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# CODES ON $m$-REPEATED SOLID BURST ERRORS

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Abstract. In coding theory, several kinds of errors due to the different behaviours of communication channels have been considered and accordingly error detecting and error correcting codes have been constructed. In general communication due to the long messages, the strings of same type of error may repeat in a vector itself. The concept of repeated bursts is introduced by Beraradi, Dass and Verma [4] which has opened a new area of study. They defined 2-repeated bursts and obtained results for detection and correction of such type of errors. The study was further extended to $m$-repeated bursts [3]. Solid burst errors are common in many communications. This paper considers a new similar kind of error which will be termed as '$m$-repeated solid burst error of length $b$'. A lower bound on the number of parity checks required for the existence of codes that detect such errors is obtained. Further, codes capable of detecting and simultaneously correcting such errors have also been dealt with.

Keywords: Parity check matrix, syndrome, standard array, solid burst error.

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## 1. Introduction

Investigations in coding theory have been made in several directions but one of the most important aspects considered has been the detection and correction of errors. The nature of errors differs from channel to channel depending upon the behaviour of channels. Solid burst errors are found to be in many memory channels (viz. semiconductor memory data [6], supercomputer storage system [1]). Therefore, to study the said type of error is one of the important areas for mathematicians. For more study on solid burst, one may refer to [9, 10, 12] and references given thereat. This paper also presents a study on solid burst error, but not occurring in ordinary way. In very busy communication channels, it is observed that errors repeat themselves. So is a situation when errors need to consider in repeated form. In this direction, an error pattern, called 2-repeated burst has been introduced by Dass, Verma and Berardi [4]. This is an extension of the idea of open-loop burst given by Fire [5]. Later on Dass and Verma [3] defined $m$-repeated bursts and obtained results regarding the number of parity-check digits required for codes detecting such errors. Some results have been obtained on weights of such errors by Sharma and Rohtagi [11].

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This paper introduces yet another kind of a repeated error, termed as '$m$-repeated solid burst of length $b$' and presents a study on such errors. The paper is organized as follows. Basic definitions, related to our study are stated in Section 2. In Section 3, a lower bound on the parity checks for a code that detects $m$-repeated solid bursts of length $b$ or less is obtained. Section 4 gives a bound on code for simultaneous detection and correction of such errors. Conclusion is given in Section 5.

In what follows a linear code will be considered as a subspace of the space of all $n$-tuples over GF(q). The distance between two vectors shall be considered in the Hamming sense.

## 2. Preliminaries

The definition of a solid burst may be given as follows:

**Definition 2.1.** *A solid burst of length $b$ is a vector whose all the $b$-consecutive components are non zero and rest are zero.*

A vector may not have just one solid burst error, it may have more than one. Putting them together into one solid burst amounts to neglecting the nature of communication and unnecessarily considering a longer solid burst. In a busy communication channel, sometimes solid bursts repeat themselves. In view of this, codes detecting and correcting 2-repeated solid burst of length $b$ or less have been studied by Das [2]. A 2-repeated solid burst of length $b$ has been defined as follows:

**Definition 2.2.** *A 2-repeated solid burst of length $b$ is a vector of length $n$ whose only nonzero components are confined consecutively to two distinct sets of $b$ consecutive components.*

In very busy communication channels, errors repeat themselves more frequently. In view of this, it is desirable to consider more than two repeated solid bursts. We call such error as $m$-repeated solid burst of length $b$. An $m$-repeated solid burst of length $b$ may be defined as follows:

**Definition 2.3.** *An $m$-repeated solid burst of length $b$ is a vector of length $n$ whose only nonzero components are confined consecutively to some $m$ distinct sets of $b$ consecutive components.*

For example, (00112002410030003100) is a 4-repeated solid burst of length 3 over GF(5).

Consider a long scratch in a compact disc. The scratch is continuous upto a certain length, then no scratch, then again a continuous scratch and so on. Such type of error falls in this category of $m$-repeated solid burst errors.

It should be noted that codes dealing with $m$-repeated bursts of length $b$ or less or single bursts of length $mb$ or less can automatically deal with these $m$-repeated solid burst of length $b$ or less. But if the error occurred in communication is of the type of repeated solid bursts of length $b$ or less, then we need to consider only these patterns rather than considering other types of errors mentioned above and wasting the capacity of the system by detecting/correcting non-errors by default.

The development of codes detecting repeated solid burst errors may be more useful for channels those are already dealing with multiple (solid) burst errors improving upon their efficiency as such errors are relatively simpler to handle.

