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AN INVESTIGATION OF PYTHAGOREAN FUZZY DIGITAL CONVEXITY

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Abstract. The aim of this study is to introduce Pythagorean fuzzy digital convex sets, Pythagorean fuzzy digital cut sets, Pythagorean fuzzy digital topological spaces, Pythagorean fuzzy digital generalized closed sets, and Pythagorean fuzzy digital generalized open sets. Furthermore, to enrich the theory of Pythagorean fuzzy digital topological concepts, certain applications of Pythagorean fuzzy digital generalized closed sets, particularly the notion of Pythagorean fuzzy digital generalized $T_{\frac{1}{2}}$ spaces, are discussed and explored in detail.

Keywords: Pythagorean fuzzy digital topology, Pythagorean fuzzy digital closed sets, Pythagorean fuzzy digital open sets, Pythagorean fuzzy digital generalized closed sets, Pythagorean fuzzy digital generalized open sets, Pythagorean fuzzy digital generalised $T_{1/2}$ space.

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1. Introduction

Zadeh [21] laid the foundation for fuzzy sets to be used. The development of intuitionistic fuzzy sets, which were fuzzy set generalisations, by K. T. Atanassov [1] in 1983 came after. The membership degree μ and non-membership degree ν in intuitionistic fuzzy sets satisfy the requirement that the sum of both membership degree and non-membership degree should be less than one. The Pythagorean fuzzy sets are recognized to be more appropriate than intuitionistic fuzzy sets and other sorts of sets, and Yager R R [20] was able to begin the Pythagorean fuzzy sets [2, 3] for under the restriction $\mu^2 + \nu^2 \leq 1$ as a result of this realization. The use of Pythagorean fuzzy sets as a tool for decision-making has been advocated by many researchers. The conceptualization of the idea originated to represent ambiguity and lack of precision in the mathematical way and to implement a formal tool for tackling ambiguity to real world problems.

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Convex and concave fuzzy sets are crucial concepts in applied mathematics, operations research, and optimization theory. Zadeh [21] proposed a significant idea of convex fuzzy sets. Katsaras and Liu [5] have since examined ideas and applications pertaining to convex (concave) fuzzy sets . In 1982, Janos and Rosenfeld [4] invented fuzzy digital convexity. The connection between fuzzy and crisp sets is further impacted by the concepts of p-cuts, or level sets. They can be thought of as a bridge linking crisp and fuzzy sets. Li. M. [9] offered the Cut sets of intuitionistic fuzzy sets.

Digital topology explores the characteristics and attributes of two-dimensional (2D) or three-dimensional (3D) digital images that relate to the topological properties (such as connectedness) or topological features (such as boundaries) of objects. The concepts and findings in digital topology contribute to the identification and justification of significant image analysis algorithms at a lower level. Kong and Kopperman [6] proposed a topological method for digital topology. Additionally, Kong et al. [7, 8] examined the digital fundamental group and demonstrated its applicability to a strongly normal digital picture space, revealing the equivalence of discrete and continuous concepts. In another study, Rosenfeld and Kak [17] explored the geometric properties and interrelationships among different parts of a digital picture through subsets obtained via various segmentation processes. To enhance this powerful methodology, Rosenfeld and Kak [17] extended the principles of digital picture geometry to encompass fuzzy subsets. While classical digital topology primarily focuses on the analysis of black and white images in the digital plane, A. Rosenfeld [14, 15] introduced the concept of fuzzy sets to represent gray scale level images.

In this work, the perceptions of Pythagorean fuzzy digital topology with some properties that will showcase its decision making ability is introduced. This paper introduces the Pythagorean fuzzy digital convex sets, Pythagorean fuzzy digital cut sets, Pythagorean fuzzy digital generalized closed and open sets with some of its basic properties.

2. Preliminaries

Definition 2.1. [3, 11, 18, 19] *A Pythagorean fuzzy subset* (PFS) Ω *of a nonempty set* T *is a pair* (μ_Ω, ν_Ω) *of a membership function* $\mu_\Omega : T \to [0,1]$ *and a non-membership function* $\nu_\Omega : T \to [0,1]$ *with* $\mu^2_\Omega(x)+\nu^2_\Omega(x)=r^2_\Omega(x)$ *for any* $x \in T$*, where* $r_\Omega : T \to [0,1]$ *is a function which is called the strength of commitment at point x. Let* $(r_\Omega(x), \theta_\Omega(x))$ *be a polar coordinates of* $(\mu_\Omega(x), \nu_\Omega(x))$ *for a point* $x \in T$*. If we define* $d_\Omega(x) = (1 - \theta_\Omega(x))\frac{\pi}{2}$*, then the function* $d_\Omega : T \to [0,1]$ *can be considered the direction of commitment at point* x*. It is obvious that the function* d_Ω *scales the first quadrant between 0 and 1 i.e., if* $\theta_\Omega = \frac{\pi}{2}$ *, then* $\mu_\Omega(x) = 0$ *and* $\nu_\Omega(x) = r_\Omega(x)$*, which means the direction* $d_\Omega = 0$*, and if* $\theta_\Omega(x) = 0$*, then* $\mu_\Omega(x) = r_\Omega(x)$ *and* $\nu_\Omega(x) = 0$*, which means the direction* $d_\Omega(x) = 1$*. Therefore, a PFS* Ω *can be expressed by either* (μ_Ω, ν_Ω) *or* $(r_\Omega(x), \theta_\Omega(x))$*.*

Definition 2.2. [3, 11, 18, 19] *Let* $\Omega_1 = (\mu_{\Omega_1}, \nu_{\Omega_1})$ *and* $\Omega_2 = (\mu_{\Omega_2}, \nu_{\Omega_2})$ *be two Pythagorean fuzzy subsets of a T set. Then,*

(i) *the complement of* Ω *is defined by* $\Omega^c = (\mu_\Omega, \nu_\Omega)$
(ii) *the intersection of* Ω_1 *and* Ω_2 *is defined by* $\Omega_1 \cap \Omega_2 = [min\{\mu_{\Omega_1}, \mu_{\Omega_2}\}, max\{\nu_{\Omega_1}, \nu_{\Omega_2}\}]$.
(iii) *the union of* Ω_1 *and* Ω_2 *is defined by* $\Omega_1 \cap \Omega_2 = [max\{\mu_{\Omega_1}, \mu_{\Omega_2}\}, min\{\nu_{\Omega_1}, \nu_{\Omega_2}\}]$.
(iv) Ω_1 *is a subset of* Ω_2 *or* Ω_2 *contained* Ω_1 *and we write* $\Omega_1 \subset \Omega_2$ *or* $\Omega_2 \supset \Omega_1$ *if* $\mu_{\Omega_1} \leq \mu_{\Omega_2}$ *and* $\nu_{\Omega_1} \geq \nu_{\Omega_2}$.

