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INTERVAL VALUED KERNEL SYMMETRIC, K-KERNEL SYMMETRIC, RANGE SYMMETRIC AND COLUMN SYMMETRIC NEUTROSOPHIC FUZZY MATRICES

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Abstract. We present the range-symmetric, interval-valued neutrosophic fuzzy matrix (RS-IVNFM) and the kernel-symmetric, interval-valued neutrosophic fuzzy matrix (KS-IVNFM), which are similar to the EP-matrices in the unitary domain. Additionally, we illustrate a graphical representation of KS, column-symmetric, and range-symmetric adjacency and incidence neutrosophic fuzzy matrices. Every adjacency NFM is symmetric, range-symmetric, column-symmetric, and kernel-symmetric, but the incidence matrix satisfies only the kernel-symmetric condition.Similarly, every range-symmetric adjacency NFM is a kernel-symmetric adjacency NFM, but a kernel-symmetric adjacency NFM need not be range-symmetric. We first present equivalent characterizations for an RS matrix. Then, we derive the equivalent condition for an IVNFM to be a KS matrix, and finally, we study the relationship between RS-IVNFM and KS-IVNFM. With suitable examples, we introduce the concept of k-KS and RS-IVNFM. We also present some primary results about KS matrices. We demonstrate that KS implies k-KS, but the converse need not apply. Numerical results illustrate the equivalent relationships between KS, the Moore-Penrose inverse of IVNFM, and k-KS.

Keywords: IVNFM, RS-IVNFM, KS-IVNFM, k- KS-IVNFM.

AMS Subject Classification: 03E72, 15B15, 15B99.

1. Introduction

Zadeh [26] first introduced Fuzzy Sets (FSs) in 1965, which are traditionally defined by their membership value or grade of membership. Assigning membership values to a fuzzy set can sometimes be challenging. To address this, Atanassov [7] introduced intuitionistic

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FSs, which also consider non-membership values. Ben Isral and Greville [8] have studied Generalized Inverse Theory and Application. Later, Smarandache [21] introduced the concept of Neutrosophic Sets (NSs) to handle indeterminate information and deal with problems involving imprecision, uncertainty, and inconsistency. Wang et al., [24,25] generalized the concept of NSs to single valued NSs and interval-valued NSs to overcome the difficulties faced during application of NS values in real-life.

If fuzzy matrices P and P^+ have a positive counterpart (P^+), then the two coincide with P's transpose. Kim and Roush [13] studied generalized fuzzy matrices and found that a fuzzy P matrix is range symmetric when the range P equals the transpose range, P^T. It is also kernel symmetric when the nullspace of P equals the nullspace of P^T. It is widely accepted that range and kernel symmetry concepts are equivalent in matrices.

The range symmetry of NFMs is $R[P] = R[P^T]$, but not the kernel symmetry $N(P) = N(P^T)$. Meenakshi [14] introduced fuzzy matrices by incorporating k, a fixed product derived from disjoint transformations. Hill and Waters [10] discussed on k- real and k-hermitian matrices. Baskett and Katz [9] explored the concepts on EPr matrixes. Schwerdtfeger [22] delved deeply into linear algebra and matrices.

Meenakshi and Jayashri [15] discussed k-Kernel Symmetric Matrices. Riyaz Ahmed Padder, Murugadas [18-20] studied idempotent intuitionistic fuzzy matrices of the T-type. They also explored the reduction of non-nilpotent IFMs using implication operators and determinant theories for IFMs. Atanassov [6], on the other hand, focused on generalized index matrices. Sumathi and Arockiarani [23] introduced new operations on fuzzy neutrosophic soft matrices. Meenakshi and Krishnamoorthy [17] presented k-EP matrixes.

Anandhkumar et.al [1-5] have studied Generalized Symmetric Neutrosophic Fuzzy Matrices, Interval Valued Secondary k-Range Symmetric Neutrosophic Fuzzy Matrices, Partial orderings, Characterizations and Generalization of k- idempotent Neutrosophic fuzzy matrices, Reverse Sharp and Left-T Right-T Partial Ordering on Neutrosophic Fuzzy Matrices, On various Inverse of Neutrosophic Fuzzy Matrices, Pseudo Similarity of Neutrosophic Fuzzy matrices. Meenakshi and Jaya Shree [16] have discussed On k -range symmetric matrices. Jaya shree [11] has characterized Secondary k-Kernel Symmetric Fuzzy Matrices. Jaya Shree [12] has present Secondary k-range symmetric fuzzy matrices.

1.1 Research gaps

Anandhkumar et al. [1] presented the concept of range and kernel-symmetry principles to NFM. We have applied these principles to IVNFM in this context. We examined some of the results and extended both concepts to IVNFMs. First, we present equivalent characterizations for an RS matrix and a KS matrix. Then, we derive the equivalent conditions that IVNFMs must meet to show kernel symmetry and explore the relationship between range symmetry and kernel symmetry. Additionally, we identify the equivalent conditions that allow various generalized inverses to exhibit kernel symmetry.

Table:1 Review of the Extension of Neutrosophic Fuzzy Matrices.

[15]	On k - kernal symmetric matrices	2009
[16]	On k -range symmetric matrices	2009
[11]	Secondary k-Kernel Symmetric Fuzzy Matrices	2014
[12]	Secondary k-range symmetric fuzzy matrices	2018
[1]	Generalized Symmetric Neutrosophic Fuzzy Matrices	2023
Proposed	Interval valued kernel symmetric, k-kernel symmetric, Range symmetric and Column symmetric Neutrosophic Fuzzy Matrices	2024

Based on the literature review, no research has been carried out on interval-valued kernel symmetric, k-kernel symmetric, Range symmetric and Column symmetric Neutrosophic Fuzzy Matrices and marge this gap.

2. PRELIMINARIES AND NOTATIONS

2.1. PRELIMINARIES.

If $K(Y) = (y_{k[1]}, y_{k[2]}, y_{k[3]}, ..., y_{k[n]}) \in F_{n\times 1}$ for $y = (y_1, y_2, ..., y_n) \in F_{[1\times n]}$ where K is involutary, then the following properties hold.
$(P_1) K = K^T, K^2 = 1$ and $K(y) = Ky$,
For every $P =< [P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U >\in (IVNFM)_n$
$(P_2) N[P_\mu, P_\lambda, P_v]_L = N([P_\mu, P_\lambda, P_v]_L K) = N(K[P_\mu, P_\lambda, P_v]_L)$
$N[P_\mu, P_\lambda, P_v]_U = N([P_\mu, P_\lambda, P_v]_U K) = N(K[P_\mu, P_\lambda, P_v]_U)$
$(P_3)([P_\mu, P_\lambda, P_v]_L K)^+ = (K[P_\mu, P_\lambda, P_v]_L)^+$ and $(K[P_\mu, P_\lambda, P_v]_L)^+ = ([P_\mu, P_\lambda, P_v]_L^+)K$ exists, if $[P_\mu, P_\lambda, P_v]_L^+$ exists.
$([P_\mu, P_\lambda, P_v]_U K)^+ = K[P_\mu, P_\lambda, P_v]_U^+$ and $(K[P_\mu, P_\lambda, P_v]_U)^+ = [P_\mu, P_\lambda, P_v]_U^+ K$ exists, if $[P_\mu, P_\lambda, P_v]_U^+$ exists.
$(P_4)[P_\mu, P_\lambda, P_v]_L^T$ is a g - inverse of $[P_\mu, P_\lambda, P_v]_L$ iff $[P_\mu, P_\lambda, P_v]_L^+$ exists
$[P_\mu, P_\lambda, P_v]_U^T$ is a g -inverse of $[P_\mu, P_\lambda, P_v]_U$ iff $[P_\mu, P_\lambda, P_v]_U^+$ exists.

