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ISOLATE DOMINATION IN THE CLASS OF UNICYCLIC GRAPHS

S. K. VAIDYA[1,*], J. B. KELAIYA[2], §

Abstract. A subset D of the vertex set V of a graph G is called a dominating set of G if every vertex in $V - D$ is adjacent to a vertex in D. A dominating set D such that $< D >$ has an isolated vertex is called an isolate dominating set and the minimum cardinality of an isolate dominating set is called the isolate domination number of G and is denoted by $\gamma_0(G)$. In this work, we investigate an isolate domination number for unicyclic graphs in the context of various transformations.

Keywords: Dominating set, isolate dominating set, isolate domination number.

AMS Subject Classification: 05C69.

1. Introduction

In graph theory, the concept of domination plays a crucial role in understanding the structure and characteristics of networks. A dominating set in a graph $G = (V, E)$ is a subset of vertices $D \subseteq V$ such that every vertex in V is either in D or adjacent to at least one vertex in D. This concept provides a powerful tool for analyzing various properties of graphs.

Domination problems arise in numerous practical contexts, such as network design, resource allocation, and social network analysis. For instance, in a communication network, a dominating set can represent a set of strategically placed nodes that ensure complete coverage of the network. In social networks, it can model influential individuals whose connections can affect the entire community.

The present work delves into the theoretical aspects of domination in graph theory, focusing on the properties and applications of dominating sets. Various types of domination, such as total domination, connected domination, independent domination and isolate

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domination are there in the existing literature.

The concept of isolate domination was introduced by Sahul Hamid and Balamurugan in [1, 2]. A dominating set D is said to be an isolate dominating set if the subgraph induced by D, that is, $< D >$ has at least one isolated vertex. The minimum cardinality of an isolate dominating set is called the isolated domination number $\gamma_0(G)$.

Isolate dominating sets have practical applications in various fields where efficient control, monitoring, and communication are essential. In security and surveillance systems, they help determine optimal placements for guards or sensors, ensuring every location is monitored directly or via adjacency, while unmonitored zones remain isolated. This reduces the risk of unauthorized activity in unobserved areas and is especially beneficial in high-security environments.

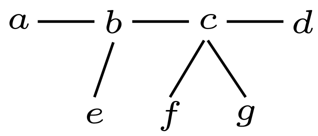


FIGURE 1. A Graph G

Consider a connected graph G as shown in FIGURE 1, where the set $D = \{b, c\}$ and the set $D' = \{a, c, e\}$ forms a dominating set and isolate dominating set respectively. Therefore, $\gamma(G) = 2$ while $\gamma_0(G) = 3$. Moreover, each node not in D' (i.e., a, c, e) is adjacent to at least one node in D', and there are no connections between nodes in D', satisfying both the domination and isolation conditions. A practical real-life application of this structure appears in hierarchical cybersecurity systems for corporate networks. In such a model, nodes in D' represent secure monitoring or control hubs, each responsible for overseeing specific operational nodes (a, c, e). These operational nodes do not communicate with each other, reducing the risk of lateral movement in the event of a cyber breach. If one node is compromised, the attacker cannot spread to others, as they are isolated and only interface with their respective secure hub. This architecture enhances threat containment, simplifies monitoring, and supports strong fault tolerance. Thus, the isolate domination number provides a refined perspective in various graph classes.

Through a detailed analysis of existing research and application scenarios, this paper aims to provide a comprehensive understanding of how isolate domination can be utilized to address the challenges in graph based models. Here, we investigate the isolate domination number of unicyclic graphs in the context of various graph transformations. Throughout the discussion, we consider finite, simple and connected graph G and for any undefined terms related to graph theory and domination we refer to West [6] and Haynes *et al.* [3, 4] respectively.

2. Preliminaries

Manian *et al.* have introduced various graph transformations in [5] which are as follows.

Definition 2.1. ***Pendant-Path (PP) transformation:***

Let G be a nontrivial connected graph and $u, v \in V(G)$, not necessarily different, such that $d(v) \geq 3$ and $P_1 : uu_1u_2 \dots u_s$ and $P_2 : vv_1v_2 \dots v_t$ be two paths in G that hang on u and v. Then $\overline{G}$ is the graph obtained from G in the way that P_1 and P_2 are being catenated, so that the tail $v_1v_2 \dots v_t$ of path P_2 is attached to u_s. Therefore the path $uu_1 \dots u_sv_1 \dots v_t$ in $\overline{G}$ as shown in Figure 2.

