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MULTI-DECOMPOSITION OF COMPLETE BIPARTITE GRAPHS AND COMPLETE GRAPHS INTO BANNERS AND STARS OF SIZE FIVE

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ABSTRACT. A decomposition of a graph G is a set of edge disjoint subgraphs $H_1, H_2, ..., H_r$ of G such that every edge of G belongs to exactly one H_i. If all the subgraphs in the decomposition of G are isomorphic to a graph H, then we say that G is H-decomposable. The graph G has an (H_1, H_2)-multi-decomposition if α copies of H_1 and β copies of H_2 decompose G, where α and β are non-negative integers. In this paper, we have obtained the multi-decomposition of complete bipartite graphs and complete graphs into banners and stars of size 5.

Keywords: Banners, Stars, Complete graphs, Complete Bipartite graphs, Multi-decomposition.

AMS Subject Classification: 05C51

1. INTRODUCTION

All graphs considered here are finite, simple and undirected. Let K_n denotes the complete graph on n vertices and $K_{m,n}$ denotes the complete bipartite graph with vertex partite sets of cardinality m and n. A *banner* is a graph in which one edge is attached to any vertex of the square C_4 and it is denoted by $C_4 + e$ and $S_5(= K_{1,5})$ denotes a *star* on 5 edges. For a graph G, if $E(G)$ can be partitioned into subsets $E_1, \cdots, E_k$ such that the subgraph of G induced by E_i is H_i for each $1 \le i \le k$, we write $G = H_1 \oplus \cdots \oplus H_k$, where $\oplus$ denotes the edge disjoint sum of the subgraph H_i. For $1 \le i \le k$, if $H_i \cong H$, we say that G has an *H-decomposition.* If G can be decomposed into α copies of H_1 and β copies of H_2, then we say that G has an $\{H_1^\alpha, H_2^\beta\}$*-decomposition* or an (H_1, H_2)*-multi-decomposition.* If the necessary conditions for the existence of an $\{H_1^\alpha, H_2^\beta\}$-decomposition of G are satisfied by the pairs (α, β), $\alpha, \beta \ge 0$, then we say that (α, β) is an admissible pair.

The study of $\{H_1^\alpha, H_2^\beta\}$ - decomposition has been introduced by Abueida and Daven [1]. In [2], the existence of multi-decomposition of $K_m(\lambda)$ into stars and cycles of same

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size has been studied. The multi-decomposition of complete graphs into kites, cycles, stars and paths has been studied in [7, 10, 11]. In [4], the kite and star multi-decomposition of complete graphs has been obtained. The banner decomposition of graphs such as complete graphs, complete equi-partite graphs and some regular graphs has been studied in [3, 5, 8]. The multi-decomposition of complete bipartite graphs into cycles, stars and paths has been studied in [6, 9]. In this paper, we investigate the multi-decomposition of $K_{m,n}$ and K_n into banners and stars of size 5, for the given positive integers $m \geq 2$ and $n \geq 5$.

Notations:

- Let $V(G) = \{v_i\},\ i = 1, 2, ..., n$. Then a banner $C_4 + e$ in a graph G is denoted by $(v_a, v_b, v_c, v_d; v_a v_f)$, where v_a, v_b, v_c, v_d is cycle C_4 and $v_a v_f$ is the pendant edge e attached to the vertex v_a of C_4.
- A star S_5 in a graph G is of the form $(v_a; v_b, v_c, v_d, v_e, v_f)$, where v_a is the center vertex (of degree 5) and v_b, v_c, v_d, v_e, v_f are the end vertices (pendant vertices). The $n+1$ stars with the same end vertices v_1, v_2, v_3, v_4, v_5 and different center vertices $a_0, a_1, \cdots, a_n$ are denoted by $(a_0, a_1, \cdots, a_n; v_1, v_2, \cdots, v_5)$. This notation is same as for two or more twins having same end vertices.
- Let $V(K_{p,q}) = (M, N)$, where $M = \{m_1, m_2, \cdots, m_p\}$ and $N = \{n_1, n_2, \cdots, n_q\}$.

Remark 1.1. *If two graphs G_1 and G_2 have an $\{H_1^{\alpha}, H_2^{\beta}\}$-decomposition, then $G_1 \oplus G_2$ also has an $\{H_1^{\alpha}, H_2^{\beta}\}$-decomposition.*

Remark 1.2. *For a graph G and a subgraph H of G, if sH denotes s edge disjoint copies of H in G, then sH has an $\{H_1^{\alpha}, H_2^{\beta}\}$-decomposition whenever H has such a decomposition.*

Observation 1.3. *If there exist two stars S_5 having the same end vertices $v_1, v_2, ...v_5$ and centers at a_0 & a_1 are denoted by $(a_0, a_1; v_1, v_2, v_3, v_4, v_5)$. Further they can be decomposed into two banner graphs given by $(a_0, v_1, a_1, v_2; a_0 v_3)$, $(a_0, v_4, a_1, v_5; a_1 v_3)$. Hence, we call this structure as Twins denoted by $\mathcal{T}$.*

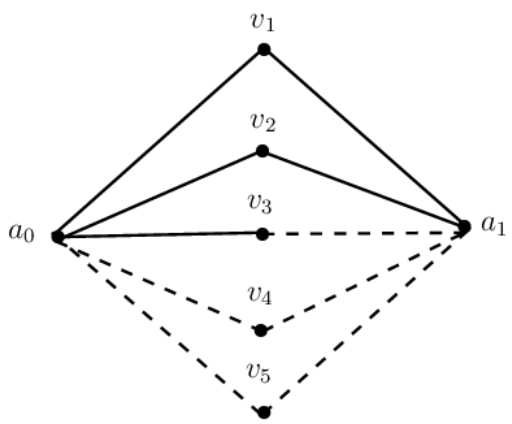

FIGURE 1. $\mathcal{T}$ (Twins)

2. $\{(C_4+e)^\alpha, S_5^\beta\}$ - DECOMPOSITION OF COMPLETE BIPARTITE GRAPHS

In this section, we prove the sufficient condition for the existence of a $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition of $K_{m,n}$ for $m \geq 2$ and $n \geq 5$. The following lemmas are useful in proving our main result.