Definition 2.3. [11] *Let τ be a family of PFS of ζ and let ζ be a set if*

(i) $1_\sim, 0_\sim \in \tau$,

(ii) For any $\{\Omega_i/i \in I, \Omega_i \in \tau\}, \cap_{i=1}^{n} \Omega_i \in \tau$

(iii) $\cup_{i=1}\Omega \in \tau$ *for any* $\{\Omega_i/i \in I, \Omega_i \in \tau\}$*, since I represents any arbitrary index set, τ is known to as a PFT in ζ.*

In such instances, the pair (ζ, τ) is termed a PFTS. Each element in τ is an open PFS, and a closed PFS is the complement connected to an open PFS. The discrete PFTS is the topology that includes all PFS.

Definition 2.4. [11] *Let Ω_1, U be two PFS in a PFTS. Then, U is said to be a neighbourhood of Ω_1 if there exists an open PFS Ω_2 such that $\Omega_1 \subset \Omega_2 \subset U$.*

Definition 2.5. [10, 14] *If A, B are the points of ζ (the rectangular array of integer-coordinate points), then a path ρ from A to B is a sequence of points $A = A_0, A_1, A_2, ..., A_n$ =B such that A_i is adjacent (either 4-adjacent or 8-adjacent) to A_{i-1}, for $1 \leq i \leq n$.*

Definition 2.6. [15] *If Ω is any subset of ζ, then the points are said to be connected in Ω if there is a path from A to B which consist entire points of Ω.*

Definition 2.7. [13] *Consider ζ as a rectangular array (RA) of integer-coordinate points in the Euclidean plane $\sum$ and let $\Omega = (\mu_\Omega, \nu_\Omega)$ be a PFS of $\sum$. Then, the degrees of membership and non-membership of the Pythagorean fuzzy digital (PFD) subset of ζ, depicted as $\Omega_\sim$ are defined as follows: $\mu_{\Omega_\sim}\eta = \max\{\mu_\Omega(R)|R \in \eta^*\}$ and $\nu_{\Omega_\sim}(R)|R \in \eta^*$. Here R is the subset of the plane and $\eta*$ is the open unit square has the center η.*

Definition 2.8. [13] *Let $\Omega = (\mu_\Omega, \nu_\Omega)$ be a Pythagorean fuzzy digital subset of $\sum$. Then Ω is said to be Pythagorean fuzzy digital regular (PFD_R) set if the sets Ω_r, defined by $\Omega_r = \{R : \mu_{\Omega r}(R) > r\}$ and $\nu_{\Omega r}(R) < r$ are Pythagorean fuzzy regular for all $r \in$ [0,1).*

3. Pythagorean Fuzzy Digital Convexity

In this section we introduce Pythagorean fuzzy digital convex set and studied some of its properties.

Definition 3.1. *Let $\Omega = (\mu_\Omega, \nu_\Omega)$ be a Pythagorean fuzzy subset of $\sum$, then Ω is said to be a Pythagorean fuzzy convex set (PF-convex set) of $\sum$, if for every pair A,B of points in $\sum$, and for all A_i, $0 \leq i \leq n$ on the line segment, $A = A_0, A_1, A_2, ..., A_n = B$ from A to B, $\mu_\Omega(A_i) \geq \min(\mu_\Omega(A), \mu_\Omega(B)); \nu_\Omega(A_i) \leq \max(\nu_\Omega(A), \nu_\Omega(B))$; The complement 1- Ω of the PFD-con set Ω is said to be an Pythagorean fuzzy concave set of $\sum$.*

Proposition 3.1. *A Pythagorean fuzzy subset $\Omega = (\mu_\Omega, \nu_\Omega)$ of $\sum$ is a PF-convex set iff for every pair A, B of points in $\sum$ and for all A_i, $0 \leq i \leq n$ on the line segment, $A = A_0, A_1, A_2, ..., A_n = B$ from A to B, $\mu_\Omega = (\mu_\Omega, \nu_\Omega)((1-\alpha)A + \alpha B) \geq \min(\mu_\Omega(A), \mu_\Omega(B)); \nu_\Omega((1-\alpha)A + \alpha B) \geq \min(\nu_\Omega(A), \nu_\Omega(B))$.*

Definition 3.2. *Let $\Omega = (\mu_\Omega, \nu_\Omega)$ be a Pythagorean fuzzy subset of $\sum$, then the level sets of $\Omega = (\mu_\Omega, \nu_\Omega)$ are defined as $\Omega^\alpha = \{A \in \sum, \mu_\Omega(A) \geq \alpha, \nu_\Omega(A) \leq \alpha\}, \alpha \in I$.*

Definition 3.3. *Let $\Omega = (\mu_\Omega, \nu_\Omega)$ be a Pythagorean fuzzy subset of $\sum$, then Ω is convex iff if its level sets are all convex.*

Definition 3.4. *Let $\Omega = (\mu_\Omega, \nu_\Omega)$ be a Pythagorean fuzzy subset of $\sum$ which is a Pythagorean fuzzy regular and convex, then $\Omega_\sim$ is said to be a Pythagorean fuzzy digital convex (or PFD convex) set, if the digital image of Ω is $\Omega_\sim$. The complement 1- Ω of the PFD convex set $\Omega_\sim$ is said to be a Pythagorean fuzzy digital concave (or PFD concave) set.*