2.2. Notations.

For $P =< [P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U >\in (IVNFM)_n$
$[P_\mu, P_\lambda, P_v]_L^T$: Transpose of $[P_\mu, P_\lambda, P_v]_L$,
$[P_\mu, P_\lambda, P_v]_U^T$: Transpose of $[P_\mu, P_\lambda, P_v]_U$,
$R([P_\mu, P_\lambda, P_v]_L)$: Row Space of $[P_\mu, P_\lambda, P_v]_L$,
$R([P_\mu, P_\lambda, P_v]_U)$: Row Space of $[P_\mu, P_\lambda, P_v]_U$,
$C([P_\mu, P_\lambda, P_v]_L)$: Column Space of $[P_\mu, P_\lambda, P_v]_L$,
$C([P_\mu, P_\lambda, P_v]_U)$: Column Space of $[P_\mu, P_\lambda, P_v]_U$,
$N([P_\mu, P_\lambda, P_v]_L)$: Null Space of $[P_\mu, P_\lambda, P_v]_L$,
$N([P_\mu, P_\lambda, P_v]_U)$: Null Space of $[P_\mu, P_\lambda, P_v]_U$,

$[P_\mu, P_\lambda, P_v]_L^\dagger$: Moore - Penrose inverse of $[P_\mu, P_\lambda, P_v]_L$,
$[P_\mu, P_\lambda, P_v]_U^\dagger$: Moore - Penrose inverse of $[P_\mu, P_\lambda, P_v]_U$,
$IVNFM$: Interval valued Neutrosophic Fuzzy Matrices.
$PNFM$: Permutation Neutrosophic Fuzzy Matrices.

3. DEFINITIONS AND THEOREMS

Definition 3.1. ($IVNFM$) : An IVNFM P of order $m \times n$ is defined as $P = [X_{ij}, < p_{ij\mu}, p_{ij\lambda}, p_{ijv} >]_{m\times n}$ where $p_{ij\mu}, p_{ij\lambda}$ and p_{ijv} are the subset of $[0,1]$ which are denoted by $p_{ij\mu} = [p_{ij\mu L}, p_{ij\mu U}]$, $p_{ij\lambda} = [p_{ij\lambda L}, p_{ij\lambda U}]$ and $p_{ijv} = [p_{ijvL}, p_{ijvU}]$ which maintaining the condition $0 \leq p_{ij\mu U} + p_{ij\lambda U} + p_{ijvU} \leq 3, 0 \leq p_{ij\mu L} + p_{ij\lambda L} + p_{ijvL} \leq 3, 0 \leq p_{\mu L} + p_{\mu U} \leq 2, 0 \leq p_{\lambda L} + p_{\lambda U} \leq 2, 0 \leq p_{vL} + p_{vU} \leq 2$, for $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$.

Definition 3.2. (Addition of two IVNFM). An IVNFM P and Q of order $m \times n$ is defined as $P = [X_{ij}, < p_{ij\mu}, p_{ij\lambda}, p_{ijv} >]_{m\times n}$ where $p_{ij\mu}, p_{ij\lambda}$ and p_{ijv} are the subset of $[0,1]$ which are denoted by $p_{ij\mu} = [p_{ij\mu L}, p_{ij\mu U}]$, $p_{ij\lambda} = [p_{ij\lambda L}, p_{ij\lambda U}]$ and $p_{ijv} = [p_{ijvL}, p_{ijvU}]$ and $Q = [Y_{ij}, < q_{ij\mu}, q_{ij\lambda}, q_{ijv} >]_{m\times n}$ where $q_{ij\mu}, q_{ij\lambda}$ and q_{ijv} are the subset of $[0,1]$ which ae denoted by $q_{ij\mu} = [q_{ij\mu L}, q_{ij\mu U}]$, $q_{ij\lambda} = [q_{ij\lambda L}, q_{ij\lambda U}]$ and $q_{ijv} = [q_{ijvL}, q_{ijvU}]$ then
P+Q$= < [min(p_{ij\mu L}, q_{ij\mu L}), min(p_{ij\lambda U}, q_{ij\lambda U})] >, < [min(p_{ijvL}, q_{ij\lambda L}), min(p_{ijvU}, q_{ijvU})] >$ $, < [max(p_{ij\mu L}, q_{ij\mu L}), max(p_{ij\mu U}, q_{ij\mu U})] >$

Example 3.1. *Consider an IVNFM*

$$P = \begin{bmatrix} <[0,0],[1,1],[1,1]> & <[0.1,0.3],[0.2,0.4],[0.2,0.5]> \\ <[0.1,0.3],[0.2,0.4],[0.2,0.5]> & <[0,0],[1,1],[1,1]> \end{bmatrix}$$

Lower limit of NFM, $[P_\mu, P_\lambda, P_v]_L = \begin{bmatrix} <0,1,1> & <0.3,0.4,0.5> \\ <0.3,0.4,0.5> & <0,1,1> \end{bmatrix}$

$$Q = \begin{bmatrix} <[0,0],[1,1],[1,1]> & <[0.2,0.4],[0.3,0.5],[0.1,0.5]> \\ <[0.2,0.4],[0.3,0.5],[0.1,0.5]> & <[0,0],[1,1],[1,1]> \end{bmatrix}$$

$$P+Q = \begin{bmatrix} <[0,0],[1,1],[1,1]> & <[0.1,0.4],[0.2,0.4],[0.1,0.5]> \\ <[0.1,0.4],[0.2,0.4],[0.1,0.5]> & <[0,0],[1,1],[1,1]> \end{bmatrix}$$

Definition 3.3. Let $P =< [P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U >$ be an IVNFM, if $R\left[[P_\mu, P_\lambda, P_v]_L\right]$ $= R\left[[P_\mu, P_\lambda, P_v]_L^T\right]$ and $R\left[[P_\mu, P_\lambda, P_v]_U\right] = R\left[[P_\mu, P_\lambda, P_v]_U^T\right]$ then
$P =< [P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U >$ is called as RS.

Example 3.2. *Consider a IVNFM*

$$P = \begin{bmatrix} <[0.4,0.5],[0,0],[1,1]> & <[1,1],[0,0],[1,1]> & <[0.2,0.5],[0.3,0.6],[0.4,0.7]> \\ <[1,1],[0,0],[1,1]> & <[1,1],[0,0],[1,1]> & <[1,1],[0,0],[1,1]> \\ <[0.2,0.5],[0.3,0.6],[0.4,0.7]> & <[1,1],[0,0],[1,1]> & <[1,1],[0,0],[1,1] \end{bmatrix},$$

Lower limit of NFM,

$$[P_\mu, P_\lambda, P_v]_L = \begin{bmatrix} <0.4,0,1> & <1,0,1> & <0.2,0.3,0.4> \\ <1,0,1> & <1,0,1> & <1,0,1> \\ <0.2,0.3,0.4> & <1,0,1> & <1,0,1> \end{bmatrix}.$$

Every symmetric matrices is range symmetric. Therefore $R\left[[P_\mu, P_\lambda, P_v]_L\right] = R\left[[P_\mu, P_\lambda, P_v]_L^T\right]$

Upper limit of NFM, $[P_\mu, P_\lambda, P_v]_U = \begin{bmatrix} <0.5,0,1> & <1,0,1> & <0.5,0.6,0.7> \\ <1,0,1> & <1,0,1> & <1,0,1> \\ <0.5,0.6,0.7> & <1,0,1> & <1,0,1>) \end{bmatrix}$

Here $R\left[[P_\mu, P_\lambda, P_v]_U\right] = R\left[[P_\mu, P_\lambda, P_v]_U^T\right]$

Therefore the given matrix is range symmetric.

The following matrices does not satisfies the range symmetric condition

$$P = \begin{bmatrix} <[0.4,0.5],[0,0],[1,1]> & <[1,1],[0,0],[1,1]> & <[0.2,0.5],[0.3,0.6],[0.4,0.7]> \\ <[1,1],[0,0],[1,1]> & <[1,1],[0,0],[1,1]> & <[1,1],[0,0],[1,1]> \\ <[0.2,0.7],[0.4,0.6],[0.4,0.7]> & <[1,1],[0,0],[1,1]> & <[1,1],[0,0],[1,1]> \end{bmatrix}$$

Lower limit of NFM,

$$[P_\mu, P_\lambda, P_v]_L = \begin{bmatrix} <0.4,0,1> & <1,0,1> & <0.2,0.3,0.4> \\ <1,0,1> & <1,0,1> & <1,0,1> \\ <0.2,0.4,0.4> & <1,0,1> & <1,0,1> \end{bmatrix}$$

$$[P_\mu, P_\lambda, P_v]_L^T = \begin{bmatrix} <0.4,0,1> & <1,0,1> & <0.2,0.3,0.4> \\ <1,0,1> & <1,0,1> & <1,0,1> \\ <0.2,0.3,0.4> & <1,0,1> & <1,0,1> \end{bmatrix}.$$

Therefore the given matrices is not symmetric.
Therefore $R\,[P_\mu, P_\lambda, P_v]_L]$ is not equal to $R\left[[P_\mu, P_\lambda, P_v]_L^T\right]$

$$[P_\mu, P_\lambda, P_v]_U = \begin{bmatrix} <0.5,0,1> & <1,0,1> & <0.5,0.6,0.7> \\ <1,0,1> & <1,0,1> & <1,0,1> \\ <0.7,0.6,0.7> & <1,0,1> & <1,0,1> \end{bmatrix}$$

Similarly, $R([P_\mu, P_\lambda, P_v]_U) \notin R([P_\mu, P_\lambda, P_v]_U^T)$. Therefore the given matrices is not range symmetric.