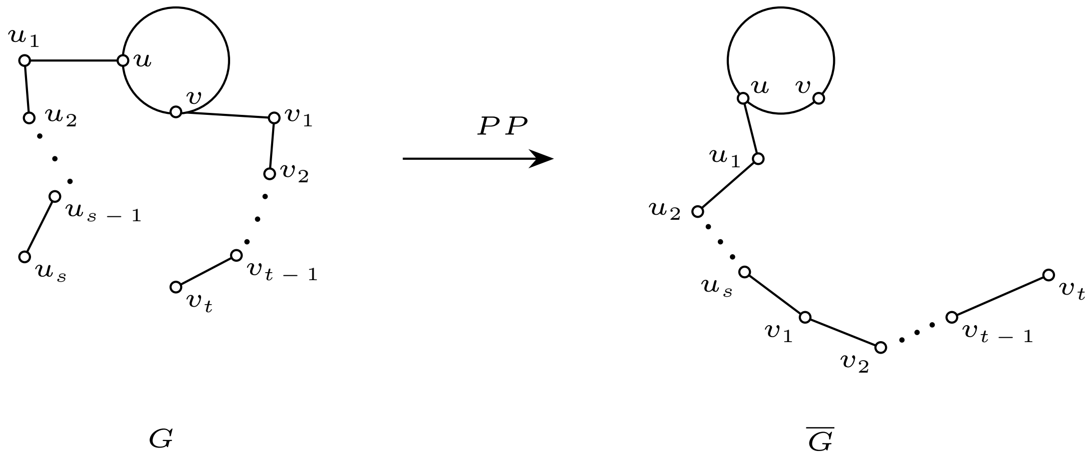


Figure 2. PP transformation

Definition 2.2. ***Contraction to Path (CP) transformation:***

Let G be a connected graph and $u \in V(G)$, $d(u) \geq 3$ and $P : uu_1u_2 \dots u_s$ be path in G of length $s \geq 1$. In addition, let v be a neighbor of u so that v is out of P, u and v have no common neighbor and $d(v) \geq 2$. Then $\overline{G}$ is the graph created from $G.uv$ by contracting an edge uv and subdividing an edge $u_{s-1}u_s$ with a vertex v as shown in Figure 3.

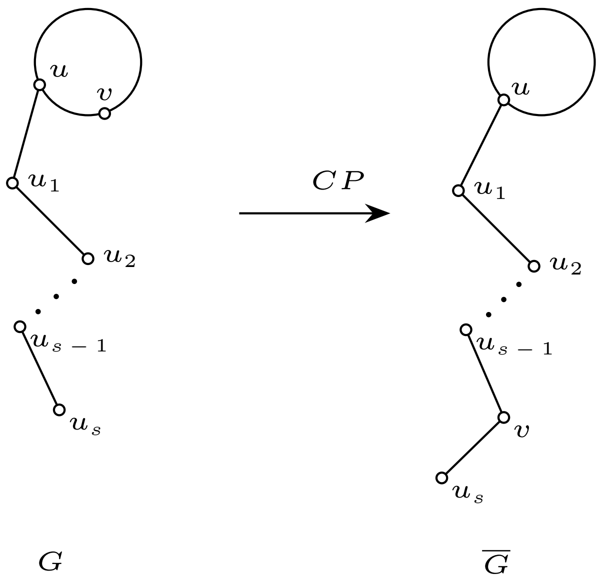


FIGURE 3. CP transformation

Definition 2.3. ***Comet ($CO_{n,i}$):***

For $n > i$, the comet $CO_{n,i}$ is a unicyclic graph with the cycle C_i on which appended a path on length $n - i$ as shown in FIGURE 4.

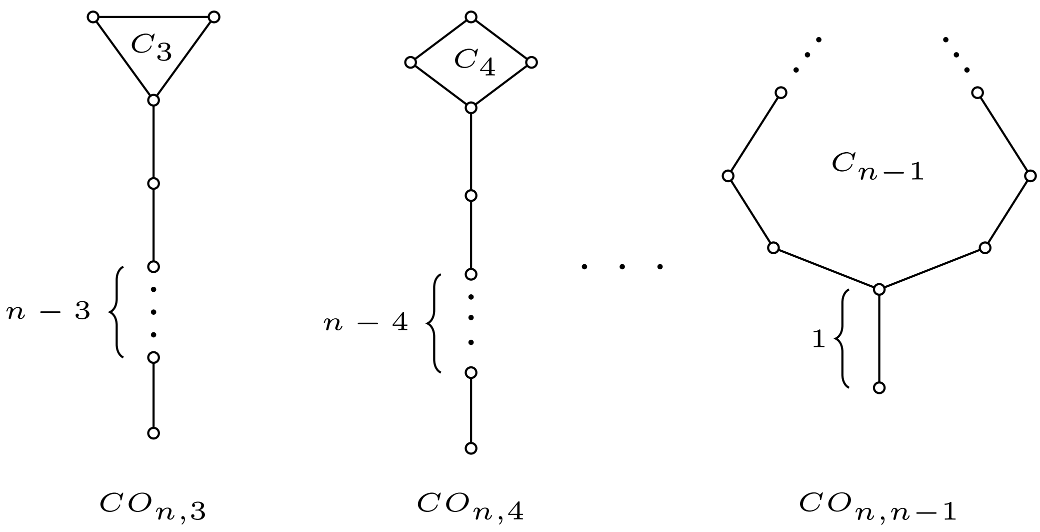


FIGURE 4. Comet

Definition 2.4. ***Contraction to Star (CS) transformation:***

Let u and v be adjacent vertices of the graph G that have no common neighbor in G, $d(u) = m$ and $d(v) = n$, where $m \geq n \geq 2$. If $e = uv$, then $\overline{G} = (G.e) + uv$. In fact, $\overline{G}$ is obtained by the contraction of the edge $e = uv$ onto the vertex u and then appending a pendant vertex v to u as shown in FIGURE 5.