Proposition 3.2. *Let $\Omega = (\mu_\Omega, \nu_\Omega)$ be a Pythagorean fuzzy subset of $\sum$ which is PFD regular and let $\Omega_\sim$ be a PFD convex set, then the sets $\Omega_r = \{R : \mu_{\Omega r}(R) > r$ and $\nu_{\Omega r}(R) < r\}$ are PFD convex for all $r \in I$.*

Proof. The proof follows from the Propositions 3.2 and 3.4. □

Proposition 3.3. *Let $\Omega = (\mu_\Omega, \nu_\Omega)$ be a Pythagorean fuzzy subset of $\sum$ which is PFD regular and let $\Omega_\sim$ be a PFD convex set, t, then then for any line, l' and any three points A,B,C $\in l$ of ζ where B lies in between A and C, $\mu_\Omega(B) \geq \min(\mu_{\Omega_\sim}(A), \mu_{\Omega_\sim}(C)); \nu_{\Omega_\sim}(B) \leq \max(\nu_{\Omega_\sim}(A), \nu_{\Omega_\sim}(C))$.*

Proof. Let $A' \in \eta^*$ and $C' \in \eta^*$ be the interior points. From the Definition 2.7, $\mu_{\Omega\sim}(A) \leq \mu_\Omega(A') + \epsilon, \nu_{\Omega_\sim}(A) \geq \nu_\Omega(A')$ *and* $\mu_{\Omega_\sim}(B) \geq \nu_\Omega(B') + \epsilon, \nu_{\Omega_\sim}(B) \geq \nu_\Omega(B') + \epsilon$ for $\epsilon > 0$. Let 'l' be the line joining A' and C'. Since B lies in between A and C, 'l' will meet B^* at some of its interior points, say B'. From Definition 2.7, $\mu_{\Omega_\sim} = \max\{\mu_\Omega(T)|T \in B^*\} \geq \mu_\Omega(B') \geq \min(\mu_\Omega(A'), \mu_\Omega(C'))$; since Ω is convex ¿ $\min(\mu_{\Omega\sim}(A) - \epsilon, \mu_{\Omega\sim}(C) - \epsilon)$
$\mu_\Omega(B) = \min(\mu_{\Omega_\sim}(A), \mu_{\Omega\sim}(C)) - \epsilon \geq \min(\mu_{\Omega_\sim}(A), \mu_\Omega(C))$.
Similarly $\nu_{\Omega_\sim}(B) \leq \max(\nu_{\Omega_\sim}(A), \nu_{\Omega_\sim}(C))$. □

Proposition 3.4. *Let $A_0, A_1, A_2, ..., A_r$ be a set of lattice points, $\Omega_\sim(A_i)$ be the PFD convex subset of those points, then $\gamma \subset \Omega_\sim$, where $\gamma = \sum_{i=0}^{r} a_{\sim i}A_{\sim i}$, $\gamma = (\mu_\gamma, \nu_\gamma)$, $a_i = (a_{i\mu}, a_{i\nu})$, $A_i = (\mu_{\Omega_\sim i}(A_i), \nu_{\Omega\sim i}(A_i))$ with $\mu_\gamma = \sum_{i=0}^{r} a_{i\mu}\mu_{\Omega\sim i}(A_i), \nu_r = \sum_{i=0}^{r} a_{i\nu}\nu_{\Omega\sim i}(A_i)$, $\sum_{i=0}^{r} a_{i\mu} + a_{i\nu} = 1$ and $0 \leq a_{i\mu} + a_{i\nu} \leq 1$ for each i.*

Proof. This proposition can be proved by mathematical induction on r. The result is obvious for r= 1 and from the definition 3.2, it is true for r= 1. By induction hypothesis, assume that the proposition is true for r= k. Now to prove for r= k+1, consider $\sum_{i=0}^{k} a_{\sim i}A_{\sim i}$. Let $\sum_{i=0}^{k} a_{\sim i} = \alpha$. Hence $a_{\sim K+1} = 1 - \alpha$. Now $\sum_{i=0}^{k} a_{\sim i}A_{\sim i} + a_{\sim K+1}A_{\sim K+1} = \alpha\left(\sum_{i=0}^{k} \frac{a_{\sim i}}{\alpha}A_{\sim i}\right) + (1-\alpha)A_{\sim K+1}$ and $\left(\sum_{i=0}^{k} \frac{a_{\sim i}}{\alpha}\right) = 1$. Hence by induction hypothesis, $\left(\sum_{i=0}^{k} \frac{a_{\sim i}}{\alpha}A_{\sim i}\right) \subset \gamma$. Since Ω is an PFD convex set containing $A_{\sim K+1}$ the result follows from the Proposition 3.2. □

4. Pythagorean Fuzzy Digital Cut Sets

Definition 4.1. *Let $\Omega = (\mu_\Omega, \nu_\Omega)$ be a Pythagorean fuzzy subset of $\sum$, $\Omega_\sim$ be the PFD subset of the lattice points ζ and $\gamma, \varpi \in [0,1]$ where γ, ϖ are real numbers such that $\gamma + \varpi \leq 1$, then Pythagorean fuzzy digital upper cut sets (or PFD upper cut sets) are defined and denoted as follows,*

(i) $\Omega_\sim\big[\gamma, \varpi\big]^+ = \{A : \mu_\Omega(A) \geq \gamma, \nu_\Omega(A) \leq \varpi\}$
(ii) $\Omega_\sim\big(\gamma, \varpi\big]^+ = \{A : \mu_\Omega(A) > \gamma, \nu_\Omega(A) \leq \varpi\}$
(iii) $\Omega_\sim\big(\gamma, \varpi\big)^+ = \{A : \mu_\Omega(A) > \gamma, \nu_\Omega(A) < \varpi\}$
(iv) $\Omega_\sim\big[\gamma, \varpi\big)^+ = \{A : \mu_\Omega(A) \geq \gamma, \nu_\Omega(A) < \varpi\}$

Here $\Omega_\sim\big[\gamma, \varpi\big]^+$ denotes the PFD upper cut set, where the superscript " + " represents the upper cut.

