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ON MONOPHONIC ECCENTRIC SEQUENCES OF GRAPHS

DIYA JAJI[1], TABITHA AGNES MANGAM[1*], H. R. DEEPIKA[1], §

Abstract. For a connected graph G and a vertex $v \in V(G)$, the eccentricity $e(v)$ is the maximum distance from v to any other vertex in G. A path P in G is called a monophonic path if it is chordless. The monophonic distance $d_m(u, v)$ between two vertices u and v is the length of a longest chordless u–v path in G. The monophonic eccentricity of a vertex v, denoted by $e_m(v)$, is defined as the maximum monophonic distance between v and any other vertex of G. In this paper, we investigate monophonic eccentric sequences for several classes of graphs. In particular, we determine these sequences for unicyclic graphs and the Petersen graph, and obtain characterizations related to monophonic eccentricities in these graph classes.

Keywords: diameter, eccentricity, eccentric sequence, monophonic distance, monophonic eccentric sequence.

AMS Subject Classification: 05C12, 05C38.

1. Introduction

The study of distance in graphs is a prominent area of research, with several definitions of distance introduced over time. Several parameters related to distance in graphs, such as eccentricity, radius, and centre, have been studied extensively. The eccentricity of the vertex v in a graph G, denoted by $e(v)$, is defined as the distance between v and a farthest vertex from it in G. The eccentric sequence of graph G, which is a nondecreasing sequence of the eccentricities of G, was first introduced by Lesniak [1] in 1975. Several properties of eccentric sequences were investigated. The monophonic distance and monophonic eccentricity were introduced in 2011, see [5]. The authors studied the realisability of graphs with certain values of monophonic radius and monophonic diameter.

In this paper, we consider simple and undirected graphs. Let G be a connected graph with vertex set $V(G)$ and edge set $E(G)$. If $u, v \in V(G)$, the distance between u and v, denoted by $d(u, v)$, is the length of a shortest path between them. For a vertex $v \in V(G)$,

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the eccentricity of v, denoted by $e(v)$, is defined as the distance between v and a vertex farthest from it in G,

$$e(v) = max\{d(v,u) : u \in V(G)\}.$$

A path P has a chord if there exists an edge joining two non-adjacent vertices in P. A path P in a connected graph G is said to be a monophonic path if it does not have a chord in it. For any two vertices $u, v \in V(G)$, the monophonic distance $d_m(u,v)$ from u to v is the length of a longest $u-v$ monophonic path in G. The maximum monophonic distance between v and all other vertices is called the monophonic eccentricity of v and is denoted by $e_m(v)$.

The monophonic convexity is introduced and the relation between monophonic paths and convex sets in graphs is analysed [4]. These ideas were extended to define the detour monophonic number of a graph. The bounds and characterizations of graphs were established in [7]. Following this, the upper detour monophonic number and connected detour monophonic number are studied in [8] and [9] respectively. Later, several types of monophonic distance were studied in [10, 11].

Identifying monophonic paths in a graph has several practical applications. Consider a network that represents a population susceptible to a disease and its contact patterns. Suppose a monophonic path exists between a pair of nodes in such a network. Then, the population at a higher risk of infection can be differentiated from that of the lower risk since the vertices at the lower end of the transmission chain, which form a monophonic path, would be at a lower risk than those on a path that contains a chord. Similarly, the spread of misinformation in social networks can be slowed by designing algorithms that allow the passage of information through monophonic paths only.

2. Main Results

Based on the definition of monophonic distance, we introduce the following definition of monophonic eccentric sequence.

Definition 2.1. *A non-decreasing sequence $S(a_1, a_2, ..., a_n)$ of non-negative integers is called a monophonic eccentric sequence if there exists a connected graph G whose vertices can be labeled $v_1, v_2, ..., v_n$ such that $e_m(v_i)$=a_i for all $1 \leq i \leq n$.*

