

A STUDY ON (c, d) $\mathcal{IF} - \mathcal{Q}$ UNIFORM SPECTRAL SPACES

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ABSTRACT. A conceptual way of approach to sober space is explored by the irreducibility of closed sets and their components in topological spaces. Sober space has been defined by both generic points and the irreducibility of closed sets. From this, the extension of a novel space which is known as spectral space is developed. Spectral space is one of the inventive extensions of sober space. Spectral space, can also be studied along with compact spaces, T_0 space and sober space in topological space. T_0 space has played a major role in spectral space. Quasi-spectral space and Semi-spectral space are also probed in addition to spectral space. In this article, the author introduces a new concept called (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set. By using it the new space called (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform sober space is introduced and studied. The extension of (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform sober space is studied as (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform spectral space. Moreover, (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform semi-spectral space and (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral space are also introduced and some of its properties are discussed.

Keywords: (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed sets, (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform sober space, (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform spectral space, (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral space and (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform semi-spectral space.

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1. INTRODUCTION

Zadeh .L [28] in 1965, proposed the novel set is called Fuzzy set from universal set X . It studies about fuzzy sset the uncertainty, vagueness etc. Each elements in a fuzzy set is represented as a membership value. The membership values are defined by membership functions from fuzzy set to $[0,1]$. It plays an important role in fuzzy sets. Later, in 1986, Atanassov K. T. [2, 3] extended fuzzy set as a new form called intuitionistic fuzzy set. An intuitionistic fuzzy set deals with both belongingness and non-belongingness of each elements in the intuitionistic fuzzy set. Topology deals with the geometrical objects. Otherwise it is called as Rubber sheet-geometry. In topological space many of the new

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ideas, geometrical interpretations has been evolved. So, one of the prominent author Chang C. L. [8] enhanced with a new idea called fuzzy topology in 1968. He framed the definition of fuzzy topological space, continuity, closed set, compactness etc. Followed by Chang, Lowen R. [23] in 1976, proposed an another defintion for fuzzy topological space. In 1977, Hutton B. [18] gives an detailed analysis on quasi uniformities in topological spaces and later it was extended to fuzzy topological spaces. The author's aim is to give an detailed explanation on quasi-uniformizable, uniformizability, complete regularity etc. Kotze W. [22] in 1986, had developed an idea of quasi-coincidence to fuzzy hausdorff space. The authors Kim Y. C. and Lee S. J. [20] in 1998, proposed a new idea called fuzzy quasi-uniform spaces amd explored some of its properties. Followed by above authors, Kim Y. C. and Ko J. M. [21] in 2006, extended to L-fuzzy quasi uniformity structure in L-fuzzy topology and some of its results has been proved. The concept of fuzzy quasi-uniformity was extended to intuitionistic fuzzy quasi-uniform topological space and it was studied by Revathi G. K. and et. al [25] in 2012. Functors, plays a huge impact in intuitionistic fuzzy quasi uniform space. Irreducibility in topological space leads to an extension called sober space along with spectral spaces.

2. LITERATURE REVIEW

Connectedness, compactness and continuous functions made a huge impact in topological spaces. Many of the novel ideas had been enhanced from these three C's. Among the three, compactness will play a major role for defining open cover from the collection of all family of open sets. Compactness has a vital role in dimension of a topological spaces. Among the authors, Chang C. L. [8], Lowen R. [23] and Gantner T. E.[15] gave the defintions for compactness in fuzzy topological spaces. Later that Coker D. [9] and Abbas S. E. [1] formed definition of intuitionistic fuzzy topological space and intuitionistic fuzzy compactness. Hochster M. [17] introduced the prime ideals structure in commutative rings in 1968. The author Johnstone P. T. [19] studied about the stone spaces in topological spaces. Later that Kotze W. [22] defined quasi-coincidence and quasi Hausdorff in topological space. David E. [11] investigated the irreduciblity components in topological sober space and also the soberfications concept are also studied. In addition the author defined soberity and explored its expansions along with the subcategories. In this article establishing, soberfications in topological space is the author's primary goal. Girez G., and et. al [16] introduced a concept called spectral theory of continuous lattices in which the author discussed about the irreduciblity, complete lattices etc. Echi O. and et. al [12, 13, 14] extended the concept to Gold-Spectral and Jacspectral spaces using quasi-homeomorphism. Additionally sober spaces and sober sets are also discussed. Belaid K. and et. al [5], extended the spectral space idea to Alexandorff space. The author gives an detailed analysis on extension theorems in that space. Bouacida E., and etal [7] gave extension theorems for sober spaces along with Goldman topology. Later in 2009 Xu L.[26, 27] discussed sober space and its decomposition by using functors and also via posets a scott topology and its soberfications had been defined. The author David Chen [10] introduced and studied sober topological spaces in Zariski topologies. The author gave a special emphasis to soberity in Zariski topology. Also David Chen [10], generalised Zarisiki topology as a new concept called spectral spaces and its functors. The sober space in topological space will make a new extension called spectral space. In spectral space, compactness, sober space, T_0 space will give an important results and ideas. In this paper, the authors expressed a new idea $(\mathfrak{C}, \mathfrak{D})$ $\mathcal{IF} - \mathcal{Q}$ uniform spectral space. Further, an $(\mathfrak{C}, \mathfrak{D})$ $\mathcal{IF} - \mathcal{Q}$ uniform spectral space can be induced from $(\mathfrak{C}, \mathfrak{D})$ $\mathcal{IF} - \mathcal{Q}$ uniform topological

space and make the article more interesting which studying the properties. In this article the author discussed $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform spectral space in $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform topological space. The ideas will give new results and some of the types of $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform spectral space is also defined.

3. MOTIVATION AND CONTRIBUTION OF THE STUDY

The structure of $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform spectral space is quite interesting and hence various fruitful results had been established already. The study of spectral space is an important extension of sober space and also it is one of the main part in topological space. Also the $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform space has deep conceptual ideas in intuitionistic fuzzy topological space. Spectral space is an extension of sober space. The sober space can be studied with the functors. Likewise, spectral space can also dealt with the properties of sober space in topological space. $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform sober spaces and $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ spectral spaces have novel properties which were listed and studied in this work. The article is to framework new concept called $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform spectral space and which paved the way to find some interesting and essential topological properties. $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform spectral space portrays the new forms of $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform semi-spectral space, $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral space, $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform q -compact spaces, $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform Γ_q -compact sets and $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform ξ -sets etc., Also various results are proved.

4. PRELIMINARIES

Throughout this paper, $(\mathfrak{c}, \mathfrak{d})$ intuitionistic fuzzy quasi uniform topological space is represented as $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform topological space.

Definition 4.1. [2, 3] Let X be a universal set. An intuitionistic fuzzy Set $\mathcal{A}$ in X is defined as $\mathcal{A} = \{\langle x, \mu_{\mathcal{A}(x)}, \nu_{\mathcal{A}(x)} \rangle : x \in X\}$ where $\mu_{\mathcal{A}(x)} : X \rightarrow [0, 1]$ and $\nu_{\mathcal{A}(x)} : X \rightarrow [0, 1]$ are the membership and non-membership functions respectively for every $x \in X$, with the condition $0 \leq \mu_{\mathcal{A}(x)} + \nu_{\mathcal{A}(x)} \leq 1$. The collection of intuitionistic fuzzy set is denoted as $\text{IF}(X)$

Definition 4.2. [9] An intuitionistic fuzzy topology τ on a non-empty set X is a collection of intuitionistic fuzzy sets with the following properties.

- (i) $0_{\sim}, 1_{\sim} \in \tau$,
- (ii) $\mathcal{A}_1 \cap \mathcal{A}_2 \in \tau$ for any $\mathcal{A}_1, \mathcal{A}_2 \in \tau$,
- (iii) $\cup \mathcal{A}_i \in \tau$ for any arbitrary family $\{\mathcal{A}_i : i \in J\} \subseteq \tau$.

An element in τ and its complement with respect to τ are called intuitionistic fuzzy open and closed sets respectively. In this case, (X, τ) is called a intuitionistic fuzzy topological space.

Definition 4.3. [9] Let $\mathcal{A} = \langle x, \mu_{\mathcal{A}(x)}, \nu_{\mathcal{A}(x)} \rangle$ and $\mathcal{B} = \langle x, \mu_{\mathcal{B}(x)}, \nu_{\mathcal{B}(x)} \rangle$ be two intuitionistic fuzzy sets in X , then $\mathcal{A}$ and $\mathcal{B}$ are said to be q -coincident ($\mathcal{A} \text{ } q \text{ } \mathcal{B}$ respectively) iff there exists an element $x \in X$ such that either $\mu_{\mathcal{A}(x)} > \nu_{\mathcal{B}(x)}$ or $\nu_{\mathcal{A}(x)} < \mu_{\mathcal{B}(x)}$.