Definition 4.2. *Let $\Omega = (\mu_\Omega, \nu_\Omega)$ be a Pythagorean fuzzy subset of $\sum$, $\Omega_\sim$ be the PFD subset of the lattice points ζ and $\gamma, \varpi \in [0,1]$ where γ, ϖ are real numbers such that $\gamma + \varpi \leq 1$, then Pythagorean fuzzy digital lower cut sets (or PFD lowercut sets)are defined and denoted as follows,*

(i) $\Omega_{\sim}\big[\gamma, \overline{\omega}\big]^{-} = \{A : \mu_{\Omega}(A) \leq \gamma, \nu_{\Omega}(A) \geq \overline{\omega}\}$
(ii) $\Omega_{\sim}\big(\gamma, \overline{\omega}\big]^{-} = \{A : \mu_{\Omega}(A) < \gamma, \nu_{\Omega}(A) \geq \overline{\omega}\}$
(iii) $\Omega_{\sim}\big(\gamma, \overline{\omega}\big)^{-} = \{A : \mu_{\Omega}(A) < \gamma, \nu_{\Omega}(A) > \overline{\omega}\}$
(iv) $\Omega_{\sim}\big[\gamma, \overline{\omega}\big)^{-} = \{A : \mu_{\Omega}(A) \leq \gamma, \nu_{\Omega}(A) < \overline{\omega}\}$

Here $\Omega_{\sim}\big[\gamma, \overline{\omega}\big]^{-}$ *denotes the PFD lower cut set, where the superscript " $-$ " represents the lower cut.*

Proposition 4.1. *Let* $\Omega = (\mu_{\Omega}, \nu_{\Omega})$ *be a Pythagorean fuzzy subset of* $\sum$, $\Omega_{\sim}$ *be the PFD subset of the lattice points* ζ, *then the following statements are equivalent.*

(i) $\Omega_{\sim}$ *is a PFD convex set*
(ii) $\Omega_{\sim}\big[\gamma, \overline{\omega}\big]^{-}$ *is a PFD convex set*
(iii) $\Omega_{\sim}\big(\gamma, \overline{\omega}\big]^{-}$ *is a PFD convex set*
(iv) $\Omega_{\sim}\big(\gamma, \overline{\omega}\big)^{-}$ *is a PFD convex set*
(v) $\Omega_{\sim}\big[\gamma, \overline{\omega}\big)^{-}$ *is a PFD convex set*

Proof. (i) $\Longrightarrow$ (ii)
Assume that (i) is true. Since $\Omega_{\sim}$ is a PFD convex set, Ω is PFD regular and convex. Then for all A,B in $\Omega_{\sim}\big[\gamma, \overline{\omega}\big]^{-}$ and for all $A_i, 0 \leq i \leq n$, on the line segment, $A = A_0, A_1, A_2, ..., A_n = B$ from A to B, we have $\mu_{\Omega}((1-\alpha)A+\alpha B) \geq \min(\mu_{\Omega}(A), \mu_{\Omega}(B)) \geq \gamma$; $\nu_{\Omega}((1-\alpha)A+\alpha B) \geq \min(\nu_{\Omega}(A), \nu_{\Omega}(B)) \leq \overline{\omega}$. Therefore $\mu_{\Omega}(A_i) \geq \gamma$ and $\nu_{\Omega}(A_i) \leq \overline{\omega}$ which implies $A_i \in \Omega\ \big[\gamma, \overline{\omega}\big]^{-}$ and so $\Omega\big[\gamma, \overline{\omega}\big]^{-}$ is convex. Hence $\Omega_{\sim}\big[\gamma, \overline{\omega}\big]^{-}$ is a PFD convex set and also IFD regular. In the similar manner, the other parts of the Proposition can also be proved. ☐

Proposition 4.2. *Let* $\Omega = (\mu_{\Omega}, \nu_{\Omega})$ *be a Pythagorean fuzzy subset of* $\sum$, $\Omega_{\sim}$ *be the PFD subset of the lattice points* ζ, *then the following statements are equivalent.*

(i) $\Omega_{\sim}$ *is a PFD concave set*
(ii) $\Omega_{\sim}\big[\gamma, \overline{\omega}\big]^{+}$ *is a PFD concave set*
(iii) $\Omega_{\sim}\big(\gamma, \overline{\omega}\big]^{+}$ *is a PFD convex set*
(iv) $\Omega_{\sim}\big(\gamma, \overline{\omega}\big)^{+}$ *is a PFD concave set*
(v) $\Omega_{\sim}\big[\gamma, \overline{\omega}\big)^{+}$ *is a PFD concave set*

Proof. The proof is similar to the proof of Proposition 4.3. ☐

5. Pythagorean Fuzzy Digital Topology

Definition 5.1. *The pair* (ζ, τ) *is called a Pythagorean Fuzzy digital topological space (PFDTS), where* ζ *is a rectangular array of coordinates that are integers points and* τ *is a family of Pythagorean fuzzy digital sets in* ζ *satisfy the following conditions:*

(i) $0, 1 \in \tau$*;*
(ii) $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$*;*
(iii) $\cup G_i \in \tau$ *for any arbitrary family.*

Any PFD Ω *set in* τ *is called a Pythagorean Fuzzy Digital Open set (PFDOS) in* ζ*. The complement* Ω^c *of a Pythagorean Fuzzy Digital Open set* Ω *in a PFDTS* (ζ, τ) *is called a Pythagorean Fuzzy Digital Closed set (PFDCS).*

Example 5.1. *Let* $\zeta = \{a, b, c\}$ *be the RA of integer coordinates points and* $\tau = \{0_{\sim}, 1_{\sim}, \Omega, \gamma\}$ *where* $0_{\sim} = \{< 0, 1 >, < 0, 1 >, < 0, 1 >\}, 1_{\sim} = \{< 1, 0 >, < 1, 0 >, < 1, 0 >\}, \Omega = \{< 0.8, 0.3 >, < 0.6, 0.5 >, < 0.3, 0.7 >, < 0.8, 0.3 >\}$ *and* $\gamma = \{< 0.4, 0.6 >$

$, < 0.1, 0.7 >, < 0.3, 0.7 >\}$ be any two PFD sets in ζ. Then τ is a Pythagorean Fuzzy digital Topology(PFDT) on ζ and the ordered pair (ζ, τ) is a topological space.