Lemma 2.1. *For $2 \leq n \leq 9$, there exists a $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition of $K_{5,n}$.*

Proof. The following table shows the existence of a $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition of $K_{5,n}$, where $2 \leq n \leq 9$ for the given admissible pairs (α,β).

n	$K_{5,n}$	Admissible pairs (α,β)
2	$1\mathfrak{T}$	$(0,2),(2,0)$
3	$1\mathfrak{T}\oplus 1S_5$	$(0,3),(2,1)$
4	$2\mathfrak{T}$	$(0,4),(2,2)(4,0),$
5	$\mathcal{D}_1$	$(5,0)$
	$2\mathfrak{T}\oplus 1S_5$	$(0,5),(2,3),(4,1)$
6	$3\mathfrak{T}$	$(6,0),(0,6),(2,4),(4,2)$
	$\mathcal{D}_2$	$(3,3)$
	$K_{5,5}\oplus 1S_5$	$(5,1)$
7	$K_{5,5}\oplus \mathcal{D}_3$	$(7,0)$
	$K_{5,6}\oplus 1S_5$	$(0,7),(2,5),(3,4),(4,3),(5,2),(6,1)$
8	$K_{5,6}\oplus \mathcal{D}_4$	$(8,0)$
	$K_{5,7}\oplus 1S_5$	$(0,8),(2,6),(3,5),(4,4),(5,3),(6,2),(7,1)$
9	$K_{5,7}\oplus \mathcal{D}_5$	$(9,0)$
	$K_{5,8}\oplus 1S_5$	$(0,9),(2,8),(3,6),(4,5),(5,4),(6,3),(7,2),(8,1)$

TABLE 1. Multi-decomposition of $K_{5,n}$ into banners and stars

where $\mathcal{D}_1 = \{\ (m_1,n_1,m_3,n_2;m_1n_4),\ (m_1,n_3,m_5,n_5;n_3m_3),\ (n_1,m_2,n_3,m_4;n_1m_5),$
$(m_2,n_2,m_5,n_4;n_2m_4),\ (m_3,n_4,m_4,n_5;n_5m_2)\},$
$\mathcal{D}_2 = \{(m_1,n_4,m_4,n_1;n_1m_2),\ (m_1,n_5,m_4,n_2;n_2m_3),\ (m_1,n_6,m_4,n_3;n_3m_5),$
$(m_2;n_2,n_3,n_4,n_5,n_6),\ (m_3;n_1,n_3,n_4,n_5,n_6),\ (m_5;n_1,n_2,n_4,n_5,n_6)\},$
$\mathcal{D}_3 = \{(n_6,m_1,n_7,m_2;n_6m_3),\ (n_6,m_4,n_7,m_5;n_7m_3)\},$
$\mathcal{D}_4 = \{(n_7,m_1,n_8,m_2;n_7m_3),\ (n_7,m_4,n_8,m_5;n_8m_3)\}$ and
$\mathcal{D}_5 = \{(n_8,m_1,n_9,m_2;n_8m_3),\ (n_8,m_4,n_9,m_5;n_9m_3)\}.$ □

Lemma 2.2. *There exists a $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition of $K_{5,10}$ for all admissible pairs (α,β) with $\alpha \neq 1$ such that $\alpha+\beta=10$.*

Proof. The decomposition of $K_{5,10}$ into α copies of C_4+e and β copies of S_5 for all admissible pairs (α,β) such that $\alpha+\beta=10$ is discussed in the following cases:
Case 1: $(\alpha,\beta) \in \{(0,10),(2,8),(4,6),(6,4),(8,2),(10,0)\}$.
We can write, $K_{5,10} = 10S_5 = 5\mathfrak{T}$. We get the required decomposition for the even pairs $(\alpha,\beta) \in \{(0,10),(2,8),(4,6),(6,4),(8,2),(10,0)\}$ by Observation 1.3.
Case 2: $(\alpha,\beta) \in \{(3,7),(5,5),(7,3)\}$.
Here, we have $K_{5,10} = 3(C_4+e)\oplus 3S_5 \oplus 2\mathfrak{T}$ as given below:
$\{(n_1,m_2,n_6,m_3;n_1m_4),\ (m_2,n_3,m_3,n_8;n_3m_5),\ (m_2,n_2,m_3,n_7;n_2m_1)\} \oplus \{(m_1;n_1,n_3,n_6,n_7,$
$n_8),\ (m_4;n_2,n_3,n_6,n_7,n_8),\ (m_5;n_1,n_2,n_6,n_7,n_8)\} \oplus \{(n_4,n_9,n_5,n_{10};m_1,m_2,m_3,m_4,m_5)\}.$

Then the required decomposition for the odd pairs $(\alpha, \beta) \in \{(3,7),(5,5),(7,3)\}$ follows from Observation 1.3.
Case 3: $(\alpha, \beta) = (9,1)$.
We have, $K_{5,10} = 9(C_4 + e) \oplus 1S_5$, as given below:
$\{(n_1,m_2,n_2,m_1;m_2n_9)$, $(m_1,n_7,m_2,n_6;m_2n_{10})$, $(n_1,m_4,n_2,m_5;n_1m_3)$, $(m_2,n_5,m_3,n_4;n_4m_1)$, $(m_1,n_9,m_3,n_{10};m_1n_5)$, $(n_4,m_5,n_5,m_4;m_4n_6)$, $(m_4,n_{10},m_5,n_9;m_5n_7)$, $(m_2,n_8,m_4,n_3;m_4n_7)$, $(n_3,m_1,n_8,m_5;m_5n_6)\} \oplus \{(m_3;n_2,n_3,n_6,n_7,n_8)\}$.
Hence, we get the desired decomposition for all admissible pairs (α, β) such that $\alpha + \beta = 10$, $\alpha \neq 1$. When $\alpha = 1$, by removing one copy of $C_4 + e$ in $K_{5,10}$, the resulting graph does not have sufficient vertex degree for the decomposition of $9S_5$. □

