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SOME FIXED POINT THEOREMS IN DISLOCATED QUASI EXTENDED B-METRIC SPACE

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Abstract. In 2017, Kamran et al. [14] introduced the notion of extended b-metric space as a generalization of b-metric space. The concept of dislocated quasi extended b-metric space was presented by Nurwahyu [15]. In this paper, Banach's contraction principle and fixed point results for other different contraction type mapping given by Bianchini [5] and Agrawal et al. [3] are established in this space. The established results broaden and generalize a number of fixed point results in the literature. Also provided are appropriate examples of the established results.

Keywords: Fixed Point, Contraction Map, b-Metric, Dislocated Quasi b-Metric Space, Cauchy Sequence.

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1. Introduction

The theory of fixed points is one of the most well-known and progressive field of research in nonlinear functional analysis. It is quite useful to study the existence and uniqueness of fixed point($S(u) = u$). In this field, one of the most discussed result was given by S. Banach in 1922 [4] for a contraction mapping in a complete metric space, well known as Banach contraction principle."Let S be a map from a complete metric space $(\mathcal{K}, \ell)$ into itself satisfying: $\ell(Su, Sv) \leq \alpha\ell(u, v)$ for $\alpha \in [0, 1)$; then S has a unique fixed point in $\mathcal{K}$." Since then many authors have extended this result in different metric spaces such as dislocated metric space, dislocated quasi metric space, partial metric space, cone metric space, G-metric space, b-metric space and so forth, by utilising various contractions and mappings.

Bakhtin [7] and Bourbaki [6] introduced the concept of b-metric space as a generalization

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of metric space. Later Czerwik [8] extended the results in b-metric space. Following Czerwik, numerous studies that included fixed point results in b-metric space were established. Rahman and Sarwar [16] in 2016 defined dislocated quasi b-metric space and well known fixed point results were established in this space. Recently, Kamran et al. [14] introduced the concept of extended b-metric space as a generalization of b-metric space.

Motivated by the idea of extended b-metric space, Nurwahyu [15] introduced dislocated quasi extended b-metric space as its generalization. Here, we have established the Banach contraction principle and other well known fixed point results by utilizing contraction condition given by Bianchini [5] and Agrawal et.al [3] in dislocated quasi extended b-metric space.

2. Preliminaries

The purpose of this section is to acquaint with the basic concepts and definitions used throughout this research paper which will further assist to understand next section.

Definition 2.1. (Bakhtin [7], Czrerwik [8]) Given that $s \geq 1$ is a real number and $\mathcal{K}$ is a nonempty set, the mapping $\ell_s : \mathcal{K} \times \mathcal{K} \to [0, \infty)$ is called b-metric if satisfies the following conditions for all $u, v, w \in \mathcal{K}$:

$(b1)$ $\ell_s(u, u) = 0$;
$(b2)$ $\ell_s(u, v) = \ell_s(v, u) = 0$ implies that $u = v$;
$(b3)$ $\ell_s(u, v) = \ell_s(v, u)$;
$(b4)$ $\ell_s(u, v) \leq s[\ell_s(u, w) + \ell_s(w, v)]$.

The pair $(\mathcal{K}, \ell_s)$ is called a b-metric space.

Remark 2.1. (1) The definition of b-metric space coincides with the definition of usual metric space if $s = 1$. Thus, every metric space is b-metric space, but the converse is not true [13].
(2) A b-metric ℓ_s is not continuous [19], however a metric ℓ is known to be continuous.
(3) If ℓ_s satisfies the conditions $(b2)$ and $(b4)$ only then it is called dislocated quasi b-metric (ℓ_{qs} b-metric), introduced by Rahman and Sarwar [16].

Recently Kamran et al. [14] introduced the notion of extended b-metric space as a generalization of b-metric space.

Definition 2.2. *(*[14]*)* Given that $\mathcal{K}$ is a non empty set and $\theta : \mathcal{K} \times \mathcal{K} \to [1, \infty)$, the mapping $\ell_\theta : \mathcal{K} \times \mathcal{K} \to [0, \infty)$ is called extended b-metric if it satisfies the following properties for all $u, v, w \in \mathcal{K}$:

$(\ell_\theta 1)$ $\ell_\theta(u, u) = 0$;
$(\ell_\theta 2)$ $\ell_\theta(u, v) = \ell_\theta(v, u) = 0$ implies that $u = v$;
$(\ell_\theta 3)$ $\ell_\theta(u, v) = \ell_\theta(v, u)$;
$(\ell_\theta 4)$ $\ell_\theta(u, v) \leq \theta(u, v)[\ell_\theta(u, w) + \ell_\theta(w, v)]$.

The pair $(\mathcal{K}, \ell_\theta)$ is called an extended b-metric space.

Remark 2.2. (1) The definition of extended b-metric space coincides with the definition of b-metric for $\theta(u, v) = s \geq 1$, for all $u, v \in \mathcal{K}$ and it reduces to be usual metric if $s = 1$.
(2) If ℓ_θ satisfies the conditions $(\ell_\theta 2)$ and $(\ell_\theta 4)$ only then it is called dislocated quasi extended b-metric ($\ell_{q\theta}$ b-metric), introduced by Nurwahyu [15].

(3) Every dislocated quasi b-metric space is dislocated quasi extended b-metric space and every extended b-metric space is dislocated quasi extended b-metric space with the same $\theta : \mathcal{K} \times \mathcal{K} \to [1, \infty)$ and zero self distance. However the converse is not true.

