

LOCATION ECCENTRIC DOMINATING SET IN SIMPLE GRAPHS

R. JAHIR HUSSAIN¹, A. FATHIMA BEGAM^{1*}, §

ABSTRACT. In this paper we define location eccentric dominating set, location eccentric domination number. The location eccentric dominating set with minimum cardinality is defined as referencing eccentric dominating set that is RED-set and its cardinality is known as location eccentric domination number $RED(G)$. Also, we have found location eccentric domination number $RED(G)$ of simple complete graphs, bipartite graphs, star graphs and also for the path.

Keywords: location eccentric dominating set, location eccentric domination number, referencing eccentric dominating set.

AMS Subject Classification: 05C12, 05C69.

1. INTRODUCTION

P. J. Slater[10] in 1975 proposed the idea of locating set that can uniquely identify all other vertices. In 1987, P. J. Slater [11, 12] extended his idea by including the concept of dominating set with locating features. Other studies that deal with the concepts of locating set and locating dominating set are located in [1, 2, 8, 9]. The detailed concepts of distance and its results are studied in [3]. In 2010, T. N. Jankiraman, M. Bhanumathi and S. Muthammai [7] introduced the concept of eccentric dominating set and eccentric domination number in graphs. The eccentric dominating set exerts control over the graph not only through its adjacent vertices but also by containing vertices at greater distances. For further results on eccentric domination number we refer [4, 5, 6]. In this paper we introduced the concept of location eccentric dominating set which are useful in safeguard application for facilities, such as identifying the location of an intruder or security breach by analyzing which sensor have been triggered and also useful to protect confidential and important objects from accidental place by replacing them to the location which is farthest from unsafe or affected zone.

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Definition 1.1. A set $D \subseteq V(G)$ is an eccentric dominating set if it serves as a dominating set for G and for every vertex v in $V - D$, there is at least one eccentric point of v in D . The minimum cardinality of an eccentric dominating set of G is known as eccentric domination number of G , denoted by $\gamma_{ed}(G)$.

Definition 1.2. A set $D \subseteq V(G)$ is a locating dominating set if D is a dominating set of G and also $N_G(u) \cap D \neq N_G(v) \cap D$ for every u, v in $V - D$ and it is denoted by LD-set. It is strictly locating dominating set if $N_G(u) \cap D \neq D$ for every u in $V - D$. The minimum cardinality of an LD-set is referred as the location domination number of G and, denoted by $RD(G)$. An LD-set containing $RD(G)$ elements is known as a referencing dominating set that is RD-set.

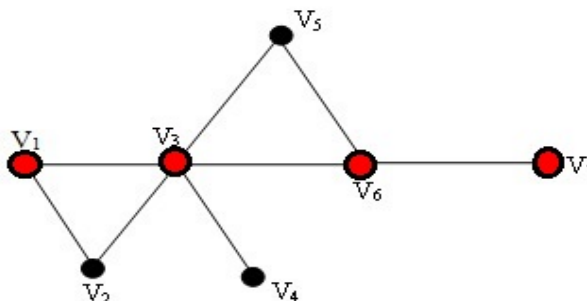
2. LOCATION ECCENTRIC DOMINATING SETS

Definition 2.1. An eccentric dominating set $L \subseteq V(G)$ is defined as a location eccentric dominating set if $N_G(x) \cap L \neq N_G(y) \cap L \forall x, y \in V(G) - L$. It is denoted by LED-set.

Definition 2.2. The minimum cardinality of a LED-set of a graph G is referred as location eccentric domination number and, denoted by $RED(G)$. The LED-set with $RED(G)$ elements is referred as referencing eccentric dominating set that is RED-set.

Example 2.1.

FIGURE 1. Graph G



$L_1 = \{V_1, V_3, V_7\}$ is an eccentric dominating set.

Since, $N_G(V_4) \cap L_1 = N_G(V_5) \cap L_1 = \{V_3\}$

So L_1 is not a location eccentric dominating set.

Let $L_2 = \{V_1, V_3, V_6, V_7\}$

Since, $N_G(V_2) \cap L_2 = \{V_1, V_3\}$, $N_G(V_4) \cap L_2 = \{V_3\}$ and

$N_G(V_5) \cap L_2 = \{V_3, V_6\}$

So, L_2 is a referencing eccentric dominating set.

