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# MODIFIED SOMBOR INDEX OF TREES

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Abstract. The modified Sombor index of a graph $G$, denoted by ${}^{m}SO(G)$, is defined as the sum of weights $\frac{1}{\sqrt{d_G^2(u)+d_G^2(v)}}$ of all edges $uv$ of $E(G)$, where $d_G(u)$ denotes the degree of a vertex $u$ in $G$. In this paper we show that for any tree $T$ of order $n$ with maximum degree $\Delta$,

$$ {}^{m}SO(T) \leq \frac{\Delta}{\sqrt{\Delta^2+4}} + \frac{(n-2\Delta-1)}{\sqrt{8}} + \frac{\Delta}{\sqrt{5}}, $$

when $\Delta \leq \frac{n-1}{2}$ and

$$ {}^{m}SO(T) \leq \frac{(2\Delta+1-n)}{\sqrt{\Delta^2+1}} + \frac{(n-\Delta-1)}{\sqrt{\Delta^2+4}} + \frac{(n-\Delta-1)}{\sqrt{5}}, $$

when $\Delta > \frac{n-1}{2}$. Also we determine the extremal trees achieve these bounds.

Keywords: Sombor index, Modified Sombor index, Upper bound, Tree.

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## 1. Introduction

Consider a simple graph $G$ of order $n$ such that $V(G)$ and $E(G)$ are the vertex and edge sets of $G$ respectively. For $x \in V(G)$, the open neighborhood of $x$ is the set $N_G(x) = \{y \mid xy \in E(G)\}$ and the degree $d_G(x)$ of $x$ in $G$ is the cardinality of $N_G(x)$. $\Delta = \Delta(G)$ is the maximum degree of $G$. The distance between two vertices of $G$ is the length of any shortest path in $G$ connecting them.

A rooted tree is a tree together with a special vertex chosen as the root of the tree. A pendant vertex (leaf) is a vertex of degree one. A support vertex is adjacent to a pendant verex and a strong support vertex is a support vertex adjacent to at least two pendant vertices. A tree with exactly one vertex of degree greater than two is called a spider. The high degree vertex of a spider $T$ is the center of $T$. A leg of a spider is a path from its center to a pendant vertex. A star is a spider such that all legs have length one. Also a path is a spider with one leg or two legs.

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The Zagreb indices are the oldest vertex–degree–based graph invariants. They were introduced in the 1970s [17, 18]. Details of their mathematical theory and chemical applications can be found in the surveys [4, 16].

In recent years, some novel variants of Zagreb indices have been put forward, such as generalized Zagreb index [2], hyper Zagreb index [11, 12, 23, 25], forgotten coindex [3], inverse sum indeg index [13], reformulated Zagreb indices [20, 27], multiplicative Zagreb indices [14, 32, 33], Lanzhou index [9, 34], entire Zagreb indices [1, 26], leap Zagreb indices [8, 28, 31], etc.

In 2021, a novel degree-based topological index was introduced by Gutman [15], called the *Sombor index*. The Sombor index is defined as:

$$SO(G) = \sum_{uv \in E(G)} \sqrt{d_G^2(u) + d_G^2(v)}.$$

Also Kulli and Gutman [22] defined the modified Sombor index as:

$$^mSO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_G^2(u) + d_G^2(v)}}.$$

Some lower and upper bounds on Sombor index in terms of graph parameters obtained by Das, et al. [5, 6]. In [35] the relations between Sombor index and other degree-based indices determined. Réti, et al. [29] were determined maximum Sombor index among all connected $k$-cyclic graphs of order $n$, where $1 \leq k \leq n-2$. Huang and Liu [19] obtained several interesting properties and bounds of the modified Sombor index, and they found its relations with some other topological indices, such as the harmonic index, the sum-connectivity index, and the geometric–arithmetic index. For more information about variants of the Sombor index see [7, 10, 21, 24, 30] and the references therein.

In this paper, we focus on the modified Sombor index. We obtain upper bounds on the modified Sombor index and characterize the extremal trees achieving these bounds.