Example 2.1. *Assuming that $\mathcal{K} = [0, \infty)$, define $\ell_{q\theta} : \mathcal{K} \times \mathcal{K} \to [0, \infty)$ as $\ell_{q\theta}(u, v) =\mid u \mid +(u - v)^2$ for all $u, v \in \mathcal{K}$. Define $\theta : \mathcal{K} \times \mathcal{K} \to [1, \infty)$ as $\theta(u, v) = u + v + 2$ for all $u, v \in \mathcal{K}$. Then $(\mathcal{K}, \ell_{q\theta})$ is dislocated quasi extended b-metric space. Since $\ell_{q\theta}(1, 1) = 1 \neq 0$, $\ell_{q\theta}(1, 2) \neq \ell_{q\theta}(2, 1)$ and $\mathcal{K}$ is infinite, hence it is neither extended b-metric space nor dislocated quasi b-metric space.*

Now, we define convergent sequence, Cauchy sequence and completeness in the context of dislocated quasi extended b-metric space.

Definition 2.3. Let $(\mathcal{K}, \ell_{q\theta})$ be a dislocated quasi extended b-metric space.

(i) A sequence $\{u_n\}$ in $\mathcal{K}$ converges to $\eta \in \mathcal{K}$ if

$$lim_{n\to\infty}\ell_{q\theta}(u_n, \eta) = 0 = lim_{n\to\infty}\ell_{q\theta}(\eta, u_n).$$

In this situation, η is referred to as $\ell_{q\theta}$ limit of $\{u_n\}$ and written as $u_n \to \eta(n \to \infty)$.

(ii) A sequence $\{u_n\}$ in $\mathcal{K}$ is called a Cauchy sequence if for given any $\epsilon > 0, \exists\, n_0 \in \mathbb{N}$ such that

$$\ell_{q\theta}(u_m, u_n) < \epsilon \, or \, \ell_{q\theta}(u_n, u_m) < \epsilon, \text{ for all } m, n \geq n_0,$$

that is

$$lim_{m,n\to\infty}\ell_{q\theta}(u_m, u_n) = 0 = lim_{m,n\to\infty}\ell_{q\theta}(u_n, u_m).$$

(iii) $(\mathcal{K}, \ell_{q\theta})$ is called complete if every Cauchy sequence in $\mathcal{K}$ converges to a point of $\mathcal{K}$.

Note that a dislocated quasi extended b-metric need not be continuous.

Example 2.2. Let $\mathcal{K} = \mathbb{N} \cup \{\infty\}$ and define $\ell_{q\theta} : \mathcal{K} \times \mathcal{K} \to \mathbb{R}$ as

$$\ell_{q\theta}(n_1, n_2) = \begin{cases} 1 & \text{if}\, n_1 = n_2 \\ \mid \frac{1}{n_1} - \frac{1}{n_2} \mid & \text{if}\, n_1, n_2 \,\text{are even or}\, n_1 n_2 = \infty \\ 5 & \text{if}\, n_1, n_2 \,\text{are odd and}\, n_1 < n_2 \\ 7 & \text{if}\, n_1, n_2 \,\text{are odd and}\, n_1 > n_2 \\ 2 & \text{otherwise} \end{cases}$$

If $\theta(u, v) = 3$ for all $u, v \in \mathcal{K}$, then $(\mathcal{K}, \ell_{q\theta})$ is a dislocated quasi extended b-metric space, but it is not continuous.
For each $n \in \mathbb{N}$, set $u_n = 2n$. Then $\ell_{q\theta}(2n, \infty) = \frac{1}{2n} \to 0$ as $n \to \infty$,
that is, $u_n \to \infty$ but $\ell_{q\theta}(u_n, 1) = 2 \nrightarrow \ell_{q\theta}(\infty, 1)$ as $n \to \infty$.

The Following Lemma as given in Kamran et al. [14] can be carried over directly from extended b-metric space to the dislocated quasi extended b-metric space.

Lemma 2.1. *([14])* Let $(\mathcal{K}, \ell_{q\theta})$ be a dislocated quasi extended b-metric space such that $\ell_{q\theta}$ is continuous, then each convergent sequence has a unique limit in $(\mathcal{K})$.

Let us recollect the main result of Bianchini [5], Kannan [12] and Agrawal et al. [3] addressing the presence and uniqueness of fixed point in complete metric space.

Theorem 2.1. *([5]) Let $(\mathcal{K}, \ell)$ be a complete metric space and let $S : \mathcal{K} \to \mathcal{K}$ be a self mapping satisfying the following condition:*

$$\ell(Su, Sv) \leq h\, max\{\ell(u, Su), \ell(v, Sv)\} \text{ for all } u, v \in \mathcal{K},$$

where $h \in [0, 1)$. Then S has a unique fixed point $\mathcal{K}$.

Theorem 2.2. *([12]) Let $(\mathcal{K}, \ell)$ be a complete metric space and let $S : \mathcal{K} \to \mathcal{K}$ be a self mapping satisfying the following condition:*

$$\ell(Su, Sv) \leq \alpha\, [\ell(u, Su) + \ell(v, Sv)] \text{ for all } u, v \in \mathcal{K},$$

where $\alpha \in [0, \frac{1}{2})$. Then S has a unique fixed point $\mathcal{K}$.

Theorem 2.3. *([3]) Let $(\mathcal{K}, \ell)$ be a complete metric space and let $S : \mathcal{K} \to \mathcal{K}$ be a self mapping satisfying the following condition:*

$$\ell(Su, Sv) \leq a\, max\{\ell(u, Su), \ell(v, Sv), \ell(u, v)\} + b\{\ell(u, Sv) + \ell(v, Su)\} \text{ for all } u, v \in \mathcal{K},$$

where $a, b > 0$ such that $a + 2b \leq 1$. Then S has a unique fixed point $\mathcal{K}$.

3. Main Results

In this section, we establish Banach's contraction principle and fixed point results for other different contraction type mapping given by Bianchini [5] and Agrawal et al. [3] in the context of dislocated quasi extended b-metric space.