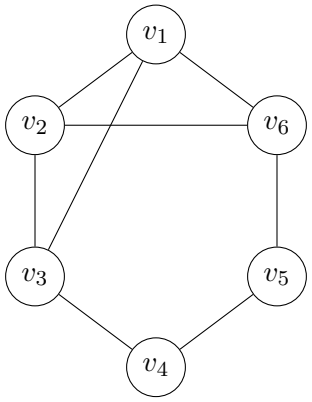


Figure 1. Graph G on six vertices

$d_m(v_i, v_j)$	v_1	v_2	v_3	v_4	v_5	v_6	$e_m(v)$
v_1	0	1	1	3	3	1	$e_m(v_1) = 3$
v_2	1	0	1	3	3	1	$e_m(v_2) = 3$
v_3	1	1	0	1	3	3	$e_m(v_3) = 3$
v_4	3	3	1	0	1	3	$e_m(v_4) = 3$
v_5	3	3	3	3	0	1	$e_m(v_5) = 3$
v_6	1	1	3	3	1	0	$e_m(v_6) = 3$

TABLE 1. Monophonic distance between each pair of vertices in G

Example 2.1. *For the graph G in Figure* 1*, the monophonic distances and the monophonic eccentricities are given in Table* 1*.*

Observation 2.1. *In a complete graph K_n, all the vertices are adjacent to each other. Hence, the monophonic eccentric sequence of a complete graph K_n is 1^n.*

Observation 2.2. *A vertex v in a graph G has $e_m(v)$=1 if and only if v is a universal vertex in G.*

Observation 2.3. *A tree is an undirected graph in which any two vertices are connected by exactly one path. Therefore, the monophonic eccentric sequence is the same as the eccentric sequence for trees.*

Proposition 2.1. *The monophonic eccentric sequence of a cycle C_n of order n is $(n-2)^n$.*

Proof. Let $u, v \in \mathrm{V}(C_n)$ where v and u are a pair of non-adjacent vertices such that $d(u, v) = p$. Then, there exist paths of length p and $n - p$ between u and v. Since $p \leq n - p$ and $n - p$ is maximum when $p = 2$, for every vertex $u \in V(C_n)$, there exists a vertex $v \in V(C_n)$ such that $d(u, v) = 2$ and $d_m(u, v) = n - 2$. Therefore, the monophonic eccentric sequence of cycle C_n of order n is $(n - 2)^n$. □

Corollary 2.1. *The monophonic eccentric sequence of a wheel $W_{1,n}$ is $1, (n - 2)^n$.*

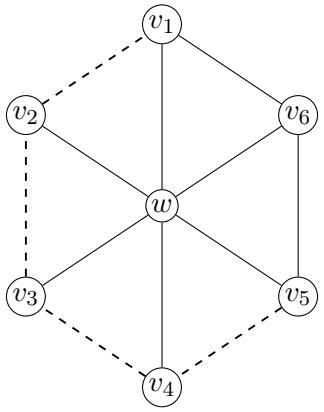


FIGURE 2. The monophonic path between v_1 and v_5 of a $W_{1,6}$

Example 2.2. *Figure 2 shows the monophonic path between the vertices v_1 and v_5 of a wheel $W_{1,6}$.*

Corollary 2.2. *The monophonic eccentric sequence of a sunlet graph $C_n \circ K_1$, where $n \geq 3$ is $(n - 1)^n, n^n$.*

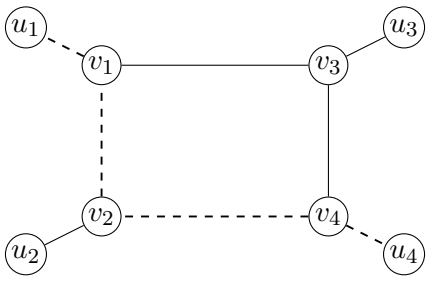


FIGURE 3. The monophonic path $u_1 - u_4$ of $C_4 \circ K_1$

Example 2.3. *Figure 3 shows the monophonic distance between the vertices u_1 and u_4 of a sunlet graph $C_n \circ K_1$.*

Corollary 2.3. *The monophonic eccentric sequence of a helm graph, H_n of order $2n+1$ is $2, (n-1)^n, n^n$.*