Definition 4.4. [4, 27] Let (X, τ) be a topological space. Let $\mathcal{A}, \mathcal{B}$ and $\mathcal{C}$ be a non-empty closed sets in (X, τ) . Then $\mathcal{A}$ is said to be irreducible closed set in (X, τ) , if either $\mathcal{A} \subseteq \mathcal{B}$ or $\mathcal{A} \subseteq \mathcal{C}$.

Definition 4.5. [13, 19] Let (X, τ) be a topological space. A topological space X is said to be a sober space, if for each irreducible closed set $\mathcal{C}$ there is a unique element $x \in X$ such that $\{\bar{x}\} = \mathcal{C}$, where $\{\bar{x}\}$ is the closure of x in X .

Definition 4.6. [25] Let Ω_X be family of intuitionistic fuzzy mappings, $\mathfrak{g} : \text{IF}(X) \rightarrow \text{IF}(X)$ then the following are:

- (i) $\mathfrak{g}(0_{\sim}) = 0_{\sim}$
- (ii) $\mathcal{A} \subseteq \mathfrak{g}(\mathcal{A}), \forall \mathcal{A} \in \text{IF}(X)$
- (iii) $\mathfrak{g}(\cup \mathcal{A}_i) = \cup \mathfrak{g}(\mathcal{A}_i), \forall \mathcal{A}_i \in \text{IF}(X), i \in J$

For $\mathfrak{g} \in \Omega_X$, the function $\mathfrak{g}^{-1}(\mathcal{A}) = \cap \{\mathcal{B} : \mathfrak{g}(\mathcal{B}) \subseteq \mathcal{A}\} \in \Omega_X$, given for all $\mathcal{A} \in \text{IF}(X)$, $\mathfrak{g} \cap \mathfrak{g}'(\mathcal{A}) = \cap \{\mathfrak{g}(\mathcal{A}_1) \cup \mathfrak{g}'(\mathcal{A}_2) : \mathcal{A}_1 \cup \mathcal{A}_2 = \mathcal{A}\}$, $(\mathfrak{g} \circ \mathfrak{g}')(\mathcal{A}) = \mathfrak{g}(\mathfrak{g}'(\mathcal{A}))$.

Definition 4.7. [25] Let $v : \Omega_X \rightarrow \mathcal{I} \times \mathcal{I}$ be a intuitionistic fuzzy mapping. Then v is said to be a intuitionistic fuzzy quasi uniformity on X , if it satisfies the following conditions:

- (i) $v(\mathfrak{g}_1 \cap \mathfrak{g}_2) \supseteq v(\mathfrak{g}_1) \cap v(\mathfrak{g}_2)$ for $\mathfrak{g}_1, \mathfrak{g}_2 \in \Omega_X$
- (ii) $\mathfrak{g} \in \Omega_X$ we have $\cup \{v(\mathfrak{g}_1) : \mathfrak{g}_1 \circ \mathfrak{g}_1 \subseteq \mathfrak{g}\} \supseteq v(\mathfrak{g})$
- (iii) $\mathfrak{g}_1 \supseteq \mathfrak{g}$ then $v(\mathfrak{g}_1) \supseteq v(\mathfrak{g})$
- (iv) $\mathfrak{g} \in \Omega_X$ then $v(\mathfrak{g}) = 1_{\sim}$.

The pair (X, v) is said to be an $\mathcal{IF} - \mathcal{Q}$ uniform space.

Definition 4.8. [25] Let (X, v) be a $\mathcal{IF} - \mathcal{Q}$ uniform space. Let $\mathfrak{c} \in (0, 1] = \mathcal{I}_0$ and $\mathfrak{d} \in [0, 1) = \mathcal{I}_1$ with $\mathfrak{c} + \mathfrak{d} \leq 1$ and $\mathcal{A} \in \text{IF}(X)$.

Define $(\mathfrak{c}, \mathfrak{d}) \text{IFQI}_v(\mathcal{A}) = \cup \{\mathcal{B} : \mathfrak{f}(\mathcal{B}) \subseteq \mathcal{A} \text{ for some cases } \mathfrak{f} \in \Omega(X) \text{ with } v(\mathfrak{f}) > (\mathfrak{c}, \mathfrak{d})\}$

Definition 4.9. [25] Let (X, v) be a $\mathcal{IF} - \mathcal{Q}$ uniform space. The function $\tau_v : \text{IF}(X) \rightarrow \mathcal{I} \times \mathcal{I}$ is defined as

$\tau_v(\mathcal{A}) = \cup \{(\mathfrak{c}, \mathfrak{d}) : (\mathfrak{c}, \mathfrak{d}) \text{IFQI}_v(\mathcal{A}) = \mathcal{A}, \mathfrak{c} \in \mathcal{I}_0, \mathfrak{d} \in \mathcal{I}_1 \text{ with } \mathfrak{c} + \mathfrak{d} \leq 1\}$. Hence, a pair (X, τ_v) is called $\mathcal{IF} - \mathcal{Q}$ uniform topological space. The elements of (X, τ_v) is called $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform open sets and its complement is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed sets.

Definition 4.10. [25] Let (X, τ_v) be a $\mathcal{IF} - \mathcal{Q}$ uniform topological space and $\mathcal{A}$ be an intuitionistic fuzzy set. Then the $\mathcal{IF} - \mathcal{Q}$ uniform interior of $\mathcal{A}$ is defined as $(\mathfrak{c}, \mathfrak{d}) \text{IFQ int}_v(\mathcal{A}) = \cup \{\mathcal{B} : \mathcal{B} \subseteq \mathcal{A} \text{ and } \mathcal{B} \text{ is a } (\mathfrak{c}, \mathfrak{d}) \text{IF} - \mathcal{Q} \text{ uniform open set where } \mathfrak{c} \in \mathcal{I}_0, \mathfrak{d} \in \mathcal{I}_1, \text{ and } \mathfrak{c} + \mathfrak{d} \leq 1\}$.

Definition 4.11. [25] Let (X, τ_v) be an $\mathcal{IF} - \mathcal{Q}$ uniform topological space and $\mathcal{A}$ be an intuitionistic fuzzy set. Then the $\mathcal{IF} - \mathcal{Q}$ uniform closure of $\mathcal{A}$ is defined as $(\mathfrak{c}, \mathfrak{d}) \text{IFQ cl}_v(\mathcal{A}) = \cap \{\mathcal{B} : \mathcal{B} \supseteq \mathcal{A} \text{ and } \mathcal{B} \text{ is an } (\mathfrak{c}, \mathfrak{d}) \text{IF} - \mathcal{Q} \text{ uniform closed set where } \mathfrak{c} \in \mathcal{I}_0, \mathfrak{d} \in \mathcal{I}_1 \text{ with } \mathfrak{c} + \mathfrak{d} \leq 1\}$

Definition 4.12. [8] Let (X, τ) be a fuzzy topological space. A family $\mathbf{A}$ of fuzzy sets is a cover of a fuzzy set $\mathcal{B}$ iff $\mathcal{B} \subset \cup \{\mathcal{A} : \mathcal{A} \in \mathbf{A}\}$. It is an open cover iff each member of $\mathbf{A}$ is an open fuzzy set. A subcover of $\mathbf{A}$ is a subfamily of $\mathbf{A}$ which is also a cover.

Definition 4.13. [9] Let (X, τ) be an intuitionistic fuzzy topological space. A family $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\}$ of intuitionistic fuzzy open sets in X satisfies the condition $\cup \{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\} = \tilde{1}$ then it is called a fuzzy open cover $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\}$ of X . A finite subfamily of a fuzzy open cover $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\}$ which is also a fuzzy open cover of X is called a finite subcover of $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\}$.

Definition 4.14. [9] An intuitionistic fuzzy topological space (X, τ) is called intuitionistic fuzzy compact space, if for each open cover of X there exists a finite subcover of X .

Definition 4.15. [9] Let (X_1, τ_{v_1}) and (X_2, τ_{v_2}) be two $\mathcal{IF} - \mathcal{Q}$ uniform topological spaces and $f : (X_1, \tau_{v_1}) \rightarrow (X_2, \tau_{v_2})$ be a function. Then f is said to $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform continuous function if the pre-image of each $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform open set in (X_2, τ_{v_2}) is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform open set in (X_1, τ_{v_1}) .

5. STRUCTURE OF THE PAPER

In this article, a new conceptual idea called $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform spectral space is defined. The major extension of $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform sober space is $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform spectral space. $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform sober space deals with the irreducibility of closed sets and its generic points in $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform topological space. Likewise, in $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform spectral space gives a detailed version of $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform compactness and $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform T_0 -space. In Section-6, $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform sober space is discussed and some of its results are proved. In Section-7, an extension from $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform sober space called $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform spectral space and $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral space is introduced. In this section, an $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform q -compact space has been discussed. Section-7.1, $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform semi-spectral space and new results are discussed.

6. $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ UNIFORM SOBER SPACE

In this section, gives an detailed version of $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform irreducible closed sets, $\mathcal{IF} - \mathcal{Q}$ uniform generic set are discussed. The definition of $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform sober space is introduced.