## 2. Upper bound

Throughout this section, $T$ denotes a rooted tree with root $x$ where $x$ is a vertex of maximum degree and $N_T(x) = \{x_1, x_2, \ldots, x_\Delta\}$. Also, $\mathcal{T}_{n,\Delta}$ denotes the set of trees with $n$ verties and maximum degree $\Delta$. We start with some lemmas.

**Lemma 2.1.** *Let $T$ be a tree of order $n$ with maximum degree $\Delta$. If $T$ has a strong support vertex of degree at least three different from $x$, then there is a tree $T' \in \mathcal{T}_{n,\Delta}$ such that $^mSO(T') > {}^mSO(T)$.*

*Proof.* Let $y \neq x$ be a strong support vertex of degree $d_T(y) = \alpha \geq 3$ in maximum distance from $x$ and let $N_T(y) = \{y_1, y_2, \ldots, y_\alpha\}$. The vertices $x$ and $y$ may or may not be adjacent. Assume that $y_\alpha$ lies on the $x,y$-path in $T$.

Since $y$ is a strong support vertex, we may assume that $d_T(y_1) = d_T(y_2) = 1$. Let $T'$ be the tree obtained by attaching the path $y_1y_2y$ to $T - \{y_1, y_2\}$. Clearly, $T' \in \mathcal{T}_{n,\Delta}$. Since

$\alpha \geq 3$, we have:

$$
\begin{aligned}
{}^{m}SO(T) - {}^{m}SO(T') = & \sum_{uv \in E(T)} \frac{1}{\sqrt{d_T^2(u) + d_T^2(v)}} - \sum_{uv \in E(T')} \frac{1}{\sqrt{d_{T'}^2(u) + d_{T'}^2(v)}} \\
= & \frac{1}{\sqrt{d_T^2(y_1) + d_T^2(y)}} + \frac{1}{\sqrt{d_T^2(y_2) + d_T^2(y)}} \\
& + \sum_{i=3}^{\alpha} \frac{1}{\sqrt{d_T^2(y) + d_T^2(y_i)}} \\
& - \frac{1}{\sqrt{d_{T'}^2(y_1) + d_{T'}^2(y_2)}} - \frac{1}{\sqrt{d_{T'}^2(y_2) + d_{T'}^2(y)}} \\
& - \sum_{i=3}^{\alpha} \frac{1}{\sqrt{(d_T(y) - 1)^2 + d_T^2(y_i)}} \\
= & \frac{2}{\sqrt{\alpha^2 + 1}} + \sum_{i=3}^{\alpha} \frac{1}{\sqrt{\alpha^2 + d_T^2(y_i)}} \\
& - \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{(\alpha - 1)^2 + 4}} - \sum_{i=3}^{\alpha} \frac{1}{\sqrt{(\alpha - 1)^2 + d_T^2(y_i)}} \\
< & \frac{2}{\sqrt{\alpha^2 + 1}} - \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{(\alpha - 1)^2 + 4}} \\
= & \left(\frac{1}{\sqrt{\alpha^2 + 1}} - \frac{1}{\sqrt{5}}\right) + \left(\frac{1}{\sqrt{\alpha^2 + 1}} - \frac{1}{\sqrt{(\alpha - 1)^2 + 4}}\right) \\
< & \frac{1}{\sqrt{10}} - \frac{1}{\sqrt{5}} < 0.
\end{aligned}
$$

This completes the proof. □

**Lemma 2.2.** *Let $T$ be a tree of order $n$ with maximum degree $\Delta$. If $T$ has a support vertex of degree at least three different from $x$, then there is a tree $T' \in \mathcal{T}_{n,\Delta}$ such that ${}^{m}SO(T') > {}^{m}SO(T)$.*

*Proof.* Let $y \neq x$ be a support vertex of degree $d_T(y) = \alpha \geq 3$ in maximum distance from $x$ and let $N_T(y) = \{y_1, y_2, \ldots, y_\alpha\}$. The vertices $x$ and $y$ may or may not be adjacent. Assume that $y_\alpha$ lies on the $x, y$-path in $T$.