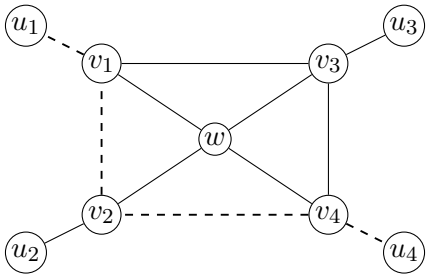


FIGURE 4. The monophonic path $u_1 - u_4$ of H_4

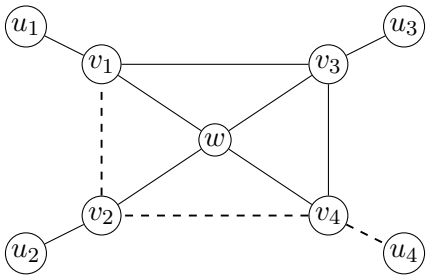


FIGURE 5. The monophonic path $v_1 - u_4$ of H_4

Example 2.4. *Figures 4 and 5 show the monophonic distance between two different pairs of vertices of a helm graph H_4.*

The gear graph G_n is a wheel graph with a vertex added between each pair of adjacent vertices of the outer cycle.

Proposition 2.2. *The monophonic eccentric sequence of G_n is given by $2, (2n-2)^{2n}$.*

Proof. Consider a Gear graph on $2n+1$ vertices. Let $v_1, v_2, v_3, \ldots, , v_n$ be the vertices of degree 3, w be the vertex of degree n and $u_1, u_2, u_3, \ldots, u_n$ be the vertices of degree 2.

The vertex v_i has a maximum monophonic distance with a vertex at a distance 2 from v_i. Since u_i cannot be at a distance 2 from it, we have $d_m(v_i, v_j) = 2n-2$ for a pair of vertices v_i and u_j such that $d(v_i, v_j) = 2$. This path cannot contain the vertex w, since w is adjacent to more than two vertices on it. Hence, we have a monophonic eccentricity of $2n-2$ for each v_i. Using a similar argument, we can show that each u_i has a monophonic eccentricity of $2n-2$.

Now, consider the vertex w of degree n. Suppose w is connected to any vertex u_i by a path P that has more than three vertices in it. Then, P has at least two $v_i's$. Since w is adjacent to each v_i, this creates a chord in P. This implies there can be only one vertex on the path P. Hence, we have a monophonic eccentricity of 2 for the vertex w. Therefore, the monophonic eccentric sequence of gear graph is 2, $(2n-2)^{2n}$. □

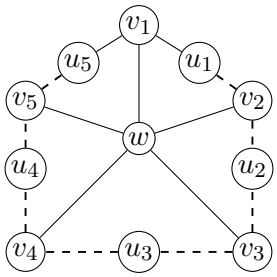


FIGURE 6. The monophonic path $u_1 - u_5$ of G_5

Example 2.5. *Figure* 6 *shows the monophonic distance between the vertices* u_1 *and* u_5 *of a gear graph* G_5.

Theorem 2.1. *The monophonic eccentric sequence of a friendship graph is* $1, 2^{2n}$.

Proof. Consider the friendship graph F_n on $2n+1$ vertices. Let $v_1, v_2, v_3, \ldots, v_{2n}$ be the vertices of degree 2 and w be the vertex of degree n. Since w is the universal vertex, its monophonic eccentricity is 1.

Now, consider each vertex v_i which is adjacent to one other vertex v_j, $j \in \{i-1, i+1\}$, $1 \leq \mathrm{i} \leq n$. Since each v_i is adjacent to w, the monophonic eccentricity of v_i is due to a monophonic path P with a non-adjacent vertex v_k. Assume that such a path P has more than three vertices on it. This implies there are at least two interior vertices v_p and v_q, $1 \leq p, q \leq n$ on P. Since w has to be a vertex in P, w is now adjacent to more than two vertices in P and will hence induce a chord. Therefore, the path P cannot have more than three vertices. Hence, we have a monophonic eccentricity of 2 for each v_i. Therefore, the monophonic eccentric sequence of a friendship graph F_n is $1, 2^{2n}$.

□

Proposition 2.3. *The monophonic eccentric sequence of the Petersen graph is* 4^{10}.

Proof. Petersen graph is a 3-regular graph on 10 vertices and has a diameter 2. Consider the following cases.
Case 1: Suppose M is a monophonic path of length 5 between a pair of non-adjacent

vertices. The diameter of a Petersen graph is 2. Hence, the distance between any pair of non-adjacent vertices is 2. Since any pair of non-adjacent vertices have a common neighbour, it can be seen that each path of length 5 between these vertices will pass through the common neighbour. This will result in a chord between non-adjacent vertices on the monophonic path M. Therefore, any pair of vertices in Petersen graph cannot have a monophonic path of length 5 and hence monophonic eccentricity 5.
Case 2: Suppose a pair of non-adjacent vertices has a monophonic path M of length greater than six between them. Since M has 7 vertices in it, at least one non-interior vertex of M is adjacent to more than two vertices of M, hence inducing a chord. Therefore, a Petersen graph cannot have a monophonic path of length more than four. Hence, we have the monophonic eccentric sequence of the Petersen graph to be 4^{10}.