Definition 5.2. *Let (ζ, τ) be a PFDTS and Ω be a PFD set in ζ. Then the interior and closure of Ω are denoted as PFD-int(Ω) and PDF-cl(Ω) are defined as,*

- *PFD-int(Ω)=$\cup$ $\{G/G$ is a PFDOS in ζ and $G \subseteq \Omega$ $\}$*
- *PFD-cl(Ω)=$\cap$ $\{K/K$ is a PFDCS in ζ and $\Omega \subseteq K$ $\}$*

Example 5.2. *Let ζ be the RA of integer coordinates points and $\tau = \{0_\sim, 1_\sim, \Omega\}$. Let $\Omega = \{X, G_1, G_2\}$ be a PFD sets in ζ where $\Omega = \{< x, 0.5, 0.7 > /x \in \zeta\}$, $G_1 = \{< x, 0.4, 0.7 >\}/x \in \zeta$, $G_2 = \{< x, 0.3, 0.8 >\}/x \in \zeta$, then PFD-(int-$\Omega$) $= G_1 \cup G_2 = G_1 = \{< x, 0.4, 0.7 > /x \in \zeta\} \in \tau$ and PFD-(cl- Ω) $= G_1 \cap G_2 = \{< x, 0.7, 0.4 >\}$.*

Proposition 5.1. *Let (ζ,τ) be a PFDTS. Ω and γ be the PFD sets in ζ. Then the Pythagorean fuzzy digital closure operator satisfies the following properties*

- *$\Omega \subseteq$ PFD - cl(Ω)*
- *PFD-int(Ω) $\subseteq \Omega$*
- *$\Omega \subseteq Y \Rightarrow$ PFD-cl(Ω) $\subseteq$ PFD-cl(Y)*
- *$A \subseteq B \Rightarrow$ PFD-int(A) $\subseteq$ PFD-int(B)*
- *PFD-cl(A $\cup$ B)=PFD-cl(A) $\cup$ PFD -cl(B)*
- *PFD-int(Ω $\cap$ Y)=PFD-int(Ω) $\cap$ PFD -int(Y)*
- *1-(PFD-cl(Ω))=PFD-int(1-Ω)*
- *1-(PFD-int(Ω))=PFD-cl(1-Ω)*

Definition 5.3. *Let (ζ, τ) be a PFDTS and Ω be a PFD set in (ζ,τ) is said to be a*

- *Pythagorean fuzzy digital regular open set (PFDROS in short) if Ω = PFD-int(PFD-cl(Ω))*
- *Pythagorean fuzzy digital regular closed set (PFDRCS in short) if Ω = PFD - cl(PFD- int(Ω))*
- *Pythagorean fuzzy digital semi closed set (PFDSCS in short) if PFD - int(PFD - cl(Ω))$\subseteq \Omega$*
- *Pythagorean fuzzy digital semi open set (PFDSOS in short) if $\Omega \subseteq$ PFD - cl(PFD-int(Ω))*
- *Pythagorean fuzzy digital pre-closed set (PFDPCS in short) if PFD-cl(PFD-int(Ω)) $\subseteq \Omega$*
- *Pythagorean fuzzy digital pre-open set (PFDPOS in short) if $\Omega \subseteq$ PFD-int(PFD-cl(Ω))*
- *Pythagorean fuzzy digital α- closed set (PFDαCS in short) if PFD- cl(PFD-int(PFD-cl(Ω))) $\subseteq \Omega$*
- *Pythagorean fuzzy digital α-open set (PFD α OS in short) if $\Omega \subseteq$ PFD-int(PFD-cl(PFD-int(Ω)))*

6. Pythagorean Fuzzy Digital generalized Closed Set

Definition 6.1. *Let (ζ, τ) be a PFDTS and Ω be a PFD set in ζ. Then Ω is said to be generalized Pythagorean fuzzy digital closed (PFDG closed) if PFD-cl(Ω)$\subseteq$ G whenever $\Omega \subseteq G$ and G is Pythagorean fuzzy digital open.*

Example 6.1. *Let $G_1 = \{< x, 0.4, 0.7 > x \in \zeta\}$ and G_2=$\{< x, 0.3, 0.8 > /x \in \zeta\}$. Consider Ω=$\{< x, 0.3, 0.9 > /x \in \zeta\}$. Now $\Omega \subseteq G_1$,G_2 $\Rightarrow$ PFD - cl(Ω)=$\{< x, 0.3, 0.8 > /x \in \zeta\} \subseteq G_1$,$G_2$.*

Definition 6.2. *Let (ζ,τ) be a PFDTS and Ω be a PFD set in ζ. Then Pythagorean fuzzy digital generalized closure and Pythagorean fuzzy digital generalized interior of Ω are defined by*

- *PFDG - cl(Ω) = $\{G/G$ is a PFDG closed set in ζ and $\Omega \subseteq G\}$*
- *PFDG - int(Ω) = $\{G/G$ is a PFDG open set in ζ and $\Omega \supseteq G\}$*

Proposition 6.1. *If Ω and γ are PFDG closed sets then $\Omega \cup Y$ is a PFDG closed set.*