Definition 6.1. Let (X, τ_v) be a $\mathcal{IF} - \mathcal{Q}$ uniform topological space. Let $\mathcal{A}, \mathcal{B}$ and $\mathcal{C}$ be a $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform closed sets on (X, τ_v) . Then $\mathcal{A}$ are said to be a $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set, if $\mathcal{A} \subseteq \mathcal{B} \cup \mathcal{C}$ such that either $\mathcal{A} \subseteq \mathcal{B}$ or $\mathcal{A} \subseteq \mathcal{C}$. The complements of a $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform irreducible closed sets are said to be $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform irreducible open sets.

Remark 6.1. Every $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform irreducible closed is $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform closed set. The converse need not be true.

Example 6.1. Let $X = \{\mathfrak{a}, \mathfrak{b}\}$ be a non empty set,

$$\mathcal{A} = \left\langle x, \left(\frac{\mathfrak{a}}{0.6}, \frac{\mathfrak{b}}{0.7} \right), \left(\frac{\mathfrak{a}}{0.4}, \frac{\mathfrak{b}}{0.1} \right) \right\rangle, \mathcal{B} = \left\langle x, \left(\frac{\mathfrak{a}}{0.1}, \frac{\mathfrak{b}}{0.2} \right), \left(\frac{\mathfrak{a}}{0.5}, \frac{\mathfrak{b}}{0.6} \right) \right\rangle,$$

$$\mathcal{C} = \left\langle x, \left(\frac{\mathfrak{a}}{0.3}, \frac{\mathfrak{b}}{0.2} \right), \left(\frac{\mathfrak{a}}{0.4}, \frac{\mathfrak{b}}{0.3} \right) \right\rangle \text{ and } \mathcal{D} = \left\langle x, \left(\frac{\mathfrak{a}}{0.2}, \frac{\mathfrak{b}}{0.2} \right), \left(\frac{\mathfrak{a}}{0.5}, \frac{\mathfrak{b}}{0.4} \right) \right\rangle$$

be intuitionistic fuzzy sets on X and $\mathcal{E}$ be also a intuitionistic fuzzy set on X .

Let $\Omega_X: \text{IF}(X) \rightarrow \text{IF}(X)$ be an intuitionistic fuzzy mapping. Let $\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \mathfrak{g}_4$ and $\mathfrak{g}_5 \in \Omega_X$ be defined as:

$$\mathfrak{g}_1(\mathcal{E}) = \begin{cases} 0_{\sim}, & \text{if } \mathcal{E} = 0_{\sim} \\ 1_{\sim}, & \text{otherwise.} \end{cases}$$

$$\mathfrak{g}_2(\mathcal{E}) = \begin{cases} 0_{\sim}, & \text{if } \mathcal{E} = 0_{\sim} \\ \mathcal{A}, & \text{if } \mathcal{E} \subseteq \mathcal{A} \\ 1_{\sim}, & \text{otherwise.} \end{cases}$$

$$\mathfrak{g}_3(\mathcal{E}) = \begin{cases} 0_{\sim}, & \text{if } \mathcal{E} = 0_{\sim} \\ \mathcal{B}, & \text{if } \mathcal{E} \subseteq \mathcal{B} \\ 1_{\sim}, & \text{otherwise.} \end{cases}$$

$$\mathfrak{g}_4(\mathcal{E}) = \begin{cases} 0_{\sim}, & \text{if } \mathcal{E} = 0_{\sim} \\ \mathcal{C}, & \text{if } \mathcal{E} \subseteq \mathcal{C} \\ 1_{\sim}, & \text{otherwise.} \end{cases}$$

$$\mathfrak{g}_5(\mathcal{E}) = \begin{cases} 0_{\sim}, & \text{if } \mathcal{E} = 0_{\sim} \\ \mathcal{D}, & \text{if } \mathcal{E} \subseteq \mathcal{D} \\ 1_{\sim}, & \text{otherwise.} \end{cases}$$

$$v(\mathfrak{g}) = \begin{cases} (1, 0), & \text{if } \mathfrak{g} = \mathfrak{g}_1 \\ (2/7, 5/8), & \text{if } \mathfrak{g} = \mathfrak{g}_2 \\ (3/6, 1/5), & \text{if } \mathfrak{g} = \mathfrak{g}_3 \\ (3/7, 1/6), & \text{if } \mathfrak{g} = \mathfrak{g}_4 \\ (3/8, 1/8), & \text{if } \mathfrak{g} = \mathfrak{g}_5 \\ (4/9, 1/9), & \text{if } \mathfrak{g} = \mathfrak{g}_2 \sqcap \mathfrak{g}_3 \\ (5/7, 5/9), & \text{if } \mathfrak{g} = \mathfrak{g}_2 \sqcap \mathfrak{g}_4 \\ (6/7, 6/9), & \text{if } \mathfrak{g} = \mathfrak{g}_2 \sqcap \mathfrak{g}_5 \\ (3/7, 8/9), & \text{if } \mathfrak{g} = \mathfrak{g}_3 \sqcap \mathfrak{g}_4 \\ (4/9, 1/10), & \text{if } \mathfrak{g} = \mathfrak{g}_3 \sqcap \mathfrak{g}_5 \\ (1/6, 4/7), & \text{if } \mathfrak{g} = \mathfrak{g}_4 \sqcap \mathfrak{g}_5 \\ (0, 1), & \text{otherwise} \end{cases}$$

Clearly, (X, v) is an $\mathcal{IF} - \mathcal{Q}$ uniform space, for $\mathfrak{c}=0.08$ and $\mathfrak{d}=0.03$. define intuitionistic fuzzy mapping, $\tau_v: \text{IF}(X) \rightarrow \mathcal{I} \times \mathcal{I}$ as

$$\tau_v(\mathcal{E}) = \begin{cases} (0, 1), & \text{if } \mathcal{E} = 0_{\sim} \\ (2/5, 3/7), & \text{if } \mathcal{E} = \mathcal{A} \\ (1/8, 2/7), & \text{if } \mathcal{E} = \mathcal{B} \\ (1/3, 3/8), & \text{if } \mathcal{E} = \mathcal{C} \\ (2/5, 3/5), & \text{if } \mathcal{E} = \mathcal{D} \\ (1, 0), & \text{otherwise} \end{cases}$$

Then $\tau_v = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, 0_{\sim}, 1_{\sim}\}$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform open sets. Clearly (X, τ_v) is a $\mathcal{IF} - \mathcal{Q}$ uniform topological space. In this example, clearly $\mathcal{B}$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed set but it is not a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible set and $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set.

Definition 6.2. A pair (X, τ_v) is $\mathcal{IF} - \mathcal{Q}$ uniform topological space and $\mathcal{A} \in \text{IF}(X)$ is said to be a $\mathcal{IF} - \mathcal{Q}$ uniform generic set if $\mathcal{A} \subseteq \mathcal{B}$ and $(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_v}(\mathcal{A}) = \mathcal{B}$.

Definition 6.3. The pair (X, τ_v) is $\mathcal{IF} - \mathcal{Q}$ uniform topological space is said to be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform sober space, if for every $(\mathfrak{c}, \mathfrak{d})$ $(\mathcal{IF} - \mathcal{Q})$ uniform irreducible closed set $\mathcal{A}$, there exists an $\mathcal{IF} - \mathcal{Q}$ uniform generic set $\mathcal{B} \in \text{IF}_G(X)$ of $\mathcal{A}$ such that $\mathcal{B} \subseteq \mathcal{A}$.

Definition 6.4. Let the pairs (X_1, τ_{v_1}) and (X_2, τ_{v_2}) be a $\mathcal{IF} - \mathcal{Q}$ uniform topological spaces and $f: (X_1, \tau_{v_1}) \rightarrow (X_2, \tau_{v_2})$ be a $\mathcal{IF} - \mathcal{Q}$ uniform continuous function. Then f is called as $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform homeomorphism, if for every $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform open set (resp. closed set) $\mathcal{A} \in (X_1, \tau_{v_1})$, there exists a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform open set (resp. closed set) $\mathcal{B} \in (X_2, \tau_{v_2})$ such that $\mathcal{A} = f^{-1}(\mathcal{B})$.

Remark 6.2. Let (X_1, τ_{v_1}) and (X_2, τ_{v_2}) be $\mathcal{IF} - \mathcal{Q}$ uniform topological spaces and the mapping $f: (X_1, \tau_{v_1}) \rightarrow (X_2, \tau_{v_2})$ be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-homeomorphisms. For any two intuitionistic fuzzy sets $\mathcal{A}, \mathcal{B} \in \text{IF}(X_2)$, if $f^{-1}(\mathcal{A}) = f^{-1}(\mathcal{B})$, then $\mathcal{A} = \mathcal{B}$.

