T$ for N distinct solutions of Lax pair in equation (2) with $\lambda_1, \lambda_2, ...\lambda_N$ are eigenvalues and $\lambda^{'}_l s$ $(l = 1, 2, ...N)$, and $\psi^{'}_{1,N}s$ and $\psi^{'}_{2,N}s$ are functions of x and t.

Then the Nth iterated DT for the system in equation (1) can be expressed as

$$r[N] = r[0] + 2\sum_{p=1}^{N} \frac{\left(\lambda^*_p - \lambda_p\right)\psi^*_{1,p}[p-1]\psi_{2,p}[p-1]}{\psi_{1,p}[p-1]\psi^*_{1,p}[p-1] + \psi_{2,p}[p-1]\psi^*_{2,p}[p-1]} \tag{5}$$

also

$$\Psi[N] = D[N]D[N-1]....D[1]\Psi,$$
$$\begin{pmatrix} \psi_{1,1}[0] \\ \psi_{2,1}[0] \end{pmatrix} = \begin{pmatrix} \psi_{1,1} \\ \psi_{2,1} \end{pmatrix},\ \begin{pmatrix} \psi_{1,p}[p-1] \\ \psi_{2,p}[p-1] \end{pmatrix} = (D[p-1]....D[p-1])\,|_{\lambda=\lambda_p} \begin{pmatrix} \psi_{1,p} \\ \psi_{2,p} \end{pmatrix} \tag{6}$$

The Darboux matrix $D[p]$ for Nth iterated DT in equation (5) is

$$D[p]=\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}-\begin{bmatrix}\psi_{1,p}[p-1] & \psi^*_{2,p}[p-1]\\ \psi_{2,p}[p-1] & -\psi^*_{1,p}[p-1]\end{bmatrix}\begin{bmatrix}\lambda_p & 0\\ 0 & \lambda^*_p\end{bmatrix}\begin{bmatrix}\psi_{1,p}[p-1] & \psi^*_{2,p}[p-1]\\ \psi_{2,p}[p-1] & -\psi^*_{1,p}[p-1]\end{bmatrix}^{-1} \tag{7}$$

By using Nth iterated DT, we can obtain soliton and breather solutions for the system in equation (1).

3. Soliton solutions

In this section, we will obtain soliton solutions for the system in equation (1). By inserting trivial seed solution $r[0]=0$ into the Lax pair in equation (2), we can obtain the solution for the Lax pair as

$$\psi_{1,1}=e^{i(2\Delta_1+\lambda_1 x)},\qquad \psi_{1,2}=e^{-i(2\Delta_1+\lambda_1 x)} \tag{8}$$

where

$$\Delta_1=\lambda_1^2\int\left[-2\lambda_1 a(t)+\lambda_1 b(t)-4\lambda_1^2 c(t)+8\lambda_1^3 d(t)\right]dt$$

By inserting the values in equation (8) along with trivial seed solution $r[0]=0$ into equation (4), we obtain one soliton solution as

$$r[1]=-\frac{2\left(\lambda_1-\lambda_1^*\right)}{e^{2i(2\Delta_1+\lambda_1 x)}+e^{2i\left(2\Delta_1^*+\lambda_1^* x\right)}} \tag{9}$$

We can obtain the intensity of r[1] by rewriting one soliton solution in equation (9) with $\lambda_1=c+id$, where a and b are real constants.

$$|r[1]|^2=4b^2\sec h^2[2b(x-Q_0Q_1+4aQ_3)] \tag{10}$$

Where $Q_0=4(a^2-b^2)$, $Q_1=\int[(-12a^2+4b^2)d(t)+4ac(t)+a(t)]dt$, $Q_3=\int[(8a^3-24ab^2)d(t)-Q_0c(t)-2a\ a(t)+b(t)]dt$ From equation (10), we can easily obtain that the amplitude of soliton is $|2b|$. From which we can understand that the amplitude of soliton is only related to the imaginary part of λ_1, which is spectral parameter.

The concept of characteristic line is introduce in order to obtain soliton velocity. From equation (10), the characteristic line can be written as $2b(x-Q_0Q_1+4aQ_3)=$constant. We can get soliton $v=4(3a^2-b^2)a(t)+32a(a^2-b^2)c(t)+16(-5a^4+10a^2b^2-b^4)d(t)-4ab(t)$ after differentiating characteristic line with respect to t to the both sides. From which we govern that soliton velocity is depend upon $\lambda_1=a+ib$ and $a(t)$, $b(t)$, $c(t)$ and $d(t)$ which are spectral parameter and coefficients of dispersion

When pulse propagate in optical fiber, then due to dispersion optical signal spread out and consequently the amplitude of optical decrease. So by controlling spectral parameter and coefficients of dispersion, we can maintain the amplitude of the pulse, hence soliton formed which are responsible for data transmission over long distance with out loss of intensity.