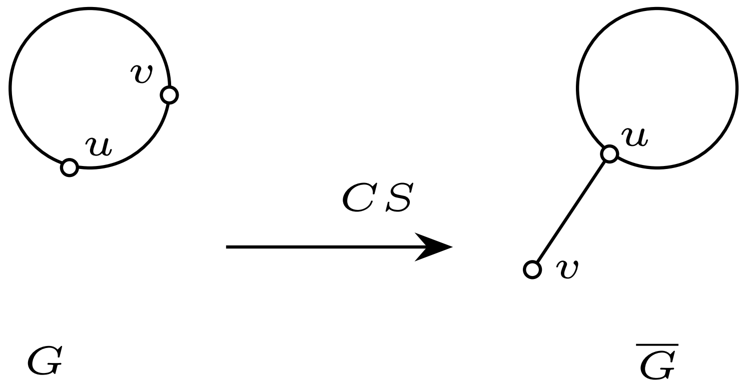


FIGURE 5. CS transformation

Definition 2.5. ***Star Translation (ST) transformation:***

Let $v \in V(G)$ and $v_1, v_2, \ldots, v_t$ be pendant vertices and neighbors of vertex v. Let $u \in V(G) \setminus \{v_1, v_2, \ldots, v_t\}$. Then $\overline{G} = (G - \{vv_1, vv_2, \ldots, vv_t\}) + \{uv_1, uv_2, \ldots, uv_t\}$. In fact $\overline{G}$ is obtained by moving the star $G[v_1, v_2, \ldots, v_t]$ from vertex v to u as shown in FIGURE 6.

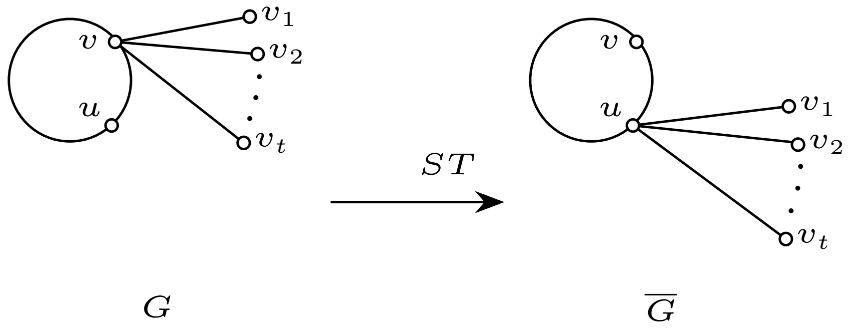


FIGURE 6. ST transformation

Definition 2.6. ***Graph C_n^k:***

Let C_n^k be the unicyclic graph with n vertices, whose cycle C_k contains a vertex adjacent to $n - k$ pendant vertices as shown in FIGURE 7.

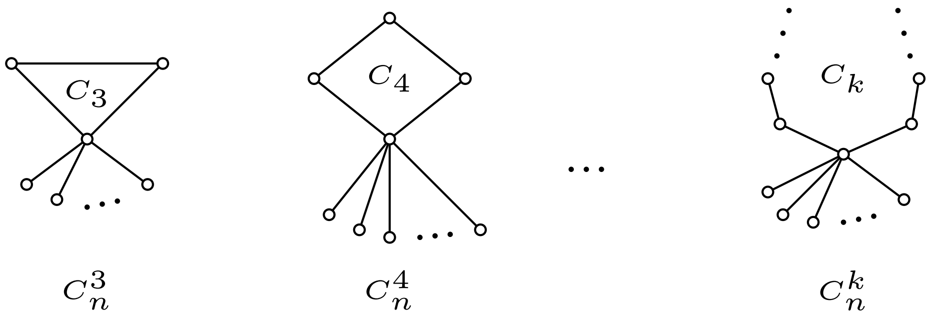


FIGURE 7. C_n^k

3. Main Results

Theorem 3.1. *Let C_n be the cycle on n vertices and $u, v \in V(C_n)$, such that u and v are neighbors and $d(u) = d(v) = 3$. Let $P_1 : uu_1u_2 \dots u_s$ and $P_2 : vv_1v_2 \dots v_t$ be two paths that hang on u and v, denote the new graph as G. Let $\overline{G}$ be the PP transformation of G. Then, either $\gamma_0(\overline{G}) = \gamma_0(G)$ or $\gamma_0(\overline{G}) = \gamma_0(G) - 1$.*

Proof. From the selection process of vertices for the dominating sets of cycles C_n and paths P_n, it is evident that every dominating set contains at least one isolate vertex. In our case, the graph G is simply a combination of a cycle C_n and paths P_n. Therefore, $\gamma(G) = \gamma_0(G)$. So it is enough to prove that $\gamma(\overline{G}) = \gamma(G)$ or $\gamma(\overline{G}) = \gamma(G) - 1$.