Theorem 2.1. *Let $m \geq 2$ and $n \geq 5$ be the given integers. Then the graph $K_{m,n}$ admits a $\{(C_4 + e)^\alpha, S_5^\beta\}$-decomposition for all admissible pairs (α, β) with $\alpha \neq 1$, if and only if $mn \equiv 0 (mod\ 5)$.*

Proof. The necessary condition is obvious. The sufficiency can be proved in two cases.
Case 1. $m = 5p$ and $n = 5q$.
When both p and q are odd, we can write, $K_{5p,5q} = \frac{p(q-1)}{2} K_{5,10} \oplus \frac{p-1}{2} K_{5,10} \oplus K_{5,5}$. From Lemma 2.2, the graph $K_{5,10}$ has a $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition and from Lemma 2.1, the graph $K_{5,5}$ has a $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition. Hence, $K_{m,n}$ has a desired decomposition in this case, as given in Remarks 1.1 and 1.2. For other values of p and q, we can write, $K_{5p,5q} = \frac{pq}{2} K_{5,10}$. Then the graph $K_{m,n}$ has a $\{(C_4 + e)^\alpha, S_5^\beta\}$-decomposition, by Lemma 2.2 and Remark 1.2.
Case 2. $m = 5p$ and $n = 5q + r$, where $r \in \{1,2,3,4\}$.
When both p and q are odd, we can write, $K_{5p,5q+r} = pK_{5,5+r} \oplus \frac{p(q-2)}{2} K_{5,10} \oplus \frac{p-1}{2} K_{5,10} \oplus K_{5,5}$. For other values of p and q, we can write, $K_{5p,5q+r} = pK_{5,5+r} \oplus \frac{p(q-1)}{2} K_{5,10}$. Then the proof follows from Lemmas 2.1, 2.2 and Remarks 1.1, 1.2. □

3. $\{(C_4 + e)^\alpha, S_5^\beta\}$ - DECOMPOSITION OF COMPLETE GRAPHS

In this section, the sufficient condition for the existence of a $\{(C_4 + e)^\alpha, S_5^\beta\}$ - decomposition of K_n for $n \geq 10$ is obtained. The following lemmas are useful in proving our main result.

Lemma 3.1. *There exists a $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition of K_{15} for all admissible pairs (α, β) such that $\alpha + \beta = 21$.*

Proof. We can write, $K_{15} = K_{10} \oplus K_5 \oplus K_{5,10}$. From Appendix 5.1, the graph K_{10} admits a $\{(C_4 + e)^\alpha, S_5^\beta\}$-decomposition for all admissible pairs (α, β) such that $\alpha + \beta = 9$. From Appendix 5.3, the graph $K_5 \oplus K_{5,10}$ admits a $\{(C_4 + e)^\alpha, S_5^\beta\}$-decomposition for all admissible pairs (α, β) such that $\alpha + \beta = 12$. Hence the proof follows from Remark 1.1. □

Lemma 3.2. *There exists a $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition of K_{16} for all admissible pairs (α, β) such that $\alpha + \beta = 24$.*

Proof. We can write, $K_{16} = K_{10} \oplus K_6 \oplus K_{6,10}$. From Appendix 5.1, the graph K_{10} admits a $\{(C_4 + e)^\alpha, S_5^\beta\}$-decomposition for all admissible pairs (α, β) such that $\alpha + \beta = 9$. From Appendix 5.4, the graph $K_6 \oplus K_{6,10}$ admits a $\{(C_4 + e)^\alpha, S_5^\beta\}$-decomposition for all admissible pairs (α, β) such that $\alpha + \beta = 15$. Hence the proof follows from Remark 1.1. □

Observation 3.1. *For $n \geq 20$, $p, q \geq 10$, let*

$$K_n = K_p \oplus K_q \oplus K_{p,q} \tag{1}$$

and assume that the graphs K_p and K_q admits a $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition. Let the graph $K_{p,q}$ admits such a decomposition for all admissible pairs (α, β) such that $\alpha+\beta = \frac{mn}{5}$ except when $\alpha = 1$. But this does not mean that the graph K_n would not have a $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition for all admissible pairs (α, β). Using a different admissible pairs (α, β) for the subgraphs K_p, K_q and $K_{p,q}$, we can get the required decomposition in K_n for all admissible pairs (α, β).

Theorem 3.1. *Let $n > 6$ be the given integer. Then the graph K_n admits a $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition for all admissible pairs (α, β), if and only if $n \equiv 0$ or $1(mod\ 5)$.*

Proof. **Necessity:** We know that the necessary condition for the existence of a $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition is $5(\alpha+\beta) = \binom{n}{2}$. Then either $n \equiv 0(mod\ 5)$ or $n \equiv 1(mod\ 5)$.
Sufficiency: Assume that $n \equiv 0$ *or* $1(mod\ 5)$. Then n can be written as $n = 10s + t$, where $s \geq 1$ and $t \in \{1, 2, 5, 6\}$. Then the sufficiency follows from the four cases as given below:
Case 1: For $t = 0$, we can write,

$$K_{10s} = sK_{10} \oplus \frac{s(s-1)}{2} K_{10,10} \tag{2}$$

Case 2: For $t = 1$, we can write,

$$K_{10s+1} = K_{11} \oplus (s-1)K_{10} \oplus (s-1)K_{11,10} \oplus \frac{(s-1)(s-2)}{2} K_{10,10}. \tag{3}$$