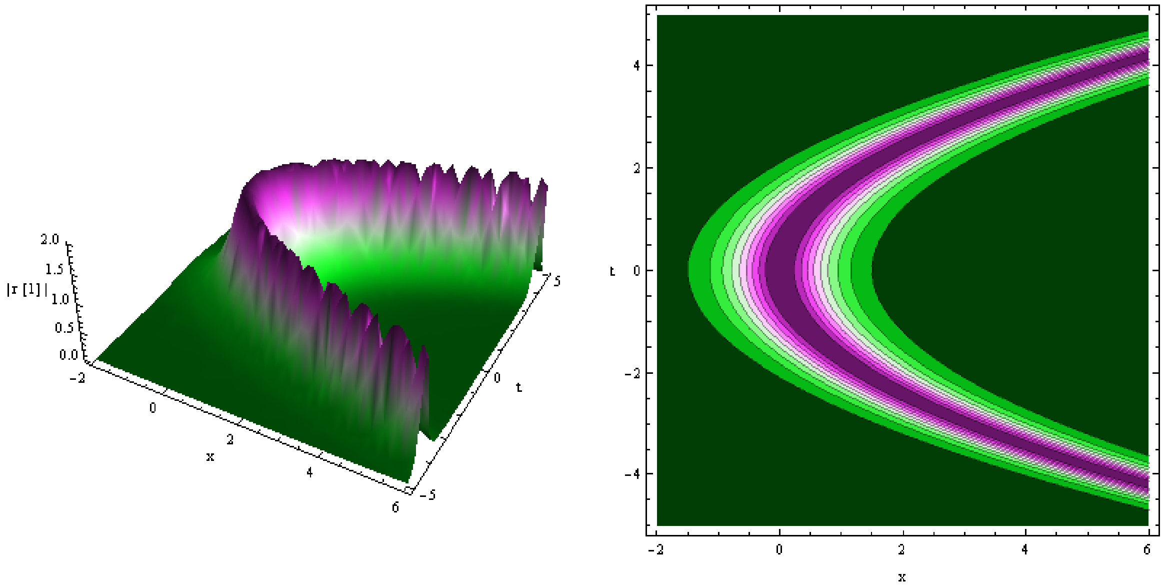

Figure: 1 One soliton solution as computed from Eq. (9) with parameters $\lambda_1 = 1 + i$, $a(t) = b(t) = c(t) = d(t) = 0.01t$

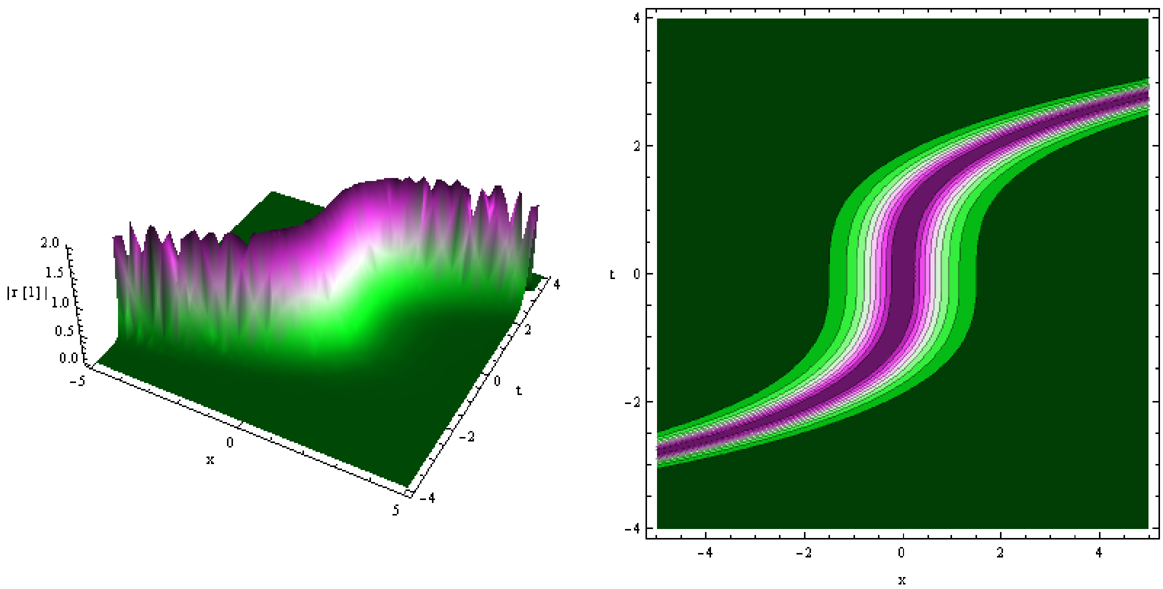

Figure: 2 One soliton solution as computed from Eq. (9) with parameters $\lambda_1 = 1 + i$, $a(t) = b(t) = c(t) = d(t) = 0.01t^2$