Therefore, $RED(G) = 4$.

Observations 2.1.

2.1.1 For any Graph G with $|V(G)| = n, 1 \leq RED(G) \leq n$.

2.1.2 $RED(G) = |V(G)|$ iff $G = \overline{K_n}$.

2.1.3 If $RED(G) = 1$ then either $G = K_1$ or $G = K_2 = P_2$.

2.1.4 For any connected graph G with order $p \geq 1$, $1 \leq RED(G) \leq p-1$.

2.1.5 For any connected graph G , $RD(G) \leq RED(G)$.

Theorem 2.1. (a) $RED(P_2) = 1$

(b) For $|V(G)| = n \geq 3$ and $n \equiv r \pmod{5}$

$$RED(P_n) = 2 \left\lfloor \frac{n}{5} \right\rfloor + 1 \text{ when } r = 0, 1$$

$$RED(P_n) = 2 \left\lceil \frac{n}{5} \right\rceil \text{ when } r = 2, 3, 4$$

Proof.

(a) In P_2 , Let $D = \{v_1\}$. Then $\{v_1\}$ is a unique neighborhood set of v_2 in D and also v_1 is an eccentric vertex of $v_2 \in V - D$.

$\therefore RED(P_2) = 1$.

(b) In P_n , for $n \geq 3$ let $|V(G)| = n$. Every positive integer n is one of the form $5r, 5r+1, 5r+2, 5r+3, 5r+4$ where $r \geq 0$.

In P_n , all non-end vertices have either one or both of the two end vertices as its eccentric vertex. So minimum LED-set D of P_n must contains it two end vertices. Let v_1 is one end vertex so add v_1 in D which dominates v_2 , then add v_4 in D which dominates v_3 and v_5 . In order to get unique neighborhood set of v_5 in D , we must include v_6 in D which also dominate v_7 . Then add v_9 in D which dominates v_8 and v_{10} . Then add v_{11} in D to get unique neighborhood set for v_{10} . Continue this process until we include the vertex v_{5r-1} vertex in D .

Therefore, $D = \{v_1, v_4, v_6, v_9, \dots, v_{5r-4}, v_{5r-1}\}$.

Generally, starting from v_1 in first five consecutive vertices we choose v_1 and v_4 to D and from next five consecutive vertices we choose v_6 and v_9 to D and from r th five consecutive vertices we choose v_{5r-4} and v_{5r-1} to D . That is, we add two vertices from every five consecutive vertices of $V(G)$ starting from v_1 . That is D contains first and fourth vertex from every five consecutive vertices starting from v_1 . Therefore, $|D| = 2r$.

Case(i): $n = 5r$, here $r > 0$

Let $V(P_n) = \{v_1, v_2, v_3, \dots, v_{5r-4}, v_{5r-3}, v_{5r-2}, v_{5r-1}, v_{5r}\}$ and we know that $D = \{v_1, v_4, v_6, v_9, \dots, v_{5r-4}, v_{5r-1}\}$. Since v_{5r} is the end vertex of P_n , we must include it in D .

$\therefore, D = \{v_1, v_4, v_6, v_9, \dots, v_{5r-4}, v_{5r-1}, v_{5r}\}$

$$\Rightarrow |D| = 2r + 1$$

$$\Rightarrow = 2 \left\lceil \frac{n}{5} \right\rceil + 1 \quad \{ \text{since } n = 5r \Rightarrow \frac{n}{5} = r \}$$

$$\text{Therefore, } RED(P_n) = 2 \left\lceil \frac{n}{5} \right\rceil + 1 \text{ when } n \geq 3 \text{ and } n = 5r. \quad (1)$$

Case(ii): $n = 5r + 1$, here $r > 0$

So, $V(P_n) = \{v_1, v_2, v_3, \dots, v_{5r-4}, v_{5r-3}, v_{5r-2}, v_{5r-1}, v_{5r}, v_{5r+1}\}$ with $D = \{v_1, v_4, v_6, v_9, \dots, v_{5r-4}, v_{5r-1}\}$. Here we have completely r number of disjoint sets of five consecutive numbers set from v_1 to v_{5r} . And we left with only one end vertex v_{5r+1} which is end vertex of P_n . So add v_{5r+1} in D .