Remark 3.1. *For NFM P with $det[P_\mu, P_\lambda, P_v]_L >< 0,0,0 >$ and $det[P_\mu, P_\lambda, P_v]_U >< 0,0,0 >$ has non - zero rows and non - columns, here after $N([P_\mu, P_\lambda, P_v]_L^T) =< 0,0,0 >= N([P_\mu, P_\lambda, P_v]_U) =< 0,0,0 >= N([P_\mu, P_\lambda, P_v]_L^T)$. Furthermore, a symmetric matrix $[P_\mu, P_\lambda, P_v]_L = [P_\mu, P_\lambda, P_v]_L^T, [P_\mu, P_\lambda, P_v]_U = [P_\mu, P_\lambda, P_v]_U^T$ that is $N([P_\mu, P_\lambda, P_v]_L) = N([P_\mu, P_\lambda, P_v]_L^T)$ and $N([P_\mu, P_\lambda, P_v]_U) = N([P_\mu, P_\lambda, P_v]_U^T)$.*

Definition 3.4. Let $P \in (IVNFM)_n$, if $N([P_\mu, P_\lambda, P_v]_L) = N([P_\mu, P_\lambda, P_v]_L^T)$ and $N([P_\mu, P_\lambda, P_v]_U) = N([P_\mu, P_\lambda, P_v]_U^T)$ then $[P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U$ is called KS - IVNFM where
$N([P_\mu, P_\lambda, P_v]_L) = \{x/x[P_\mu, P_\lambda, P_v] = (0,0,0) and x \in F_{1\times n}$
$N([P_\mu, P_\lambda, P_v]_U) = \{x/x[P_\mu, P_\lambda, P_v] = (0,0,0) and x \in F_{1\times n}$

Example 3.3. *Consider an IVNFM*

$$P = \begin{bmatrix} <[0.3,0.4],[0.3,0.4],[0.4,0.6]> & <[0.5,0.6],[0.3,0.7],[0.6,0.7]> & <[0.2,0.5],[0.3,0.6],[0.4,0.7]> \\ <[0.4,0.6],[0.7,0.8],[0.6,0.7]> & <[0.4,0.7],[0.8,0.9],[0.1,0.2]> & <[0.2,0.3],[0.5,0.7],[0.1,0.2]> \\ <[0.2,0.7],[0.4,0.6],[0.4,0.7]> & <[0.4,0.5],[0.4,0.6],[0.5,0.6]> & <[0.4,0.5],[0.5,0.7],[0.6,0.8]> \end{bmatrix}$$

Lower limit of NFM, $[P_\mu, P_\lambda, P_v]_L = \begin{bmatrix} <0.3,0.3,0.4> & <0.5,0.3,0.6> & <0.2,0.3,0.4> \\ <0.4,0.7,0.6> & <0.4,0.8,0.1> & <0.2,0.5,0.1> \\ <0.2,0.4,0.4> & <0.4,0.4,0.5> & <0.4,0.5,0.6> \end{bmatrix}$

Upper limit of NFM, $[P_\mu, P_\lambda, P_v]_U = \begin{bmatrix} <0.4,0.4,0.6> & <0.6,0.7,0.7> & <0.5,0.6,0.7> \\ <0.6,0.8,0.7> & <0.7,0.9,0.2> & <0.3,0.7,0.2> \\ <0.7,0.6,0.7> & <0.5,0.6,0.6> & <0.5,0.5,0.6> \end{bmatrix}$

$N([P_\mu, P_\lambda, P_v]_L) = N([P_\mu, P_\lambda, P_v]_L^T) = (0,0,0), N([P_\mu, P_\lambda, P_v]_U) = N([P_\mu, P_\lambda, P_v]_U^T) = (0,0,0).$

Definition 3.5. Symmetric IVNFM. If $P \in (IVNFM)_n$ is said to be symmetric NFM if $p_{ij} = p_{ji}$.

Example 3.4. Consider a IVNFM

$$P = \begin{bmatrix} <[0.4,0.5],[0,0],[1,1]> & <[1,1],[0,0],[1,1]> & <[0.2,0.5],[0.3,0.6],[0.4,0.7]> \\ <[1,1],[0,0],[1,1]> & <[1,1],[0,0],[1,1]> & <[1,1],[0,0],[1,1]> \\ <[0.2,0.5],[0.3,0.6],[0.4,0.7]> & <[1,1],[0,0],[1,1]> & <[1,1],[0,0],[1,1]> \end{bmatrix}$$

Lower limit of NFM, $[P_\mu, P_\lambda, P_v]_L = \begin{bmatrix} <0.4,0,1> & <1,0,1> & <0.2,0.3,0.4> \\ <1,0,1> & <1,0,1> & <1,0,1> \\ <0.2,0.3,0.4> & <1,0,1> & <1,0,1> \end{bmatrix}$

Here, $[P_\mu, P_\lambda, P_v]_L = [P_\mu, P_\lambda, P_v]_L^T$.

Upper limit of NFM, $[P_\mu, P_\lambda, P_v]_U = \begin{bmatrix} <0.5,0,1> & <1,0,1> & <0.5,0.6,0.7> \\ <1,0,1> & <1,0,1> & <1,0,1> \\ <0.5,0.6,0.7> & <1,0,1> & <1,0,1> \end{bmatrix}$

Here, $[P_\mu, P_\lambda, P_v]_U = [P_\mu, P_\lambda, P_v]_U^T$.

Definition 3.6. Every row single $(1,1,0)$ with $(0,0,1)$'s elsewhere is called NFPM.

Example 3.5. *Consider a NFPM,* $K = \begin{bmatrix} (1,1,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,1,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (1,1,0) \end{bmatrix}$

4. Graphical Representation of Range symmetric, Column symmetric and kernel symmetric Adjacency NFM

Definition 4.1. Adjacency NFM : An adjacency NFM is a square matrix that serves as a representation of a finite graph. The matrix's elements convey information regarding whether pairs of vertices within the graph are connected or not. In the specific scenario of a finite simple graph, the adjacency matrix can be described as a binary matrix, often denoted as a $(1, 1, 0)$ and $(0, 0, 1)$ matrix, where the diagonal elements are uniformly set to $(0, 0, 1).LetG(V, E)$ denote a simple graph with n vertices. The adjacency matrix $A = [a_{ij}]$ is a symmetric matrix defined

$$A = [a_j] = \begin{cases} (1, 1, 0) \ when \ v_i \ is \ adiancent \ to \ V_j \\ (0, 0, 1) \ otherwise \end{cases}$$

denoted by $A(G)orA_G$.

Example 4.1. Consider an adjacency NFM and a corresponding graph

$$A = \begin{bmatrix} & a & b & c & d \\ a & (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) \\ b & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) \\ c & (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) \\ d & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) \end{bmatrix}$$

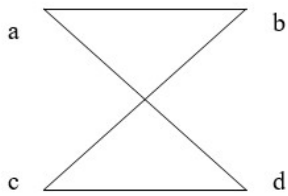

Figure 1

Definition 4.2. Incidency NFM:
If $G(V, E)$ represent a simple graph with n vertices. Let$V = V_1, V_2, \ldots, V_n$ and $E = e_1, e_2, ..., e_n$. Then, the incidence NFM $I = [m_{ij}]$ is a $n \times m$ matrix defined by

$$I = [m_{ij}] = \begin{cases} (1, 1, 0) \ when \ v, is \ ncidence \ to \ e_j \\ (0, 0, 1) \ otherwise \end{cases}$$

denoted by $A(G)orA_G$.