By the definitions of the graph G and its pendant path transformation $\overline{G}$, it follows that G consists of the union of P_s, C_n, and P_t, while $\overline{G}$ consists of the union of P_{s+t} and C_n. Consequently, the domination numbers satisfy

$$\gamma(G) \leq \left\lceil \frac{s}{3} \right\rceil + \left\lceil \frac{n}{3} \right\rceil + \left\lceil \frac{t}{3} \right\rceil \quad \text{and} \quad \gamma(\overline{G}) \leq \left\lceil \frac{s+t}{3} \right\rceil + \left\lceil \frac{n}{3} \right\rceil.$$

Therefore, from the properties of the ceiling function we have $\gamma(\overline{G}) \leq \gamma(G)$.

Next, we show that $\gamma(\overline{G}) \geq \gamma(G) - 1$. Consider D be a dominating set of G and $|D| = k$. Therefore, $\gamma(G) = k$. Now suppose, if possible, $\gamma(\overline{G}) = k - 2$. Let $\overline{D}$ be a dominating set of $\overline{G}$, then $|\overline{D}| < k - 1$. Observe that if $u_s \in \overline{D}$, then each vertex of G is either in $\overline{D} \cup \{v_1\}$ or has a neighbor in $\overline{D} \cup \{v_1\}$ and if $u_s \notin \overline{D}$, then each vertex of G is either in $\overline{D} \cup \{u_s\}$ or has a neighbor in $\overline{D} \cup \{u_s\}$. So, $\gamma(G) \leq k - 1 < k$, which is a contradiction. Hence, $\gamma(\overline{G}) = k - 1$. As $\gamma(G) = \gamma_0(G)$, we have either $\gamma_0(\overline{G}) = \gamma_0(G)$ or $\gamma_0(\overline{G}) = \gamma_0(G) - 1$. □

Illustration 3.1. *In* FIGURE *8, the graph G and its PP transformation $\overline{G}$ are shown in which the sets of solid vertices are the isolate dominating sets of respective graphs. Here, $\gamma_0(\overline{G}) = \gamma_0(G)$.*

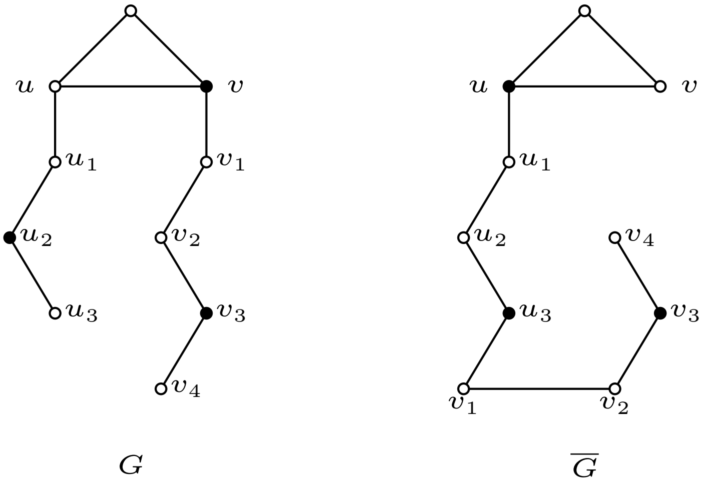


FIGURE 8. A graph G with $\gamma_0(\overline{G}) = \gamma_0(G)$

Illustration 3.2. *In* FIGURE *9, the graph G and its PP transformation $\overline{G}$ are shown in which the sets of solid vertices are the isolate dominating sets of respective graphs. Here, $\gamma_0(\overline{G}) = \gamma_0(G) - 1$.*

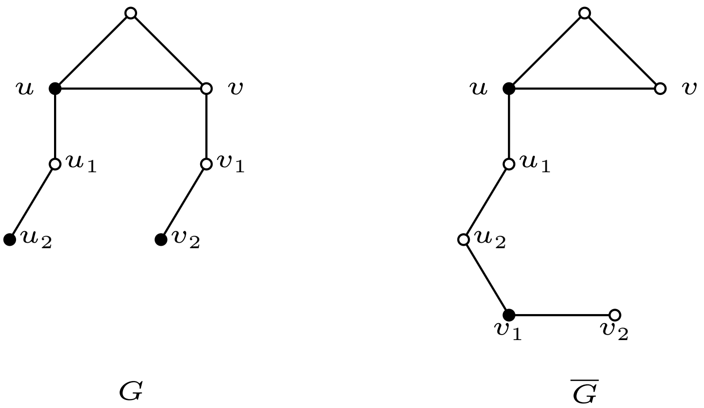


FIGURE 9. A graph G with $\gamma_0(\overline{G}) = \gamma_0(G) - 1$

Theorem 3.2. *Let C_n be the cycle on $n \geq 4$ vertices, $u \in V(C_n)$, $d(u) = 3$ and $P : uu_1u_2 \dots u_s$ be a path of length $s > 1$ that hang on u. In addition, let $v \in V(C_n)$ be a neighbor of u so that v is out of P, u and v have no common neighbor and $d(v) = 2$, denote the graph as G and let $\overline{G}$ be the CP transformation of G. Then, we have the following*