Case 3: For $t = 5$, we can write,

$$K_{10s+5} = K_{15} \oplus (s-1)K_{10} \oplus (s-1)K_{15,10} \oplus \frac{(s-1)(s-2)}{2} K_{10,10} \tag{4}$$

Case 4: For $t = 6$, we can write,

$$K_{10s+6} = K_{16} \oplus (s-1)K_{10} \oplus (s-1)K_{16,10} \oplus \frac{(s-1)(s-2)}{2} K_{10,10} \tag{5}$$

From Appendix 5.1, the graph K_{10} admits a $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition for all admissible pairs (α, β) such that $\alpha + \beta = 9$. From Appendix 5.2, the graph K_{11} admits a $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition for all admissible pairs (α, β) such that $\alpha+\beta = 11$. From Lemma 3.1, the graph K_{15} admits a $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition for all admissible pairs (α, β) such that $\alpha + \beta = 21$. From Lemma 3.2, the graph K_{16} admits a $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition for all admissible pairs (α, β) such that $\alpha+\beta = 24$. From Theorem 2.1, the graphs $K_{10,10}$, $K_{10,11}$, $K_{15,10}$ and $K_{16,10}$ admit a $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition for all admissible pairs except $(\alpha, \beta) \in (1, \beta)$. Hence, the graph K_{10s+t} where $t \in \{1, 2, 5, 6\}$ admits the required decomposition for all admissible pairs by Remarks 1.1, 1.2 and Observation 3.1. □

4. Conclusions

In this paper, it is proved that there exists a $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition in $K_{m,n}$ for all admissible pairs (α, β) with $\alpha \neq 1$, whenever $mn \equiv 0(mod\ 5)$, for $m \geq 2$ and $n \geq 5$. Also, it is proved that there exists a $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition in K_n for all admissible pairs (α, β), whenever $n \equiv 0$ *or* $1(mod\ 5)$, for $n > 6$.

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5. Appendix

If the desired $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition exists in G, then we write, $G=\alpha(C_4+e)\oplus\beta S_5$. Here, we discuss the existence of such a decomposition in G for all admissible pairs (α,β).

5.1. A $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition of K_{10}.

Case 1: $(\alpha,\beta)=(9,0)$
$K_{10}=9(C_4+e)=\{(v_1,v_2,v_4,v_9;v_1v_8),\ (v_1,v_3,v_4,v_6;v_3v_7),\ (v_1,v_5,v_9,v_7;v_9v_3),$
$(v_2,v_3,v_5,v_{10};v_2v_8),\ (v_2,v_6,v_8,v_9;v_9v_{10}),\ (v_3,v_6,v_7,v_{10};v_7v_5),\ (v_4,v_5,v_6,v_{10};v_6v_9),$
$(v_2,v_5,v_8,v_7;v_7v_4),(v_1,v_4,v_8,v_{10};v_8v_3)\}.$
Case 2: $(\alpha,\beta)=(8,1)$
$K_{10}=8(C_4+e)\oplus 1S_5=\{(v_3,v_4,v_9,v_8;v_3v_{10}),\ (v_4,v_6,v_5,v_8;v_8v_2),\ (v_4,v_5,v_9,v_{10};v_9v_2),$
$(v_2,v_3,v_7,v_6;v_7v_{10}),\ (v_3,v_5,v_{10},v_6;v_{10}v_2),\ (v_2,v_4,v_7,v_5;v_7v_9),\ (v_1,v_9,v_6,v_8;v_9v_3),$
$(v_1,v_7,v_8,v_{10};v_7v_2)\}\oplus\{(v_1;v_2,v_3,v_4,v_5,v_6)\}.$
Case 3: $(\alpha,\beta)=(7,2)$
$K_{10}=7(C_4+e)\oplus 2S_5\ \{(v_1,v_7,v_8,v_{10};v_{10}v_2),\ (v_2,v_8,v_4,v_9;v_4v_{10}),\ (v_1,v_3,v_{10},v_9;v_{10}v_7),$
$(v_5,v_6,v_9,v_7;v_6v_{10}),\ (v_3,v_5,v_9,v_8;v_3v_7),\ (v_4,v_5,v_8,v_6;v_5v_{10}),\ (v_3,v_4,v_7,v_6;v_3v_9)\}\oplus$
$\{(v_1;v_2,v_4,v_5,v_6,v_8),\ (v_2;v_3,v_4,v_5,v_6,v_7)\}.$
Case 4: $(\alpha,\beta)=(6,3)$
$K_{10}=6(C_4+e)\oplus 3S_5=\{(v_2,v_9,v_3,v_{10};v_2v_8),\ (v_1,v_9,v_4,v_{10};v_9v_7),\ (v_4,v_5,v_9,v_8;v_8v_6),$
$(v_1,v_7,v_5,v_8;v_7v_{10}),\ (v_6,v_7,v_8,v_{10};v_7v_4),\ (v_5,v_6,v_9,v_{10};v_6v_4)\}\oplus\{(v_1;v_2,v_3,v_4,v_5,v_6),$
$(v_2;v_3,v_4,v_5,v_6,v_7),\ (v_3;v_4,v_5,v_6,v_7,v_8)\}.$
Case 5: $(\alpha,\beta)=(5,4)$
$K_{10}=5(C_4+e)\oplus 4S_5=\{(v_1,v_2,v_5,v_7;v_5v_{10}),\ (v_6,v_7,v_9,v_8;v_8v_{10}),\ (v_5,v_6,v_{10},v_9;v_5v_8),$
$(v_1,v_8,v_7,v_{10};v_{10}v_4),\ (v_2,v_9,v_3,v_{10};v_9v_6)\}\oplus\{(v_1;v_3,v_4,v_5,v_6,v_9),\ (v_2;v_3,v_4,v_6,v_7,v_8),$
$(v_3;v_4,v_5,v_6,v_7,v_8),\ (v_4;v_5,v_6,v_7,v_8,v_9)\}.$
Case 6: $(\alpha,\beta)=(4,5)$
$K_{10}=4(C_4+e)\oplus 5S_5=\{(v_1,v_2,v_8,v_7;v_2v_9),\ (v_2,v_3,v_6,v_5;v_3v_8),\ (v_3,v_4,v_{10},v_9;v_9v_5),$

$(v_4,v_5,v_8,v_9;v_5v_3)\} \oplus \{(v_1;v_3,v_5,v_6,v_8,v_9), (v_4;v_1,v_2,v_6,v_7,v_8), (v_6;v_2,v_7,v_8,v_9,v_{10}), (v_7;v_2,v_3,v_5,v_9,v_{10}), (v_{10};v_1,v_2,v_3,v_5,v_8)\}$.