Lemma 3.1. Consider a dislocated quasi extended b-metric space $(\mathcal{K}, \ell_{q\theta})$ such that $\ell_{q\theta}$ is continuous. Let $\{u_n\}$ be a sequence in $\mathcal{K}$ satisfying

$$0 < \ell_{q\theta}(u_n, u_{n+1}) \leq \alpha\, \ell_{q\theta}(u_{n-1}, u_n). \tag{1}$$

for $n = 1, 2, 3, ...$ and $\alpha \in [0, 1)$ is a real number such that $lim_{n,m\to\infty}\theta(u_n, u_m) < \frac{1}{\alpha}$, where $\theta : \mathcal{K} \times \mathcal{K} \to [1, \infty)$ is defined in $\ell_{q\theta}$ b-metric space. Then, in $\mathcal{K}$, $\{u_n\}$ is a Cauchy sequence.

Proof. Assume $m > n \geq 1$, applying triangular inequality $(\ell_{q\theta}4)$ and (1), we get

$$\begin{aligned}
\ell_{q\theta}(u_n, u_m) &\leq \theta(u_n, u_m)[\ell_{q\theta}(u_n, u_{n+1}) + \ell_{q\theta}(u_{n+1}, u_m)] \\
&\leq \theta(u_n, u_m)\alpha^n \ell_{q\theta}(u_0, u_1) + \theta(u_n, u_m)\theta(u_{n+1}, u_m) \\
&[\ell_{q\theta}(u_{n+1}, u_{n+2}) + \ell_{q\theta}(u_{n+2}, u_m)] \\
&\leq \theta(u_n, u_m)\alpha^n \ell_{q\theta}(u_0, u_1) + \theta(u_n, u_m)\theta(u_{n+1}, u_m)\alpha^{n+1}\ell_{q\theta}(u_0, u_1) \\
&+ \cdots\cdots + \theta(u_n, u_m)\theta(u_{n+1}, u_m)\theta(u_{n+2}, u_m)\cdots \\
&\cdots\theta(u_{m-2}, u_m)\theta(u_{m-1}, u_m)\alpha^{m-1}\ell_{q\theta}(u_0, u_1) \\
&\leq \ell_{q\theta}(u_0, u_1)[\theta(u_1, u_m)\theta(u_2, u_m)\cdots\theta(u_{n-1}, u_m)\theta(u_n, u_m)\alpha^n \\
&+ \theta(u_1, u_m)\theta(u_2, u_m)\cdots\theta(u_n, u_m)\theta(u_{n+1}, u_m)\alpha^{n+1} \\
&+ \cdots + \theta(u_1, u_m)\theta(u_2, u_m)\cdots\theta(u_n, u_m)\theta(u_{n+1}, u_m)\cdots \\
&\cdots\theta(u_{m-2}, u_m)\theta(u_{m-1}, u_m)\alpha^{m-1}]\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (*)
\end{aligned}$$

As, $lim_{n,m\to\infty}\theta(u_{n+1}, u_m) < \frac{1}{\alpha}$, therefore the series $\sum_{n=1}^{\infty}\alpha^n\prod_{i=1}^{n}\theta(u_i, u_m)$ converges by ratio test for each $m \in \mathbb{N}$.
Let $\mathbb{S} = \sum_{n=1}^{\infty}\alpha^n\prod_{i=1}^{n}\theta(u_i, u_m)$, with the partial sum

$$\mathbb{S}_n = \sum_{j=1}^{n}\alpha^j\prod_{i=1}^{j}\theta(u_i, u_m) \tag{2}$$

using (2) in (*), it follows that

$$\ell_{q\theta}(u_n, u_m) \leq \ell_{q\theta}(u_0, u_1)[\mathbb{S}_{m-1} - \mathbb{S}_{n-1}], for\, m > n \tag{3}$$

Letting $n \to \infty$ in (3), we get that $lim_{m,n\to\infty}\ell_{q\theta}(u_n, u_m) = 0$ i.e., the constructed sequence $\{u_n\}$ is a Cauchy sequence in $(\mathcal{K}, \ell_{q\theta})$. □

Next theorem resembles Banach contraction principle in the setting of dislocated quasi extended b-metric space.

Theorem 3.1. Given that $\ell_{q\theta}$ is continuous, let $(\mathcal{K}, \ell_{q\theta})$ be a complete dislocated quasi extended b-metric space. Let $S : \mathcal{K} \to \mathcal{K}$ be a self mapping that satisfies the following condition:

$$\ell_{q\theta}(Su, Sv) \leq \lambda\, \ell_{q\theta}(u, v) \text{ for all } u, v \in \mathcal{K} \tag{4}$$

where $\lambda \in [0, 1)$ is such that for each $u_0 \in \mathcal{K}$, $lim_{n,m\to\infty}\theta(u_n, u_m) < \frac{1}{\lambda}$, where $u_n = S^n u_0, n = 1, 2, ...$Then S has a unique fixed point η. Additionally for each $u \in \mathcal{K}$, $S^n u \to \eta$.

Proof. Let $u_0 \in \mathcal{K}$ be arbitrary, let us define a sequence $\{u_n\}$ in $\mathcal{K}$ as follows:
$u_0, u_1 = Su_0, u_2 = Su_1, ..., u_{n+1} = Su_n, ...$
If $u_{n+1} = u_n$ for any n, then u_n is a fixed point and further investigation is not necessary.
Therefore, assume that $u_{n+1} \neq u_n$ for any n.
We have

$$\begin{aligned} \ell_{q\theta}(u_n, u_{n+1}) &= \ell_{q\theta}(Su_{n-1}, Su_n) \\ &\leq \lambda\, \ell_{q\theta}(u_{n-1}, u_n) \end{aligned}$$