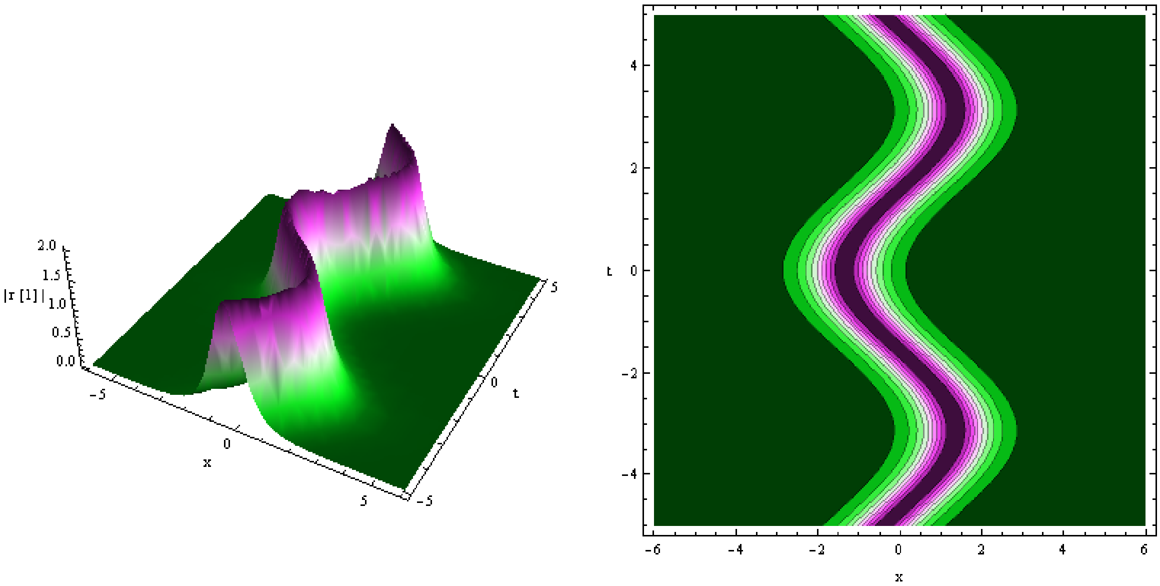

Figure: 3 One soliton solution as computed from Eq. (9) with parameters $\lambda_1 = 1 + i$, $a(t) = b(t) = c(t) = d(t) = 0.02\sin(t)$

In Figure: 1, by choosing parameters as describe in caption the one soliton solution in equation (9) shows a parabolic soliton. When we change the parameters from $a(t) = b(t) = c(t) = d(t) = 0.01t^2$ to $a(t) = b(t) = c(t) = d(t) = 0.02\sin(t)$, we obtain cubic and periodical oscillating soliton respectively as shown in Figure: 2 and 3.

By repeating above method and inserting $\psi_{2,1}$ and $\psi_{2,2}$ with $\lambda = \lambda_2$ into equation (5) we obtain two soliton solution as

$$r[2] = \frac{2F}{G} \tag{11}$$

where

$$\begin{aligned}
F &= E_0 e^{2i(2\Delta_1+\lambda_1 x)} + E_1 e^{2i(2\Delta_2+\lambda_2 x)} + E_2 e^{2i\left(2\Delta_2^*+\lambda_2^* x\right)} + E_3 e^{2i\left(2\Delta_1^*+\lambda_1^* x\right)}, \\
G &= E_4 e^{2i\left[2\left(\Delta_1+\Delta_1^*\right)+\left(\lambda_1+\lambda_1^*\right)x\right]} + E_5 e^{2i\left[2\left(\Delta_2+\Delta_1^*\right)+\left(\lambda_2+\lambda_1^*\right)x\right]} + E_6 e^{2i\left[2\left(\Delta_1+\Delta_2^*\right)+\left(\lambda_1+\lambda_2^*\right)x\right]} \\
&\quad + E_7 e^{2i\left[2\left(\Delta_2+\Delta_2^*\right)+\left(\lambda_2+\lambda_2^*\right)x\right]} + E_8 e^{2i[2(\Delta_1+\Delta_2)+(\lambda_1+\lambda_2)x]} + E_9 e^{2i\left[2\left(\Delta_1^*+\Delta_2^*\right)+\left(\lambda_1^*+\lambda_2^*\right)x\right]}, \\
\Delta_2 &= \lambda_2^2 \int \left[-2\lambda_2 a(t) + \lambda_2 b(t) - 4\lambda_2^2 c(t) + 8\lambda_2^3 d(t)\right] dt, \\
E_0 &= \left(\lambda_1^* - \lambda_2^*\right)\left(\lambda_2 - \lambda_1^*\right)\left(\lambda_2 - \lambda_2^*\right), \; E_1 = -\left(\lambda_1^* - \lambda_2^*\right)\left(\lambda_1 - \lambda_1^*\right)\left(\lambda_1 - \lambda_2^*\right), \\
E_2 &= \left(\lambda_1 - \lambda_2\right)\left(\lambda_1 - \lambda_1^*\right)\left(\lambda_2 - \lambda_1^*\right), \; E_3 = -\left(\lambda_1 - \lambda_2\right)\left(\lambda_1 - \lambda_2^*\right)\left(\lambda_2 - \lambda_2^*\right), \\
E_4 &= E_7 = \left(\lambda_1 - \lambda_1^*\right)\left(\lambda_2 - \lambda_2^*\right), \; E_5 = E_6 = -\left(\lambda_2 - \lambda_1^*\right)\left(\lambda_1 - \lambda_2^*\right), \\
E_8 &= E_9 = \left(\lambda_1 - \lambda_1\right)\left(\lambda_1^* - \lambda_2^*\right).
\end{aligned}$$

In multimode optical fiber, multiple modes of light propagate. When optical signals propagate in the form of soliton, than during propagation soliton keeps its shape unchanged even before and after interactions, which is very helpful for huge amount of data transmission without distortion of signals.