(i) $\gamma_0(G) - 1 \leq \gamma_0(\overline{G}) \leq \gamma_0(G) + 1$

(ii) $$\gamma_0(G) = \begin{cases} \left\lceil \frac{s}{3} \right\rceil + \gamma_0(C_n) - 1 \ ; \ s = 3k+1, \ k \in \mathbb{N} \\ \\ \left\lceil \frac{s}{3} \right\rceil + \gamma_0(C_n) \ ; \text{otherwise} \end{cases}$$

Proof. From the selection process of vertices for the dominating sets of cycles C_n and paths P_n, it is evident that every dominating set contains at least one isolate vertex. In our case, the graph G is simply a combination of a cycle C_n and a path P_n. Therefore, $\gamma(G) = \gamma_0(G)$. So, for the proof of the assertion (i), it is enough to prove $\gamma(G) - 1 \leq \gamma(\overline{G}) \leq \gamma(G) + 1$.

Let D be a dominating set of G. Observe that if $u_{s-1} \in D$, then each vertex of $\overline{G}$ is either in $D \cup \{u_s\}$ or has neighbor in $D \cup \{u_s\}$ as after subdivision, the new vertex v and possibly u_s may not be dominated by D, because u_{s-1} is no longer directly connected to u_s. So to ensure domination of the whole graph $\overline{G}$, we need to add at least one more vertex, either v or u_s. This means the new dominating set for $\overline{G}$ is at most $D \cup \{u_s\}$, and has size at most $k+1$. By similar arguments, if $u_{s-1} \notin D$, then each vertex of $\overline{G}$ is either in $D \cup \{u_s\}$ or has a neighbor in $D \cup \{u_s\}$. This gives $\gamma(\overline{G}) \leq k+1$.

Suppose, if possible, $\gamma(\overline{G}) < k-1$. Let $\overline{D}$ be a dominating set of $\overline{G}$. Then $|\overline{D}| < k-1$. Observe that if $v \in \overline{D}$, then each vertex of G is either in $\overline{D} \cup \{u_s\}$ or has a neighbor in $\overline{D} \cup \{u_s\}$. If $v \notin \overline{D}$, then each vertex of G is either in $\overline{D} \cup \{v\}$ or has neighbor in $\overline{D} \cup \{v\}$. So, $\gamma(G) \leq k-1 < k$, which is a contradiction. So, $\gamma(\overline{G}) \geq k-1$.

Thus, $\gamma(G) - 1 \leq \gamma(\overline{G}) \leq \gamma(G) + 1$. As, $\gamma(G) = \gamma_0(G)$. We have, $\gamma_0(G) - 1 \leq \gamma_0(\overline{G}) \leq \gamma_0(G) + 1$.

Next, we discuss the proof of the assertion (ii) with the following cases:

Case 1. If $s = 3k$ for some $k \in \mathbb{N}$, then

$$\begin{aligned}
\gamma_0(G) = \gamma(G) &= \gamma(C_n) + \left\lceil \frac{s-1}{3} \right\rceil \\
&= \gamma_0(C_n) + \left\lceil \frac{3k-1}{3} \right\rceil \\
&= \gamma_0(C_n) + k \\
&= \gamma_0(C_n) + \left\lceil \frac{3k}{3} \right\rceil \\
&= \gamma_0(C_n) + \left\lceil \frac{s}{3} \right\rceil .
\end{aligned}$$

Case 2. If $s = 3k+1$ for some $k \in \mathbb{N}$, then

$$\begin{aligned}
\gamma_0(G) = \gamma(G) &= \gamma(C_n) + \left\lceil \frac{s-1}{3} \right\rceil \\
&= \gamma_0(C_n) + \left\lceil \frac{3k}{3} \right\rceil \\
&= \gamma_0(C_n) + k \\
&= \gamma_0(C_n) + \left\lceil \frac{s}{3} \right\rceil - 1.
\end{aligned}$$

Case 3. If $s = 3k+2$ for some $k \in \mathbb{N}$, then

$$\begin{aligned}
\gamma_0(G) = \gamma(G) &= \gamma(C_n) + \left\lceil \frac{s-1}{3} \right\rceil \\
&= \gamma_0(C_n) + \left\lceil \frac{3k+1}{3} \right\rceil \\
&= \gamma_0(C_n) + k + 1 \\
&= \gamma_0(C_n) + \left\lceil \frac{s}{3} \right\rceil .
\end{aligned}$$