Similarly $\ell_{q\theta}(u_{n-1}, u_n) \leq \lambda\, \ell_{q\theta}(u_{n-2}, u_{n-1})$
Proceeding like this, we get

$$\ell_{q\theta}(u_n, u_{n+1}) \leq \lambda^n \ell_{q\theta}(u_0, u_1)$$

Using Lemma 3.1, in complete $\ell_{q\theta}$ b-metric space $\mathcal{K}$, $\{u_n\}$ is a Cauchy sequence. As a result, there is a point $\eta \in \mathcal{K}$ such that $lim_{n\to\infty}u_n = \eta$. Consider

$$\begin{aligned} \ell_{q\theta}(S\eta, \eta) &\leq \theta(S\eta, \eta)[\ell_{q\theta}(S\eta, u_n) + \ell_{q\theta}(u_n, \eta)] \\ &\leq \theta(S\eta, \eta)[\lambda\, \ell_{q\theta}(\eta, u_{n-1}) + \ell_{q\theta}(u_n, \eta)] \\ \ell_{q\theta}(S\eta, \eta) &\leq 0\, as\, n \to \infty \\ \ell_{q\theta}(S\eta, \eta) &= 0 \end{aligned}$$

Similarly $\ell_{q\theta}(\eta, S\eta) = 0$. So by $(\ell_{q\theta}1)$ we have $S\eta = \eta$. Thus η is a fixed point of S.

Uniqueness: Assume η, ζ to be two distinct fixed points of S in $\mathcal{K}$, we have

$$\ell_{q\theta}(\eta, \zeta) = \ell_{q\theta}(S\eta, S\zeta) \leq \lambda\, \ell_{q\theta}(\eta, \zeta).$$

The aforementioned inequality is feasible if $\ell_{q\theta}(\eta, \zeta) = 0$ since $0 \leq \lambda < 1$. Similar to this, we demonstrate that $\ell_{q\theta}(\eta, \zeta) = 0$. By $(\ell_{q\theta}2)$ we get $\eta = \zeta$. Thus S has a unique fixed point in $\mathcal{K}$. □

Corollary 3.1. Let $(\mathcal{K}, \ell_{qs})$ be a complete dislocated quasi b-metric space where $s \geq 1$ is a constant such that ℓ_{qs} is continuous and $S : \mathcal{K} \to \mathcal{K}$ be a self map satisfying the condition:

$$\ell_{qs}(Su, Sv) \leq \lambda\, \ell_{qs}(u, v), \text{ for all } u, v \in \mathcal{K}$$

where $0 \leq \lambda < 1$ satisfies $0 \leq \lambda s < 1$, with $s \geq 1$. Then S has a unique fixed point in $\mathcal{K}$.

Proof. Let $\theta(u,v) = s \geq 1,$ for all $u, v \in \mathcal{K}$ in Theorem 3.1, we see that

$$lim_{n,m\to\infty}\theta(u_n, u_m) < \frac{1}{\lambda}$$

$\Rightarrow 0 \leq \lambda s < 1.$ Hence the corollary. □

Remark 3.1. Corollary 3.1 is the main result of Rahman and Sarwar [16].

Following theorems prove the existence and uniqueness of fixed point for contraction type mapping given by Bianchini [5] and Agrawal et al. [3] in the setting of dislocated quasi extended b-metric space.

Theorem 3.2. Let $(\mathcal{K}, \ell_{q\theta})$ be a complete dislocated quasi extended b-metric space such that $\ell_{q\theta}$ is continuous. Let $S : \mathcal{K} \to \mathcal{K}$ be a continuous self mapping that satisfies the following condition:

$$\ell_{q\theta}(Su, Sv) \leq h\, max\{\ell_{q\theta}(u, Su), \ell_{q\theta}(v, Sv)\}\, \text{for all}\, u, v \in \mathcal{K} \tag{5}$$

where $h \in [0, 1)$ is such that for each $u_0 \in \mathcal{K}$, $lim_{n,m\to\infty}\theta(u_n, u_m) < \frac{1}{h}$, where $u_n = S^n u_0, n = 1, 2, ...$ Then S has a unique fixed point.

Proof. Let $u_0 \in \mathcal{K}$ be arbitrary, let us define a sequence $\{u_n\}$ in $\mathcal{K}$ as follows:
$u_0, u_1 = Su_0, u_2 = Su_1, ..., u_{n+1} = Su_n, ...$
If $u_{n+1} = u_n$ for any n, then u_n is a fixed point and further investigation is not necessary. Therefore, assume that $u_{n+1} \neq u_n$ for any n.
From (5), we have

$$\begin{aligned} \ell_{q\theta}(u_n, u_{n+1}) &= \ell_{q\theta}(Su_{n-1}, Su_n) \\ &\leq h\, max\{\ell_{q\theta}(u_{n-1}, Su_{n-1}), \ell_{q\theta}(u_n, Su_n)\} \\ &\leq h\, max\{\ell_{q\theta}(u_{n-1}, u_n), \ell_{q\theta}(u_n, u_{n+1})\} \cdots\cdots\cdots\cdots (**) \end{aligned}$$

For refining the inequality above, consider the following cases:
Case(i). If $max\{\ell_{q\theta}(u_{n-1}, u_n), \ell_{q\theta}(u_n, u_{n+1})\} = \ell_{q\theta}(u_n, u_{n+1})$
then $\ell_{q\theta}(u_n, u_{n+1}) \leq h\, \ell_{q\theta}(u_n, u_{n+1})$, which is a contradiction.