## 3. Detection of $m$-repeated solid burst errors

Consider the linear codes that are capable of detecting any $m$-repeated solid burst of length $b$ or less. Clearly, the patterns to be detected should not be code words. In other words, codes are considered which have no $m$-repeated solid burst of length $b$ or less as a code word. In this section, a lower bound over the number of parity-check digits required for such a code is obtained .

**Theorem 3.1.** *Any $(n, k)$ linear code over GF(q) that detects any $m$-repeated solid burst of length $b$ or less must have at least $m \log_q \left\{ \sum_{i=0}^{b} (q-1)^i \right\}$ parity-check digits.*

*Proof.* The result will be proved on the basis that no detectable error vector can be a code word.
Let V be an $(n, k)$ linear code over GF(q). Let X be the set of all vectors such that all non zero components are confined consecutively in some $m$ distinct fixed sets of $b$ consecutive components, first component of each set being non zero.
We claim that no two vectors of the set X can belong to the same coset of the standard array; else a code word shall be expressible as a sum or difference of two error vectors.
Assume on the contrary that there is a pair, say $x_1$, $x_2$ in X belonging to the same coset of the standard array. Their difference viz. $x_1$-$x_2$ must be a code vector. But $x_1$-$x_2$ is a vector all of whose non-zero components are confined consecutively to the same $m$ fixed sets of $b$ consecutive components and so is a member of X, i.e., $x_1$-$x_2$ is an $m$-repeated solid bursts of length $b$ or less, which is a contradiction. Thus all the vectors in X must belong to distinct cosets of the standard array. The number of such vectors, including all zero vector, over GF(q) is clearly

$$\Big(\sum_{i=0}^{b}(q-1)^i\Big)^m.$$

The theorem follows since there must be at least this number of cosets. □

**Remark 3.1.** *For $m = 2$, this result coincides with Theorem 1, Das[2] when bursts considered are 2-repeated solid bursts of length $b$ or less.*

## 4. Simultaneous detection and correction of $m$-repeated solid burst errors

This section determines extended Reiger's bound (refer [8]; also Theorem 4.15, Peterson and Weldon[7]) for simultaneous detection and correction of $m$-repeated solid bursts of length $b$ or less. The following theorem gives a bound on the number of parity-check digits for a linear code that simultaneously detects and corrects such errors.

**Theorem 4.1.** *An $(n, k)$ linear code over GF(q) that corrects all $m$- repeated solid bursts of length $b$ or less must have at least $m \log_q \{ \sum_{i=0}^{2b} (q-1)^i \}$ parity-check digits. Further, if the code corrects all $m$-repeated solid bursts of length $b$ or less and simultaneously detects $m$-repeated solid bursts of length $d$ $(d > b)$ or less then the code must have at least $m \log_q \{ \sum_{i=0}^{b+d} (q-1)^i \}$ parity-check digits.*

*Proof.* To prove the first part, consider an $m$-repeated solid burst of length $2b$ or less. Such a vector is expressible as a sum or difference of two vectors, each of which is an $m$-repeated solid burst of length $b$ or less. These component vectors must belong to different cosets of the standard array because both such errors are correctable errors. Accordingly, such a vector viz. an $m$-repeated solid burst of length $2b$ or less can not be a code vector. In view of Theorem 1, such a code must have at least $m\log_q\left\{\sum_{i=0}^{2b}(q-1)^i\right\}$ parity-check digits.

Further, consider an $m$-repeated solid burst of length $(b+d)$ or less. Such a vector is also expressible as a sum or difference of two vectors, one of which is an $m$-repeated solid burst of length $b$ or less and the other is an $m$-repeated solid burst of length $d$ or less. Both such component vectors, one being a detectable error and the other being a correctable error, can not belong to the same coset of the standard array. Therefore such a vector can not be a code vector, i.e., an $m$-repeated solid burst of length $b+d$ or less can not be a code vector. Hence the code must have at least $m\log_q\left\{\sum_{i=0}^{b+d}(q-1)^i\right\}$ parity-check digits. □

**Remark 4.1.** *For $m = 2$, this result reduces to a result due to Das [2] (Theorem 3) when bursts considered are 2-repeated solid bursts of length $b$ or less.*

## 5. Conclusion

The paper presents lower bound for a code detecting $m$-repeated solid burst of length $b$ or less, also presents the case when simultaneous detection and correction of such errors is required. Correction of such errors will remain a further study.

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