Example 4.2. Consider an incidency NFM and a corresponding graph. The incidence NFM is

$$I = \begin{bmatrix} & e_1 & e_2 & e_3 & e_4 & e_5 \\ a & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) & (1,1,0) \\ b & (1,1,0) & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1) \\ c & (0,0,1) & (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1) \\ d & (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1) & (1,1,0) \end{bmatrix}$$

Corresponding Grap

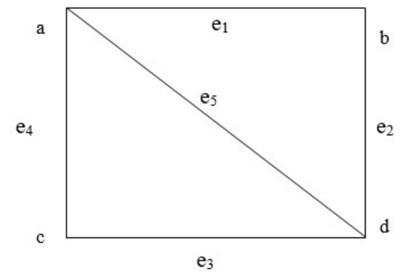

FIGURE 2

4.1. **Range symmetric, Column symmetric and kernel symmetric Adjacency NFM.** Graph A

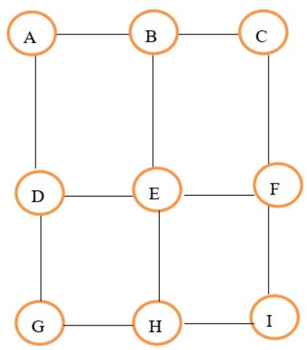

FIGURE 3

Adjacency NFM

Example 4.3.

$$A = \begin{bmatrix} & A & B & C & D & E & F & I & G & H \\ A & <0,0,1> & <1,1,0> & <0,0,1> & <1,1,0> & <0,0,1> & <0,0,1> & <0,0,1> & <0,0,1> & <0,0,1> \\ B & <1,1,0> & <0,0,1> & <1,1,0> & <0,0,1> & <1,1,0> & <0,0,1> & <0,0,1> & <0,0,1> & <0,0,1> \\ C & <0,0,1> & <1,1,0> & <0,0,1> & <0,0,1> & <0,0,1> & <1,1,0> & <0,0,1> & <0,0,1> & <0,0,1> \\ D & <1,1,0> & <0,0,1> & <0,0,1> & <0,0,1> & <1,1,0> & <0,0,1> & <1,1,0> & <0,0,1> & <0,0,1> \\ E & <0,0,1> & <1,1,0> & <0,0,1> & <1,1,0> & <0,0,1> & <1,1,0> & <0,0,1> & <1,1,0> & <0,0,1> \\ F & <0,0,1> & <0,0,1> & <1,1,0> & <0,0,1> & <1,1,0> & <0,0,1> & <0,0,1> & <0,0,1> & <1,1,0> \\ I & <0,0,1> & <0,0,1> & <0,0,1> & <1,1,0> & <0,0,1> & <0,0,1> & <0,0,1> & <1,1,0> & <0,0,1> \\ G & <0,0,1> & <0,0,1> & <0,0,1> & <0,0,1> & <1,1,0> & <0,0,1> & <1,1,0> & <0,0,1> & <1,1,0> \\ H & <0,0,1> & <0,0,1> & <0,0,1> & <0,0,1> & <0,0,1> & <1,1,0> & <0,0,1> & <1,1,0> & <0,0,1> \end{bmatrix}$$

The given Graph is range symmetric NFM $R(A) = R(A^T)$.
Graph B

Adjacency NFM

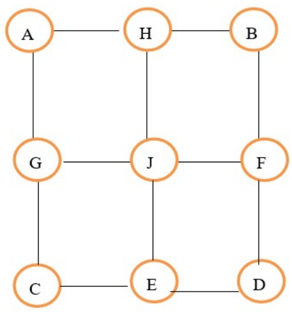

FIGURE 4

$B =$

	A	B	C	D	E	F	I	G	H
A	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<1,1,0>$	$<0,0,1>$	$<1,1,0>$	$<0,0,1>$
B	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<1,1,0>$	$<1,1,0>$	$<0,0,1>$
C	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<1,1,0>$	$<1,1,0>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$
D	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<1,1,0>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$
E	$<0,0,1>$	$<0,0,1>$	$<1,1,0>$	$<1,1,0>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<1,1,0>$
F	$<0,0,1>$	$<1,1,0>$	$<0,0,1>$	$<1,1,0>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<1,1,0>$
I	$<1,1,0>$	$<0,0,1>$	$<1,1,0>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<1,1,0>$
G	$<1,1,0>$	$<1,1,0>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<1,1,0>$
H	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<1,1,0>$	$<1,1,0>$	$<1,1,0>$	$<1,1,0>$	$<0,0,1>$

The given Graph is column symmetric NFM $C(B) = C(B^T)$

Graph C

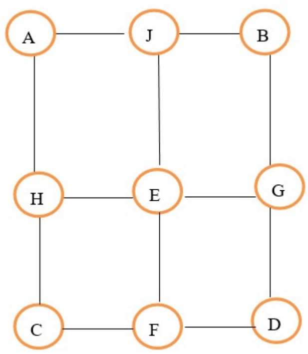

FIGURE 5

Adjacency Matrix

$C =$

	A	B	C	D	E	F	I	G	H
A	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<1,1,0>$	$<1,1,0>$
B	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<1,1,0>$	$<0,0,1>$	$<1,1,0>$
C	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<1,1,0>$	$<0,0,1>$	$<1,1,0>$	$<0,0,1>$
D	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<1,1,0>$	$<1,1,0>$	$<0,0,1>$	$<0,0,1>$
E	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<1,1,0>$	$<1,1,0>$	$<1,1,0>$	$<1,1,0>$
F	$<0,0,1>$	$<0,0,1>$	$<1,1,0>$	$<1,1,0>$	$<1,1,0>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$
I	$<0,0,1>$	$<1,1,0>$	$<0,0,1>$	$<1,1,0>$	$<1,1,0>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$
G	$<1,1,0>$	$<0,0,1>$	$<1,1,0>$	$<0,0,1>$	$<1,1,0>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$
H	$<1,1,0>$	$<1,1,0>$	$<0,0,1>$	$<0,0,1>$	$<1,1,0>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$

The given Graph is kernel symmetric Fuzzy Matrix $N(C) = N(C^T)$.

4.2. **Incidency matrix.**

$$A = \begin{bmatrix} <1,1,0> & <0,0,1> & <0,0,1> & <0,0,1> & <0,0,1> \\ <0,0,1> & <1,1,0> & <1,1,0> & <1,1,0> & <0,0,1> \\ <1,1,0> & <1,1,0> & <0,0,1> & <0,0,1> & <1,1,0> \\ <0,0,1> & <0,0,1> & <1,1,0> & <1,1,0> & <1,1,0> \end{bmatrix}$$

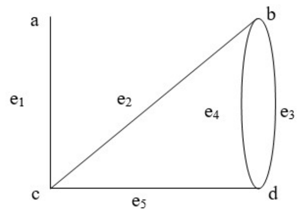

FIGURE 6

The given Graph is kernel symmetric Fuzzy Matrix but not Range and Column symmetric.
Note:4.1 "Every adjacency NFM is symmetric, range symmetric, column symmetric, and kernel symmetric, while the incidence matrix satisfies only the kernel symmetric conditions."
Note:4.2 "Every range symmetric NFM is also a kernel symmetric NFM, but a kernel symmetric NFM need not be a range symmetric NFM.".

5. Theorems and Resuts

Theorem 5.1. For an IVNFM $P =< [P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U >$,
$Q =< [Q_\mu, Q_\lambda, Q_v]_L, [Q_\mu, Q_\lambda, Q_v]_U > \in IVNFM_m$ and K be a NFPM if
$N([P_\mu, P_\lambda, P_v]_L) = N([Q_\mu, Q_\lambda, Q_v]_L) \Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_L K^T) = N(K[Q_\mu, Q_\lambda, Q_v]_L K^T)$.
$N([P_\mu, P_\lambda, P_v]_U) = N([Q_\mu, Q_\lambda, Q_v]_U) \Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_U K^T) = N(K[Q_\mu, Q_\lambda, Q_v]_U K^T)$.
Proof: Let $w \in N(K[P_\mu, P_\lambda, P_v]_L)K^T)$
$\Rightarrow w(K[P_\mu, P_\lambda, P_v]_L)K^T) = (0.0, 0.0, 0.0)$
$\Rightarrow yK^T = (0, 0, 0)$ where $y = wK([P_\mu, P_\lambda, P_v]_L)$
$\Rightarrow y \in N(K^T)$
$detK = detK^T > (0.0, 0.0, 0.0)$ (By Note 3.1)
Therefore, $N(K^T) = (0.0, 0.0, 0.0)$
Hence, $y = (0.0, 0.0, 0.0)$
$\Rightarrow wK([P_\mu, P_\lambda, P_v]_L) = (0.0, 0.0, 0.0)$
$\Rightarrow wK \in N([P_\mu, P_\lambda, P_v]_L) = N([Q_\mu, Q_\lambda, Q_v]_L)$
$\Rightarrow wK([Q_\mu, Q_\lambda, Q_v]_L)K^T = (0.0, 0.0, 0.0)$
$\Rightarrow w \in N(K([Q_\mu, Q_\lambda, Q_v]_L)K^T)$
$N(K[P_\mu, P_\lambda, P_v]_L K^T) \subseteq N(K[Q_\mu, Q_\lambda, Q_v]_L K^T)$
Similarly, $N(K[Q_\mu, Q_\lambda, Q_v]_L K^T) \subseteq N(K[P_\mu, P_\lambda, P_v]_L K^T)$
Therefore,
$N([P_\mu, P_\lambda, P_v]_L) = N([Q_\mu, Q_\lambda, Q_v]_L) \Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_L K^T) = N(K[Q_\mu, Q_\lambda, Q_v]_L K^T)$
Therefore,
$N([P_\mu, P_\lambda, P_v]_U) = N([Q_\mu, Q_\lambda, Q_v]_U) \Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_U K^T) = N(K[Q_\mu, Q_\lambda, Q_v]_U K^T)$