Case(ii). If $max\{\ell_{q\theta}(u_{n-1}, u_n), \ell_{q\theta}(u_n, u_{n+1})\} = \ell_{q\theta}(u_{n-1}, u_n)$
then the inequality (**) turns into the inequality below:
$\ell_{q\theta}(u_n, u_{n+1}) \leq h\, \ell_{q\theta}(u_{n-1}, u_n)$, which holds.
Similarly, $\ell_{q\theta}(u_{n-1}, u_n) \leq h\, \ell_{q\theta}(u_{n-2}, u_{n-1})$.
Applying it recursively, we get

$$0 < \ell_{q\theta}(u_n, u_{n+1}) \leq h^n\, \ell_{q\theta}(u_0, u_1)\, \text{for}\, n = 1, 2, 3, ...$$

Since $0 \leq h < 1$, so by Lemma 3.1, $\{u_n\}$ is a Cauchy sequence in complete $\ell_{q\theta}$ b-metric space $\mathcal{K}$. Thus there is a point $\eta \in \mathcal{K}$ such that $lim_{n\to\infty}u_n = \eta$.
As S is continuous, we have $lim_{n\to\infty}Su_n = S\eta \Rightarrow lim_{n\to\infty}u_{n+1} = S\eta$
Hence, $S\eta = \eta$ and η is a fixed point of S.

Uniqueness: Let η, ζ to be two distinct fixed points of S in $\mathcal{K}$, we have

$$\begin{aligned} \ell_{q\theta}(\eta, \eta) &= \ell_{q\theta}(S\eta, S\eta) \\ &\leq h\, max\{\ell_{q\theta}(\eta, S\eta), \ell_{q\theta}(\eta, S\eta)\} \\ &= h\, max\{\ell_{q\theta}(\eta, \eta), \ell_{q\theta}(\eta, \eta)\} \\ \ell_{q\theta}(\eta, \eta) &\leq h\, \ell_{q\theta}(\eta, \eta) \end{aligned}$$

Since $0 \leq h < 1$ and $\ell_{q\theta}(\eta, \eta) \geq 0$, so the above inequality is possible if $\ell_{q\theta}(\eta, \eta) = 0$
Thus, $\ell_{q\theta}(\eta, \eta) = 0$, if η is any fixed point of S.
Now consider

$$\begin{aligned}\ell_{q\theta}(\eta, \zeta) &= \ell_{q\theta}(S\eta, S\zeta) \\ &\leq h\, max\{\ell_{q\theta}(\eta, S\eta), \ell_{q\theta}(\zeta, S\zeta)\}\end{aligned}$$

We get $\ell_{q\theta}(\eta, \zeta) = 0$.
Similarly, we can show that $\ell_{q\theta}(\zeta, \eta) = 0$. So by $(\ell_{q\theta}2)$, we have $\eta = \zeta$.
Thus, S has a unique fixed point in $\mathcal{K}$. □

Corollary 3.2. Let $(\mathcal{K}, \ell_{q\theta})$ be a complete dislocated quasi extended b-metric space such that $\ell_{q\theta}$ is continuous. Let $S : \mathcal{K} \to \mathcal{K}$ be a continuous self mapping satisfying the condition:

$$\ell_{q\theta}(Su, Sv) \leq a_1 \ell_{q\theta}(u, Su) + a_2 \ell_{q\theta}(v, Sv), \text{ for all}\, u, v \in \mathcal{K}$$

where $a_i \in [0, 1), i = 1, 2$ with $0 < a_1 + a_2 < 1$ and for each $u_0 \in \mathcal{K}, lim_{n\to\infty}\theta(u_n, u_m) < \frac{1}{a_1+a_2}$, where $u_n = S^n u_0, n = 1, 2, 3, ...$ Then S has a unique fixed point in $\mathcal{K}$.

Proof. The proof follows from Theorem 3.2 by taking $h = a_1 + a_2$. Indeed we have

$$\begin{aligned}\ell_{q\theta}(Su, Sv) &\leq a_1 \ell_{q\theta}(u, Su) + a_2 \ell_{q\theta}(v, Sv) \\ &\leq (a_1 + a_2)\, max\{\ell_{q\theta}(u, Su), \ell_{q\theta}(v, Sv)\}\end{aligned}$$

□

Corollary 3.3. Let $(\mathcal{K}, \ell_{qs})$ be a complete dislocated quasi b-metric space with constant $s \geq 1$ such that ℓ_{qs} is continuous. Let $S : \mathcal{K} \to \mathcal{K}$ be a continuous self mapping satisfying the condition:

$$\ell_{qs}(Su, Sv) \leq h\, max\{\ell_{qs}(u, Su), \ell_{qs}(v, Sv)\} \text{ for all}\, u, v \in \mathcal{K}$$

where $h \in [0, 1)$ is such that $0 \leq sh < 1$. Then S has a unique fixed point in $\mathcal{K}$.

Proof. Let $\theta(u, v) = s \geq 1$, for all $u, v \in \mathcal{K}$ in Theorem 3.2, we see that $\lim_{n\to\infty} \theta(u_n, u_m) < \frac{1}{h} \Rightarrow 0 \leq sh < 1$.

Hence the corollary. □

Theorem 3.3. Let $(\mathcal{K}, \ell_{q\theta})$ be a complete dislocated quasi extended b-metric space such that $\ell_{q\theta}$ is continuous. Let $S : \mathcal{K} \to \mathcal{K}$ be a continuous self mapping that satisfies the following condition:

$$\ell_{q\theta}(Su, Sv) \leq a\, max\{\ell_{q\theta}(u, Su), \ell_{q\theta}(v, Sv), \ell_{q\theta}(u, v)\} + b\, \{\ell_{q\theta}(u, Sv) + \ell_{q\theta}(v, Su)\} \quad (6)$$

for all $u, v \in \mathcal{K}$, where $a, b > 0$ with $a + 2b < 1$ and for each $u_0 \in \mathcal{K}$,

$$lim_{n,m\to\infty} \frac{\{(k - (1 - a)) + b[\theta(u_n, u_{n+2}) + \theta(u_{n+1}, u_{n+1})]\}\theta(u_{n+1}, u_m)}{k - b[\theta(u_n, u_{n+2}) + \theta(u_{n+1}, u_{n+1})]} < 1$$

,
for $k = 1, 1 - a$ where $u_n = S^n u_0, n = 1, 2, ...$ Then S has a unique fixed point in $\mathcal{K}$.

