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APPROXIMATE CONTROLLABILITY OF SEMILINEAR CONTROL SYSTEMS IN HILBERT SPACES

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Abstract. This paper deals with the approximate controllability of semilinear evolution systems in Hilbert spaces. Sufficient condition for approximate controllability have been obtained under natural conditions.

Keywords: Controllability, stochastic systems, fixed point.

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1. Introduction

We are given a probability space $(\Omega, \Im, P)$ together with a normal filtration $(\Im_t)_{t\geq 0}$. We consider three real separable spaces K, X and U, and Q-Wiener process on $(\Omega, \Im, P)$ with covariance linear bounded operator Q such that tr$Q < \infty$. We assume that there exists a complete orthonormal system $\{e_k\}_{k\geq 1}$ in K, a bounded sequence of nonnegative real numbers λ_k such that $Qe_k = \lambda_k e_k,\ \ k = 1, 2, ...$, and a sequence $\{\beta_k\}_{k\geq 1}$ of independent Brownian motions such that

$$\langle w\left(t\right), e\rangle = \sum_{k=1}^{\infty} \sqrt{\lambda_k} \left\langle e_k, e\right\rangle \beta_k\left(t\right),\ e \in K,\ t \in [0, b],$$

and $\Im_t = \Im_t^w$, where $\Im_t^w$ is the sigma algebra generated by $\{w\left(s\right) : 0 \leq s \leq t\}$. Let $L_2^0 = L_2\left(Q^{1/2}K; X\right)$ be the space of all Hilbert-Schmidt operators from $Q^{1/2}K$ to X with the inner product $\langle \psi, \phi\rangle_{L_2^0} = tr\left[\psi Q\phi\right]$. $L^p\left(\Im_b, X\right)$ is the Banach space of all $\Im_b-$measurable square integrable variables with values in X. $L_{\Im}^p(0, b; X)$ is the Banach space of all p-square integrable and $\Im_t-$adapted processes with values in X. Let $C\left(0, b; L^p\left(\Im, X\right)\right)$ be the Banach space of continuous maps from $[0, b]$ into $L^p\left(\Im, X\right)$ satisfying the condition $\sup\left\{\mathbf{E}\left\|\varphi\left(t\right)\right\|^p : t \in [0, b]\right\} < \infty$. $\mathfrak{C}_p\left(0, b; X\right)$ is the closed subspace of $C\left(0, b; L^p\left(\Im, X\right)\right)$ consisting of measurable and $\Im_t-$adapted X-valued processes $\varphi \in C\left(0, b; L^p\left(\Im, X\right)\right)$ endowed with the norm $\left\|\varphi\right\|_{\mathfrak{C}_p} = \left(\sup\limits_{0\leq t\leq b} \mathbf{E}\left\|\varphi(t)\right\|_X^p\right)^{\frac{1}{p}}$.

Abstract semilinear differential equation serves as a formulation for many control systems described by partial or functional differential equations.Controllability theory for

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abstract linear control systems in infinite-dimensional spaces is well-developed, and extensively investigated in the literature, see [1], [6], [17] and [23] and the references therein. Several authors have extended controllability concepts to infinite-dimensional systems represented by nonlinear evolution equations. The approximate controllability for the systems of differential equations has been investigated by several authors, see for instance [2]- [24].

This paper is devoted to the approximate controllability problems of the following semilinear control system

$$\begin{cases} dy(t) = [Ay(t) + (Bu)(t) + f(t, y(t))]\, dt + \int_0^t \sigma(r, y(r))\, dw(r), \\ y(0) = \xi, \quad 0 \le t \le b, \end{cases} \tag{1}$$

in a real Hilbert space $(X, \|\cdot\|)$. The meaning of all notations are listed in the following: A is the infinitesimal generator of a C_0-semigroup $\{S(t) : t \ge 0\}$, $u \in L^2_{\Im}(0, b; U)$ is a control function, U is a Hilbert space, B is a linear bounded operator from $L^2_{\Im}(0, b; U)$ to $L^2_{\Im}(0, b; X)$, $f : [0, b] \times X \to X$, $\sigma : [0, b] \times X \to L_2^0$.