Definition 6.5. Let (X_1, τ_{v_1}) and (X_2, τ_{v_2}) be $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform topological spaces. A function $f: (X_1, \tau_{v_1}) \rightarrow (X_2, \tau_{v_2})$ is said to be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform injective function (one-to-one), if for each intuitionistic fuzzy set $\mathcal{A}$ in (X_1, τ_{v_1}) there exists an intuitionistic fuzzy set $\mathcal{B}$ in (X_2, τ_{v_2}) such that $f(\mathcal{A}) = f(\mathcal{B})$ then $\mathcal{A} = \mathcal{B}$.

Definition 6.6. Let (X_1, τ_{v_1}) and (X_2, τ_{v_2}) be $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform topological spaces. A function $f: (X_1, \tau_{v_1}) \rightarrow (X_2, \tau_{v_2})$ is said to be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform surjective function (onto), if for every $\mathcal{B} \in (X_2, \tau_{v_2})$, there exists atleast one $\mathcal{A} \in (X_1, \tau_{v_1})$ such that $f(\mathcal{B}) = \mathcal{A}$.

Definition 6.7. Let (X_1, τ_{v_1}) and (X_2, τ_{v_2}) be $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform topological spaces. A map $g: (X_1, \tau_{v_1}) \rightarrow (X_2, \tau_{v_2})$ is said to be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform bijective function, if it is both $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform injective function and $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform surjective function.

Proposition 6.1. The pairs (X_1, τ_{v_1}) and (X_2, τ_{v_2}) be any two $\mathcal{IF} - \mathcal{Q}$ uniform topological spaces and $f: (X_1, \tau_{v_1}) \rightarrow (X_2, \tau_{v_2})$ be an $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform surjective mapping and $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-homeomorphism. If $\mathcal{A}$ is a intuitionistic fuzzy set in (X_2, τ_{v_2}) , then the following statements are equivalent:

- (i) $\mathcal{A}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X_2, τ_{v_2}) ;
- (ii) $f^{-1}(\mathcal{A})$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X_1, τ_{v_1}) .

Proof. (i) $\implies$ (ii) Let $\mathcal{A} \in \text{IF}(X_2)$ be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X_2, τ_{v_2}) . Let $\mathcal{B}_1, \mathcal{B}_2 \in \text{IF}(X_1)$ be any two $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed set in (X_1, τ_{v_1}) such that $f^{-1}(\mathcal{A}) \subseteq \mathcal{B}_1 \cup \mathcal{B}_2$. As f is an $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-homeomorphism, there exists two $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed sets $\mathcal{C}_1$ and $\mathcal{C}_2 \in \text{IF}(X_2)$ such that $\mathcal{B}_1 = f^{-1}(\mathcal{C}_1)$ and $\mathcal{B}_2 = f^{-1}(\mathcal{C}_2)$. Thus, $f^{-1}(\mathcal{A}) \subseteq \mathcal{B}_1 \cup \mathcal{B}_2 = f^{-1}(\mathcal{C}_1) \cup f^{-1}(\mathcal{C}_2) = f^{-1}(\mathcal{C}_1 \cup \mathcal{C}_2)$. Hence $f^{-1}(\mathcal{A}) \subseteq f^{-1}(\mathcal{C}_1 \cup \mathcal{C}_2)$. Also, $f(f^{-1}(\mathcal{A})) \subseteq f(f^{-1}(\mathcal{C}_1 \cup \mathcal{C}_2))$. Since f is $(\mathfrak{c}, \mathfrak{d})$ intuitionistic fuzzy quasi-uniform surjective with $\mathcal{A} \subseteq \mathcal{C}_1 \cup \mathcal{C}_2$, $\mathcal{A}$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform space irreducible closed set in (X_2, τ_{v_2}) , and $\mathcal{A} \subseteq \mathcal{C}_1$ or $\mathcal{A} \subseteq \mathcal{C}_2$. Thus $\mathcal{B}_1 = f^{-1}(\mathcal{C}_1) \subseteq f^{-1}(\mathcal{A})$ and hence $f^{-1}(\mathcal{A}) \subseteq \mathcal{B}_1$. Similarly $\mathcal{B}_2 = f^{-1}(\mathcal{C}_2) \subseteq f^{-1}(\mathcal{A})$ implies that $f^{-1}(\mathcal{A}) \subseteq \mathcal{B}_2$. Therefore $f^{-1}(\mathcal{A}) \subseteq \mathcal{B}_1$ or $f^{-1}(\mathcal{A}) \subseteq \mathcal{B}_2$. Hence f^{-1} is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X_1, τ_{v_1}) .

(ii) $\implies$ (i)

Suppose that f^{-1} is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X_1, τ_{v_1}) . Let $\mathcal{D}_1$ and $\mathcal{D}_2 \in \text{IF}(X_2)$ be an $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed sets in (X_2, τ_{v_2}) such that $\mathcal{A} \subseteq \mathcal{D}_1 \cup \mathcal{D}_2$. Then $f^{-1}(\mathcal{A}) \subseteq f^{-1}(\mathcal{D}_1 \cup \mathcal{D}_2) = f^{-1}(\mathcal{D}_1) \cup f^{-1}(\mathcal{D}_2)$. Since f^{-1} is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set, $f^{-1}(\mathcal{A}) \subseteq f^{-1}(\mathcal{D}_1)$ or $f^{-1}(\mathcal{A}) \subseteq f^{-1}(\mathcal{D}_2)$. Also $f(f^{-1}(\mathcal{A})) \subseteq f(f^{-1}(\mathcal{D}_1))$ or $f(f^{-1}(\mathcal{A})) \subseteq f(f^{-1}(\mathcal{D}_2))$. Since f is an $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform surjective function, $\mathcal{A} \subseteq \mathcal{D}_1$ or $\mathcal{A} \subseteq \mathcal{D}_2$. Thus $\mathcal{A}$ is an $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X_2, τ_{v_2}) . $\square$

Corollary 6.1. Let (X, τ_v) be $\mathcal{IF} - \mathcal{Q}$ uniform topological space. If $\mathcal{A} \in \text{IF}(X)$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X, τ_v) , then $(\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_v}(\mathcal{A})$ is also a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X, τ_v) .

Proof. Let $\mathcal{A} \in \text{IF}(X)$ be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X, τ_v) and let $\mathcal{B}, \mathcal{C} \in \text{IF}(X)$ be any two $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed sets in (X, τ_v) . Let $(\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_v}(\mathcal{A}) \subseteq \mathcal{B} \cup \mathcal{C}$ and also $\mathcal{B} \subseteq (\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_v}(\mathcal{A})$, $\mathcal{C} \subseteq (\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_v}(\mathcal{A})$. Thus $\mathcal{A} \cap (\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_v}(\mathcal{A}) \subseteq \mathcal{A} \cap (\mathcal{B} \cup \mathcal{C})$. So, $\mathcal{A} \cap (\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_v}(\mathcal{A}) \subseteq (\mathcal{A} \cap \mathcal{B}) \cup (\mathcal{A} \cap \mathcal{C}) \implies \mathcal{A} \subseteq (\mathcal{A} \cap \mathcal{B}) \cup (\mathcal{A} \cap \mathcal{C})$. Since $\mathcal{A}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed sets, either $\mathcal{A} \subseteq (\mathcal{A} \cap \mathcal{B})$ or $\mathcal{A} \subseteq (\mathcal{A} \cap \mathcal{C})$. Thus $\mathcal{A} \subseteq \mathcal{B}$ or $\mathcal{A} \subseteq \mathcal{C}$. Therefore $(\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_v}(\mathcal{A}) \subseteq (\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_v}(\mathcal{B})$ or $(\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_v}(\mathcal{A}) \subseteq (\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_v}(\mathcal{C})$. Since $\mathcal{B}$ and $\mathcal{C}$ are $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$

uniform closed sets such that $(\mathfrak{c}, \mathfrak{d}) \text{ IFQcl}_{\tau_v}(\mathcal{A}) \subseteq \mathcal{B}$ or $(\mathfrak{c}, \mathfrak{d}) \text{ IFQcl}_{\tau_v}(\mathcal{A}) \subseteq \mathcal{C}$. Hence $(\mathfrak{c}, \mathfrak{d}) \text{ IFQcl}_{\tau_v}(\mathcal{A})$ is a $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X, τ_v) . $\square$

7. $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ UNIFORM SPECTRAL SPACE AND $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ UNIFORM QUASI-SPECTRAL SPACE

The concept of $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform q -compactness is defined in this section. Also the $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform q -compact open cover and $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform q -compact space also discussed. This section defines the concept termed as $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform spectral space and $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral space. In addition, it explores a few of its important properties.