□

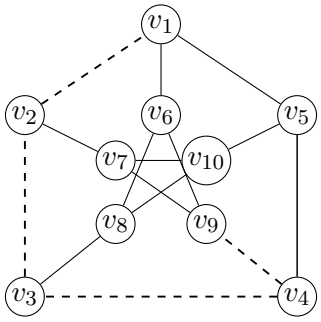


FIGURE 7. The monophonic path between v_1 and v_9 of Petersen graph

Example 2.6. *Figure* 7 *shows the monophonic distance between the pair of vertices in the Petersen graph.*

Proposition 2.4. *Consider a cycle C_n on $n \geq 6$ vertices. Let G_n denote the resulting graph after adding an edge between every pair of vertices at distance* 2 *in C_n. Then, the monophonic eccentric sequence of G_n is p^n, where*

$$p = \begin{cases} 2k-2 & n = 3k \\ 2k-1 & n = 3k+1, 3k+2 \end{cases}$$

for all $k > 1$.

Proof. Consider a pair of non-adjacent vertices v_i and v_j such that $d(v_i, v_j) = 3$ in G_n. Suppose $v_i = v_1$ and $v_j = v_{n-2}$. Then, any three consecutive vertices $v_i - v_{i+1} - v_{i+2}, 1 \leq i \leq n-4$ cannot be on the monophonic path between v_i and v_j since there would be a chord between v_i and v_{i+2} in G_n. Therefore, $v_1 - v_2 - v_4 - v_5 - v_7 - \cdots - v_{n-2}$ is the longest monophonic path between v_1 and v_{n-2} in G_n. This path is obtained by not including the vertices whose index is a multiple of 3. Hence, we have the following cases.

Case 1: $n = 3k$
Then $d(v_1, v_{n-2}) = 3k-3$ in G_n and the number of vertices excluded from the monophonic path whose index is a multiple of 3 is $k-1$.
Hence, $d_m(v_1, v_{n-2}) = 3k - 3 - (k-1) = 2k-2$.

Case 2: $n = 3k+1$
Then $d(v_1, v_{n-2}) = 3k-2$ in G_n and the number of vertices excluded from the monophonic

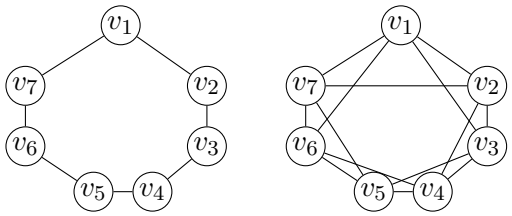


FIGURE 8. Construction of a graph with monophonic eccentric sequence 3^7 from C_7.

path whose index is a multiple of 3 is $k-1$.
Hence, $d_m(v_1, v_{n-2}) = 3k-2-(k-1) = 2k-1$.

Case 3: $n = 3k+2$
Then $d(v_1, v_{n-2}) = 3k-2$ in G_n and the number of vertices excluded from the monophonic path whose index is a multiple of 3 is k.
Hence $d_m(v_1, v_{n-2}) = 3k-1-k = 2k-1$. □

Example 2.7. *Figure* 8 *shows the construction of a graph with monophonic eccentric sequence* 2^7 *from* C_7.

Theorem 2.2. *For a unicyclic graph with a cycle of order* k, $d_m(u,v) \leq d(u,v) + k - 2$.

Proof. Let C be the cycle of order k in the unicyclic graph G. Then, we have the following cases.

Case 1: Let u and v be two vertices of C.
The maximum monophonic distance between any vertices u and v on a cycle C of order k is $k-2$.

Subcase 1: Consider k to be even. The length of all monophonic paths between u and v is the same. Hence, we have $d_m(u,v) = k-2$ and $d(u,v) \geq 1$ which implies $d_m(u,v) \leq d(u,v) + k - 2$.