Denote the solution of (1) corresponding to a control u by $y(\cdot; u)$. Then $y(b; u)$ is the state value at the terminal time b. Introduce the set

$$R_b(f) = \left\{ y(b; u) : u \in L^2_{\Im}(0, b; U) \right\},$$

which is called the reachable set of system (1) at terminal time b, its closure in $L^2(\Im_b, X)$ is denoted by $\overline{R_b(f)}$.

Definition 1. *System (1) is said to be approximately controllable on* $[0, b]$ *if* $\overline{R_b(f)} = L^2(\Im_b, X)$.

2. Assumptions

Throughout the paper we impose the following assumptions:

: (A1) $(f, \sigma) : [0, b] \times X \to X \times L_2^0$ is locally Lipschitz continuous in y uniformly in $t \in [0, b]$: there exists a constant $L > 0$ such that

$$\|f(t, y_1) - f(t, y_2)\| + \|\sigma(t, y_1) - \sigma(t, y_2)\|_{L_2^0} \le L \|y_1 - y_2\|$$

for any $t \in [0, b]$.

: (A2) There exists $L_1 > 0$ such that for all $(t, y) \in [0, b] \times X$

$$\|f(t, y)\| + \|\sigma(t, y)\|_{L_2^0} \le L_1 (1 + \|y\|)$$

: (A3) For any $p \in L^2_{\Im}(0, b; X)$, there exists a function $q \in \overline{\mathrm{Im}(B)}$ such that $\Xi p = \Xi q$, where $\Xi : L^2_{\Im}(0, b; X) \to L_2^0$ is defined as follows

$$\Xi p = \int_0^b S(b - s)\, p(s)\, ds, \quad p \in L^2_{\Im}(0, b; X).$$

The assumption (A3) was introduced by Naito in [15]. Let $N = \ker \Xi = \left\{ p \in L^2_{\Im}(0, b; X) : \Xi p = 0 \right\}$ and let G be an orthogonal projection operator from $L^2_{\Im}(0, b; X)$ into $N^{\perp}$ and $\mathrm{Im}\, B$ be the range of B. It follows from (A3) that $\{x + N\} \cap \overline{\mathrm{Im}\, B} \ne \emptyset$ for any $x \in N^{\perp}$. Therefore, the operator $P : N^{\perp} \to \overline{\mathrm{Im}\, B}$ defined by

$$Px = x^*,$$

where $x^* \in \{x + N\} \cap \overline{\mathrm{Im}\, B}$ and $\|x^*\| = \min \left\{ \|y\| : y \in \{x + N\} \cap \overline{\mathrm{Im}\, B} \right\}$ is well defined. The operator P is bounded [15].

3. Approximate controllability

This section provides the main results and several lemmas that will be used to prove the main results.

Under the assumptions (A1) and (A2), for any control $u \in L^2_{\Im}(0,b;U)$ the system (1) has a unique mild solution. This mild solution is defined as a solution of the following integral equation:

$$y(t;u) = S(t)\xi + \int_0^t S(t-s)\left[(Bu)(s) + f(s,y(s))\right] ds$$
$$+ \int_0^t S(t-s)\int_0^s \sigma(r,y(r))\,dw(r)\,ds, \quad 0 \le t \le b. \tag{2}$$

Similarly, for any $z \in L^2_{\Im}(0,b;X)$, the following integral equation

$$x(t;z) = x(t) = S(t)\xi + \int_0^t S(t-s)\left[z(s) + f(s,x(s))\right] ds$$
$$+ \int_0^t S(t-s)\int_0^s \sigma(r,x(r))\,dw(r)\,ds, \quad 0 \le t \le b \tag{3}$$

has a unique mild solution $x(\cdot;z)$. Therefore, the following operator $W : L^2_{\Im}(0,b;X) \to \mathfrak{C}_2(0,b;X)$ can be defined $(Wz)(\cdot) = x(\cdot;z)$.