Conversely, if $N(K[P_\mu, P_\lambda, P_v]_L K^T) = N(K[Q_\mu, Q_\lambda, Q_v]_L K^T)$,
$N([P_\mu, P_\lambda, P_v]_L) = N(K^T(K([P_\mu, P_\lambda, P_v]_L)K^T)K) = N(K^T(K([Q_\mu, Q_\lambda, Q_v]_L)K^T)K)$
$N([P_\mu, P_\lambda, P_v]_L) = N([Q_\mu, Q_\lambda, Q_v]_L)$
Similarly,
$N(K[P_\mu, P_\lambda, P_v]_U K^T) = N(K[Q_\mu, Q_\lambda, Q_v]_U K^T) \Leftrightarrow N([P_\mu, P_\lambda, P_v]_U) = N([Q_\mu, Q_\lambda, Q_v]_U)$.

Example 5.1. *Consider an IVNFM*

$$P = \begin{bmatrix} <[0.3,0.4],[0.3,0.4],[0.4,0.6]> & <[0.5,0.6],[0.3,0.7],[0.6,0.7]> & <[0.2,0.5],[0.3,0.6],[0.4,0.7]> \\ <[0.4,0.6],[0.7,0.8],[0.6,0.7]> & <[0.4,0.7],[0.8,0.9],[0.1,0.2]> & <[0.2,0.3],[0.5,0.7],[0.1,0.2]> \\ <[0.2,0.7],[0.4,0.6],[0.4,0.7]> & <[0.4,0.5],[0.4,0.6],[0.5,0.6]> & <[0.4,0.5],[0.5,0.7],[0.6,0.8]> \end{bmatrix}$$

Lower limit of NFM, $[P_\mu, P_\lambda, P_v]_L = \begin{bmatrix} <0.3,0.3,0.4> & <0.5,0.3,0.6> & <0.2,0.3,0.4> \\ <0.4,0.7,0.6> & <0.4,0.8,0.1> & <0.2,0.5,0.1> \\ <0.2,0.4,0.4> & <0.4,0.4,0.5> & <0.4,0.5,0.6> \end{bmatrix}$

Upper limit of NFM, $[P_\mu, P_\lambda, P_v]_U = \begin{bmatrix} <0.4,0.4,0.6> & <0.6,0.7,0.7> & <0.5,0.6,0.7> \\ <0.6,0.8,0.7> & <0.7,0.9,0.2> & <0.3,0.7,0.2> \\ <0.7,0.6,0.7> & <0.5,0.6,0.6> & <0.5,0.5,0.6> \end{bmatrix}$

$$K = \begin{bmatrix} ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([0,0],[0,0],[1,1]) \\ ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) \\ ([0,0],[0,0],[1,1]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) \end{bmatrix}$$

$$K_L = K_U = \begin{bmatrix} (1,1,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,1,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (1,1,0) \end{bmatrix}$$

$$Q = \begin{bmatrix} <[0.5,0.6],[0.3,0.4],[0.7,0.9]> & <[0.4,0.5],[0.3,0.4],[0.6,0.7]> & <[0.3,0.5],[0.4,0.6],[0.4,0.8]> \\ <[0.5,0.6],[0.7,0.8],[0.5,0.7]> & <[0.4,0.5],[0.8,0.9],[0.2,0.2]> & <[0.2,0.3],[0.5,0.7],[0.2,0.2]> \\ <[0.2,0.3],[0.4,0.5],[0.4,0.7]> & <[0.4,0.6],[0.4,0.7],[0.5,0.6]> & <[0.4,0.6],[0.5,0.5],[0.6,0.9]> \end{bmatrix}$$

Lower limit of NFM, $[Q_\mu, Q_\lambda, Q_v]_L = \begin{bmatrix} <0.5,0.3,0.7> & <0.4,0.3,0.6> & <0.3,0.4,0.4> \\ <0.5,0.7,0.5> & <0.4,0.8,0.2> & <0.2,0.5,0.2> \\ <0.2,0.4,0.4> & <0.4,0.4,0.5> & <0.4,0.5,0.6> \end{bmatrix}$

Upper limit of NFM, $[Q_\mu, Q_\lambda, Q_v]_U = \begin{bmatrix} <0.6,0.4,0.9> & <0.5,0.4,0.7> & <0.5,0.6,0.8> \\ <0.6,0.8,0.7> & <0.5,0.9,0.2> & <0.3,0.7,0.2> \\ <0.3,0.5,0.7> & <0.6,0.7,0.6> & <0.6,0.5,0.9> \end{bmatrix}$

Theorem 5.2. For an IVNFM $P =< [P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U >\in IVNFM_m$ and K be a NFPM if
$N([P_\mu, P_\lambda, P_v]_L) = N([P_\mu, P_\lambda, P_v]_L) \Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_L K^T) = N(K[P_\mu, P_\lambda, P_v]_L K^T)$.
and $N([P_\mu, P_\lambda, P_v]_U) = N([P_\mu, P_\lambda, P_v]_U) \Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_U K^T) = N(K[P_\mu, P_\lambda, P_v]_U K^T)$.
Proof: Let $x \in N(K[P_\mu, P_\lambda, P_v]_L)K^T)$
$\Rightarrow x(K[P_\mu, P_\lambda, P_v]_L)K^T) = (0.0, 0.0, 0.0)$
$\Rightarrow wK^T = (0, 0, 0)$ where $w = xK([P_\mu, P_\lambda, P_v]_L)$
$\Rightarrow w \in N(K^T)$
$detK = detK^T > (0.0, 0.0, 0.0)$ (By Note 3.1)
$N(K^T) = (0.0, 0.0, 0.0)$
Here, $w = (0.0, 0.0, 0.0)$
$\Rightarrow xK([P_\mu, P_\lambda, P_v]_L) = (0.0, 0.0, 0.0)$
$\Rightarrow xK \in N([P_\mu, P_\lambda, P_v]_L) = N([P_\mu, P_\lambda, P_v]_L^T)$
$\Rightarrow xK([P_\mu, P_\lambda, P_v]_L^T)K^T = (0.0, 0.0, 0.0)$
$\Rightarrow x \in N(K([P_\mu, P_\lambda, P_v]_L)K^T)$
$N(K[P_\mu, P_\lambda, P_v]_L K^T) \subseteq N(K[P_\mu, P_\lambda, P_v]_L K^T)$
Similarly, $N(K[P_\mu, P_\lambda, P_v]_L K^T) \subseteq N(K[P_\mu, P_\lambda, P_v]_L K^T)$
Therefore,
$N(K[P_\mu, P_\lambda, P_v]_L K^T) = N([P_\mu, P_\lambda, P_v]_L^T K^T)$
Conversely, if $N(K[P_\mu, P_\lambda, P_v]_L K^T) = N(K[P_\mu, P_\lambda, P_v]_L K^T)$,
$N([P_\mu, P_\lambda, P_v]_L) = N(K^T(K([P_\mu, P_\lambda, P_v]_L)K^T)K) = N(K^T(K([P_\mu, P_\lambda, P_v]_L)K^T)K)$
$N([P_\mu, P_\lambda, P_v]_L) = N([P_\mu, P_\lambda, P_v]_L)$
$N([P_\mu, P_\lambda, P_v]_L) = N([P_\mu, P_\lambda, P_v]_L^T) \Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_{LK^T}) = N(K^T[P_\mu, P_\lambda, P_v]_L^T K^T)$

Similarly,
$N(K[P_\mu, P_\lambda, P_v]_U K^T) = N(K[P_\mu, P_\lambda, P_v]_U K^T)$
$\Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_U K^T) = N(K^T[P_\mu, P_\lambda, P_v]_U^T K^T)$.