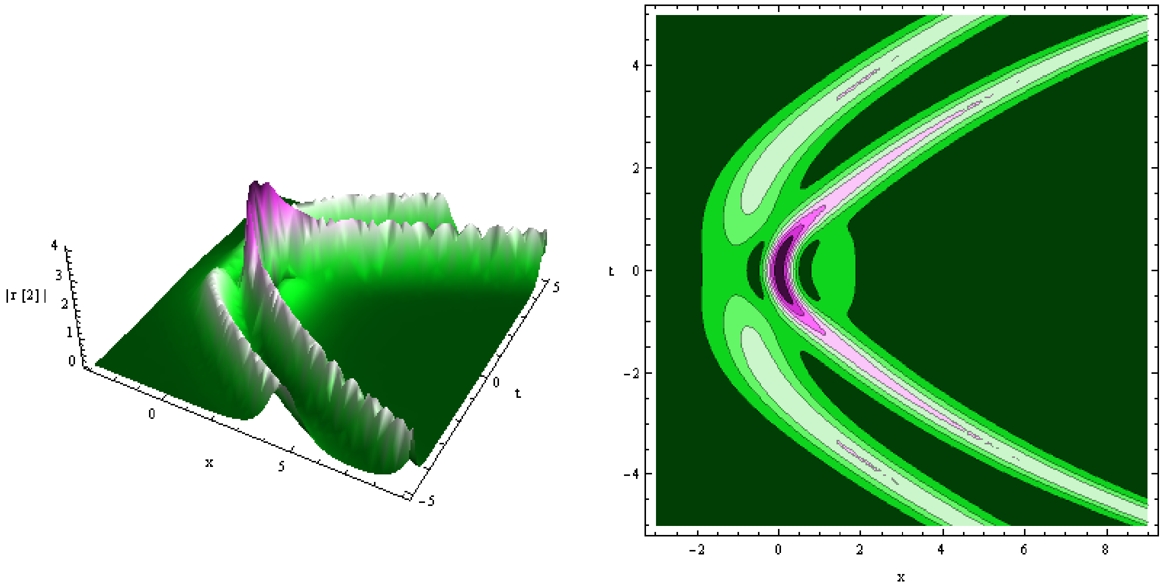

Figure: 4 Interaction of two soliton solution as computed from Eq. (11) with parameters $\lambda_1 = 1 + i$, $\lambda_2 = \frac{4}{5} + i$, $a(t) = b(t) = c(t) = d(t) = 0.01t$

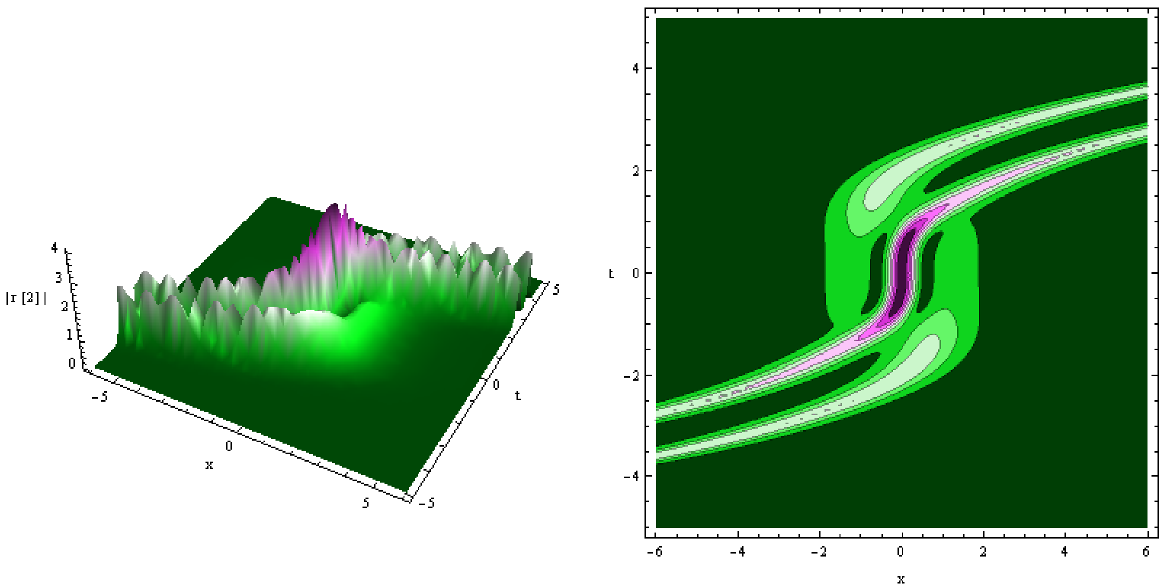

Figure: 5 Interaction of two soliton solution as computed from Eq. (11) with parameters $\lambda_1 = 1 + i$, $\lambda_2 = \frac{4}{5} + i$, $a(t) = b(t) = c(t) = d(t) = 0.01t^2$