Proof. Given Ω and γ are PFDG closed set. So PFD - cl(Ω) $\subseteq$ U whenever $\Omega \subseteq$ U and PFD - cl(Y) $\subseteq$ U whenever Y $\subseteq$ U . Here $\Omega \cup$ Y $\subseteq$ U. Now PFD - cl($\Omega \cup$ Y)= PFD - cl(Ω) $\cup$ PFD - cl(Y) $\subseteq \Omega \cup$ Y $\subseteq$ U. Hence A $\cup$ B is a PFDG closed set. □

Example 6.2. *Consider G_1=$\{< x, 0.6, 0.5 > /x \in \zeta\}$, G_2=$\{< x, 0.4, 0.6 > /x \in \zeta\}$, G_3=$\{< x, 0.7, 0.2 > /x \in \zeta\}$, G_4=$\{< x, 0.3, 0.8 > /x \in \zeta\}$. Let $\Omega = \{< x, 0.5, 0.6 > /x \in \zeta\}$ & $Y = \{< x, 0.6, 0.4 > /x \in \zeta\}$ $\Omega \subset G_1, G_3$; PFD -cl(Ω) = $\{< x, 0.5, 0.6 > x \in \zeta\}$ $\subset G_1, G_3 \Longleftrightarrow \Omega$ is PFDG closed set. $Y \subseteq G_3$; PFD -cl(Y) = $\{< x, 0.6, 0.4 > /x \in \zeta\}$ $\subseteq G_3 \Longrightarrow Y$ is PFDG closed set. $\Omega \cup Y = \{< x, 0.6, 0.4 > /x \in \zeta\}$, PFD - cl($\Omega \cup Y$) = $\{¡x, 0.6, 0.4 > /x \in \zeta\} \subseteq G_3 \Longrightarrow \Omega \cup Y$ is PFDG closed set.*

Note: The intersection of two PFDG closed sets need not be PFDG closed set.

Proposition 6.2. *Every PFD closed set is a PFDG closed set but not conversely.*

Proof. Let Ω be a PFD closed set and let $\Omega \subseteq U$ and U is a PFD open set in (ζ,τ). PFD - cl(Ω)=$\Omega \subseteq U$. Therefore Ω is a PFDG closed set in ζ □

Example 6.3. *Let $\Omega = \{< x, 0.6, 0.7 > /x \in \zeta\}$; $G = \{< x, 0.6, 0.6 > /x \in \zeta\}$. Here $\Omega \subseteq G \Longrightarrow PFD - cl(\Omega) = \{< x, 0.6, 0.6 > /x \in \zeta\} \subset G$. So Ω is a PFDG closed set. But PFD -cl(Ω) $\neq \Omega$. Hence Ω is not a PFD closed set.*

Proposition 6.3. *Every PFD regular closed set is a PFDG closed set but not conversely.*

Proof. Let Ω be a PFD regular closed set in ζ. By the hypothesis, $\Omega = PFD - cl(PFD - int(\Omega))$. So PFD - cl($\Omega$) = PFD - cl(PFD - int($\Omega$)). Therefore PFD - cl($\Omega$) = Ω so Ω is a PFD closed set in ζ. Hence Ω is a PFDG closed set. □

Example 6.4. *Let $\Omega = \{< x, 0.6, 0.7 > /x \in \zeta\}$; $G_1 = \{< x, 0.4, 0.7 > /x \in \zeta\}$; Here $\Omega \subset G_1$, $\Longrightarrow$ PFD - cl(Ω) = $\{< x, 0.6, 0.6 > /x \in \zeta\} \subset G_1$. So, Ω is a GPFD closed set. But PFD – int(Ω)= $\{¡x, 0.4, 0.7 > /x \in \zeta\}$, PFD - cl(PFD - int($\Omega$)) = $\{¡x, 0.6, 0.6 > /x \in \zeta\} \neq \Omega$. Hence Ω is not a PFD regular closed set.*

Proposition 6.4. *PFD semi closed set and PFDG closed set are independent to each other.*

Example 6.5. *Let $\Omega = \{< x, 0.6, 0.6 > /x \in \zeta\}$; $G_1 = \{< x, 0.5, 0.7/x \in \zeta >\}$; $G_2 = \{< x, 0.6, 0.3 > /x \in \zeta\}$;*

Now, PFD - cl(Ω) = $\{< x, 0.7, 0.5 > \ xin\zeta\}$ and PFD - int(PFD - cl(Ω)) = $\{< x, 0.5, 0.7 > /x \in \zeta\} \subseteq G_2$. So Ω is a PFD semi closed set. But $\Omega \subset G_2$, $\Longrightarrow$ PFD - cl(Ω) = $\{< x, 0.7, 0.5 > /x \in \zeta\} \not\subset \Omega$. Hence Ω is not a PFDG closed set.

Conversely, let $\Omega = \{< x, 0.6, 0.7 > /x \in \zeta\}$; $G = \{< x, 0.6, 0.6 > /x \in \zeta\}$; if $\Omega \subset G$, $\Longrightarrow$ PFD -cl(Ω) = $\{< x, 0.6, 0.6 > /x \in \zeta\} \subset G$. So Ω is a PFDG closed set. But PFD - int(PFD - cl(Ω)) $\not\subset \Omega$. Hence Ω is not a PFD semi closed set.