$$\begin{aligned}
\text{Therefore, } |D| &= |\{v_1, v_4, v_6, v_9, \dots, v_{5r-4}, v_{5r-1}, v_{5r+1}\}| \\
&= 2r + 1 \quad \left\{ \text{since } n = 5r + 1 \Rightarrow \frac{n-1}{5} = r \Rightarrow \left\lfloor \frac{n}{5} \right\rfloor = r \right\} \\
&= 2 \left\lfloor \frac{n}{5} \right\rfloor + 1 \\
\therefore RED(P_n) &= 2 \left\lfloor \frac{n}{5} \right\rfloor + 1 \quad \text{when } n \geq 3 \text{ and } n = 5r + 1.
\end{aligned} \tag{2}$$

From (1) and (2)

$$\therefore RED(P_n) = 2 \left\lfloor \frac{n}{5} \right\rfloor + 1 \quad \text{when } n \geq 3 \text{ and } n = 5r, 5r + 1. \tag{3}$$

Case(iii): $n = 5r + 2, r > 0$

$V(P_n) = \{v_1, v_2, v_3, \dots, v_{5r-4}, v_{5r-3}, v_{5r-2}, v_{5r-1}, v_{5r}, v_{5r+1}, v_{5r+2}\}$
The set $D = \{v_1, v_4, v_6, v_9, \dots, v_{5r-4}, v_{5r-1}\}$ where v_{5r-1} dominates both v_{5r-2} and v_{5r} . So, we must include v_{5r+1} in D to maintain the uniqueness of neighborhood set of vertices which is in $V-D$. Now we left only with v_{5r+2} which is an end vertex of P_n , so it must be included in D.

$$\begin{aligned}
\text{Therefore, } |D| &= |\{v_1, v_4, v_6, v_9, \dots, v_{5r-4}, v_{5r-1}, v_{5r+1}, v_{5r+2}\}| \\
&= 2r + 2 \\
&= 2(r + 1) \\
&= 2 \left\lceil \frac{n}{5} \right\rceil \quad \left\{ \text{since } \frac{n}{5} = \frac{5r+2}{5} = r + \frac{2}{5} < r + 1 \Rightarrow \left\lceil \frac{n}{5} \right\rceil = r + 1 \right\} \\
\therefore RED(P_n) &= 2 \left\lceil \frac{n}{5} \right\rceil \quad \text{when } n \geq 3 \text{ and } n = 5r + 2.
\end{aligned} \tag{4}$$

Case(iv): $n = 5r + 3, r \geq 0$

$V(P_n) = \{v_1, v_2, v_3, \dots, v_{5r-4}, v_{5r-3}, v_{5r-2}, v_{5r-1}, v_{5r}, v_{5r+1}, v_{5r+2}, v_{5r+3}\}$
The set $D = \{v_1, v_4, v_6, v_9, \dots, v_{5r-4}, v_{5r-1}\}$. We must include v_{5r+1} and v_{5r+3} in D to get a RED-set of P_n .

$$\begin{aligned}
\text{Therefore, } |D| &= |\{v_1, v_4, v_6, v_9, \dots, v_{5r-4}, v_{5r-1}, v_{5r+1}, v_{5r+3}\}| \\
&= 2r + 2 \\
&= 2(r + 1) \\
&= 2 \left\lceil \frac{n}{5} \right\rceil \quad \left\{ \text{since } \frac{n}{5} = \frac{5r+3}{5} = r + \frac{3}{5} < r + 1 \Rightarrow \left\lceil \frac{n}{5} \right\rceil = r + 1 \right\} \\
\therefore RED(P_n) &= 2 \left\lceil \frac{n}{5} \right\rceil \quad \text{when } n \geq 3 \text{ and } n = 5r + 3.
\end{aligned} \tag{5}$$

Case(v): $n = 5r + 4, r \geq 0$

$V(P_n) = \{v_1, v_2, v_3, \dots, v_{5r-4}, v_{5r-3}, v_{5r-2}, v_{5r-1}, v_{5r}, v_{5r+1}, v_{5r+2}, v_{5r+3}, v_{5r+4}\}$
The set $D = \{v_1, v_4, v_6, v_9, \dots, v_{5r-4}, v_{5r-1}\}$. We must include v_{5r+1} and v_{5r+4} in D to form a RED-set of P_n .