Lemma 2. *For any $z_1, z_2 \in L^2_{\Im}(0,b;X)$ the following inequality holds:*

$$\mathbf{E}\left\|(Wz_1)(t) - (Wz_1)(t)\right\|^2 \le 3M\exp\left(3MLb^2(b+1)\right)\int_0^t \mathbf{E}\left\|z_1(s) - z_2(s)\right\|^2 ds.$$

Proof. Let $z_1, z_2 \in L^2_{\Im}(0,b;X)$. Then

$$\mathbf{E}\left\|(Wz_1)(t) - (Wz_2)(t)\right\|^2 \le 3M\int_0^t \mathbf{E}\left\|z_1(s) - z_2(s)\right\|^2 ds$$
$$+ 3MLb(b+1)\int_0^t \mathbf{E}\left\|(Wz_1)(s) - (Wz_2)(s)\right\|^2 ds,$$

where $M = \sup\{\|S(t)\| : 0 \le t \le b\}$. By the Gronwall inequality we have

$$\mathbf{E}\left\|(Wz_1)(t) - (Wz_2)(t)\right\|^2$$
$$\le 3M\int_0^t \mathbf{E}\left\|z_1(s) - z_2(s)\right\|^2 ds$$
$$+ 3MLb(b+1)\int_0^t\int_0^s 3M\mathbf{E}\left\|z_1(\tau) - z_2(\tau)\right\|^2 d\tau \exp\left(3MLb(b+1)(t-s)\right) ds$$
$$= 3M\int_0^t \mathbf{E}\left\|z_1(s) - z_2(s)\right\|^2 ds - \int_0^t\int_0^s 3M\mathbf{E}\left\|z_1(\tau) - z_2(\tau)\right\|^2 d\tau d_s \exp\left(3MLb(b+1)(t-s)\right)$$
$$= 3M\int_0^t \mathbf{E}\left\|z_1(s) - z_2(s)\right\|^2 ds - \int_0^s 3M\mathbf{E}\left\|z_1(\tau) - z_2(\tau)\right\|^2 d\tau \exp\left(3MLb(b+1)(t-s)\right)\Big|_{s=0}^{s=t}$$
$$+ 3M\int_0^t \exp\left(3MLb(b+1)(t-s)\right)\mathbf{E}\left\|z_1(s) - z_2(s)\right\|^2 ds$$
$$\le 3M\exp\left(3MLb^2(b+1)\right)\int_0^t \mathbf{E}\left\|z_1(s) - z_2(s)\right\|^2 ds.$$

$\square$

By the definition of reachable set $R_b(0)$, for any $h \in R_b(0)$ there exists $u \in L^2_{\Im}(0,b;U)$ such that

$$h = S(b)\xi + \int_0^b S(b-s)(Bu)(s)\,ds.$$

Define an operator $\mathcal{J} : N^\perp \to N^\perp$ as follows

$$\mathcal{J}v = GBu - G\Gamma Pv, \qquad v \in N^\perp, \tag{4}$$

where $\Gamma : L^2_{\Im}(0,b;X) \to L^2_{\Im}(0,b;X)$ is the operator defined by

$$(\Gamma z)(t) = f(t,(Wz)(t)) + \int_0^t \sigma(r,(Wz)(r))\,dw(r).$$

For any $v \in N^\perp$, we have $Pv \in L^2_{\Im}(0,b;X)$, $\Gamma Pv \in L^2_{\Im}(0,b;X)$, and $G\Gamma Pv \in N^\perp$. Therefore, $\mathcal{J}$ is well defined.

Lemma 3. *The operator $\mathcal{J}$ defined by (4) has a unique fixed point in $N^\perp$.*

Proof. The proof is based on the classical Banach fixed point theorem for contractions. It is clear that $\mathcal{J}$ maps $N^\perp$ into itself. Let $v_1, v_2 \in N^\perp$. We show that there exists a natural number n such that$\mathcal{J}^n$ is a contraction mapping. Indeed,

$$\begin{aligned}
&\mathbf{E}\left\|\mathcal{J}v_1(t) - \mathcal{J}v_2(t)\right\|^2 \\
&\le \mathbf{E}\left\|(\Gamma Pv_1)(t) - (\Gamma Pv_2)(t)\right\|^2 \\
&\le L^2\mathbf{E}\left\|(WPv_1)(t) - (WPv_2)(t)\right\|^2 + L\int_0^t \mathbf{E}\left\|(WPv_1)(s) - (WPv_2)(s)\right\|^2 ds \\
&\le 3\left(L^2+L\right)bM\exp\left(3MLb^2(b+1)\right)\int_0^t \mathbf{E}\left\|(Pv_1)(s) - (Pv_2)(s)\right\|^2 ds \\
&\le 3\left(L^2+L\right)bM\exp\left(3MLb^2(b+1)\right)\|P\|^2\int_0^t \mathbf{E}\left\|v_1(s) - v_2(s)\right\|^2 ds \\
&= l\int_0^t \mathbf{E}\left\|v_1(s) - v_2(s)\right\|^2 ds.
\end{aligned}$$