Subcase 2: Consider k to be odd. There is only one monophonic path between the vertices u and v of length $k-2$. Hence, we have $d_m(u,v) = k-2$ and $d(u,v) \geq 1$ which implies $d_m(u,v) \leq d(u,v) + k - 2$.

Case 2: Let u and v be two vertices that do not belong to C but include the vertices of C in the monophonic path M between them.

Subcase 1:

Let the path M have exactly two vertices x, y of C in it. Then, these two vertices x and y are adjacent. Any path of length greater than that of M would induce the chord $x-y$. Therefore, $d_m(u,v) = d(u,v) \leq d(u,v) + k - 2$.

Subcase 2: Let the path M have more than two vertices of C in it.

Then, the monophonic path can have at most $k-1$ vertices in it, since inclusion of k vertices would induce a chord. Therefore, the cycle C can contribute at most $k-2$ to the length of M. Thus, $d_m(u,v) \leq d(u,v) + k - 2$.

Case 3: Let u be a vertex of C and v be the vertex that does not belong to C. Let w be a vertex on C such that $d(v,w) < d(v,x)$ for some vertex x in the cycle C.

Subcase 1: If $d(w,u)=1$, then any path of greater length between v and u would induce the chord $w-u$. Therefore, $d_m(u,v)=d(u,v)\leq d(u,v)+k-2$.

Subcase 2: If $d(w,u)>1$, then there are two paths between vertices u and w on C, with the largest possible length being $k-2$. Thus, $d_m(u,v)\leq d(u,v)+k-2$. □

Observation 2.4. *For a unicyclic graph G, $d(u,v)=d_m(u,v)$ if the order of the cycle is even and G has no pendant vertices.*

Corollary 2.4. *For a unicycle graph G with a cycle of order k, we have $e_m\leq e+k-2$ for any vertex v of G.*

Theorem 2.3. *A vertex $v\in V(G)$ is a cut vertex in a connected graph G if and only if there exist vertices u and w in distinct components of $G-v$ such that v lies on the $u-w$ monophonic path in G.*

Proof. Let v be a cut-vertex of a connected graph G. Let u and w be the vertices in distinct components of $G-v$, such that u and w are neighbors of v. Then, v lies on the $u-w$ monophonic path in G.

Conversely, suppose that vertices u and w exist in distinct components of $G-v$ such that v lies on the $u-w$ monophonic path in G. If v is not a cut vertex of G, then removing v will not disconnect the graph G. Thus, there exists a path between u and w in $G-v$, which contradicts our assumption that u and w are vertices in distinct components of $G-v$. Therefore, v is a cut-vertex of G. □

References

[1] Lesniak, L., (1975), Eccentric sequences in graphs, Periodica Mathematica Hungarica, 6(4), pp. 287–293.

[2] Buckley, F., Harary, F., (1990), Distance in graphs, Addison-Wesley, Redwood City.

[3] West, D. B., et al., (2001), Introduction to graph theory, Prentice Hall, Upper Saddle River.

[4] Dourado, M. C., et al., (2008), Algorithmic aspects of monophonic convexity, Electronic Notes in Discrete Mathematics, 30, pp. 177–182.

[5] Santhakumaran, A., Titus, P., (2011), Monophonic distance in graphs, Discrete Mathematics, Algorithms and Applications, 3(2), pp. 159–169.

[6] Santhakumaran, A., et al., (2014), On the monophonic number of a graph, Journal of Applied Mathematics and Informatics, 32(1–2), pp. 255–266.

[7] Titus, P., et al., (2016), On the detour monophonic number of a graph, Ars Combinatoria, 129, pp. 33–42.

[8] Titus, P., Ganesamoorthy, K., (2016), Upper detour monophonic number of a graph, Electronic Notes in Discrete Mathematics, 53, pp. 331–342.

[9] Titus, P., et al., (2016), The connected detour monophonic number of a graph, TWMS Journal of Applied and Engineering Mathematics, 6(1), pp. 75–86.

[10] Kinsley, et al., (2020), An edge rm-distance monophonic number of a graph, AIP Conference Proceedings, 2261.

[11] Titus, P., et al., (2024), Monophonic-triangular distance in graphs, Proyecciones (Antofagasta), 43(1), pp. 275–292.



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