Definition 7.1. Let (X, τ_v) be $\mathcal{IF} - \mathcal{Q}$ uniform topological space. A family $\mathfrak{S} = \{\mathcal{B}_i : \mathcal{B}_i \text{ } q \text{ } \mathcal{A}_i \text{ where } \mathcal{B}_i \neq \tilde{1} \text{ and } \mathcal{A}_i \text{ is a } (\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q} \text{ uniform irreducible open sets and } i \in I\}$ is said to be a $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform q -open cover of an intuitionistic fuzzy sets $\mathcal{B}$, if $\mathcal{B} \subseteq \cup \{\mathcal{A} : \mathcal{A} \in \mathfrak{S}\}$

Note 7.1. A $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform q -open subcover of $\mathcal{A}$ is a subfamily of $\mathfrak{S}$ which is also a subset of $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform q -open cover.

Definition 7.2. Let (X, τ_v) be an $\mathcal{IF} - \mathcal{Q}$ uniform topological space and $\mathcal{A} \in \text{IF}(X)$. Then $\mathcal{A}$ is said to be a $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform q -compact set, if for every $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform q -open cover of $\mathfrak{S}$, there exists a $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform q -open subcover such that $\mathcal{A} \subseteq \cup \{\mathcal{B}_i : \mathcal{B}_i \in \mathfrak{S} \text{ where } i=1,2,3,\dots,n\}$

Definition 7.3. Let (X, τ_v) be a $\mathcal{IF} - \mathcal{Q}$ uniform topological space. Then (X, τ_v) is said to be a $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform q -compact space, if for each $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform q -open cover of $\mathfrak{S}$ there exists a $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform q -finite subcover.

Note 7.2. From the Example 6.1, Then $\tau_v = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, 0_{\sim}, 1_{\sim}\}$ is a $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform topology and (X, τ_v) is a $\mathcal{IF} - \mathcal{Q}$ uniform topological space. Here $\mathcal{A}$ is a $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform irreducible open set. Clearly (X, τ_v) is a $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ q -compact space.

Definition 7.4. Let (X, τ_v) be a $\mathcal{IF} - \mathcal{Q}$ uniform topological space. Then (X, τ_v) is said to be a $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform T_0 space, if $\mathfrak{a}, \mathfrak{b} \in X$ and $\mathfrak{a} \neq \mathfrak{b}$, there exists $\mathcal{A} = (\mu_{\mathcal{A}}, \nu_{\mathcal{A}})$ and $\mathcal{B} = (\mu_{\mathcal{B}}, \nu_{\mathcal{B}}) \in \tau_v$ such that $\mu_{\mathcal{A}}(\mathfrak{a})=1, \mu_{\mathcal{B}}(\mathfrak{b})=1, \nu_{\mathcal{A}}(\mathfrak{a})=0, \nu_{\mathcal{B}}(\mathfrak{b})=0$, and $\mathcal{A} \cap \mathcal{B} = 0_{\sim}$.

Definition 7.5. Let (X, τ_v) be a $\mathcal{IF} - \mathcal{Q}$ uniform topological space. A base in (X, τ_v) is a subcollection $\mathbf{B}$ of τ_v such that each member of $\mathcal{A}$ of τ_v can be written as $\mathcal{A} = \cup_{j \in J} \mathcal{A}_j$ where each $\mathcal{A}_j \in \mathbf{B}$

Definition 7.6. Let (X, τ_v) be an $\mathcal{IF} - \mathcal{Q}$ uniform topological space and let $\mathfrak{S}$ be the collection of all $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform q -compact open sets in (X, τ_v) . Then (X, τ_v) is said to be a $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform spectral space, if the following axioms are to be satisfied:

- (i) (X, τ_v) is a $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform T_0 space.
- (ii) (X, τ_v) is a $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform q -compact space and it has a base, contains a collection of $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform open sets.
- (iii) If $\mathcal{A}, \mathcal{B} \in \mathfrak{S}$ then $\mathcal{A} \cap \mathcal{B} \in \mathfrak{S}$.
- (iv) (X, τ_v) is a $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform sober space.

Proposition 7.1. Let (X_1, τ_{v_1}) and (X_2, τ_{v_2}) be any two $\mathcal{IF} - \mathcal{Q}$ uniform topological spaces. Let the function $\varphi: (X_1, \tau_{v_1}) \rightarrow (X_2, \tau_{v_2})$ be $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform quasi-homeomorphism and $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform surjective function and $\mathcal{A}$ be a $(\mathfrak{c}, \mathfrak{d}) \mathcal{IF} - \mathcal{Q}$ uniform open set. Then the following conditions hold:

- (i) $\mathcal{A}$ is a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform q -compact in (X_2, τ_{v_2}) .
- (ii) $\varphi^{-1}(\mathcal{A})$ is (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform q -compact in (X_1, τ_{v_1})

Proof. (i) $\implies$ (ii)

Let $\mathcal{A} \in \text{IF}(X_2)$ be a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform q -compact set in (X_2, τ_{v_2}) . The collection $\mathfrak{S}$ be a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform open covering of $\varphi^{-1}(\mathcal{A}) \in \text{IF}(X_1)$. Then $\varphi^{-1}(\mathcal{A}) \subseteq \bigcup_{i \in J} \mathcal{B}_i$. Since φ is a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform quasi-homeomorphism, and by Definition 6.4, For every $\mathcal{B}_i \in \text{IF}(X_1)$ where $i \in J$, there exists a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform open set in $\mathcal{C}_i \in \text{IF}(X_2)$ such that $\mathcal{B}_i = \varphi^{-1}(\mathcal{C}_i)$, $i \in J$. Then $\varphi^{-1}(\mathcal{A}) \subseteq \bigcup_{i \in J} \mathcal{B}_i$. $\varphi^{-1}(\mathcal{A}) = \bigcup_{i \in J} \varphi^{-1}(\mathcal{C}_i) = \varphi^{-1}(\bigcup_{i \in J} \mathcal{C}_i)$. Thus $\varphi^{-1}(\mathcal{A}) = \varphi^{-1}(\bigcup_{i \in J} \mathcal{C}_i)$. Since φ is a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform quasi-homeomorphism, $\mathcal{A} \subseteq \bigcup_{i \in J} \mathcal{C}_i$. Since $\mathcal{A}$ is a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform q -compact set, there exists a finite subset J_0 of J such that $\mathcal{A} \subseteq \bigcup_{i \in J_0} \mathcal{C}_i$. $\varphi^{-1}(\mathcal{A}) \subseteq \varphi^{-1}(\bigcup_{i \in J_0} \mathcal{C}_i) \subseteq \bigcup_{i \in J_0} \varphi^{-1}(\mathcal{C}_i) \subseteq \bigcup_{i \in J_0} \mathcal{B}_i$. Therefore $\varphi^{-1}(\mathcal{A}) \subseteq \bigcup_{i \in J_0} \mathcal{B}_i$. Hence $\varphi^{-1}(\mathcal{A})$ is a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform q -compact set in (X_1, τ_{v_1}) .

(ii) $\implies$ (i)

Assume that $\varphi^{-1}(\mathcal{A}) \in \text{IF}(X_1)$ is a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform q -compact set in (X, τ_{v_1}) . The collection $\mathfrak{S}$ be a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform open covering of $\varphi^{-1}(\mathcal{A}) \in \text{IF}(X_2)$. Then $\mathcal{A} \subseteq \bigcup_{i \in J} \mathcal{C}_i$. Thus $\varphi^{-1}(\mathcal{A}) \subseteq \varphi^{-1}(\bigcup_{i \in J} \mathcal{C}_i) = \bigcup_{i \in J} \varphi^{-1}(\mathcal{C}_i)$. Therefore $\varphi^{-1}(\mathcal{A}) \subseteq \varphi^{-1}(\mathcal{C}_i)$. Since φ^{-1} is a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform q -compact set, there exists a finite subset $J_0 \in J$ such that $\varphi^{-1}(\mathcal{A}) \subseteq \bigcup_{i \in J_0} \varphi^{-1}(\mathcal{C}_i) = \varphi^{-1}(\bigcup_{i \in J_0} \mathcal{C}_i)$. $\varphi^{-1}(\mathcal{A}) \subseteq \varphi^{-1}(\bigcup_{i \in J_0} \mathcal{C}_i)$. $\varphi(\varphi^{-1}(\mathcal{A})) \subseteq \varphi(\varphi^{-1}(\bigcup_{i \in J_0} \mathcal{C}_i))$. Thus $\mathcal{A} \subseteq \bigcup_{i \in J_0} \mathcal{C}_i$. Since φ is a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform onto. Hence $\mathcal{A}$ is a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform q -compact set in (X_2, τ_{v_2}) . $\square$

Definition 7.7. Let (X_1, τ_{v_1}) and (X_2, τ_{v_2}) be any two $\mathcal{IF} - \mathcal{Q}$ uniform topological space and $\varphi: (X_1, \tau_{v_1}) \rightarrow (X_2, \tau_{v_2})$ be any intuitionistic fuzzy function. An intuitionistic fuzzy set $\mathcal{A} \in \text{IF}(X_1)$ is said to be (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform saturated under φ , if $\varphi^{-1}(\varphi(\mathcal{A})) = \mathcal{A}$

Remark 7.1. Let (X_1, τ_{v_1}) and (X_2, τ_{v_2}) be any two $\mathcal{IF} - \mathcal{Q}$ uniform topological space and $\varphi: (X_1, \tau_{v_1}) \rightarrow (X_2, \tau_{v_2})$ be any intuitionistic fuzzy function. If $\mathcal{A} \in \text{IF}(X_1)$ is a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform saturated set in (X_1, τ_{v_1}) under φ . $\mathcal{A}$ is also a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform saturated under φ . Then $\bar{\mathcal{A}}$ is also a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform saturated under φ .