Since $y$ is support vertex, we may assume that $d_T(y_1) = 1$ and by Lemma 2.1, $d_T(y_i) = 2$ for $2 \leq i \leq \alpha - 1$. Let $yz_1z_2 \ldots z_t$ be a path in $T$ such that $t \geq 2$ and $y_2 = z_1$.

Assume that $T'$ be the tree obtained from $T - \{y_1\}$ by attaching the path $z_t y_1$. Since $\alpha \geq 3$, we have:

$$
\begin{aligned}
{}^{m}SO(T) - {}^{m}SO(T') &= \sum_{uv \in E(T)} \frac{1}{\sqrt{d_T^2(u) + d_T^2(v)}} - \sum_{uv \in E(T')} \frac{1}{\sqrt{d_{T'}^2(u) + d_{T'}^2(v)}} \\
&= \frac{1}{\sqrt{d_T^2(y_1) + d_T^2(y)}} + \frac{1}{\sqrt{d_T^2(z_t) + d_T^2(z_{t-1})}} \\
&\quad + \sum_{i=2}^{\alpha} \frac{1}{\sqrt{d_T^2(y) + d_T^2(y_i)}} \\
&\quad - \frac{1}{\sqrt{d_{T'}^2(y_1) + d_{T'}^2(z_t)}} - \frac{1}{\sqrt{d_{T'}^2(z_t) + d_{T'}^2(z_{t-1})}} \\
&\quad - \sum_{i=2}^{\alpha} \frac{1}{\sqrt{(d_T(y) - 1)^2 + d_T^2(y_i)}} \\
&= \frac{1}{\sqrt{\alpha^2 + 1}} + \frac{1}{\sqrt{5}} + \sum_{i=2}^{\alpha} \frac{1}{\sqrt{\alpha^2 + d_T^2(y_i)}} \\
&\quad - \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{8}} - \sum_{i=2}^{\alpha} \frac{1}{\sqrt{(\alpha - 1)^2 + d_T^2(y_i)}} \\
&< \frac{1}{\sqrt{\alpha^2 + 1}} - \frac{1}{\sqrt{8}} \\
&\leq \frac{1}{\sqrt{10}} - \frac{1}{\sqrt{8}} < 0.
\end{aligned}
$$

This completes the proof. □

**Lemma 2.3.** *Let $T$ be a tree of order $n$ with maximum degree $\Delta$. If $T$ has a vertex of degree at least three different from $x$, then there is a tree $T' \in \mathcal{T}_{n,\Delta}$ such that ${}^{m}SO(T') > {}^{m}SO(T)$.*

*Proof.* Let $y \neq x$ be a support vertex of degree $d_T(y) = \alpha \geq 3$ in maximum distance from $x$ and let $N_T(y) = \{y_1, y_2, \ldots, y_\alpha\}$. The vertices $x$ and $y$ may or may not be adjacent. Assume that $y_\alpha$ lies on the $x,y$-path in $T$.

By Lemmas 2.1 and 2.2, $d_T(y_i) = 2$ for $1 \leq i \leq \alpha - 1$. Let $yz_1z_2 \ldots z_t$ and $yw_1w_2 \ldots w_k$ be two paths in $T$ for $t, k \geq 2$ such that $y_1 = w_1$ and $y_2 = z_1$.