□

Illustration 3.3. *In* FIGURE *10, the graph G and its CP transformation $\overline{G}$ are shown in which the sets of solid vertices are the isolate dominating sets of respective graphs. Here, $\gamma_0(\overline{G}) = \gamma_0(G) - 1$.*

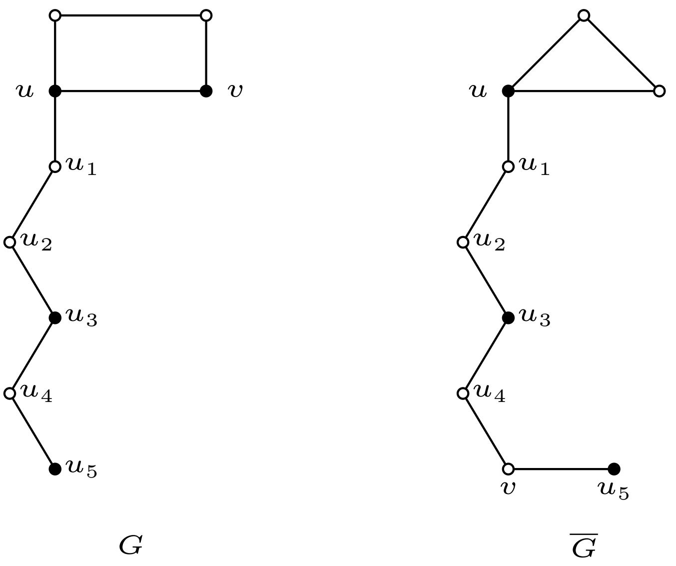


FIGURE 10. A graph G with $\gamma_0(\overline{G}) = \gamma_0(G) - 1$

Illustration 3.4. *In* FIGURE *11, the graph G and its CP transformation $\overline{G}$ are shown in which the sets of solid vertices are the isolate dominating sets of respective graphs. Here, $\gamma_0(\overline{G}) = \gamma_0(G)$.*

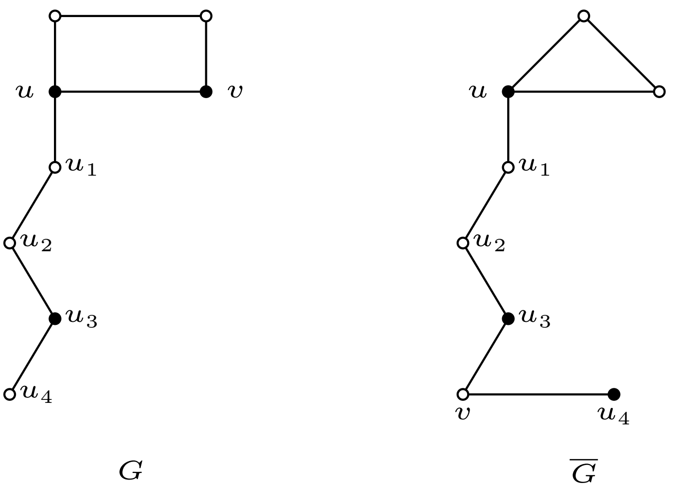


FIGURE 11. A graph G with $\gamma_0(\overline{G}) = \gamma_0(G)$

Illustration 3.5. *In* FIGURE *12, the graph G and its CP transformation $\overline{G}$ are shown in which the sets of solid vertices are the isolate dominating sets of respective graphs. Here, $\gamma_0(\overline{G}) = \gamma_0(G) + 1$.*

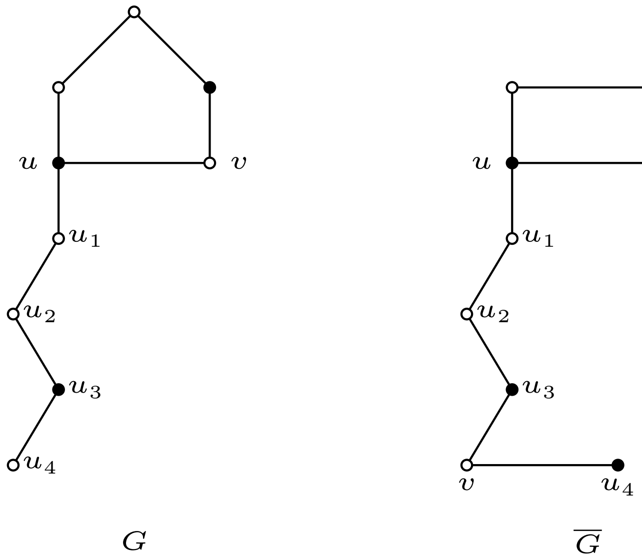


FIGURE 12. A graph G with $\gamma_0(\overline{G}) = \gamma_0(G) + 1$

Theorem 3.3. *For Comets $CO_{n,i}$,*

$$\gamma_0(CO_{n,i}) = \begin{cases} \gamma_0(C_i) \ ; \ n-i=1 \\ \left\lceil \dfrac{n-i}{3} \right\rceil + \gamma_0(C_i) - 1 \ ; \ n-i = 3k+1, \ \textit{for } k \in \mathbb{N} \\ \left\lceil \dfrac{n-i}{3} \right\rceil + \gamma_0(C_i) \ ; \ \text{otherwise} \end{cases}$$

Proof. Observe that when $n - i = 1$, the path appended on the cycle C_i is of length 1. Therefore, any isolate dominating set of C_i is also an isolate dominating set for $CO_{n,i}$. Thus, $\gamma_0(CO_{n,i}) = \gamma_0(C_i)$, when $n - i = 1$.