Definition 7.8. Let (X_1, τ_{v_1}) and (X_2, τ_{v_2}) be any two $\mathcal{IF} - \mathcal{Q}$ uniform topological spaces. A mapping $\varphi: (X_1, \tau_{v_1}) \rightarrow (X_2, \tau_{v_2})$ is said to be (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform open function (resp. closed function), if for every (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform open set (resp. closed set) $\mathcal{A} \in (X_1, \tau_{v_1})$ there exists a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform open set (resp. closed set) $\varphi(\mathcal{A})$ in (X_2, τ_{v_2}) .

Proposition 7.2. Let (X_1, τ_{v_1}) and (X_2, τ_{v_2}) be any two $\mathcal{IF} - \mathcal{Q}$ uniform topological spaces. Let the mapping $\varphi: (X_1, \tau_{v_1}) \rightarrow (X_2, \tau_{v_2})$ be a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform surjective function. Then the following statements are equivalent:

- (i) φ is a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform open function and each (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform open set in (X_1, τ_{v_1}) is a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform saturated set under φ .
- (ii) φ is a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform closed function and each (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform closed set in (X_1, τ_{v_1}) is a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform saturated set under φ .

Proof. (i) $\implies$ (ii)

Let $\mathcal{A} \in \text{IF}(X_1)$ is a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform closed set in (X_1, τ_{v_1}) , then $\bar{\mathcal{A}}$ is a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform open set in (X_1, τ_{v_1}) . Since each (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform open set in (X_1, τ_{v_1}) is (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform saturated set under φ . By Remark 7.9, $\bar{\mathcal{A}}$ is also a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform saturated set under φ and $\varphi(\bar{\mathcal{A}}) = \mathcal{A}$. Further as φ is a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform open function, $\varphi(\bar{\mathcal{A}}) = \mathcal{A}$ is a (c, d) $\mathcal{IF} - \mathcal{Q}$ uniform open set in (X_2, τ_{v_2}) . Thus $\varphi(\mathcal{A})$ is

a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed set in (X_2, τ_{v_2}) . Therefore φ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed function.

(ii) $\implies$ (i)

This proof is simple. $\square$

Definition 7.9. Let (X_1, τ_{v_1}) and (X_2, τ_{v_2}) be any two $\mathcal{IF} - \mathcal{Q}$ uniform topological spaces and $\varphi: (X_1, \tau_{v_1}) \rightarrow (X_2, \tau_{v_2})$ be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform surjective and $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform continuous function. Then φ is said to be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform special morphism, if φ satisfies one of the following axioms:

- (i) φ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform open function and each $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform open set in (X_1, τ_{v_1}) is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform saturated under φ .
- (ii) φ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed function and each $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed set in (X_1, τ_{v_1}) is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform saturated under φ .

Proposition 7.3. Let (X_1, τ_{v_1}) and (X_2, τ_{v_2}) be any two $\mathcal{IF} - \mathcal{Q}$ uniform topological space. Let the mapping $\varphi: (X_1, \tau_{v_1}) \rightarrow (X_2, \tau_{v_2})$ be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform special morphism. If (X_1, τ_{v_1}) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform T_0 space, then φ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform homeomorphism.

Proof. Since φ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform special morphism. By Definition 7.12, φ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed and continuous function. It is enough to prove that φ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform injective. Let $\mathcal{A}, \mathcal{B} \in \text{IF}(X_1)$ be such that $\varphi(\mathcal{A}) = \varphi(\mathcal{B})$. Then $(\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_2}}(\varphi(\mathcal{A})) = (\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_2}}(\varphi(\mathcal{B}))$. Since φ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed function.

$$(\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_2}}(\varphi(\mathcal{A})) \subseteq \varphi((\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_1}}(\mathcal{A})) \quad (1)$$

and

$$(\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_2}}(\varphi(\mathcal{B})) \subseteq \varphi((\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_1}}(\mathcal{B})). \quad (2)$$

Also, φ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform continuous function,

$$(\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_2}}(\varphi(\mathcal{A})) \supseteq \varphi((\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_1}}(\mathcal{A})) \quad (3)$$

and

$$(\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_2}}(\varphi(\mathcal{B})) \supseteq \varphi((\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_1}}(\mathcal{B})). \quad (4)$$

From equations (1) and (2). $(\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_2}}(\varphi(\mathcal{A})) \subseteq \varphi((\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_1}}(\mathcal{A}))$ and the equations (3) and (4) $(\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_2}}(\varphi(\mathcal{B})) \subseteq \varphi((\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_1}}(\mathcal{B}))$. Therefore $(\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_2}}(\varphi(\mathcal{A})) = (\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_2}}(\varphi(\mathcal{B}))$ implies that $\varphi((\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_1}}(\mathcal{A})) = \varphi((\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_1}}(\mathcal{B}))$. By Proposition 7.2, it is known that every $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed set in (X_1, τ_{v_1}) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform saturated set under φ . Since φ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform saturated, by the Definition 7.7, $(\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_1}}(\varphi(\mathcal{A})) = (\mathfrak{c}, \mathfrak{d})\text{IFQcl}_{\tau_{v_1}}(\varphi(\mathcal{B}))$. Since (X_1, τ_{v_1}) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform T_0 space and also by $\mathcal{A} = \mathcal{B}$. Thus φ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform injective. Therefore φ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform homeomorphism. $\square$

Definition 7.10. Let (X, τ_v) be a $\mathcal{IF} - \mathcal{Q}$ uniform topological space. Then is said to be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral space, if it satisfies the following axioms:

- (i) (X, τ_v) be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact space and that has a base which contains a collection of $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact open sets.
- (ii) If $\mathcal{A}, \mathcal{B} \in \mathfrak{S}$, then $\mathcal{A} \cap \mathcal{B} \in \mathfrak{S}$
- (iii) (X, τ_v) be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform sober space.

Proposition 7.4. Let (X_1, τ_{v_1}) and (X_2, τ_{v_2}) be any two $\mathcal{IF} - \mathcal{Q}$ uniform topological spaces. Let $\varphi: (X_1, \tau_{v_1}) \rightarrow (X_2, \tau_{v_2})$ be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-homeomorphisms and (X_2, τ_{v_2}) be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral. Then the following statements hold:

- (i) (X_1, τ_{v_1}) be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact and has a base of $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact open sets.
- (ii) If $\mathcal{B}, \mathcal{C} \in \text{IF}(X_1)$ are any two $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact open sets in (X_1, τ_{v_1}) , then $\mathcal{B} \cap \mathcal{C}$ is also $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact in (X_1, τ_{v_1}) .

Proof. Let $\varphi: (X_1, \tau_{v_1}) \rightarrow (X_2, \tau_{v_2})$ be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-homeomorphism and let (X_2, τ_{v_2}) be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral space.

(i) Given that (X_2, τ_{v_2}) is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral space. The collection $\mathfrak{S}$ contains $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact sets. Since φ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-homeomorphism, there exists a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform open sets $\mathcal{B}_i \in \text{IF}(X_1)$ where $i \in J$ such that $\mathcal{B}_i = \varphi^{-1}(\mathcal{C}_i)$, $i \in J$. Let $\mathcal{B} \in \text{IF}(X_1)$ be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform open set in (X_1, τ_{v_1}) . Since φ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-homeomorphism, then there exists a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform open set $\mathcal{C} \in \text{IF}(X_2)$ in $\text{IF}(X_2)$ in (X_2, τ_{v_2}) such that $\mathcal{B} = \varphi^{-1}(\mathcal{C})$. Since $\mathcal{C}_i$ $i \in J$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact in (X_2, τ_{v_2}) and by the Proposition 7.1, $\varphi^{-1}(\mathcal{C}_i)$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact in (X_1, τ_{v_1}) . Thus $\varphi^{-1}(\mathcal{C}_i) = \mathcal{B}_i$, $i \in J$ is also $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact. Therefore there exists a finite subset J_0 of J such that $\mathcal{C} \subseteq \bigcup_{i \in J_0} \mathcal{C}_i$, hence $\mathcal{B} = \varphi^{-1}(\mathcal{C}) \subseteq \varphi^{-1}(\bigcup_{i \in J_0} \mathcal{C}_i) \subseteq \bigcup_{i \in J_0} \varphi^{-1}(\mathcal{C}_i)$. Hence $\mathcal{B} \subseteq \bigcup_{i \in J_0} \mathcal{B}_i$. Thus (X_1, τ_{v_1}) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact. Therefore the collection $\mathfrak{S}$ is a base for (X_1, τ_{v_1}) which is the collection of $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact open sets in (X_1, τ_{v_1}) .