Proposition 6.5. *PFD pre closed set and PFDG closed set are independent to each other.*

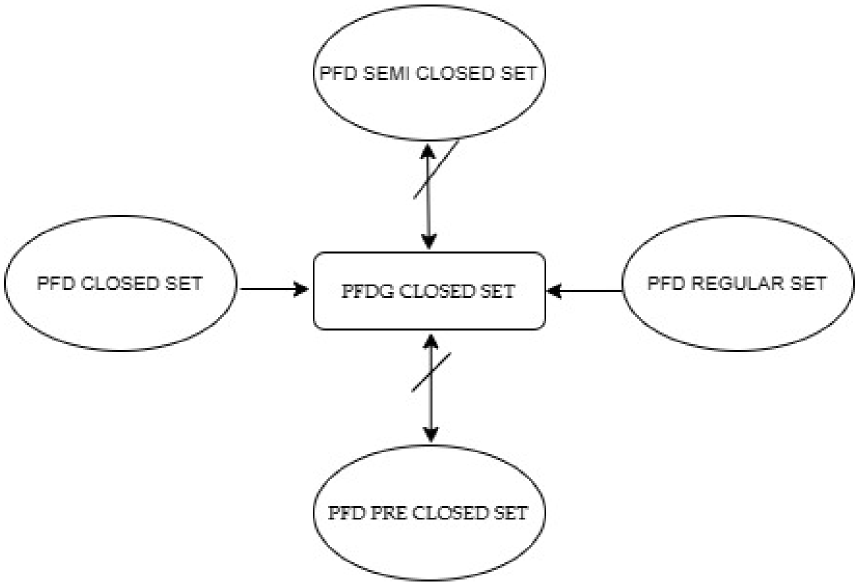

Figure 1

Example 6.6. *Let* $\Omega = \{< x, 0.6, 0.6 > /x \in \zeta\}$*;* $G_1 = \{< x, 0.5, 0.6/x \in \zeta >\}$*;* $G_2 = \{< x, 0.6, 0.3 > /x \in \zeta\}$*;*

Here $\Omega \subseteq G_2$, $\implies$ *PFD -cl(*Ω*)* $= \{< x, 0.6, 0.5 > /x \in \zeta\} \subseteq G_2$*. So* Ω *is a GPFD closed set. But PFD - int(*Ω*)* $= \{< x, 0.5, 0.6 > /x \in \zeta\}$ *and PFD - cl(PFD - int(*Ω*))* $= \{< x, 0.6, 0.5 > /x \in \zeta\} \not\subset \Omega$*. Hence* Ω *is not a PFD pre closed set. Conversely, now,* $\Omega = \{< x, 0.43, 0.5 > x \in \zeta\}$*;* $G_1 = \{< x, 0.6, 0.42 > /x \in \zeta\}$*;* $G_2 = \{< x, 0.42, 0.6 > /x \in \zeta\}$*; PFD -cl(PFD -int(*$\Omega$*))* $= \{< x, 0.42, 0.9 > /x \in \zeta\} \subset \Omega$*. Hence* Ω *is a PFD pre closed set. But PDF -cl(*Ω*)* $= \{< x, 0.9, 0.42 > /x \in \zeta\} \not\subset G_1$ *if* $\Omega \subseteq G_1$*. So* Ω *is not a PFDG closed set.*

Proposition 6.6. *Let (*ζ*,* τ*) be a PFDTS .If* Ω *is a PFD open set and PFDG closed then* Ω *is a PFD closed.*

Proof. Let $\Omega \subseteq \Omega$ and Ω is a PFDG closed set in ζ. Therefore, PFD - cl(Ω) $\subseteq \Omega$. But Ω $\subseteq$ PFD - cl(Ω) $\Rightarrow$ Ω= PFD - cl(Ω).Hence Ω is a PFD closed. □

Proposition 6.7. *In a Pythagorean fuzzy digital topological space (*ζ*,*τ*),* τ *= T ((the family of all Pythagorean fuzzy digital closed sets)) iff every Pythagorean fuzzy digital subset of (*ζ*,*τ*) is a PFDG closed set.*

Proof. Let Ω be a PFD set in (ζ,τ). Let γ be a PFD open set in (ζ,τ) such that $\Omega \subseteq$ Y. By the hypothesis, γ is PFD closed. By the definition, PFD-cl(Ω) $\subseteq \gamma$. Therefore Ω is PFDG closed. □

Proposition 6.8. *Let (*ζ*,*τ*) be a PFDTS. If* γ *is a PFDG closed set and* $Y \subseteq \Omega \subseteq$ *PFD - cl(Y), then* Ω *is a PFDG closed set.*

Proof. Let G be a PFD open set in (ζ,τ), such that $\zeta \subseteq$ G. Since Y $\subseteq \Omega$, Y $\subseteq \Omega$. So γ is a PFDG closed set and PFD-cl(Y) $\subseteq$ G. But PFD - cl(Ω) $\subseteq$ PFD - cl(Y). Because PFD - cl(Ω) $\subseteq$ PFD - cl(Y) $\subseteq$ G. Therefore Ω is a PFDG closed set. □

Proposition 6.9. *Let (*ζ*,*τ*) be a PFDTS. A PFD set* Ω *is a PFDG open set iff* $Y \subseteq$ *PFD - int(*Ω*) whenever* γ *is a PFD closed set and* $Y \subseteq \Omega$*.*

Proof. Let Ω be a PFDG open set and γ be a PFD closed set such that Y $\subseteq \Omega$. If Y $\subseteq$ Ω then $\overline{\Omega} \subseteq \overline{Y}$ and since $\overline{\Omega}$ is a PFDG closed set, then PFDG -cl($\overline{\Omega}$) $\subseteq \overline{Y}$. That is, Y= $(\overline{\overline{Y}}) \subseteq \overline{PFD - cl(\overline{\Omega})}$. But $\overline{PFD - cl(\overline{\Omega})}$ = PFD - int(Ω). Therefore Y $\subseteq$ PFD -int(Ω). Conversely, if Ω is PFD set such that Y $\subseteq$ PFD - int(Ω), whenever γ is a PFDCS and D $\subseteq$ C. Let $\overline{\Omega} \subseteq$ Y whenever γ is a PFD open set. If $\overline{\Omega} \subseteq$ Y then $\overline{Y} \subseteq \Omega$. By assumption, $\overline{Y} \subseteq$ PFD - int(Ω). That is,$\overline{PFD - int(\overline{\Omega})} \subseteq$ Y. But $\overline{PFD - int(\Omega)}$ = PFD -cl($\overline{\Omega}$). Therefore PFD -cl($\overline{\Omega}$) $\subseteq$ Y. Hence $\overline{\Omega}$ is a PFDG closed set. Therefore Ω is a PFDG open set. ☐

