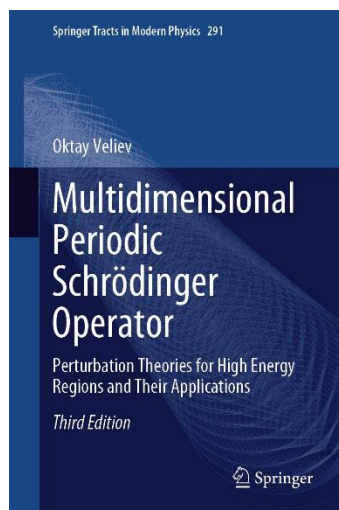


## Book Review



**Oktay Veliev. Multidimensional Periodic Schrödinger Operator (Perturbation Theories in High Energy Regions and Their Applications). Springer, Switzerland, 424 pp., 2024, Springer Tracts in Modern Physics, STMP, Vol. 291.**

This monograph is devoted to the spectral theory of multidimensional Schrödinger operators with periodic potentials, a class of operators that plays a central role in solid-state physics by describing the motion of quantum particles in crystalline media.

A major contribution of the book is the development, for the first time, of a systematic classification of large Bloch eigenvalues into two distinct groups, together with the derivation of asymptotic formulas for each class. On the basis of these asymptotic expansions, the author constructs sets of quasimomenta for which the corresponding Bloch eigenvalues are simple and derives asymptotic representations for the associated Bloch functions. These results provide the foundation for a new perturbation theory for multidimensional Schrödinger operators with periodic potentials.

Using the developed framework, the author determines and estimates the measure of isoenergetic surfaces in the high-energy region, thereby establishing the validity of the Bethe–Sommerfeld conjecture. Furthermore, the monograph introduces new spectral invariants that enable the constructive determination of periodic potentials from Bloch eigenvalues and presents an explicit algorithm for the unique reconstruction of the potential.

The book offers a systematic and comprehensive exposition of both direct and inverse spectral problems for multidimensional periodic Schrödinger operators. Owing to the breadth of the material, the mathematical rigor of the presentation, and the originality of the methods developed, the monograph constitutes a valuable resource for graduate students, researchers, and specialists working in spectral theory, mathematical physics, quantum mechanics, and related areas of analysis.

**Prof. Elman HASANOGLU**