Example 5.2. *Consider an IVNFM*

$$P = \begin{bmatrix} <[0.7,0.8],[0.3,0.4],[0.5,0.9]> & <[0.5,0.7],[0.3,0.7],[0.6,0.9]> & <[0.2,0.3],[0.3,0.6],[0.4,0.8]> \\ <[0.4,0.6],[0.7,0.9],[0.6,1]> & <[0.4,0.5],[0.8,0.9],[0.1,0.3]> & <[0.2,0.3],[0.5,0.8],[0.1,0.1]> \\ <[0.2,0.2],[0.4,0.6],[0.4,0.5]> & <[0.4,0.5],[0.4,0.7],[0.5,0.8]> & <[0.4,0.5],[0.5,0.5],[0.6,0.9]> \end{bmatrix}$$

$$K = \begin{bmatrix} ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([0,0],[0,0],[1,1]) \\ ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) \\ ([0,0],[0,0],[1,1]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) \end{bmatrix}$$

$$K_L = K_U = \begin{bmatrix} (1,1,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,1,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (1,1,0) \end{bmatrix}$$

$$\textit{Lower limit of NFM, } [P_\mu, P_\lambda, P_v]_L = \begin{bmatrix} <0.7,0.3,0.5> & <0.5,0.3,0.6> & <0.2,0.3,0.4> \\ <0.4,0.7,0.6> & <0.4,0.8,0.1> & <0.2,0.5,0.1> \\ <0.2,0.4,0.4> & <0.4,0.4,0.5> & <0.4,0.5,0.6> \end{bmatrix}$$

$$\textit{Upper limit of NFM,} [P_\mu, P_\lambda, P_v]_U = \begin{bmatrix} <0.8,0.4,0.9> & <0.7,0.7,0.9> & <0.3,0.6,0.8> \\ <0.6,0.9,0.1> & <0.5,0.9,0.3> & <0.3,0.8,0.1> \\ <0.2,0.6,0.5> & <0.5,0.7,0.8> & <0.5,0.5,0.9> \end{bmatrix}$$

Theorem 5.3. *For $P =< [P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U > \in IVNFM_m$ is kernel symmetric IVNFM, then $N([P_\mu, P_\lambda, P_v]_L [P_\mu, P_\lambda, P_v]_L^T) = N([P_\mu, P_\lambda, P_v]_L)$ $= N([P_\mu, P_\lambda, P_v]_L^T [P_\mu, P_\lambda, P_v]_L)$ and $N([P_\mu, P_\lambda, P_v]_U [P_\mu, P_\lambda, P_v]_U^T) = N([P_\mu, P_\lambda, P_v]_U)$ $= N([P_\mu, P_\lambda, P_v]_U^T [P_\mu, P_\lambda, P_v]_U)$.*

Proof: Let $x \in N([P_\mu, P_\lambda, P_v]_L)$
$\Leftrightarrow x[P_\mu, P_\lambda, P_v]_L = (0.0, 0.0, 0.0)$
$\Leftrightarrow x[P_\mu, P_\lambda, P_v]_L [P_\mu, P_\lambda, P_v]_L^T = (0.0, 0.0, 0.0)$
$\Leftrightarrow X \in N([P_\mu, P_\lambda, P_v]_L [P_\mu, P_\lambda, P_v]_L^T)$
$\Leftrightarrow N([P_\mu, P_\lambda, P_v]_L) \subseteq N([P_\mu, P_\lambda, P_v]_L [P_\mu, P_\lambda, P_v]_L^T)$
Similarly, $N([P_\mu, P_\lambda, P_v]_L [P_\mu, P_\lambda, P_v]_L^T) \subseteq N([P_\mu, P_\lambda, P_v]_L)$
Therefore, $N([P_\mu, P_\lambda, P_v]_L) = N([P_\mu, P_\lambda, P_v]_L [P_\mu, P_\lambda, P_v]_L^T)$
Similarly, $N([P_\mu, P_\lambda, P_v]_L) = N([P_\mu, P_\lambda, P_v]_L^T [P_\mu, P_\lambda, P_v]_L)$
Therefore, $N([P_\mu, P_\lambda, P_v]_L [P_\mu, P_\lambda, P_v]_L^T) = N([P_\mu, P_\lambda, P_v]_L)$
$= N([P_\mu, P_\lambda, P_v]_L^T [P_\mu, P_\lambda, P_v]_L)$
Similarly, $N([P_\mu, P_\lambda, P_v]_U [P_\mu, P_\lambda, P_v]_U^T) = N([P_\mu, P_\lambda, P_v]_U)$
$= N([P_\mu, P_\lambda, P_v]_U^T [P_\mu, P_\lambda, P_v]_U)$.

Example 5.3. *Consider a IVNFM*

$$P = \begin{bmatrix} <[0.1,0.2],[0.3,0.4],[0.5,0.9]> & <[0.4,0.6],[0.3,0.8],[0.6,0.7]> & <[0.2,0.9],[0.3,0.9],[0.4,0.9]> \\ <[0.4,0.5],[0.7,1],[0.6,1]> & <[0.4,0.7],[0.7,0.8],[0.1,0.2]> & <[0.2,0.2],[0.5,0.7],[0.1,0.1]> \\ <[0.2,0.2],[0.3,0.6],[0.5,0.5]> & <[0.4,0.5],[0.3,0.7],[0.5,0.7]> & <[0.4,0.5],[0.2,0.5],[0.1,0.9]> \end{bmatrix}$$

$$\textit{Lower limit of NFM } [P_\mu, P_\lambda, P_v]_L = \begin{bmatrix} <0.1,0.3,0.5> & <0.4,0.3,0.6> & <0.2,0.3,0.4> \\ <0.4,0.7,0.6> & <0.4,0.7,0.1> & <0.2,0.5,0.1> \\ <0.2,0.3,0.5> & <0.4,0.3,0.5> & <0.4,0.2,0.1> \end{bmatrix}$$

$$\textit{Upper limit of NFM } [P_\mu, P_\lambda, P_v]_U = \begin{bmatrix} <0.2,0.4,0.9> & <0.6,0.8,0.7> & <0.9,0.9,0.9> \\ <0.5,1,1> & <0.7,0.8,0.2> & <0.2,0.7,0.1> \\ <0.2,0.6,0.5> & <0.5,0.7,0.7> & <0.5,0.5,0.9> \end{bmatrix}$$

Theorem 5.4. *For $P =< [P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U >, < [Q_\mu, Q_\lambda, Q_v]_L,$ $[Q_\mu, Q_\lambda, Q_v]_L > \in IVNFM_{nm}$ and KNFM,*
$R([P_\mu, P_\lambda, P_v]_L) = R([Q_\mu, Q_\lambda, Q_v]_L) \Leftrightarrow R(K[P_\mu, P_\lambda, P_v]_L K^T) = R(K[Q_\mu, Q_\lambda, Q_v]_L K^T)$ *and*
$R([P_\mu, P_\lambda, P_v]_U) = R([Q_\mu, Q_\lambda, Q_v]_U) \Leftrightarrow R(K[P_\mu, P_\lambda, P_v]_U K^T) = R(K[Q_\mu, Q_\lambda, Q_v]_U K^T)$

Proof: Let $R([P_\mu, P_\lambda, P_v]_L) = R([Q_\mu, Q_\lambda, Q_v]_L)$
Then,$R([P_\mu, P_\lambda, P_v]_L K^T) = R([Q_\mu, Q_\lambda, Q_v]_L K^T)$