Proof. Let $u_0 \in \mathcal{K}$ be arbitrary, let us define a sequence $\{u_n\}$ in $\mathcal{K}$ as follows:
$u_0, u_1 = Su_0, u_2 = Su_1, ..., u_{n+1} = Su_n, ...$
If $u_{n+1} = u_n$ for any n, then u_n is a fixed point and further investigation is not necessary.

Therefore, assume that $u_{n+1} \neq u_n$ for any n.
From (6), we have

$$\begin{aligned}
\ell_{q\theta}(u_n, u_{n+1}) &= \ell_{q\theta}(Su_{n-1}, Su_n) \\
&\leq a\,max\{\ell_{q\theta}(u_{n-1}, Su_{n-1}), \ell_{q\theta}(u_n, Su_n), \ell_{q\theta}(u_{n-1}, u_n)\} \\
&+ b\{\ell_{q\theta}(u_{n-1}, Su_n) + \ell_{q\theta}(u_n, Su_{n-1})\} \\
&= a\,max\{\ell_{q\theta}(u_{n-1}, u_n), \ell_{q\theta}(u_n, u_{n+1})\} + b\{\ell_{q\theta}(u_{n-1}, u_{n+1}) + \ell_{q\theta}(u_n, u_n)\} \\
\ell_{q\theta}(u_n, u_{n+1}) &\leq aM + b\{\ell_{q\theta}(u_{n-1}, u_{n+1}) + \ell_{q\theta}(u_n, u_n)\}
\end{aligned}$$

where $M = max\{\ell_{q\theta}(u_{n-1}, u_n), \ell_{q\theta}(u_n, u_{n+1}\}$
For refining the inequality above, consider the following cases:
Case(i). If $M = \ell_{q\theta}(u_n, u_{n+1})$ then we have

$$\begin{aligned}
\ell_{q\theta}(u_n, u_{n+1}) &\leq a\ell_{q\theta}(u_n, u_{n+1}) + b\{\ell_{q\theta}(u_{n-1}, u_{n+1}) + \ell_{q\theta}(u_n, u_n)\} \\
&\leq a\ell_{q\theta}(u_n, u_{n+1}) + b\theta(u_{n-1}, u_{n+1})[\ell_{q\theta}(u_{n-1}, u_n) + \ell_{q\theta}(u_n, u_{n+1})] \\
&+ b\theta(u_n, u_n)[\ell_{q\theta}(u_n, u_{n+1}) + \ell_{q\theta}(u_{n-1}, u_n)] \\
&= [a + b(\theta(u_{n-1}, u_{n+1}) + \theta(u_n, u_n))]\ell_{q\theta}(u_n, u_{n+1}) \\
&+ b[\theta(u_{n-1}, u_{n+1}) + \theta(u_n, u_n)]\ell_{q\theta}(u_{n-1}, u_n)
\end{aligned}$$

We get,

$$\ell_{q\theta}(u_n, u_{n+1}) \leq \frac{b[\theta(u_{n-1}, u_{n+1}) + \theta(u_n, u_n)]}{1 - [a + b(\theta(u_{n-1}, u_{n+1}) + \theta(u_n, u_n))]}\ell_{q\theta}(u_{n-1}, u_n)$$

By Induction, it follows that

$$\ell_{q\theta}(u_n, u_{n+1}) \leq \delta_n\, \ell_{q\theta}(u_0, u_1)\, \text{for}\, n = 1, 2, 3.... \tag{7}$$

where $\delta_n = \prod_{j=1}^{n}\{\frac{b[\theta(u_{j-1},u_{j+1})+\theta(u_j,u_j)]}{1-[a+b(\theta(u_{j-1},u_{j+1})+\theta(u_j,u_j))]}\}$

Case(ii). If $M = \ell_{q\theta}(u_{n-1}, u_n)$ then we have

$$\begin{aligned}
\ell_{q\theta}(u_n, u_{n+1}) &\leq a\ell_{q\theta}(u_{n-1}, u_n) + b\{\ell_{q\theta}(u_{n-1}, u_{n+1}) + \ell_{q\theta}(u_n, u_n)\} \\
&\leq a\ell_{q\theta}(u_{n-1}, u_n) + b\theta(u_{n-1}, u_{n+1})[\ell_{q\theta}(u_{n-1}, u_n) + \ell_{q\theta}(u_n, u_{n+1})] \\
&+ b\theta(u_n, u_n)[\ell_{q\theta}(u_n, u_{n+1}) + \ell_{q\theta}(u_{n-1}, u_n)] \\
&= b[\theta(u_{n-1}, u_{n+1}) + \theta(u_n, u_n)]\ell_{q\theta}(u_n, u_{n+1}) \\
&+ [a + b[\theta(u_{n-1}, u_{n+1}) + \theta(u_n, u_n)]\ell_{q\theta}(u_{n-1}, u_n)
\end{aligned}$$

We get,

$$\ell_{q\theta}(u_n, u_{n+1}) \leq \frac{[a + b(\theta(u_{n-1}, u_{n+1}) + \theta(u_n, u_n))]}{1 - b[\theta(u_{n-1}, u_{n+1}) + \theta(u_n, u_n)]}\ell_{q\theta}(u_{n-1}, u_n)$$

By Induction, it follows that

$$\ell_{q\theta}(u_n, u_{n+1}) \leq \delta_n\, \ell_{q\theta}(u_0, u_1)\, \text{for}\, n = 1, 2, 3.... \tag{8}$$

where $\delta_n = \prod_{j=1}^{n}\{\frac{[a+b(\theta(u_{j-1},u_{j+1})+\theta(u_j,u_j))]}{1-b(\theta(u_{j-1},u_{j+1})+\theta(u_j,u_j))}$