7. Pythagorean Fuzzy Digital generalized open set

Definition 7.1. *An PFD set Ω is said to be a Pythagorean Fuzzy Digital generalized open set (PFDGOS) in (ζ,τ) if the complement Ω^c is a PFDG closed set in ζ.*

Example 7.1. *Let G_1=$\{< x, 0.4, 0.7 > /x \in \zeta\}$ and G_2=$\{< x, 0.8, 0.3 > /x \in \zeta\}$ Consider Ω =$\{< x, 0.9, 0.3 > /x \in \zeta\}$ is said to be a Pythagorean Fuzzy Digital generalized open set (PFDGOS) in (ζ,τ) if the complement Ω^c is a PFDG closed set in (ζ, τ).*

Theorem 7.1. *For any PFDTS (ζ,τ), we have the following results:*

(i) Every PFD open set is PFDG open but not conversely.
(ii) Every PFD semi open set is PFDG open set but not conversely.
(iii) Every PFD pre open set is PFDG open set but not conversely.
Since the proof is straight forward, we will see the counter examples.

Example 7.2. *(i) Let Ω=$\{< x, 0.7, 0.6 > /x \in \zeta\}$; G=$\{< x, 0.6, 0.6 > /x \in \zeta\}$; Here Ω is a PFDG open set. But Ω is not a PFD open set.*
(ii) Let Ω=$\{< x, 0.7, 0.6 > /x \in \zeta\}$; G=$\{< x, 0.6, 0.6 > /x \in \zeta\}$; Here Ω is a PFDG open set. But Ω is not a PFD semi open set.
(iii) Let Ω=$\{< x, 0.6, 0.6 > /x \in \zeta\}$; G_1=$\{< x, 0.5, 0.6 > /x \in \zeta\}$; G_2=$\{< x, 0.6, 0.3 > /x \in \zeta\}$; Here Ω is a PFDG open set. But Ω is not a PFD pre open set.

Theorem 7.2. *Let (ζ,τ) be a PFDTS. If Ω is a PFDG open set in ζ then V $\subseteq$ PFD - int(PFD -cl(Ω)) whenever V $\subseteq \Omega$ and V is a PFD closed set in ζ.*

Proof. Let if Ω is a PFDG open set. Then Ω^c is a PFDG closed set in ζ. So PFD - cl(Ω^c) $\subseteq$ U whenever $\Omega^c \subseteq$ U and U is a PFD open set in ζ. That is PFD - cl(PFD - int(Ω^c)) $\subseteq$ U. This implies that $U^c \subseteq$ PFD - int(PFD - cl(Ω^c)) whenever $U^c \subseteq \Omega$ and U^c is PFD closed set in ζ. Replacing U^c by V, we have V $\subseteq$ PFD - int(PFD-cl(Ω)) whenever V $\subseteq \Omega$ and V is a PFD closed set in ζ. ☐

Theorem 7.3. *Let (ζ,τ) be a PFDTS. If Ω is a PFDG open set in ζ and for every γ in ζ which is a PFD set, PFD -int(Ω) $\subseteq$ Y $\subseteq \Omega$ implies γ is a PFDG open set in ζ.*

Proof. By hypothesis, $\Omega^c \subseteq Y^c \subseteq (PFD - int(\Omega))^c$. Let $Y^c \subseteq$ U and U be a PFD open set.

Since $\Omega^c \subseteq Y^c$, $Y^c \subseteq$ U. But Ω^c is a PFDG closed set, PFD - cl(Ω^c) $\subseteq$ U. Also $Y^c \subseteq$ (PFD - int(Ω))^c. So PFD - cl(Y^c) $\subseteq$ PFD -cl(Ω^c) $\subseteq$ U. Hence Y^c is a PFDG closed set which shows that Y is a PFDG open set in ζ. ☐

8. Applications of PFDG closed sets

Definition 8.1. *A PFDTS (ζ,τ) is said to be a PFDG $T_{1/2}$ space ($PFDGT_{1/2}$) space if every PFDGCS in ζ is a PFDCS in ζ.*

Proposition 8.1. *Let (ζ,τ) be a $PFDGT_{1/2}$ space. Then*

(i) Any union of PFDG closed set is a PFDG closed set in ζ.
(ii) Any intersection of PFDG open set is a PFDG open set in ζ.

Proof. (i) Let $\{\Omega_i\}$ be the collection of PFDG closed set in ζ. Since (ζ,τ) is a $PFDGT_{1/2}$ space, every PFDG closed set is a PFD closed set. And hence each Ω_i,i $\in$ J is a PFDCS in (ζ,τ). But any union of PFD closed set is a PFD closed set,$\cup\Omega_i \in J$ for every i $\in$ J is a PFD closed set. Since every PFD closed is a PFDG closed set, $\cup\Omega_i$ is also a PFDG closed set in ζ.
(ii) can be proved by taking complement in (i).

□

Proposition 8.2. *If Ω is both a PFD open set and a PFDG closed set in ζ and if ζ is a $PFDGT_{1/2}$ space, then*
(i) Ω is a PFD regular open set.
(ii) Ω is a PFD regular closed set.

Proof. Let Ω be a PFDG closed set in ζ. Then by definition, ζ is a PFD closed set in ζ. Now
(i) PFD - int(PFD - cl(Ω))=PFD-int(Ω)= Ω and so Ω is a PFD regular open set in ζ.
(ii) PFD - cl(PFD - int(Ω))= PFD-cl(Ω)= Ω and so Ω is a PFD regular close set in ζ.

□

9. Conclusion

Digital topology and geometry are valuable in real-life medical imaging applications, benefiting clinical and research studies. The Pythagorean fuzzy set (PFS) effectively handles impreciseness and vagueness in decision-making. This paper introduces Pythagorean fuzzy digital convex sets, Pythagorean fuzzy digital cut sets, Pythagorean fuzzy digital topological spaces, extending the concept of Pythagorean fuzzy digital generalized closed and open sets, and explores their properties. These concepts are fundamental for future research and open doors for advancing applications in Pythagorean fuzzy digital topology.

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