Similarly,

$$\begin{aligned}
\mathbf{E}\left\|\mathcal{J}^2v_1(t) - \mathcal{J}^2v_2(t)\right\|^2 &\le l\int_0^t \mathbf{E}\left\|\mathcal{J}v_1(s) - \mathcal{J}v_2(s)\right\|^2 ds \\
&\le l^2\int_0^t\int_0^s \mathbf{E}\left\|v_1(r) - v_2(r)\right\|^2 drds \le l^2t\int_0^t \mathbf{E}\left\|v_1(s) - v_2(s)\right\|^2 ds.
\end{aligned}$$

Thus, it is obvious that

$$\begin{aligned}
\mathbf{E}\left\|\mathcal{J}^{n+1}v_1(t) - \mathcal{J}^{n+1}v_2(t)\right\|^2 &\le l\int_0^t \mathbf{E}\left\|\mathcal{J}^nv_1(s) - \mathcal{J}^nv_2(s)\right\|^2 ds \\
&\le l^{n+1}\int_0^t \frac{s^{n-1}}{(n-1)!}\int_0^s \mathbf{E}\left\|v_1(r) - v_2(r)\right\|^2 drds \\
&\le l^{n+1}\frac{t^n}{n!}\int_0^t \mathbf{E}\left\|v_1(s) - v_2(s)\right\|^2 ds,
\end{aligned}$$

and, consequently

$$\mathbf{E}\left\|\mathcal{J}^{n+1}v_1-\mathcal{J}^{n+1}v_2\right\|^2=\int_0^b\mathbf{E}\left\|\mathcal{J}^{n+1}v_1\left(t\right)-\mathcal{J}^{n+1}v_2\left(t\right)\right\|^2dt$$
$$\leq l^{n+1}\frac{b^{n+1}}{n!}\int_0^b\mathbf{E}\left\|v_1\left(s\right)-v_2\left(s\right)\right\|^2ds=l^{n+1}\frac{b^{n+1}}{n!}\mathbf{E}\left\|v_1-v_2\right\|^2.$$

It is known that $l^{n+1}\frac{b^{n+1}}{n!}<1$ for sufficiently large n. This results that $\mathcal{J}^{n+1}$ is a contraction mapping for sufficiently large n. Then $\mathcal{J}$ has a unique fixed point in $N^{\perp}$.

Similarly

$$\mathbf{E}\left\|\mathcal{J}v\left(t\right)\right\|^2\leq 2\mathbf{E}\left\|\left(Bu\right)\left(t\right)\right\|^2+2\mathbf{E}\left\|\left(\Gamma Pv\right)\left(t\right)\right\|^2$$
$$\leq 2\mathbf{E}\left\|\left(Bu\right)\left(t\right)\right\|^2+L_1\left(1+\mathbf{E}\left\|\left(WPv\right)\left(t\right)\right\|^2\right).$$

Now we state and prove the main result. □

Theorem 4. *Assume the assumptions (A1), (A2), (A3). Then the system (1) is approximately controllable on* $[0,b]$.

Proof. Note that the assumption (A3) implies the approximate controllability of the linear system associated with (1). Then $\overline{R_b\left(0\right)}=L^2\left(\Im_b,X\right)$ and to prove the approximate controllability of (1) it suffices to show that

$$R_b\left(0\right)\subset\overline{R_b\left(f\right)}.$$

In other words, we need to show that for any $\varepsilon>0$ and for any $h\in R_b\left(0\right)$, there exists $y_\varepsilon\in R_b\left(f\right)$ such that $\mathbf{E}\left\|y_\varepsilon-h\right\|^2<\varepsilon$. By Lemma 3 the operator $\mathcal{J}$ has a fixed point in $N^{\perp}$. So there exists $v^*\in N^{\perp}$ such that