Assume that $T'$ be the tree obtained from $T$ by deleting the edge $yy_1$ and adding the edge $z_t y_1$. If $\alpha \geq 4$, then

$$\begin{aligned}
{}^{m}SO(T) - {}^{m}SO(T') &= \sum_{uv\in E(T)} \frac{1}{\sqrt{d_T^2(u)+d_T^2(v)}} - \sum_{uv\in E(T')} \frac{1}{\sqrt{d_{T'}^2(u)+d_{T'}^2(v)}} \\
&= \frac{1}{\sqrt{d_T^2(y_1)+d_T^2(y)}} + \frac{1}{\sqrt{d_T^2(z_t)+d_T^2(z_{t-1})}} \\
&\quad + \sum_{i=2}^{\alpha} \frac{1}{\sqrt{d_T^2(y)+d_T^2(y_i)}} \\
&\quad - \frac{1}{\sqrt{d_{T'}^2(y_1)+d_{T'}^2(z_t)}} - \frac{1}{\sqrt{d_{T'}^2(z_t)+d_{T'}^2(z_{t-1})}} \\
&\quad - \sum_{i=2}^{\alpha} \frac{1}{\sqrt{(d_T(y)-1)^2+d_T^2(y_i)}} \\
&= \frac{1}{\sqrt{\alpha^2+4}} + \frac{1}{\sqrt{5}} + \sum_{i=2}^{\alpha} \frac{1}{\sqrt{\alpha^2+d_T^2(y_i)}} \\
&\quad - \frac{2}{\sqrt{8}} - \sum_{i=2}^{\alpha} \frac{1}{\sqrt{(\alpha-1)^2+d_T^2(y_i)}} \\
&< \frac{1}{\sqrt{\alpha^2+4}} + \frac{1}{\sqrt{5}} - \frac{2}{\sqrt{8}} \\
&\leq \frac{1}{\sqrt{20}} + \frac{1}{\sqrt{5}} - \frac{2}{\sqrt{8}} < 0.
\end{aligned}$$

Now let $\alpha = 3$. Hence

$$\begin{aligned}
{}^{m}SO(T) - {}^{m}SO(T') &= \sum_{uv\in E(T)} \frac{1}{\sqrt{d_T^2(u)+d_T^2(v)}} - \sum_{uv\in E(T')} \frac{1}{\sqrt{d_{T'}^2(u)+d_{T'}^2(v)}} \\
&= \frac{1}{\sqrt{d_T^2(y_1)+d_T^2(y)}} + \frac{1}{\sqrt{d_T^2(z_t)+d_T^2(z_{t-1})}} \\
&\quad + \frac{1}{\sqrt{d_T^2(y_2)+d_T^2(y)}} + \frac{1}{\sqrt{d_T^2(y)+d_T^2(y_\alpha)}} \\
&\quad - \frac{1}{\sqrt{d_{T'}^2(y_1)+d_{T'}^2(z_t)}} - \frac{1}{\sqrt{d_{T'}^2(z_t)+d_{T'}^2(z_{t-1})}} \\
&\quad - \frac{1}{\sqrt{d_{T'}^2(y_2)+d_{T'}^2(y)}} - \frac{1}{\sqrt{(d_T(y)-1)^2+d_T^2(y_\alpha)}}
\end{aligned}$$

$$\begin{aligned}
&=\frac{1}{\sqrt{\alpha^2+4}}+\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{\alpha^2+4}}+\frac{1}{\sqrt{\alpha^2+d_T^2(y_\alpha)}}\\
&\quad-\frac{2}{\sqrt{8}}-\frac{1}{\sqrt{(\alpha-1)^2+4}}-\frac{1}{\sqrt{(\alpha-1)^2+d_T^2(y_\alpha)}}\\
&<\frac{2}{\sqrt{\alpha^2+4}}+\frac{1}{\sqrt{5}}-\frac{2}{\sqrt{8}}-\frac{1}{\sqrt{(\alpha-1)^2+4}}\\
&=\frac{2}{\sqrt{13}}+\frac{1}{\sqrt{5}}-\frac{3}{\sqrt{8}}<0.
\end{aligned}$$

This completes the proof. □

**Proposition 2.1.** *Let $T$ be a spider of order $n$ with $k \geq 3$ legs. If $T$ has a leg of length 1 and a leg of length at least 3, then there is a spider $T'$ of order $n$ with $k$ legs such that ${}^{m}SO(T') > {}^{m}SO(T)$.*