Now for both the cases we show that $\{u_n\}$ is a Cauchy sequence in $\mathcal{K}$. Let $m > n \geq 1$, applying triangular inequality ($\ell_{q\theta}4$) and using (7)/(8), we get

$$\begin{aligned}\ell_{q\theta}(u_n,u_m) &\leq \theta(u_n,u_m)[\ell_{q\theta}(u_n,u_{n+1})+\ell_{q\theta}(u_{n+1},u_m)]\\ &\leq \theta(u_n,u_m)[\delta_n\ell_{q\theta}(u_0,u_1)+\ell_{q\theta}(u_{n+1},u_m)]\\ &\leq \ell_{q\theta}(u_0,u_1)[\theta(u_1,u_m)\theta(u_2,u_m)\cdots\theta(u_{n-1},u_m)\theta(u_n,u_m)\delta_n\\ &+\theta(u_1,u_m)\theta(u_2,u_m)\cdots\theta(u_n,u_m)\theta(u_{n+1},u_m)\delta_{n+1}\\ &+\cdots+\theta(u_1,u_m)\theta(u_2,u_m)\cdots\theta(u_n,u_m)\theta(u_{n+1},u_m)\cdots\\ &\cdots\theta(u_{m-2},u_m)\theta(u_{m-1},u_m)\delta_{m-1}]\cdots\cdots\cdots\cdots\cdots\cdots(***)\end{aligned}$$

Since $lim_{n,m\to\infty}\frac{\{(k-(1-a))+b[\theta(u_n,u_{n+2})+\theta(u_{n+1},u_{n+1})]\}\theta(u_{n+1},u_m)}{k-b[\theta(u_n,u_{n+2})+\theta(u_{n+1},u_{n+1}]}<1$,

(note $k=1-a$ for case(i) and $k=1$ for case(ii)) therefore, the series $\sum_{n=1}^{\infty}\delta_n\prod_{i=1}^{n}\theta(u_i,u_m)$ converges by ratio test for each $m\in\mathbb{N}$.

Let $S=\sum_{n=1}^{\infty}\delta_n\prod_{i=1}^{n}\theta(u_i,u_m)$ with the partial sum

$$S_n=\sum_{j=1}^{n}\delta_j\prod_{i=1}^{j}\theta(u_i,u_m)\,\text{for each}\,m\in\mathbb{N}. \tag{9}$$

Using (9) in (***), it follows that

$$\ell_{q\theta}(u_n,u_m)\leq\ell_{q\theta}(u_0,u_1)[S_{m-1}-S_{n-1}]\,\text{for}\,m>n. \tag{10}$$

Letting $n\to\infty$ in (10), we get that $\ell_{q\theta}(u_n,u_m)=0$ that is, the sequence $\{u_n\}$ is Cauchy in the complete dislocated quasi extended b-metric space $(\mathcal{K},\ell_{q\theta})$.
So there is a point $\eta\in\mathcal{K}$ such that $lim_{n\to\infty}u_n=\eta$.
Since S is continuous, so $lim_{n\to\infty}Su_n=S\eta\Rightarrow lim_{n\to\infty}u_{n+1}=S\eta$.
Thus, $S\eta=\eta$ and η is a fixed point of S.

Uniqueness: Let η,ζ to be two distinct fixed points of S in $\mathcal{K}$, we have

$$\begin{aligned}\ell_{q\theta}(\eta,\eta) &= \ell_{q\theta}(S\eta,S\eta)\\ &\leq a\,max\{\ell_{q\theta}(\eta,S\eta),\ell_{q\theta}(\eta,S\eta),\ell_{q\theta}(\eta,\eta)\}\\ &+ b\{\ell_{q\theta}(\eta,S\eta)+\ell_{q\theta}(\eta,S\eta)\}\\ &= a\ell_{q\theta}(\eta,\eta)+2b\eta_{q\theta}(\eta,\eta)\\ \ell_{q\theta}(\eta,\eta) &\leq (a+2b)\ell_{q\theta}(\eta,\eta)\end{aligned}$$

Since $a+2b<1$ and $\ell_{q\theta}(\eta,\eta)\geq0$, so the above inequality is possible if $\ell_{q\theta}(\eta,\eta)=0$
Thus, $\ell_{q\theta}(\eta,\eta)=0$, if η is any fixed point of S.
Now consider

$$\begin{aligned}\ell_{q\theta}(\eta,\zeta) &= \ell_{q\theta}(S\eta,S\zeta)\\ &\leq a\,max\{\ell_{q\theta}(\eta,S\eta),\ell_{q\theta}(\zeta,S\zeta),\ell_{q\theta}(\eta,\zeta)\}\\ &+ b\{\ell_{q\theta}(\eta,S\zeta)+\ell_{q\theta}(\zeta,S\eta)\}\\ &= a\ell_{q\theta}(\eta,\zeta)+b\{\ell_{q\theta}(\eta,\zeta)+\ell_{q\theta}(\zeta,\eta)\}\end{aligned}$$

We get,

$$\ell_{q\theta}(\eta,\zeta)\leq\frac{b}{(1-(a+b))}\ell_{q\theta}(\zeta,\eta) \tag{11}$$

Similarly, we can show that

$$\ell_{q\theta}(\zeta,\eta) \leq \frac{b}{(1-(a+b))}\ell_{q\theta}(\eta,\zeta) \tag{12}$$

Subtracting (11) and (12), we get

$$\mid \ell_{q\theta}(\eta,\zeta) - \ell_{q\theta}(\zeta,\eta) \mid \leq \frac{b}{(1-(a+b))} \mid \ell_{q\theta}(\zeta,\eta) - \ell_{q\theta}(\eta,\zeta) \mid$$