$$\mathcal{J}v^*=GBu-G\Gamma Pv^*.$$

Recalling that $Pv^*\in\left(v^*+N\right)\cap\overline{\operatorname{Im}B}$, and G is the projection from $L^2\left(0,b;X\right)$ into $N^{\perp}$, we have

$$\int_0^bS\left(b-s\right)\left(Pv^*\right)\left(s\right)ds=\int_0^bS\left(b-s\right)v^*\left(s\right)ds,$$
$$\int_0^bS\left(b-s\right)Gp\left(s\right)ds=\int_0^bS\left(b-s\right)p\left(s\right)ds,$$
$$\int_0^bS\left(b-s\right)\left(Bu\right)\left(s\right)ds$$
$$=\int_0^bS\left(b-s\right)\left[\int_0^s\sigma\left(r,x\left(r;Pv^*\right)\right)dw\left(r\right)+f\left(s,x\left(s;Pv^*\right)\right)+v^*\left(s\right)\right]ds$$
$$=\int_0^bS\left(b-s\right)\left[\int_0^s\sigma\left(r,x\left(r;Pv^*\right)\right)dw\left(r\right)+f\left(s,x\left(s;Pv^*\right)\right)+\left(Pv^*\right)\left(s\right)\right]ds.$$

Finally,

$$h=S\left(b\right)\xi+\int_0^bS\left(b-s\right)\left[\int_0^s\sigma\left(r,x\left(r;Pv^*\right)\right)dw\left(r\right)+f\left(s,x\left(s;Pv^*\right)\right)+\left(Pv^*\right)\left(s\right)\right]ds$$
$$=x\left(b;Pv^*\right).$$

On the other hand there exists a sequence $u_n\in L^2_{\Im}\left(0,b;U\right)$ such that $Bu_n\to Pv^*$ as $n\to\infty$. This implies that

$$x\left(b;Bu_n\right)\to x\left(b;Pv^*\right)=h$$

as $n \to \infty$. Since $x(b; Bu_n) = y(b; u_n) \in R_b(f)$, we obtain that $h \in \overline{R_b(f)}$. This completes the proof of the theorem. $\square$

4. Example

Let $X = L^2(0,\pi)$ and $e_n(x) = \sin(nx)$ for $n \geq 1$. Define $A : X \to X$ by $Ay = y''$ with domain

$$D(A) = \left\{ y \in X : y \text{ and } y' \text{ are absolutely continuous, } y'' \in X, y(0) = y(\pi) = 0 \right\}.$$

Then the operator

$$Ay = -\sum_{n=1}^{\infty} n^2 \langle y, e_n \rangle e_n, \quad y \in D(A),$$

and A generates strongly continuous semigroup $\{S(t) : t \geq 0\}$ defined by

$$S(t) = \sum_{n=1}^{\infty} e^{-n^2 t} \langle y, e_n \rangle e_n, \quad y \in X.$$

Define the space U by

$$U = \left\{ u : u = \sum_{n=2}^{\infty} u_n e_n, \ \|u\|^2 = \sum_{n=2}^{\infty} u_n^2 < \infty \right\}.$$

Define an operator $B : U \to X$ as follows:

$$Bu = 2u_2 e_1 + \sum_{n=2}^{\infty} u_n e_n.$$

Consider the following semilinear heat equation

$$\begin{cases} \dfrac{\partial y(t,x)}{\partial t} = \dfrac{\partial^2 y(t,x)}{\partial x^2} + Bu(t,x) + f(t, y(t,x)) + \int_0^t \sigma(s, y(s,x))\, dw(s), \quad 0 < t < b, 0 < x < \pi, \\ y(t,0) = y(t,\pi) = 0, \quad 0 \leq t \leq b, \\ y(t,x) = \xi(x), \quad 0 \leq x \leq \pi. \end{cases} \tag{5}$$

System (5) can be written in the abstract form (1). It follows from [16] that (A3) holds and the corresponding linear system of (5) is approximately controllable on $[0,b]$. Assuming that f and σ satisfy Lipschitz and growth conditions we may see that (A1) and (A2) are satisfied. It follows from Theorem 4 that system (5) is approximately controllable on $[0,b]$.

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