*Proof.* Let $x$ be the center of $T$ and $N_T(x) = \{x_1, \dots, x_k\}$. Root $T$ at $x$. We may assume that $d(x_1) = 1$ and let $x_2y_1y_2 \dots y_t$, $t \geq 2$ be a longest leg of $T$. Let $T'$ be the tree obtained from $T$ be deleting the edge $y_ty_{t-1}$ and adding the pendant edge $x_1y_t$. By definition we have:

$$\begin{aligned}
{}^{m}SO(T)-{}^{m}SO(T') &= \sum_{uv\in E(T)}\frac{1}{\sqrt{d_T^2(u)+d_T^2(v)}}-\sum_{uv\in E(T')}\frac{1}{\sqrt{d_{T'}^2(u)+d_{T'}^2(v)}}\\
&=\frac{1}{\sqrt{\Delta^2+1}}+\frac{1}{\sqrt{8}}-\frac{1}{\sqrt{\Delta^2+4}}-\frac{1}{\sqrt{5}}<0.
\end{aligned}$$

This complete the proof. □

Now we prove the main theorem of this section.

**Theorem 2.1.** *For any tree $T \in \mathcal{T}_{n,\Delta}$ of order $n \geq 3$,*

$${}^{m}SO(T) \leq \frac{\Delta}{\sqrt{\Delta^2+4}}+\frac{(n-2\Delta-1)}{\sqrt{8}}+\frac{\Delta}{\sqrt{5}},$$

*when $\Delta \leq \frac{n-1}{2}$ and*

$${}^{m}SO(T) \leq \frac{(2\Delta+1-n)}{\sqrt{\Delta^2+1}}+\frac{(n-\Delta-1)}{\sqrt{\Delta^2+4}}+\frac{(n-\Delta-1)}{\sqrt{5}},$$

*when $\Delta > \frac{n-1}{2}$. With equality if and only if $T$ is a spider whose all legs have length at most two or all legs have length at least two.*

*Proof.* If $\Delta = 2$, then $T = P_n$ and ${}^{m}SO(P_n) = \frac{2}{\sqrt{5}} + \frac{n-3}{\sqrt{8}}$. Therefore let $3 \leq \Delta \leq n-1$. Assume that $T' \in \mathcal{T}_{n,\Delta}$ is a tree with $n \geq 3$ and ${}^{m}SO(T') \leq {}^{m}SO(T)$ for every tree $T \in \mathcal{T}_{n,\Delta}$. Let $x$ be a vertex with $d_{T'}(x) = \Delta$ and root $T'$ at $x$. By the choice of $T'$ and Lemmas 2.1, 2.2 and 2.3, we conclude that $T'$ is a spider with center $x$. Then by Proposition 2.1, $T'$ is a spider such that all legs of $T'$ either have length at most two or have length at least two. If all legs of $T'$ have length at least two, then $\Delta \leq \frac{n-1}{2}$ and

$${}^{m}SO(T') = \frac{\Delta}{\sqrt{\Delta^2+4}}+\frac{(n-2\Delta-1)}{\sqrt{8}}+\frac{\Delta}{\sqrt{5}}.$$

Now let all legs of $T'$ have length at most two. Considering above case, we may assume that $T'$ has a leg of length 1. If $\Delta = n-1$, then $T'$ is a star and the result is immediate. Assume $\Delta < n-1$. Then the number of leaves adjacent to $x$ is $2\Delta + 1 - n$ and then

$$^{m}SO(T^*) = \frac{(2\Delta + 1 - n)}{\sqrt{\Delta^2 + 1}} + \frac{(n - \Delta - 1)}{\sqrt{\Delta^2 + 4}} + \frac{(n - \Delta - 1)}{\sqrt{5}}.$$

□

## 3. Conclusions

In this paper we calculated the upper bounds of the modified Sombor index of the tree $T$ of order $n$ and the maximum degree $\Delta$. We proved that the modified Sombor index of a spider with all legs have length at most two or all legs have length at least two is exactly these bounds.

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