Since $a + 2b < 1$, so the above inequality is possible if $\mid \ell_{q\theta}(\eta,\zeta) - \ell_{q\theta}(\zeta,\eta) \mid = 0 \Rightarrow \ell_{q\theta}(\eta,\zeta) = \ell_{q\theta}(\zeta,\eta)$.
Using above in (11) and (12), we get $\ell_{q\theta}(\eta,\zeta) = \ell_{q\theta}(\zeta,\eta) = 0 \Rightarrow \eta = \zeta$
Therefore, fixed point of S in $\mathcal{K}$ is unique. □

Corollary 3.4. Let $(\mathcal{K}, \ell_{q\theta})$ be a complete dislocated quasi extended b-metric space such that $\ell_{q\theta}$ is continuous. Let $S : \mathcal{K} \to \mathcal{K}$ be a continuous self mapping that satisfies the following condition:

$$\ell_{q\theta}(Su,Sv) \leq a\{\ell_{q\theta}(u,Su) + \ell_{q\theta}(v,Sv)\} + b\{\ell_{q\theta}(u,Sv) + \ell_{q\theta}(v,Su)\}$$

for all $u, v \in \mathcal{K}$, where $a, b > 0$ with $2a + 2b < 1$ and for each $u_0 \in \mathcal{K}$,

$$lim_{n,m\to\infty} \frac{\{a + b[\theta(u_n,u_{n+2}) + \theta(u_{n+1},u_{n+1})]\}\theta(u_{n+1},u_m)}{1 - a - b[\theta(u_n,u_{n+2}) + \theta(u_{n+1},u_{n+1})]} < 1$$

,
where $u_n = S^n u_0, n = 1, 2, ...$ Then S has a unique fixed point in $\mathcal{K}$.

Proof. Proof is based on similar steps as in Theorem 3.3. □

Corollary 3.5. Let $(\mathcal{K}, \ell_{qs})$ be a complete dislocated quasi b-metric space with constant $s \geq 1$ such that ℓ_{qs} is continuous. Let $S : \mathcal{K} \to \mathcal{K}$ be a continuous self mapping that satisfies the following condition:

$$\ell_{qs}(Su,Sv) \leq a\,max\{\ell_{qs}(u,Su), \ell_{qs}(v,Sv), \ell_{qs}(u,v)\} + b\{\ell_{qs}(u,Sv) + \ell_{qs}(v,Su)\}$$

for all $u, v \in \mathcal{K}$, where $a, b > 0$ with $a + 4bs < 1$. Then S has a unique fixed point in $\mathcal{K}$.

Proof. Let $\theta(u,v) = s \geq 1$, for all $u, v \in \mathcal{K}$ in Theorem 3.3. Consider the following two cases:
Case(i): For $k = 1, lim_{n,m\to\infty} \frac{\{a+b[\theta(u_n,u_{n+2})+\theta(u_{n+1},u_{n+1})]\}\theta(u_{n+1},u_m)}{1-b[\theta(u_n,u_{n+2})+\theta(u_{n+1},u_{n+1})]} < 1$

$\Rightarrow \frac{1-2bs}{(a+2bs)s} > 1$

$\Rightarrow 1 - 2bs > (a + 2bs)s \geq a + 2bs \Rightarrow a + 4bs < 1.$

Case(ii): For $k = 1 - a, lim_{n,m\to\infty} \frac{\{b[\theta(u_n,u_{n+2})+\theta(u_{n+1},u_{n+1})]\}\theta(u_{n+1},u_m)}{1-a-b[\theta(u_n,u_{n+2})+\theta(u_{n+1},u_{n+1})]} < 1$

$\Rightarrow \frac{1-a-2bs}{(2bs)s} > 1 \Rightarrow a + 4bs < 1.$ □

Example 3.1. *Consider a complete dislocated quasi extended b-metric space $(\mathcal{K}, \ell_{q\theta})$, where $\mathcal{K} = [0, \infty)$ and $\ell_{q\theta} : \mathcal{K} \times \mathcal{K} \to [0, \infty)$ and $\theta : \mathcal{K} \times \mathcal{K} \to [1, \infty)$ defined as:*

$$\ell_{q\theta}(u,v) = \mid u \mid + (u-v)^2, \theta(u,v) = u + v + 1$$

Define $S : \mathcal{K} \to \mathcal{K}$ by $Su = \frac{u}{3}$, for all $u \in \mathcal{K}$.

We observe that

$$\begin{aligned}\ell_{q\theta}(Su,Sv) &= \ell_{q\theta}(\frac{u}{3},\frac{v}{3})\\ &= \frac{\mid u\mid}{3}+\frac{(u-v)^2}{9}\\ &\leq \frac{1}{3}[\mid u\mid + (u-v)^2]\\ &= \lambda\ell_{q\theta}(u,v),\ for\ all\ u,v\in\mathcal{K}.\end{aligned}$$

Note that for each $u \in \mathcal{K}$, $S^n u = \frac{u}{3^n}$.
Hence,

$$\begin{aligned}lim_{n,m\to\infty}\theta(u_n,u_m) &= lim_{n,m\to\infty}\theta(S^n u, S^m u)\\ &= lim_{n,m\to\infty}(\frac{u}{3^n}+\frac{u}{3^m}+1) < 3.\end{aligned}$$

As, all requirements of Theorem 1 are satisfied, therefore S *has a unique fixed point* $u = 0$ *in* $\mathcal{K}$.

4. Conclusions

We can show that the analogous findings in the setting of dislocated quasi b-metric space and dislocated quasi usual metric space may be inferred from our results by assuming $\theta(u,v) = s \geq 1$ and $\theta(u,v) = 1$, respectively. Any interested researcher can conduct their thesis work on this subject by looking for the presence and uniqueness of fixed points for maps meeting various contraction conditions in dislocated quasi extended b-metric space or any other generalisation of metric space.

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