Case 7: $(\alpha,\beta) = (3,6)$
$K_{10} = 3(C_4+e) \oplus 6S_5 = \{(v_2,v_3,v_9,v_{10};v_{10}v_4), (v_6,v_9,v_8,v_{10};v_{10}v_3), (v_1,v_8,v_2,v_9;v_8v_6)\} \oplus \{(v_1;v_3,v_4,v_5,v_6,v_{10}), (v_2;v_1,v_4,v_5,v_6,v_7), (v_4;v_5,v_6,v_7,v_8,v_9), (v_3;v_4,v_5,v_6,v_7,v_8), (v_5;v_6,v_7,v_8,v_9,v_{10}), (v_7;v_1,v_6,v_8,v_9,v_{10})\}$.

Case 8: $(\alpha,\beta) = (2,7)$
$K_{10} = 2(C_4+e) \oplus 7S_5 = \{(v_1,v_6,v_7,v_2;v_1v_{10}), (v_6,v_8,v_7,v_9;v_8v_3)\} \oplus \{(v_1;v_3,v_4,v_5,v_7,v_8), (v_2;v_4,v_5,v_6,v_8,v_{10}), (v_3;v_2,v_4,v_5,v_6,v_7), (v_4;v_5,v_6,v_7,v_8,v_9), (v_5;v_6,v_7,v_8,v_9,v_{10}), (v_9;v_1,v_2,v_3,v_8,v_{10}), (v_{10};v_3,v_4,v_6,v_7,v_8)\}$.

Case 9: $(\alpha,\beta) = (1,8)$
$K_{10} = 1(C_4+e) \oplus 8S_5 = \{(v_1,v_8,v_9,v_2;v_8v_6)\} \oplus \{(v_1;v_3,v_4,v_5,v_6,v_7), (v_2;v_4,v_5,v_6,v_7,v_8), (v_3;v_2,v_5,v_6,v_7,v_8), (v_4;v_3,v_5,v_6,v_8,v_{10}), (v_5;v_6,v_7,v_8,v_9,v_{10}), (v_7;v_4,v_6,v_8,v_9,v_{10}), (v_9;v_1,v_3,v_4,v_6,v_{10}), (v_{10};v_1,v_2,v_3,v_6,v_8)\}$.

Case 10: $(\alpha,\beta) = (0,9)$
$K_{10} = 9S_5 = \{(v_1;v_2,v_3,v_4,v_5,v_6), (v_2;v_3,v_4,v_5,v_6,v_7), (v_3;v_4,v_5,v_6,v_7,v_8), (v_4;v_5,v_6,v_7,v_8,v_9), (v_5;v_6,v_7,v_8,v_9,v_{10}), (v_7;v_1,v_6,v_8,v_9,v_{10}), (v_8;v_1,v_2,v_6,v_9,v_{10}), (v_9;v_1,v_2,v_3,v_6,v_{10}), (v_{10};v_1,v_2,v_3,v_4,v_6)\}$.

5.2. **A $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition of K_{11}.**

Case 1: $(\alpha,\beta) = (11,0)$
$K_{11} = 11(C_4+e) = \{(v_1,v_2,v_9,v_8;v_1v_4), (v_2,v_3,v_{10},v_{11};v_2v_8), (v_3,v_4,v_{10},v_9;v_4v_7), (v_4,v_5,v_1,v_{11};v_5v_2), (v_5,v_6,v_1,v_{10};v_6v_2), (v_6,v_7,v_1,v_3;v_6v_9), (v_7,v_8,v_5,v_3;v_7v_{10}), (v_2,v_4,v_9,v_7;v_9v_1), (v_5,v_7,v_{11},v_9;v_{11}v_8), (v_3,v_8,v_6,v_{11};v_{11}v_5), (v_4,v_6,v_{10},v_8;v_{10}v_2)\}$.

Case 2: $(\alpha,\beta) = (10,1)$
$K_{11} = 10(C_4+e) \oplus 1S_5 = \{(v_1,v_2,v_9,v_8;v_1v_4), (v_5,v_6,v_1,v_{10};v_{10}v_{11}), (v_3,v_4,v_{10},v_9;v_4v_7), (v_4,v_5,v_1,v_{11};v_5v_2), (v_4,v_6,v_{10},v_8;v_{10}v_3), (v_6,v_7,v_1,v_3;v_6v_9), (v_7,v_8,v_5,v_3;v_7v_{10}), (v_2,v_4,v_9,v_7;v_9v_1), (v_5,v_7,v_{11},v_9;v_{11}v_8), (v_3,v_8,v_6,v_{11};v_{11}v_5)\} \oplus \{(v_2;v_3,v_5,v_6,v_8,v_{11})\}$.

Case 3: $(\alpha,\beta) = (9,2)$
$K_{11} = 9(C_4+e) \oplus 2S_5 = \{(v_1,v_2,v_{10},v_9;v_9v_5), (v_2,v_3,v_9,v_8;v_8v_4), (v_3,v_8,v_{10},v_4;v_4v_6), (v_4,v_5,v_{10},v_{11};v_{11}v_6), (v_5,v_6,v_1,v_{11};v_1v_7), (v_6,v_7,v_{11},v_9;v_7v_4), (v_7,v_8,v_1,v_{10};v_{10}v_6), (v_2,v_4,v_9,v_7;v_4v_1), (v_2,v_6,v_8,v_{11};v_2v_9)\} \oplus \{(v_3;v_1,v_6,v_7,v_{10},v_{11}), (v_5;v_1,v_2,v_3,v_7,v_8)\}$.