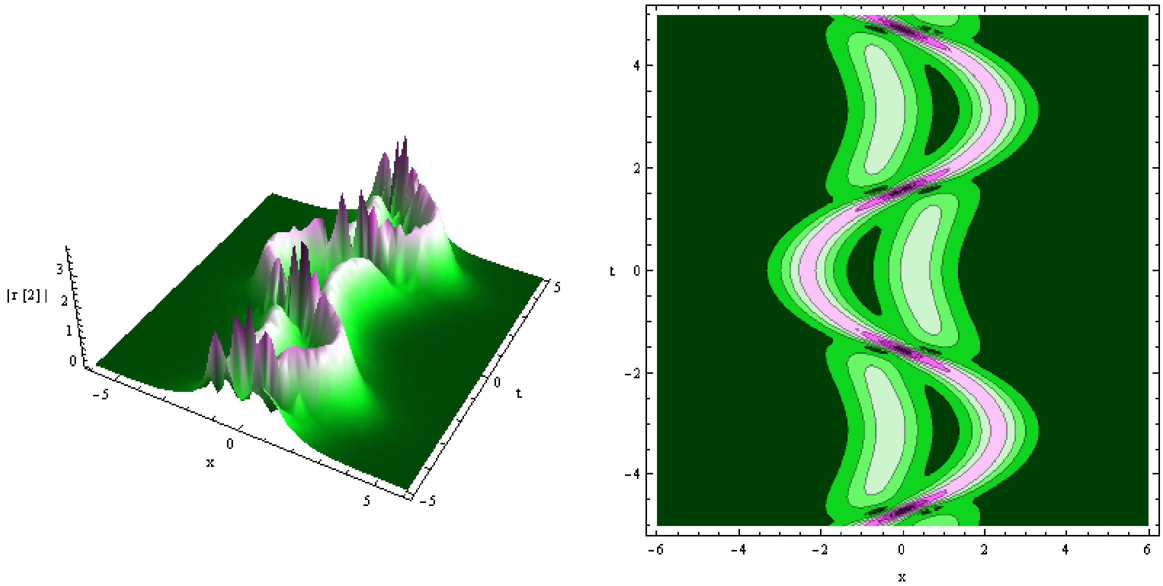

Figure: 6 Interaction of two soliton solution as computed from Eq. (11) with parameters $\lambda_1 = 1 + i$, $\lambda_2 = \frac{4}{5} + i$, $a(t) = b(t) = c(t) = d(t) = 0.02\sin(t)$

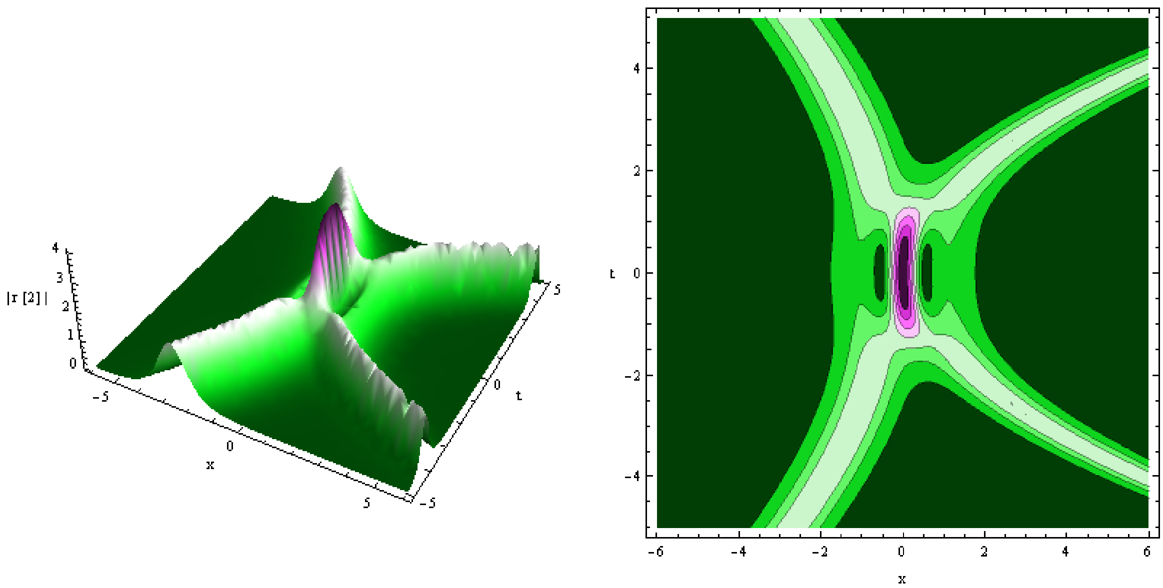

Figure: 7 Interaction of two soliton solution as computed from Eq. (11) with parameters $\lambda_1 = 1 + i$, $\lambda_2 = \frac{1}{100} + i$, $a(t) = b(t) = c(t) = d(t) = 0.01t$

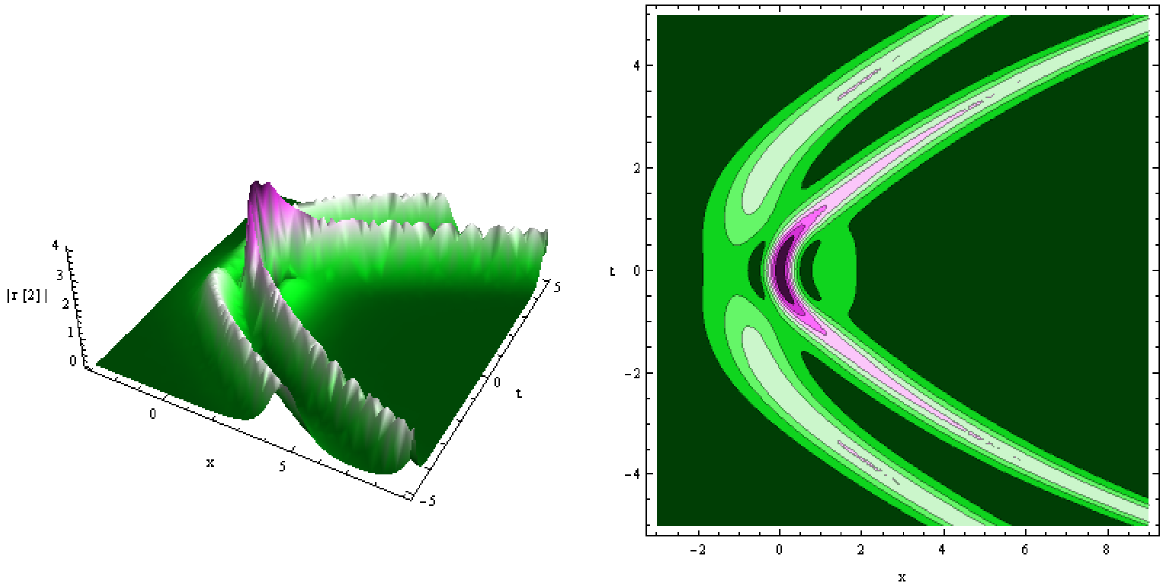

Figure: 8 Interaction of two soliton solution as computed from Eq. (11) with parameters $\lambda_1 = 1 + i$, $\lambda_2 = \frac{4}{5} + i$, $a(t) = b(t) = c(t) = d(t) = 0.01t$