$= R([P_\mu, P_\lambda, P_v]_L)K^T$
$= R([P_\mu, P_\lambda, P_v]_L K^T)$
Let $z \in \{R([P_\mu, P_\lambda, P_v]_L K^T)\}$
$z = w(K[P_\mu, P_\lambda, P_v]_L K^T)$ *for some* $w \in V^n$
$z = r[P_\mu, P_\lambda, P_v]_L K^T, r = wK$
$z \in R(R([P_\mu, P_\lambda, P_v]_L)K^T) = R([Q_\mu, Q_\lambda, Q_v]_L)(K^T)$
$z = u[P_\mu, P_\lambda, P_v]_L K^T$ *for some* $u \in V^n$
$z = (uK^T)K[Q_\mu, Q_\lambda, Q_v]_L K^T$
$z = vK[Q_\mu, Q_\lambda, Q_v]_L K^T$ *for some* $v \in V^n$
$z \in R(K([Q_\mu, Q_\lambda, Q_v]_L K^T)$
Therefore, $R(K([P_\mu, P_\lambda, P_v]_L K^T) \subseteq R(K([Q_\mu, Q_\lambda, Q_v]_L K^T)$
Similarly, $R(K([Q_\mu, Q_\lambda, Q_v]_L K^T) \subseteq R(K([P_\mu, P_\lambda, P_v]_L K^T)$
Therefore, $R(K([P_\mu, P_\lambda, P_v]_L K^T) = R(K([Q_\mu, Q_\lambda, Q_v]_L K^T)$
Conversely, Let $R(K([P_\mu, P_\lambda, P_v]_L K^T) \subseteq R(K([Q_\mu, Q_\lambda, Q_v]_L K^T)$
$= R(K^T([Q_\mu, Q_\lambda, Q_v]K^T)K$
$= R([Q_\mu, Q_\lambda, Q_v])$
$R([P_\mu, P_\lambda, P_v]_L) = R([Q_\mu, Q_\lambda, Q_v]_L)$
Similarly,
$R([P_\mu, P_\lambda, P_v]_U) = R([Q_\mu, Q_\lambda, Q_v]_U) \Leftrightarrow R(K[P_\mu, P_\lambda, P_v]_U K^T) = R(K[Q_\mu, Q_\lambda, Q_v]_U K^T)$.

Example 5.4. *Let us consider IVNFM*

$$P = \begin{bmatrix} <[0.1,0.2],[0.3,0.4],[0.5,0.9]> & <[0.4,0.6],[0.3,0.8],[0.6,0.7]> & <[0.2,0.9],[0.3,0.9],[0.4,0.9]> \\ <[0.4,0.6],[0.3,0.8],[0.6,0.7]> & <[0.4,0.7],[0.7,0.8],[0.1,0.2]> & <[0.2,0.2],[0.5,0.7],[0.1,0.1]> \\ <[0.2,0.9],[0.3,0.9],[0.4,0.9]> & <[0.2,0.2],[0.5,0.7],[0.1,0.1]> & <[0.4,0.5],[0.2,0.5],[0.1,0.9]> \end{bmatrix}$$

$$K = \begin{bmatrix} ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([0,0],[0,0],[1,1]) \\ ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) \\ ([0,0],[0,0],[1,1]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0] \end{bmatrix},$$

$$K_L = K_U \begin{bmatrix} (1,1,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,1,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (1,1,0) \end{bmatrix}$$

$$\textit{Lower limit of NFM } [P_\mu, P_\lambda, P_v]_L) = \begin{bmatrix} <0.1,0.3,0.5> & <0.4,0.3,0.6> & <0.2,0.3,0.4> \\ <0.4,0.3,0.6> & <0.4,0.7,0.1> & <0.2,0.5,0.1> \\ <0.2,0.3,0.4> & <0.2,0.5,0.1> & <0.4,0.2,0.1> \end{bmatrix}$$

$$\textit{Upper limit of NFM } [P_\mu, P_\lambda, P_v]_U) = \begin{bmatrix} <0.2,0.4,0.9> & <0.6,0.8,0.7> & <0.9,0.9,0.9> \\ <0.6,0.8,0.7> & <0.9,0.9,0.9> & <0.2,0.7,0.1> \\ <0.9,0.9,0.9> & <0.2,0.7,0.1> & <0.5,0.5,0.9> \end{bmatrix}$$

$$Q = \begin{bmatrix} <[0.2,0.9],[0.3,0.9],[0.4,0.9]> & <[0.2,0.2],[0.5,0.7],[0.1,0.1]> & <[0.4,0.5],[0.2,0.5],[0.1,0.9]> \\ <[0.4,0.6],[0.3,0.8],[0.6,0.7]> & <[0.4,0.7],[0.7,0.8],[0.1,0.2]> & <[0.2,0.2],[0.5,0.7],[0.1,0.1]> \\ <[0.1,0.2],[0.3,0.4],[0.5,0.9]> & <[0.4,0.6],[0.3,0.8],[0.6,0.7]> & <[0.2,0.9],[0.3,0.9],[0.4,0.9]> \end{bmatrix}$$

$$\textit{Lower limit of NFM } [Q_\mu, Q_\lambda, Q_v]_L) = \begin{bmatrix} <0.2,0.3,0.4> & <0.2,0.5,0.1> & <0.4,0.2,0.1> \\ <0.4,0.3,0.6> & <0.4,0.7,0.1> & <0.2,0.5,0.1> \\ <0.1,0.3,0.5> & <0.4,0.3,0.6> & <0.2,0.3,0.4> \end{bmatrix}$$

$$\textit{Upper limit of NFM } [Q_\mu, Q_\lambda, Q_v]U) = \begin{bmatrix} <0.9,0.9,0.9> & <0.2,0.7,0.1> & <0.5,0.5,0.9> \\ <0.6,0.8,0.7> & <0.7,0.8,0.2> & <0.2,0.7,0.1> \\ <0.2,0.4,0.9> & <0.6,0.8,0.7> & <0.9,0.9,0.9> \end{bmatrix}$$

$R([P_\mu, P_\lambda, P_v]_L) = R([Q_\mu, Q_\lambda, Q_v]_L) \Leftrightarrow R(K[P_\mu, P_\lambda, P_v]_L K^T) = R(K[Q_\mu, Q_\lambda, Q_v]_L K^T)$ *and*
$R([P_\mu, P_\lambda, P_v]_U) = R([Q_\mu, Q_\lambda, Q_v]_U) \Leftrightarrow R(K[P_\mu, P_\lambda, P_v]_U K^T) = R(K[Q_\mu, Q_\lambda, Q_v]_U K^T)$.

6. K - KERNAL SYMMETRIC IVNFM

Definition 6.1. *Let* $P =< [P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U > \in IVNFM_{nn}$ *is called k - Kernal Symmetric IVNFM if* $N(K[P_\mu, P_\lambda, P_v]_L K^T) = N(K[P_\mu, P_\lambda, P_v]_L K^T)$ *and* $N(K[P_\mu, P_\lambda, P_v]_L K^T) = N(K[P_\mu, P_\lambda, P_v]_U K^T)$.

Remark 6.1. *Let* $P =< [P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U >$ *is k- Symmetric IVNFM implies it is k=KS-IVNFM for* $[P_\mu, P_\lambda, P_v]_L = K([P_\mu, P_\lambda, P_v]_L^T)K, [P_\mu, P_\lambda, P_v]_U = K([P_\mu, P_\lambda, P_v]_U^T)K$

spontaneously implies $N([P_\mu, P_\lambda, P_v]_L) = N(K[P_\mu, P_\lambda, P_v]_L^T K)$ *and* $N([P_\mu, P_\lambda, P_v]_U)$ $= N(K[P_\mu, P_\lambda, P_v]_U^T K)$.

Example 6.1. *shows that if and only if need not be true.*

$$P = \begin{bmatrix} <[0,0.1],[0,0.4],[0.5,0.9]> & <[0,0.5],[0,0.8],[0.4,0.7]> & <[0.3,0.4],[0.4,0.5],[0.5,1]> \\ <[0.5,0.5],[0.4,0.5],[0.6,0.7]> & <[0.1,0.7],[0.4,0.8],[0.6,0.2]> & <[0,0.6],[0,0.5],[0.4,0.2]> \\ <[0.4,0.4],[0.5,0.6],[0.3,0.7]> & <[0.3,0.7],[0.4,0.7],[0.5,0.2]> & <[0,0.4],[0,0.1],[0.3,0.2]> \end{bmatrix}$$

$$K = \begin{bmatrix} ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([0,0],[0,0],[1,1]) \\ ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) \\ ([0,0],[0,0],[1,1]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) \end{bmatrix},$$

$$K_L = K_U \begin{bmatrix} (1,1,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,1,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (1,1,0) \end{bmatrix}$$

$$[P_\mu, P_\lambda, P_v]_L) = \begin{bmatrix} (0,0,0.5) & (0,0,0.4) & (0.3,0.4,0.5) \\ (0.5,0.4,0.6) & (0.1,0.4,0.6) & (0,0,0.4) \\ (0.4,0.5,0.3) & (0.3,0.4,0.5) & (0,0,0.3) \end{bmatrix}$$

$$K[P_\mu, P_\lambda, P_v]_L K^T = \begin{bmatrix} (0,0,0.3) & (0,0,0.4) & (0.3,0.4,0.5) \\ (0.3,0,0.5) & (0.1,0,0.6) & (0,0.4,0.4) \\ (0.4,0,0.3) & (0.5,0,0.6) & (0,0.4,0.5) \end{bmatrix}$$

Therefore $[P_\mu, P_\lambda, P_v]_L \neq K[P_\mu, P_\lambda, P_v]_L K^T$
But, $N([P_\mu, P_\lambda, P_v]_L) = N(K[P_\mu, P_\lambda, P_v]_L K^T) = (0.0, 0.0, 0.0)$
Similarly,
Therefore, $[P_\mu, P_\lambda, P_v]_U \neq K([P_\mu, P_\lambda, P_v]_U^T)K$
But, $N([P_\mu, P_\lambda, P_v]_U) = N(K[P_\mu, P_\lambda, P_v]_U^T K) = (0.0, 0.0, 0.0)$.