The proof of remaining cases is analogous to the proof of assertion (ii) of Theorem 3.2. □

Illustration 3.6. *In* FIGURE *13, the graph $CO_{8,4}$ is shown in which the set of solid vertices is an isolate dominating set. Here, $\gamma_0(CO_{8,4}) = \left\lceil \dfrac{8-4}{3} \right\rceil + \gamma_0(C_4) - 1 = 3$.*

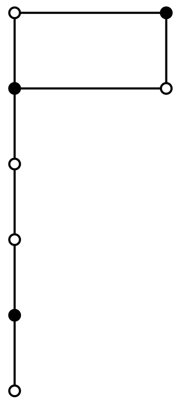


FIGURE 13. A graph $CO_{8,4}$ with $\gamma_0(CO_{8,4}) = 3$

Illustration 3.7. *In FIGURE 14, the graph $CO_{8,6}$ is shown in which the set of solid vertices is an isolate dominating set. Here,* $\gamma_0(CO_{8,6}) = \left\lceil \frac{8-6}{3} \right\rceil + \gamma_0(C_6) = 3$.

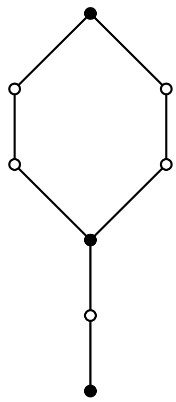


FIGURE 14. A graph $CO_{8,6}$ with $\gamma_0(CO_{8,6}) = 3$

Theorem 3.4. *Let G be a unicyclic graph with cycle $C_n (n \geq 4)$, u and v be adjacent vertices of C_n that have no common neighbor, $d(u) \geq 2$ and $d(v) = 2$, $d(w) = 2$ where $w \in V(C_n) \setminus \{u, v\}$ and $u_1, u_2, \ldots, u_t$ where $t \geq 1$ are pendant vertices appended to u. Let $\overline{G}$ be the CS transformation of G. Then, either $\gamma_0(\overline{G}) = \gamma_0(G) - 1$ or $\gamma_0(\overline{G}) = \gamma_0(G)$.*

Proof. Consider a unicyclic graph G containing a cycle C_n with $n \geq 4$. Let u and v be adjacent vertices of C_n such that they have no common neighbor, with $d(u) \geq 2$ and $d(v) = 2$. Additionally, let $w \in V(C_n) \setminus \{u, v\}$ be any other vertex of C_n with $d(w) = 2$. Suppose $u_1, u_2, \ldots, u_t$ (where $t \geq 1$) are pendant vertices appended to u.

Under these conditions, the isolate domination number of G satisfies

$$\gamma_0(G) = \gamma(G) = \gamma(C_n) = \left\lceil \frac{n}{3} \right\rceil.$$

Now, let $\overline{G}$ be the graph obtained from G by applying the CS transformation. Then,

$$\gamma_0(\overline{G}) = \gamma(\overline{G}) = \gamma(C_{n-1}) = \left\lceil \frac{n-1}{3} \right\rceil.$$

Thus, either $\gamma_0(\overline{G}) = \gamma_0(G) - 1$ or $\gamma_0(\overline{G}) = \gamma_0(G)$. □

Illustration 3.8. *In* FIGURE *15, the graph G and its CS transformation $\overline{G}$ are shown in which the sets of solid vertices are the isolate dominating sets of respective graphs. Here, $\gamma_0(\overline{G}) = \gamma_0(G) - 1$.*

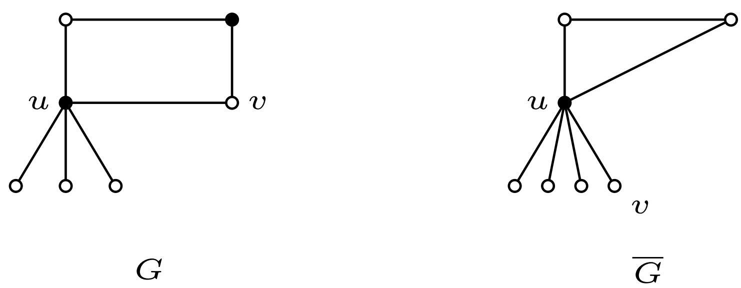


FIGURE 15. A graph G with $\gamma_0(\overline{G}) = \gamma_0(G) - 1$

Illustration 3.9. *In* FIGURE *16, the graph G and its CS transformation $\overline{G}$ are shown in which the sets of solid vertices are the isolate dominating sets of respective graphs. Here, $\gamma_0(\overline{G}) = \gamma_0(G)$.*