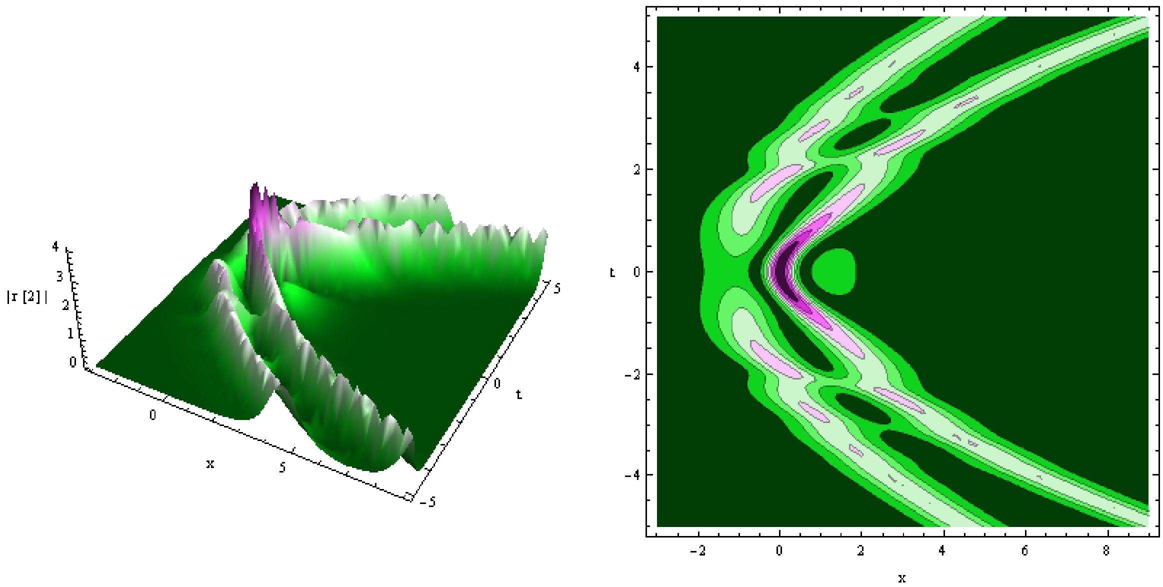

Figure: 9 Interaction of two soliton solution as computed from Eq. (11) with parameters $\lambda_1 = 1 + i$, $\lambda_2 = \frac{7}{5} + i$, $a(t) = b(t) = c(t) = d(t) = 0.01t$

In Figure: 4, the interaction between two parabolic soliton can easily be seen. By controlling parameters $a(t) = b(t) = c(t) = d(t) = 0.01t$ which are coefficients of dispersion with $\lambda_2 = \frac{4}{5} + i$, we can see that the soliton shape remain unchanged after interaction except phase shift, which describe that the interaction is elastic. Similarly in Figure: 5 and 6 the interactions can be seen between two cubic and periodic oscillating soliton, when we change the coefficients of dispersion from $0.01t^2$ to $0.02\sin(t)$. By controlling different values of spectral parameter $\lambda_i = (i = 1, 2, 3, ...)$ we can change the propagation direction of soliton. In Figure: 7, with $a(t) = b(t) = c(t) = d(t) = 0.01t$ and $\lambda_2 = \frac{1}{100} + i$, we can see that the direction of two parabolic soliton are opposite and head-on interaction. Similarly in Figure: 8 and 9, with $\lambda_2 = \frac{4}{5} + i$ and $\lambda_2 = \frac{7}{5} + i$ the interactions between two cubic and periodical oscillating soliton is overtaking and parallel respectively.

4. Breather solution

For obtaining breather solution, we begin with plane wave seed solution $r[0] = e^{i\int 2[(t)+3c(t)]dt}$ and inserting this solution into Lax pair in equation (2) with $\lambda = ih$, where h represent real constant, we can obtain solution of Lax pair as

$$\widetilde{\Omega}(h) = \begin{pmatrix} \psi_{1,1} \\ \psi_{1,2} \end{pmatrix} = \begin{pmatrix} i\left(c_1 e^{[\xi(x+\zeta(t))]}\right) - \left(c_2 e^{[-\xi(x+\zeta(t))]}\right) e^{-i\int [b(t)+3c(t)]dt} \\ \left(c_2 e^{[\xi(x+\zeta(t))]}\right) - \left(c_1 e^{[-\xi(x+\zeta(t))]}\right) e^{i\int [b(t)+3c(t)]dt} \end{pmatrix} \tag{12}$$

where

$$c_1 = \frac{\sqrt{h-\xi}}{\xi}, \quad c_2 = \frac{\sqrt{\xi+h}}{\xi}, \quad \xi = \sqrt{h^2-1},$$
$$\zeta(t) = 2\int \left[8h^4 d(t) + 4ih^3 c(t) + 2h^2 a(t) + 4h^2 d(t) + ihb(t) + 2ihc(t) + a(t) + 3d(t)\right] dt.$$