Case 4: $(\alpha,\beta) = (8,3)$
$K_{11} = 8(C_4+e) \oplus 3S_5 = \{(v_1,v_2,v_9,v_8;v_2v_6), (v_2,v_3,v_{10},v_{11};v_{10}v_8), (v_3,v_4,v_{10},v_9;v_3v_{11}), (v_4,v_5,v_1,v_{11};v_1v_9), (v_5,v_6,v_1,v_{10};v_5v_3), (v_6,v_7,v_{11},v_9;v_{11}v_8), (v_6,v_8,v_2,v_{10};v_6v_{11}), (v_1,v_3,v_8,v_7;v_3v_6)\} \oplus \{(v_4;v_1,v_2,v_6,v_8,v_9), (v_5;v_2,v_7,v_8,v_9,v_{11}), (v_7;v_2,v_3,v_4,v_9,v_{10})\}$.

Case 5: $(\alpha,\beta) = (7,4)$
$K_{11} = 7(C_4+e) \oplus 4S_5 = \{(v_1,v_2,v_9,v_8;v_2v_5), (v_2,v_3,v_8,v_7;v_3v_{10}), (v_3,v_4,v_{10},v_9;v_3v_{11}), (v_4,v_5,v_{10},v_{11};v_5v_7), (v_5,v_6,v_1,v_{11};v_{11}v_9), (v_2,v_6,v_7,v_{10};v_2v_{11}), (v_3,v_5,v_9,v_7;v_7v_{11})\} \oplus \{(v_1;v_3,v_5,v_7,v_9,v_{10}), (v_4;v_1,v_2,v_6,v_7,v_9), (v_6;v_3,v_8,v_9,v_{10},v_{11}), (v_8;v_2,v_4,v_5,v_{10},v_{11})\}$.

Case 6: $(\alpha,\beta) = (6,5)$
$K_{11} = 6(C_4+e) \oplus 5S_5 = \{(v_1,v_2,v_9,v_8;v_1v_4), (v_2,v_3,v_9,v_{10};v_9v_{11}), (v_3,v_4,v_8,v_7;v_4v_9), (v_4,v_5,v_{10},v_{11};v_5v_8), (v_2,v_5,v_7,v_{11};v_{11}v_8), (v_2,v_4,v_{10},v_8;v_8v_6)\} \oplus \{(v_1;v_6,v_7,v_9,v_{10},v_{11}), (v_3;v_1,v_6,v_8,v_{10},v_{11}), (v_5;v_1,v_3,v_6,v_9,v_{11}), (v_6;v_2,v_4,v_9,v_{10},v_{11}), (v_7;v_2,v_4,v_6,v_9,v_{10})\}$.

Case 7: $(\alpha,\beta) = (5,6)$
$K_{11} = 5(C_4+e) \oplus 6S_5 = \{(v_1,v_6,v_7,v_{11};v_7v_{10}), (v_1,v_2,v_8,v_7;v_7v_9), (v_2,v_3,v_9,v_{10};v_{10}v_8), (v_4,v_5,v_{10},v_{11};v_{11}v_9), (v_3,v_4,v_9,v_8;v_8v_{11})\} \oplus \{(v_1;v_4,v_5,v_8,v_9,v_{10}), (v_2;v_5,v_6,v_7,v_9,v_{11}), (v_3;v_1,v_6,v_7,v_{10},v_{11}), (v_4;v_2,v_{10},v_8,v_7,v_6), (v_5;v_3,v_7,v_8,v_9,v_{11}), (v_6;v_5,v_8,v_9,v_{10},v_{11})\}$.

Case 8: $(\alpha,\beta) = (4,7)$

$K_{11} = 4(C_4+e) \oplus 7S_5 = \{(v_1,v_2,v_9,v_8;v_8v_{10}),\ (v_2,v_3,v_7,v_8;v_7v_9),\ (v_3,v_4,v_9,v_{10};v_{10}v_7),\ (v_4,v_5,v_{10},v_{11};v_4v_6)\} \oplus \{(v_1;v_5,v_6,v_7,v_9,v_{10}),\ (v_2;v_5,v_6,v_7,v_{10},v_{11}),\ (v_3;v_1,v_6,v_8,v_9,v_{11}),\ (v_4;v_1,v_2,v_7,v_8,v_{10}),\ (v_5;v_3,v_6,v_7,v_8,v_9),\ (v_6;v_7,v_8,v_9,v_{10},v_{11}),\ (v_{11};v_1,v_5,v_7,v_8,v_9)\}$.

Case 9: $(\alpha,\beta) = (3,8)$

$K_{11} = 3(C_4+e) \oplus 8S_5 = \{(v_1,v_2,v_8,v_9;v_9v_{11}),\ (v_2,v_3,v_9,v_{10};v_{10}v_{11}),\ (v_3,v_4,v_7,v_8;v_3v_1)\} \oplus \{(v_1;v_6,v_7,v_8,v_{10},v_{11}),\ (v_2;v_5,v_6,v_7,v_9,v_{11}),\ (v_3;v_5,v_6,v_7,v_{10},v_{11}),\ (v_4;v_1,v_2,v_9,v_{10},v_{11}),\ (v_5;v_1,v_4,v_9,v_{10},v_{11}),\ (v_6;v_4,v_5,v_9,v_{10},v_{11}),\ (v_7;v_5,v_6,v_9,v_{10},v_{11}),\ (v_8;v_4,v_5,v_6,v_{10},v_{11})\}$.