Theorem 6.1. *The subsequence conditions are equivalent for* $P =< [P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U >\in IVNFMnn$

$(i) N(K[P_\mu, P_\lambda, P_v]_L K^T) = N(K[P_\mu, P_\lambda, P_v]_L K^T), N(K[P_\mu, P_\lambda, P_v]_U K^T)$ $= N(K[P_\mu, P_\lambda, P_v]_U K^T)$

$(ii) N(K[P_\mu, P_\lambda, P_v]_L) = N((K[P_\mu, P_\lambda, P_v]_L)^T), N(K[P_\mu, P_\lambda, P_v]_U)$ $= N((K[P_\mu, P_\lambda, P_v]_L)^T)$

$(iii) N(K[P_\mu, P_\lambda, P_v]_L) = N((K[P_\mu, P_\lambda, P_v]_L)^T), N([P_\mu, P_\lambda, P_v]_U K)$ $= N(([P_\mu, P_\lambda, P_v]_U K)^T)$

$(iv) N([P_\mu, P_\lambda, P_v]_L^T) = N(K[P_\mu, P_\lambda, P_v]_L), N[P_\mu, P_\lambda, P_v]_U^T = N(K[P_\mu, P_\lambda, P_v]_U)$

$(v) N([P_\mu, P_\lambda, P_v]_L) = N(([P_\mu, P_\lambda, P_v]_L K)^T), N([P_\mu, P_\lambda, P_v]_U) = N(([P_\mu, P_\lambda, P_v]_U K)^T)$

$(vi) [P_\mu, P_\lambda, P_v]_L^\dagger is k - KSIVNFM, [P_\mu, P_\lambda, P_v]_U^\dagger is k - KSIVNFM$

$(vii) N([P_\mu, P_\lambda, P_v]_L) = N[P_\mu, P_\lambda, P_v]_L^\dagger K), N[P_\mu, P_\lambda, P_v]_U = N[P_\mu, P_\lambda, P_v]_U^\dagger K$

$(viii) K[P_\mu, P_\lambda, P_v]_L^\dagger [P_\mu, P_\lambda, P_v]_L = [P_\mu, P_\lambda, P_v]_L [P_\mu, P_\lambda, P_v]_L^\dagger K,$
$K[P_\mu, P_\lambda, P_v]_U^\dagger [P_\mu, P_\lambda, P_v]_U = [P_\mu, P_\lambda, P_v]_U [P_\mu, P_\lambda, P_v]_U^\dagger K$

$(ix) [P_\mu, P_\lambda, P_v]_L^\dagger [P_\mu, P_\lambda, P_v]_L K = K[P_\mu, P_\lambda, P_v]_L [P_\mu, P_\lambda, P_v]_L^\dagger,$
$[P_\mu, P_\lambda, P_v]_U^\dagger [P_\mu, P_\lambda, P_v]_U K = K[P_\mu, P_\lambda, P_v]_U [P_\mu, P_\lambda, P_v]_U^\dagger.$

Proof: $(i) \Leftrightarrow (ii)$
$\Leftrightarrow N([P_\mu, P_\lambda, P_v]_L) = N(K[P_\mu, P_\lambda, P_v]_L^T K)$
$\Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_L) = N([P_\mu, P_\lambda, P_v]_L^T K)$ *(By* P_2*)*$(K^2 = I)$
$\Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_L) = N((K[P_\mu, P_\lambda, P_v]_L)^T)$ *(Because* $(KP)^T = P^T(K^T = P^T K)$
$K[P_\mu, P_\lambda, P_v]_L$ *is KS,*
Similarly,
$N([P_\mu, P_\lambda, P_v]_U) = N(K[P_\mu, P_\lambda, P_v]_U^T K) \Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_U) = N((K[P_\mu, P_\lambda, P_v]_U)^T)$
Condition(ii) *is true*
Condition $(i) \Leftrightarrow (iii)$
$\Leftrightarrow N([P_\mu, P_\lambda, P_v]_L) = N(K[P_\mu, P_\lambda, P_v]_L^T K)$
$\Leftrightarrow N([P_\mu, P_\lambda, P_v]_L K) = N(K[P_\mu, P_\lambda, P_v]_L^T)$ *(By* P_2*)*$(K^2 = I)$
$\Leftrightarrow N([P_\mu, P_\lambda, P_v]_L K) = N(([P_\mu, P_\lambda, P_v]_L)K)^T)$ *(Because* $(KP)^T = P^T(K^T = P^T K)$

$K[P_\mu, P_\lambda, P_v]_L K$ *is KS,*
Similarly,
$N([P_\mu, P_\lambda, P_v]_U) = N([P_\mu, P_\lambda, P_v]_U^T K) \Leftrightarrow N([P_\mu, P_\lambda, P_v]_U K) = N(([P_\mu, P_\lambda, P_v]_U K)^T)$
Therefore, (iii) *holds*
$(ii) \Leftrightarrow (iv)$
$\Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_L) = N(K[P_\mu, P_\lambda, P_v]_U^T) = N(([P_\mu, P_\lambda, P_v]_U)^T K)$
$\Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_L) = N([P_\mu, P_\lambda, P_v]_U^T)$ $(By P_2)$
Similarly,$N(K[P_\mu, P_\lambda, P_v]_U) = N(K[P_\mu, P_\lambda, P_v]_U)^T \Leftrightarrow N([P_\mu, P_\lambda, P_v]_L)^T)$
$= N(K[P_\mu, P_\lambda, P_v]_U)$
Therefore, (iv) *holds*
$(iii) \Leftrightarrow (iv)$
$\Leftrightarrow N([P_\mu, P_\lambda, P_v]_L K) = N(([P_\mu, P_\lambda, P_v]_L K)^T)$
$\Leftrightarrow N([P_\mu, P_\lambda, P_v]_L) = N(([P_\mu, P_\lambda, P_v]_L K)^T)(By P_2)$
Similarly, $N([P_\mu, P_\lambda, P_v]_U K) = N(K[P_\mu, P_\lambda, P_v]_U)K^T \Leftrightarrow N([P_\mu, P_\lambda, P_v]_L)$
$= N([P_\mu, P_\lambda, P_v]_U K)^T.$
$(ii) \Leftrightarrow (vi)$ *holds*
Condition $(i) \Leftrightarrow (vii)$ *holds.*
$(i) \Leftrightarrow (viii)$ *holds.*
$(viii) \Leftrightarrow (ix)$. *Therefore* (ix) *are holds.*

7. Conclusion

The theorems explain the properties of RS and KS IVNFMs. With relevant examples, we introduce the concepts of KS and k-KS IVNFMs. We also explore various results related to KS-IVNFMs and provide illustrative examples to clarify these findings. Additionally, we present a graphical representation of KS, column symmetric, and range symmetric adjacency and incidence IVNFM.

Every adjacency IVNFM is symmetric, range symmetric, column symmetric, and kernel symmetric, but the incidence matrix satisfies only kernel symmetric conditions. Similarly, every range symmetric adjacency IVNFM is also a kernel symmetric adjacency IVNFM, but a kernel symmetric adjacency IVNFM need not be a range symmetric IVNFM.

In the future, we plan to prove additional properties related to g-inverses of k-Kernel Symmetric IVNFMs.

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