$$\begin{aligned}
\text{Therefore, } |D| &= |\{v_1, v_4, v_6, v_9, \dots, v_{5r-4}, v_{5r-1}, v_{5r+1}, v_{5r+4}\}| \\
&= 2r + 2 \\
&= 2(r + 1) \\
&= 2 \left\lceil \frac{n}{5} \right\rceil \quad \left\{ \text{since } \frac{n}{5} = \frac{5r+4}{5} = r + \frac{4}{5} < r + 1 \Rightarrow \left\lceil \frac{n}{5} \right\rceil = r + 1 \right\}
\end{aligned}$$

$$\therefore RED(P_n) = 2 \left\lceil \frac{n}{5} \right\rceil \text{ when } n \geq 3 \text{ and } n = 5r + 4. \quad (6)$$

From (4), (5), and (6)

$$RED(P_n) = 2 \left\lceil \frac{n}{5} \right\rceil \text{ when } n \geq 3 \text{ and } n = 5r + 2, 5r + 3, 5r + 4. \quad (7)$$

From (3) and (7)

For $|V(G)| = n \geq 3$ and $n \equiv r \pmod{5}$

$$RED(P_n) = 2 \left\lfloor \frac{n}{5} \right\rfloor + 1 \text{ when } r = 0, 1$$

$$RED(P_n) = 2 \left\lceil \frac{n}{5} \right\rceil \text{ when } r = 2, 3, 4$$

□

Theorem 2.2. For $n \geq 2$, $RED(K_n) = n-1$.

Proof. For K_2 , $V(K_2) = \{v_1, v_2\}$. Let $D = \{v_1\}$. Then D is a RED-set in K_2 .

$\therefore RED(K_2) = 1$.

Suppose, $RED(K_n) = n-2$. Therefore, the minimum location eccentric dominating set D contains $n-2$ vertices of K_n . Then $V-D$ contains exactly two vertices of K_n say x, y in $V-D$ where $N(x) \cap D = N(y) \cap D = D$. This implies that D cannot be a LED-set. So, we have to include either x or y in D . Now $|D| = n-1$, and D is a RED-set.

$\therefore RED(K_n) = n-1$. □

Theorem 2.3. For $n, m \geq 2$, $RED(K_{m,n}) = m + n - 2$.

Proof. For $K_{2,2}$, Let $V(K_{2,2}) = V_1 \cup U_1$, where $V_1 = \{v_1, v_2\}$ and $U_1 = \{u_1, u_2\}$. Let $D = \{v_1, u_1\}$. Then D is a referencing eccentric dominating set.

Therefore, $RED(K_{2,2}) = 2$.

Let $m > 2, n > 2$. Suppose $RED(K_{m,n}) = m + n - 3$. Then referencing eccentric dominating set D contains $m + n - 3$ vertices of $K_{m,n}$. Then $|V-D| = 3$. Without affecting generality, we let $V-D = \{v_{m-1}, v_m, u_n\}$. Then $N(v_{m-1}) \cap D = N(v_m) \cap D = U_1 \cap D = \{u_1, u_2, u_3, \dots, u_{n-1}\}$. Therefore, D is not RED-set. So, we have to add either v_{m-1} or v_m to D . Therefore, $|D| = m + n - 2$.

Hence, $RED(K_{m,n}) = m + n - 2$, for $n, m \geq 2$. □

Theorem 2.4. $RED(K_{1,n}) = n$ for $n > 1$.

Proof. Let u be the central vertex in $K_{1,n}$. Every non-central vertex has only this central vertex as its neighbor. So, the RED-set must contain n vertices of $K_{1,n}$.

Therefore, $RED(K_{1,n}) = n$ for $n > 1$. □

3. CONCLUSIONS

In this article locating set and eccentric domination are merged and defined as location eccentric domination number and denoted as $RED(G)$ which is useful to identify the location at where the security is breached and also useful to maintain the security of confidential matters by distancing them from that location. Also the location eccentric domination number $RED(G)$ for path graph, complete graph, complete bipartite graph and star graph is determined.

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