Case 10: $(\alpha,\beta) = (2,9)$

$K_{11} = 2(C_4+e) \oplus 9S_5 = \{(v_1,v_2,v_9,v_{10};v_{10}v_{11}),\ (v_2,v_3,v_7,v_8;v_8v_9)\} \oplus \{(v_1;v_6,v_7,v_8,v_9,v_{11}),\ (v_2;v_5,v_6,v_7,v_{10},v_{11}),\ (v_3;v_1,v_8,v_9,v_{10},v_{11}),\ (v_4;v_1,v_2,v_3,v_{10},v_{11}),\ (v_5;v_1,v_3,v_4,v_{10},v_{11}),\ (v_6;v_3,v_4,v_5,v_{10},v_{11}),\ (v_7,v_8;v_4,v_5,v_6,v_{10},v_{11}),\ (v_9;v_4,v_5,v_6,v_7,v_{11})\}$.

Case 11: $(\alpha,\beta) = (1,10)$

$K_{11} = 1(C_4+e) \oplus 10S_5 = \{(v_1,v_2,v_9,v_{10};v_9v_{11})\} \oplus \{(v_1;v_6,v_7,v_8,v_9,v_{11}),\ (v_2;v_6,v_7,v_8,v_{10},v_{11}),\ (v_3;v_1,v_2,v_9,v_{10},v_{11}),\ (v_4;v_1,v_2,v_3,v_{10},v_{11}),\ (v_5;v_1,v_2,v_3,v_4,v_{11}),\ (v_6;v_3,v_4,v_5,v_8,v_{11}),\ (v_7;v_3,v_4,v_5,v_6,v_{11}),\ (v_8;v_3,v_4,v_5,v_7,v_{11}),\ (v_9;v_4,v_5,v_6,v_7,v_8),\ (v_{10};v_5,v_6,v_7,v_8,v_{11})\}$.

Case 12: $(\alpha,\beta) = (0,11)$

$K_{11} = 11S_5 = \{(v_1;v_2,v_3,v_4,v_5,v_6),\ (v_2;v_3,v_4,v_5,v_6,v_7),\ (v_3;v_4,v_5,v_6,v_7,v_8),\ (v_4;v_5,v_6,v_7,v_8,v_9),\ (v_5;v_6,v_7,v_8,v_9,v_{10}),\ (v_6;v_7,v_8,v_9,v_{10},v_{11}),\ (v_7;v_1,v_8,v_9,v_{10},v_{11}),\ (v_8;v_1,v_2,v_9,v_{10},v_{11}),\ (v_9;v_1,v_2,v_3,v_{10},v_{11}),\ (v_{10};v_1,v_2,v_3,v_4,v_{11}),\ (v_{11};v_1,v_2,v_3,v_4,v_5)\}$.

5.3. **A $\{(C_4+e)^\alpha, S_5^\beta\}$-decomposition of $K_5 \oplus K_{5,10}$.**

Case 1: $(\alpha,\beta) \in \{(0,12),(2,10),(4,8),(6,6)\}$

$K_5 \oplus K_{5,10} = 6S_5 \oplus 3\mathcal{T} = \{(m_1;m_4,m_5,n_8,n_9,n_{10}),\ (m_2;m_1,m_5,n_8,n_9,n_{10}),\ (m_3;m_1,m_2,n_8,n_9,n_{10}),\ (m_4;m_2,m_3,n_8,n_9,n_{10}),\ (m_5;m_3,m_4,n_8,n_9,n_{10}),\ (n_7;m_1,m_2,m_3,m_4,m_5)\} \oplus \{(n_1,n_2,n_3,n_4,n_5,n_6;m_1,m_2,m_3,m_4,m_5)\}$.

From Observation 1.3, this gives the required decomposition for $(\alpha,\beta) \in \{(2,10),(4,8),(6,6)\}$.

Case 2: $(\alpha,\beta) \in \{(3,9),(5,7),(7,5),(9,3)\}$

$K_5 \oplus K_{5,10} = 1(C_4+e) \oplus 3S_5 \oplus 4\mathcal{T} = \{(m_2,m_3,m_4,m_5;m_2m_1)\} \oplus \{(m_1;m_3,n_7,n_8,n_9,n_{10}),\ (m_2;m_4,n_7,n_8,n_9,n_{10}),\ (m_3;m_5,n_7,n_8,n_9,n_{10})\} \oplus \{(m_4,m_5;m_1,n_7,n_8,n_9,n_{10}),\ (n_1,n_2,n_3,n_4,n_5,n_6;m_1,m_2,m_3,m_4,m_5)\}$.

From Observation 1.3, this gives the required decomposition for $(\alpha,\beta) \in \{(3,9),(5,7),(7,5),(9,3)\}$.

Case 3: $(\alpha,\beta) = (12,0)$

$K_5 \oplus K_{5,10} = 12(C_4+e) = \{(n_2,m_3,n_7,m_4;n_7m_2),\ (n_2,m_1,n_7,m_5;n_2m_2),\ (n_1,m_3,n_6,m_4;n_6m_1),\ (m_1,m_3,m_5,n_1;m_5m_2),\ (m_2,m_4,m_5,n_6;m_5m_1),\ (n_3,m_1,n_8,m_5;n_3m_3),\ (n_3,m_2,n_8,m_4;n_8m_3),\ (n_4,m_1,n_9,m_2;n_4m_4),\ (n_4,m_3,n_9,m_5;n_9m_4),\ (n_5,m_1,n_{10},m_4;n_{10}m_5),\ (n_5,m_2,n_{10},m_3;n_5m_5),\ (m_1,m_2,m_3,m_4;m_2n_1)\}$.

From this, first two copies of banners can be written as $(n_2,n_7;m_1,m_2,m_3,m_4,m_5)$. Then, this gives the required decomposition for $(\alpha,\beta) \in (10,2)$.