By inserting the value of $\psi_{1,1}$ and $\psi_{1,2}$ from equation (12) along with $r[0] = e^{i\int [b(t)+3c(t)]dt}$ into equation (4), we get first order breather solution as

$$r[1] = e^{i\int 2[b(t)+3c(t)]dt} \frac{R}{S} \tag{13}$$

where

$$R = -h + e^{2\xi[x+2\zeta_1(t)]} + e^{2\xi[x+2\zeta(t)]} - e^{4\xi[x+\zeta(t)+\zeta_1(t)]},$$
$$S = h + \left(1 - 2h^2 + 2h\xi\right) e^{2\xi[x+2\zeta_1(t)]} + \left(1 - 2h^2 - 2h\xi\right) e^{2\xi[x+2\zeta(t)]} + he^{4\xi[x+\zeta(t)+\zeta_1(t)]},$$
$$\zeta_1(t) = \int \left[8h^4 d(t) - 4ih^3 c(t) + 2h^2 a(t) + 4h^2 d(t) - ihb(t) - 2ihc(t) + a(t) + 3d(t)\right] dt.$$

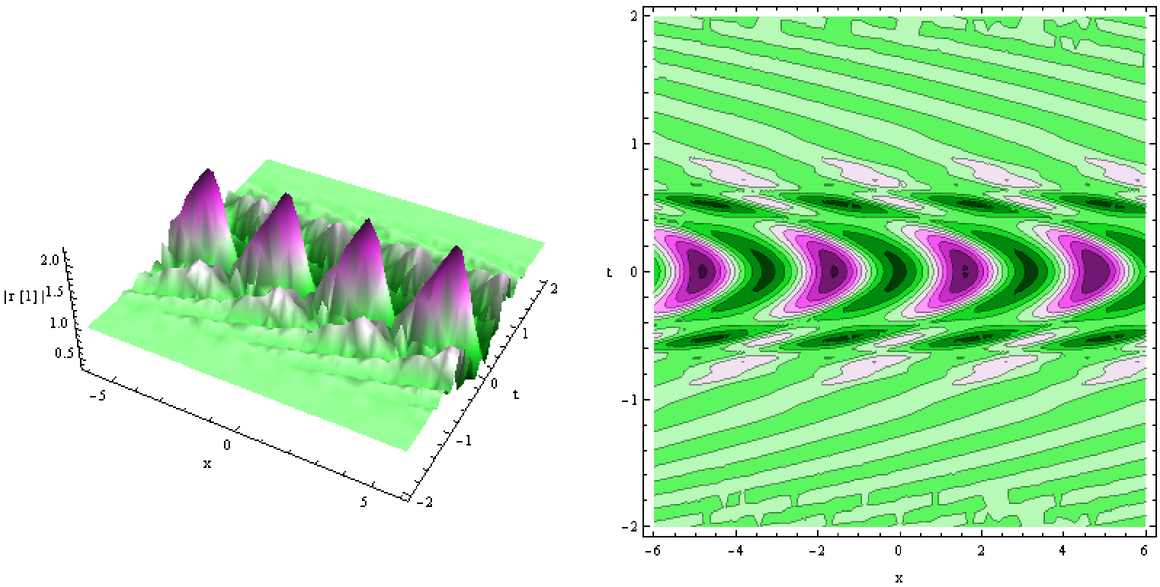

Figure: 10 First-order breather solution as computed from Eq. (13) with parameters $\lambda = 0.2i$, $a(t) = b(t) = c(t) = d(t) = t$

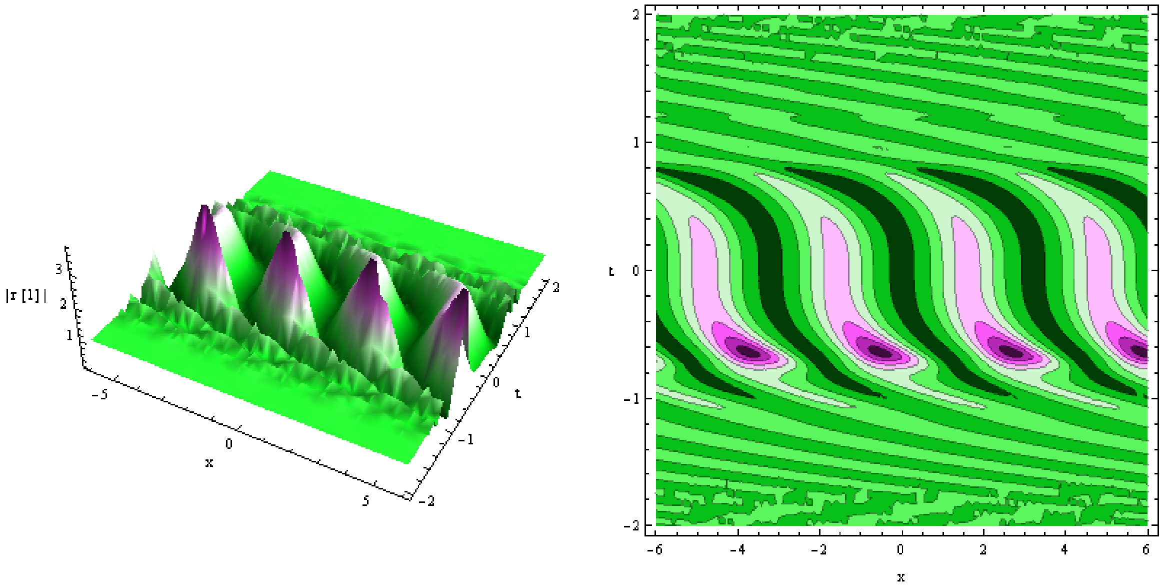

Figure: 11 First-order breather solution as computed from Eq. (13) with parameters $\lambda = 0.2i$, $a(t) = b(t) = c(t) = d(t) = t^2$