(ii): Let $\mathcal{B}, \mathcal{C} \in \text{IF}(X_1)$ be any two $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact open sets in (X_1, τ_{v_1}) . Since φ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-homeomorphism, there exists two $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact open sets $\mathcal{C}_1, \mathcal{B}_1 \in \text{IF}(X_2)$ such that $\mathcal{B} = \varphi^{-1}(\mathcal{B}_1)$ and $\mathcal{C} = \varphi^{-1}(\mathcal{C}_1)$. Since (X_2, τ_{v_2}) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral, $\mathcal{B}_1 \cap \mathcal{C}_1$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact in (X_2, τ_{v_2}) . Also, $\mathcal{B} \cap \mathcal{C} = \varphi^{-1}(\mathcal{B}_1) \cap \varphi^{-1}(\mathcal{C}_1) = \varphi^{-1}(\mathcal{B}_1 \cap \mathcal{C}_1)$. Since $\mathcal{B}_1 \cap \mathcal{C}_1$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact in (X_2, τ_{v_2}) and by Proposition 7.1, $\varphi^{-1}(\mathcal{B}_1 \cap \mathcal{C}_1)$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact in (X_1, τ_{v_1}) . Therefore, $\mathcal{B} \cap \mathcal{C}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact in (X_1, τ_{v_1}) . $\square$

Proposition 7.5. Let (X_1, τ_{v_1}) and (X_2, τ_{v_2}) be any two $\mathcal{IF} - \mathcal{Q}$ uniform topological spaces. Let $\varphi: (X_1, \tau_{v_1}) \rightarrow (X_2, \tau_{v_2})$ be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-homeomorphism. If (X_1, τ_{v_1}) is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral, then (X_2, τ_{v_2}) $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral.

Proof. It known that (X_1, τ_{v_1}) is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral. Let the collection $\mathfrak{S}$ has $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact open sets in (X_1, τ_{v_1}) . Since φ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-homeomorphism, there exists a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform open set $\mathcal{C}_i \in \text{IF}(X_2)$ where $i \in J$, such that $\mathcal{B}_i = \varphi^{-1}(\mathcal{C}_i)$, where each $\mathcal{B}_i \in \text{IF}(X_1)$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact open sets in (X_1, τ_{v_1}) . Since $\mathcal{C}_i$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact and $\mathcal{B}_i = \varphi^{-1}(\mathcal{C}_i)$. $\varphi^{-1}(\mathcal{C}_i)$ is also $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact in (X_1, τ_{v_1}) . By Proposition 7.1, $\mathcal{C}_i$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact set in (X_2, τ_{v_2}) . Let $\mathcal{C} \in \text{IF}(X_2)$ be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform open set in (X_2, τ_{v_2}) . Since φ is also a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform continuous function,

$\varphi^{-1}(\mathcal{C})$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform open set in (X_1, τ_{v_1}) , there exists a finite subset J_0 of J such that $\varphi^{-1}(\mathcal{C}) \subseteq \cup_{i \in J_0} \mathcal{B}_i \subseteq \cup_{i \in J_0} \varphi^{-1}(\mathcal{C}_i) \subseteq \varphi^{-1}(\cup_{i \in J_0} \mathcal{C}_i)$. Since φ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-homeomorphism and by Remark 6.2, $\mathcal{C} \subseteq \cup_{i \in J_0} \mathcal{C}_i$. Then the collection $\mathfrak{S}$ has a base contains the collections of $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform compact open sets in (X_2, τ_{v_2}) . Let $\mathcal{B}, \mathcal{C} \in \text{IF}(X_2)$ be any two $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact open sets in (X_2, τ_{v_2}) . By Proposition 7.1, $\varphi^{-1}(\mathcal{C})$ and $\varphi^{-1}(\mathcal{B})$ are two $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact open sets of (X_1, τ_{v_1}) . Also $\varphi^{-1}(\mathcal{B}) \cap \varphi^{-1}(\mathcal{C}) = \varphi^{-1}(\mathcal{C} \cap \mathcal{B})$. Since (X_1, τ_{v_1}) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral, $\varphi^{-1}(\mathcal{C} \cap \mathcal{B})$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact in (X_1, τ_{v_1}) . Hence by Proposition 7.7, $\mathcal{B} \cap \mathcal{C}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact set in (X_2, τ_{v_2}) . Let $\delta \in \text{IF}(X_2)$ be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X_2, τ_{v_2}) . Then by Proposition 6.9, $\varphi^{-1}(\delta)$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X_1, τ_{v_1}) . Since (X_1, τ_{v_1}) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral space, (X_1, τ_{v_1}) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform sober space. Since $\varphi^{-1}(\delta)$ is (X_1, τ_{v_1}) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X_1, τ_{v_1}) , there exists an $\mathcal{IF} - \mathcal{Q}$ uniform generic set $\mathcal{H} \in \text{IF}_G(X_1)$ of $\varphi^{-1}(\delta)$ such that $\mathcal{H} \subseteq \varphi^{-1}(\delta)$. Since $\mathcal{H}$ is an $\mathcal{IF} - \mathcal{Q}$ uniform generic set of $\varphi^{-1}(\delta)$, $\varphi^{-1}(\delta) = (\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_1}}(\mathcal{H})$. Such that $\delta \subseteq (\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_1}}(\mathcal{H})$ and $(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_1}}(\mathcal{H}) \subseteq \delta$. This proves that $(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_1}}(\mathcal{H}) = \delta$. Therefore for any $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set $\delta \in \text{IF}(X_2)$, there exists an $\mathcal{IF} - \mathcal{Q}$ uniform generic set $\varphi(\mathcal{H})$ of $\text{IF}(X_2)$ of δ such that $\varphi(\mathcal{H}) \subseteq \delta$. Also since $\varphi(\mathcal{H})$ is a $\mathcal{IF} - \mathcal{Q}$ uniform generic set of δ , such that $(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_1}}(\mathcal{H}) = \delta$. Hence (X_2, τ_{v_2}) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform sober space. Therefore (X_2, τ_{v_2}) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral space. $\square$

Proposition 7.6. Let (X_1, τ_{v_1}) and (X_2, τ_{v_2}) be any two $\mathcal{IF} - \mathcal{Q}$ uniform topological spaces. Let $\varphi: (X_1, \tau_{v_1}) \rightarrow (X_2, \tau_{v_2})$ be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-homeomorphism. If (X_1, τ_{v_1}) is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral and (X_2, τ_{v_2}) $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform spectral, then φ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform surjective function.

Proof. Given that (X_1, τ_{v_1}) is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral and $\mathcal{B} \in \text{IF}(X_2)$ be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X_2, τ_{v_2}) . Then by the Corollary 6.1, $(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\mathcal{B})$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X_2, τ_{v_2}) . Since $(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\mathcal{B})$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set and by Proposition 6.1, $(\varphi^{-1}(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\mathcal{B}))$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X_1, τ_{v_1}) . Since (X_1, τ_{v_1}) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform spectral space, also it is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform sober space. Then there exists an $\mathcal{IF} - \mathcal{Q}$ uniform generic set $\mathcal{H} \in \text{IF}_G(X_1)$ of $\varphi^{-1}((\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\mathcal{B}))$ such that $\mathcal{H} \subseteq \varphi^{-1}((\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\mathcal{B}))$. Thus $\varphi(\mathcal{H}) \subseteq \varphi(\varphi^{-1}(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\mathcal{B})) \subseteq (\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\mathcal{B})$. Also that,

$$\varphi^{-1}((\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\mathcal{B})) \supseteq (\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\varphi(\mathcal{H})) \quad (5)$$

Since (X_1, τ_{v_1}) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform sober space and by Proposition 6.1 there exists an $\mathcal{IF} - \mathcal{Q}$ uniform generic set $\mathcal{H} \in \text{IF}_G(X_1)$ of $(\varphi^{-1}(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\mathcal{B}))$ such that $(\varphi^{-1}(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\mathcal{B})) \subseteq (\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\mathcal{H})$. Thus $(\varphi^{-1}(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\mathcal{B})) \subseteq (\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\mathcal{H}) \subseteq \varphi^{-1}(\varphi(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\mathcal{H}))$. Thus

$$(\varphi^{-1}(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\mathcal{B})) \subseteq (\varphi^{-1}(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}})(\varphi(\mathcal{H})) \quad (6)$$

From the equations (5) and (6) $(\varphi^{-1}(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\mathcal{B})) = (\varphi^{-1}(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}})(\varphi(\mathcal{H}))$. Since φ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-homeomorphism. $(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\mathcal{B})) = (\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\varphi(\mathcal{H}))$. Since (X_2, τ_{v_2}) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral space,

it is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform T_0 space and so $\varphi(\mathcal{H}) = \mathcal{B}$. Thus φ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform surjective. $\square$

Proposition 7.7. Let (X_1, τ_{v_1}) and (X_2, τ_{v_2}) be any two $\mathcal{IF} - \mathcal{Q}$ uniform topological spaces. Let $\varphi: (X_1, \tau_{v_1}) \rightarrow (X_2, \tau_{v_2})$ be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed function and $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-homeomorphism. If (X_2, τ_{v_2}) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform spectral and φ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform surjective, then (X_1, τ_{v_1}) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform sober space.