Case 4: $(\alpha,\beta) = (11,1)$

$K_5 \oplus K_{5,10} = 11(C_4+e) \oplus 1S_5 = \{(m_2,m_3,m_4,m_5;m_2m_1),\ (n_1,m_2,n_6,m_3;m_3m_5),\ (n_1,m_4,n_6,m_5;m_4m_2),\ (n_2,m_1,n_7,m_3;n_2m_2),\ (n_2,m_4,n_7,m_5;n_7m_2),\ (n_3,m_1,n_8,m_2;n_3m_3),\ (n_3,m_4,n_8,m_5;n_8m_3),\ (n_4,m_1,n_9,m_2;n_4m_4),\ (n_4,m_3,n_9,m_5;n_9m_4),\ (n_5,m_1,n_{10},m_2;n_5m_5),\ (n_5,m_3,n_{10},m_4;n_{10}m_5)\} \oplus \{(m_1;n_1,n_6,m_3,m_4,m_5)\}$.

Case 5: $(\alpha, \beta) = \{(10, 2), (8, 4)\}$
$K_5 \oplus K_{5,10} = 8(C_4 + e) \oplus 2S_5 \oplus 1\mathcal{T} = \{(m_1,m_3,m_5,n_6;m_5m_2),$
$(m_1,m_2,m_3,m_4;m_2n_6)$, $(m_2,m_4,m_5,n_7;m_2n_8)$, $(m_2,n_9,m_5,n_{10};m_5n_8)$,
$(n_1,m_1,n_2,m_2;n_1m_3)$, $(n_1,m_4,n_2,m_5;n_2m_3)$, $(n_3,m_1,n_4,m_2;n_3m_3)$,
$(n_3,m_4,n_4,m_5;n_4m_3)\} \oplus \{(m_1;m_5,n_7,n_8,n_9,n_{10})$, $(n_5;m_1,m_2,m_3,m_4,m_5)\}$
$\oplus \{(m_3,m_4;n_6,n_7,n_8,n_9,n_{10})\}$.

5.4. **A $\{(C_4 + e)^{\alpha}, S_5^{\beta}\}$-decomposition of $K_6 \oplus K_{6,10}$.**
Case 1: $(\alpha, \beta) = (15, 0)$
$K_6 \oplus K_{6,10} = 15(C_4 + e) = \{(n_1,m_1,n_6,m_2;m_2m_4),$
$(n_2,m_1,n_7,m_2;m_1m_3)$, $(n_2,m_3,n_7,m_4;m_3m_6)$, $(n_2,m_5,n_7,m_6;m_5m_2)$,
$(n_3,m_5,n_8,m_6;m_5m_4)$, $(n_3,m_2,n_8,m_4;m_4m_3)$, $(n_3,m_1,n_8,m_3;m_1m_2)$,
$(n_1,m_3,n_6,m_4;m_3m_2)$, $(n_1,m_5,n_6,m_6;m_5m_3)$, $(n_4,m_1,n_9,m_4;m_1m_6)$,
$(n_4,m_2,n_9,m_3;m_2m_6)$, $(n_4,m_5,n_9,m_6;m_6m_4)$, $(n_5,m_1,n_{10},m_3;m_1m_5)$,
$(n_5,m_2,n_{10},m_6;m_6m_5)$, $(n_5,m_4,n_{10},m_5;m_4m_1)\}$.
Case 2: $(\alpha, \beta) = (14, 1)$
$K_6 \oplus K_{6,10} = 14(C_4 + e) \oplus 1S_5 = \{(n_1,m_1,n_6,m_2;m_2m_3),$
$(n_2,m_1,n_7,m_2;m_1m_4)$, $(n_2,m_3,n_7,m_4;m_4n_5)$, $(n_2,m_5,n_7,m_6;m_5m_2)$,
$(n_3,m_5,n_8,m_6;m_5m_4)$, $(n_3,m_2,n_8,m_4;m_4m_3)$, $(n_3,m_1,n_8,m_3;m_1m_2)$,
$(n_1,m_3,n_6,m_4;m_3m_1)$, $(n_1,m_5,n_6,m_6;m_5m_3)$, $(n_4,m_1,n_9,m_4;m_1m_5)$,
$(n_4,m_2,n_9,m_3;m_2m_4)$, $(n_4,m_5,n_9,m_6;m_5n_5)$, $(n_5,m_1,n_{10},m_3;n_{10}m_5)$,
$(n_5,m_2,n_{10},m_6;n_{10}m_4)\} \oplus \{(m_6;m_1,m_2,m_3,m_4,m_5)\}$.
Case 3: $(\alpha, \beta) = (13, 2)$
$K_6 \oplus K_{6,10} = 13(C_4 + e) \oplus 2S_5 = \{(n_1,m_1,n_6,m_2;m_2m_3),$
$(n_2,m_1,n_7,m_2;m_2n_5)$, $(n_2,m_3,n_7,m_4;m_4m_1)$, $(n_2,m_5,n_7,m_6;m_5n_{10})$,
$(n_3,m_5,n_8,m_6;m_6n_{10})$, $(n_3,m_2,n_8,m_4;m_4m_3)$, $(n_3,m_1,n_8,m_3;m_1m_2)$,
$(n_1,m_3,n_6,m_4;m_4m_2)$, $(n_1,m_5,n_6,m_6;m_6n_5)$, $(n_4,m_1,n_9,m_4;m_1m_3)$,
$(n_4,m_2,n_9,m_3;m_2n_{10})$, $(n_4,m_5,n_9,m_6;n_{10}m_4)$, $(n_5,m_1,n_{10},m_3;n_5m_4)\}$
$\oplus \{(m_5;m_1,m_2,m_3,m_4,n_5)$, $(m_6;m_1,m_2,m_3,m_4,m_5)\}$.

For all other admissible pairs, we can write $K_6 \oplus K_{6,10} = K_5 \oplus K_{5,10} + 3S_5$. From Appendix 5.3, the graph $K_5 \oplus K_{5,10}$ admits $\{(C_4+e)^{\alpha}, S_5^{\beta}\}$-decomposition for all admissible pairs (α, β).

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