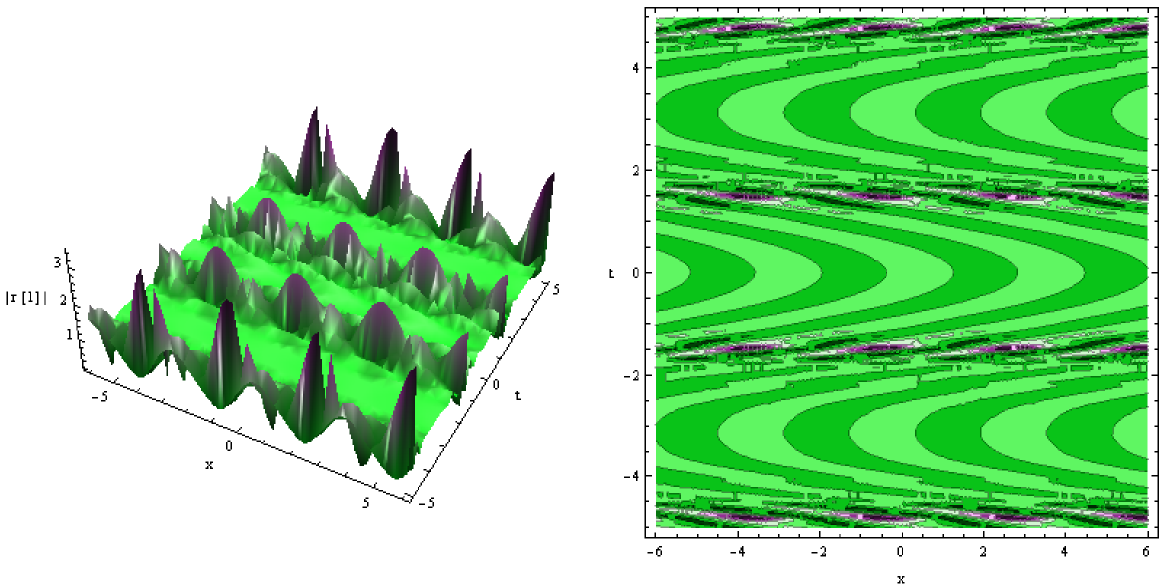

Figure: 12 First-order breather solution as computed from Eq. (13) with parameters $\lambda = 0.2i$, $a(t) = b(t) = c(t) = d(t) = \sin(t)$

In Figure: 10, we can see that the first order breather has periodic property but not symmetrical in t direction. With $(0 < h < 1)$ and $\lambda = 0.2i$, we can see that parabolic breather in Figure: 10. By changing coefficients of dispersion from $a(t) = b(t) = c(t) = d(t) = t^2$ to $a(t) = b(t) = c(t) = d(t) = \sin(t)$, we can see the cubic and periodical oscillating breathers respectively. The breathers have one peak and two valleys at each period. They are localized on parabolic, cubic and periodical oscillating curves and not located at same propagation variable x. Thus the pattern of breathers can be control by coefficients of dispersion. We can also be obtain second order breather solution by using Nth order DT in equation (5) with plane wave seed solution $r[0] = e^{i\int 2[b(t)+3c(t)]dt}$.

With different values $\lambda = ih$ and coefficients of dispersion, we can find different types of interactions. We will try to obtain higher order soliton and breathers solutions in our next publications.

5. Conclusion

We have investigated fifth-order variable coefficient nonlinear Schrödinger equation, which describe the pulse propagation in fiber optics. By applying Darboux transformation, the soliton and breather solutions have been obtained. In nonlinear theory, optical soliton and breather have significant role and an attractive research area. Optical solitons have potential applications in various nonlinear media. Soliton biased transmission are frequently affected by various in-homogeneous effects (attenuation, dispersion) and nonlinear effects (self phase modulation, cross phase modulation, stimulated Raman scattering). These effects cause broadening of soliton, soliton shape and soliton control. In order to minimize the effects with the help of controlling parameters is great importance for transmission of soliton pulse in optical fiber links. The effects of coefficients of dispersion and spectral parameters λ on the soliton and breather solutions have been demonstrated. In Figure: 1, 2 and 3, we have plotted one soliton solution from equation (9). By controlling coefficients of dispersion, we have obtained parabolic, cubic and periodical oscillating solitons. We have plotted two soliton solution from equation (11) and by controlling spectral parameter λ values, the interactions between solitons have described, which are head on, overtaking and parallel as shown in Figure: 4-9. From equation (13), we have plotted first order breather solution and the effects of coefficients of dispersion have been described in Figure: 10, 11 and 12.

So, by controlling parameters in solutions, we can change the shape, phase, amplitude, interactions and direction of solitons, which is very helpful to control pulse transmission in fiber optics. For long and stable propagation of optical signal in the form of solitons can be achieved by controlling parameters.

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