Proof. Let $\mathcal{B} \in \text{IF}(X_1)$ be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X_1, τ_{v_1}) . Since φ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-homeomorphism, for each $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed set $\mathcal{B} \in \text{IF}(X_1)$ in (X_1, τ_{v_1}) , there exists a unique $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed set $\mathcal{A} \in \text{IF}(X_2)$ in (X_2, τ_{v_2}) , such that $\varphi^{-1}(\mathcal{A}) = \mathcal{B}$. Since $\varphi^{-1}(\mathcal{A}) = \mathcal{B}$, where $\mathcal{B}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X_1, τ_{v_1}) . So $\varphi^{-1}(\mathcal{A})$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X_1, τ_{v_1}) . By proposition 6.1, $\mathcal{A} \in \text{IF}(X_2)$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X_2, τ_{v_2}) . Since (X_2, τ_{v_2}) is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform spectral space, then it is well known that it is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform sober space. Since $\mathcal{A}$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X_2, τ_{v_2}) , there exists a intuitionistic fuzzy generic set $\mathcal{H} \in \text{IF}_G(X_2)$ of $\mathcal{A}$ such that $\mathcal{A} = (\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\mathcal{H})$. Since φ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform surjective, for every $\mathcal{H} \in \text{IF}(X_2)$, there exists a $\mathcal{G} \in \text{IF}_G(X_1)$ such that $\varphi(\mathcal{G}) = \mathcal{H}$.

$\mathcal{B} = \varphi^{-1}(\mathcal{A}) = \varphi^{-1}((\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\mathcal{H})) = \varphi^{-1}((\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_2}}(\varphi(\mathcal{G}))) = \varphi^{-1}(\varphi((\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_1}}(\mathcal{G})))$. $\mathcal{B} = (\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_1}}(\mathcal{G})$. Therefore there exists a intuitionistic fuzzy generic set $\mathcal{G}$ of a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set $\mathcal{B}$ such that $\mathcal{B} = (\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_{v_1}}(\mathcal{G})$. Hence proves that (X_1, τ_{v_1}) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform sober space. $\square$

8. $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ UNIFORM SEMI-SPECTRAL SPACE

This section defines a new idea called $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform semi-spectral space. From this space, a new concepts called $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform Γ_q -compact open sets, $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform Γ_q -compact closed sets, $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform Γ_q -compact clopen sets, $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform ξ -sets are defined and some inter-relations are investigated.

Proposition 8.1. Let (X, τ_v) be a $\mathcal{IF} - \mathcal{Q}$ uniform topological space and $Y \subseteq X$. Let (Y, τ_{v_Y}) be a $\mathcal{IF} - \mathcal{Q}$ uniform topological subspace of (X, τ_v) . If $\mathcal{C} \in \text{IF}(Y)$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (Y, τ_{v_Y}) , then $(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau}(\mathcal{C})$ is also a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X, τ_v) .

Proof. Let (Y, τ_{v_Y}) be a $\mathcal{IF} - \mathcal{Q}$ uniform topological subspace of (X, τ_v) . Since $\mathcal{C} \in \text{IF}(Y)$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (Y, τ_{v_Y}) , for all $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed sets $\delta, \gamma \in \text{IF}(Y)$ in (Y, τ_{v_Y}) such that $\mathcal{C} \subseteq (\delta \cup \gamma)$, then either $\mathcal{C} \subseteq \gamma$ or $\mathcal{C} \subseteq \delta$. Thus $\mathcal{C} \subseteq (\delta \cup \gamma)$ implies that $(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_v}(\mathcal{C}) \subseteq (\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_v}(\gamma \cup \delta)$ and so $(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_v}(\mathcal{C}) \subseteq (\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_v}(\gamma) \subseteq (\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_v}(\delta)$. Thus either $(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_v}(\mathcal{C}) \subseteq (\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_v}(\gamma)$ or $(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_v}(\mathcal{C}) \subseteq (\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_v}(\delta)$. Hence it proves that $(\mathfrak{c}, \mathfrak{d}) \text{IFQcl}_{\tau_v}(\mathcal{C})$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set in (X, τ_v) . $\square$

Definition 8.1. A pair (X, τ_v) be $\mathcal{IF} - \mathcal{Q}$ uniform topological space is said to be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform semi-spectral space, if $\mathcal{A}_1, \mathcal{A}_2 \in \text{IF}(X)$ are any two $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact open sets in (X, τ_v) , then $\mathcal{A}_1 \cap \mathcal{A}_2$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact in (X, τ_v) .

Definition 8.2. A pair (X, τ_v) be a $\mathcal{IF} - \mathcal{Q}$ uniform topological space and $\delta \in \text{IF}(X)$. Then

- (i) δ is said to be $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform Γ_q -compact open, if for each $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact open set $\mu \in \text{IF}(X)$, $\delta \cap \mu$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact.
- (ii) δ is said to be $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform Γ_q -compact closed, if for each $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact closed set $\mu \in \text{IF}(X)$, then $\delta \cap \mu$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact.
- (iii) δ is said to be $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform Γ_q -compact clopen, if it is both $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact open $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact closed.
- (iv) δ is said to be $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform co-compact if $\bar{\delta}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform compact.

Definition 8.3. Let (X, τ_v) be $\mathcal{IF} - \mathcal{Q}$ uniform topological space. $\mathcal{A} \in \text{IF}(X)$ is said to be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform ξ -set, if $\mathcal{A}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed, $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact and $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform Γ_q -compact open in (X, τ_v) .

Proposition 8.2. Let (X, τ_v) be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform semi-spectral space with a base of $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ q -compact open sets and $\mathcal{A} \in \text{IF}(X)$. Then the following conditions are equivalent:

- (i) $\mathcal{A}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact and $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed.
- (ii) $\mathcal{A}$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform ξ -set in (X, τ_v) ;
- (iii) $\mathcal{A}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed and there exists two $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact open sets $\alpha, \bar{\beta} \in \text{IF}(X)$ such that $\mathcal{A} = \alpha \cap \bar{\beta}$.

Proof. (i) $\implies$ (ii)

Let $\mathcal{A} \in \text{IF}(X)$ be $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact. Since (X, τ_v) has a base of $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact open sets, $\mathcal{B} \in \text{IF}(X)$ is also $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact open. Since $\mathcal{A}$ and $\mathcal{B}$ are $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact and (X, τ_v) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact semi-spectral space, $\mathcal{A} \cap \mathcal{B}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact. Since $\mathcal{B}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact open and $\mathcal{A} \cap \mathcal{B}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact, by Definition 8.2, $\mathcal{A}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform Γ_q -compact open set. Therefore $\mathcal{A}$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform ξ set in (X, τ_v) .

(ii) $\implies$ (iii)

Since $\mathcal{A}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform ξ set, $\mathcal{A}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed and $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ co- Γ_q -compact open. Thus $\bar{\mathcal{A}}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform Γ_q -compact open. It known that in (X, τ_v) , there exists a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform compact open set $\alpha \in \text{IF}(X)$. Thus $\alpha \cap \bar{\mathcal{A}} = \beta$. Since $\bar{\mathcal{A}}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform Γ_q -compact open and by Definition 8.2, β is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact. Therefore $\alpha \cap \bar{\mathcal{A}} = \beta$ which implies that $\alpha - \mathcal{A} = \beta$ and so $\alpha - \beta = \mathcal{A}$. Hence $\alpha \cap \bar{\beta} = \mathcal{A}$. Hence $\alpha \cap \bar{\beta} = \mathcal{A}$.

(iii) $\implies$ (i)

Suppose $\mathcal{A}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed and there exists two $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact open sets $\alpha, \bar{\beta} \in \text{IF}(X)$ such that $\mathcal{A} = \alpha \cap \bar{\beta}$. Since (X, τ_v) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform semi-spectral and $\alpha, \bar{\beta}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact, $\mathcal{A}$ is also $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact. Hence $\mathcal{A}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform q -compact and $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform closed in (X, τ_v) . $\square$

9. CONCLUSION

In this article, $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform spectral space is introduced. $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform sober space and significant results are discussed. An $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform spectral space can redefines a new version it can be specified as $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform quasi-spectral space and $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform semi-spectral space are also discussed. This study pave the

way to find some interesting extensions in Gold-spectral space and Jac-spectral topological space.

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