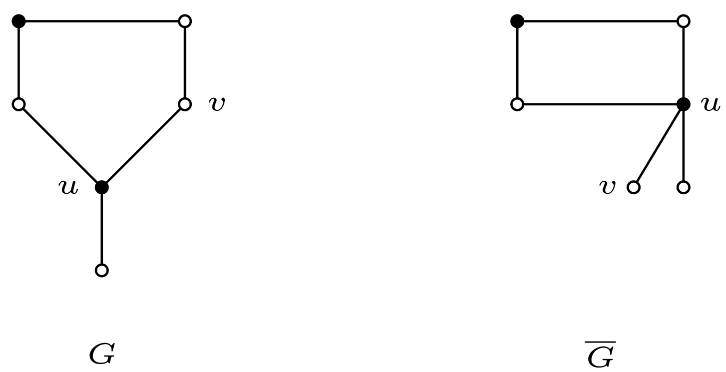


FIGURE 16. A graph G with $\gamma_0(\overline{G}) = \gamma_0(G)$

Theorem 3.5. *Let G be a unicyclic graph with cycle C_n, $v \in V(C_n)$ and $v_1, v_2, \ldots, v_t$ be pendant vertices and neighbors of vertex v. For any $w \in V(C_n) \setminus \{v\}$, $d(w) = 2$. Let $\overline{G}$ be the ST transformation of G. Then, $\gamma_0(\overline{G}) = \gamma_0(G)$.*

Proof. Let G be a unicyclic graph containing a cycle C_n, and let $v \in V(C_n)$ be a vertex with pendant neighbors $v_1, v_2, \ldots, v_t$. For any $w \in V(C_n) \setminus \{v\}$, we have $d(w) = 2$. Then, the isolate domination number of G satisfies

$$\gamma_0(G) = \gamma(G) = \gamma(C_n) = \left\lceil \frac{n}{3} \right\rceil.$$

Now, let $\overline{G}$ be the graph obtained by applying the ST transformation to G. Since the structure of dominating set remains unchanged, we again have

$$\gamma_0(\overline{G}) = \gamma(\overline{G}) = \gamma(C_n) = \left\lceil \frac{n}{3} \right\rceil.$$

Thus, we conclude that $\gamma_0(\overline{G}) = \gamma_0(G)$. □

Illustration 3.10. *In* FIGURE *17, the graph G and its ST transformation $\overline{G}$ are shown in which the sets of solid vertices are the isolate dominating sets of respective graphs. Here, $\gamma_0(\overline{G}) = \gamma_0(G)$.*

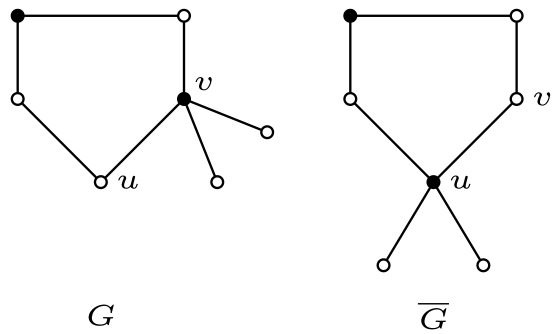


FIGURE 17. A graph G with $\gamma_0(\overline{G}) = \gamma_0(G)$

Theorem 3.6. *For every $n \geq 3$,*

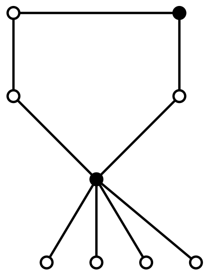
$$\gamma_0(C_n^k) = \gamma_0(C_k) = \left\lceil \frac{k}{3} \right\rceil.$$

Proof. The graph C_n^k contains the cycle C_k adjacent to $n-k$ pendant vertices. From the structure of the graph C_n^k, it is clear that any isolate dominating set of C_n^k has same number of vertices as the isolate dominating set of C_k. Thus,

$$\gamma_0(C_n^k) = \gamma_0(C_k) = \left\lceil \frac{k}{3} \right\rceil.$$

□

Illustration 3.11. *In* FIGURE *18, the graph C_9^5 is shown in which the set of solid vertices is an isolate dominating set. Here, $\gamma_0(C_9^5) = \left\lceil \frac{5}{3} \right\rceil = 2$.*



C_9^5

FIGURE 18. A graph C_9^5 with $\gamma_0(C_9^5) = 2$

4. Concluding Remarks

The present study has elucidated the properties of isolate domination in unicyclic graphs through the lens of various graph transformations. By analyzing how these transformations impact $\gamma_0(G)$, we have gained a clear understanding of the structural nuances within unicyclic graphs. This research not only contributes to the theoretical framework surrounding graph domination but also sets the stage for further investigation into more complex graph classes. Future work could build on these findings to explore additional graph properties and their interrelations.

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