ON GRAY IMAGES OF CONSTACYCLIC CODES OVER THE FINITE RING $F_2 + u_1F_2 + u_2F_2$

MUSTAFA ÖZKAN¹, ABDULLAH DERTLI², YASEMIN CENGELLENMIS¹, §

ABSTRACT. We introduce the finite ring $F_2 + u_1F_2 + u_2F_2$, $u_1^2 = u_1$, $u_2^2 = 0$, $u_1u_2 = u_2u_1 = 0$ which is not a finite chain ring. We focus on $(1 + u_2)$-constacyclic codes over $F_2 + u_1F_2 + u_2F_2$ of odd length. We prove that the Gray image of a linear $(1 + u_2)$-constacyclic code over $F_2 + u_1F_2 + u_2F_2$ of odd length $n$ is a quasi-cyclic code of index 4 and length $4n$ over $F_2$.

Keywords: Constacyclic code, Gray image.

AMS Subject Classification: 94B15

1. Introduction

In [6], Wolfman showed that the Gray image of a linear negacyclic code over $Z_4$ of length $n$ is distance invariant (not necessarily linear) cyclic code. It was shown that, for odd $n$, the Gray image of a linear cyclic code over $Z_4$ of length $n$ is equivalent to a binary cyclic code. In 2006, J.F. Qian et al. introduced linear $(1 + u)$-constacyclic codes and cyclic codes over $F_2 + uF_2$ and characterized codes over $F_2$ which are the Gray images of $(1 + u)$-constacyclic codes or cyclic codes over $F_2 + uF_2$ in [4]. In [1], they extended the result of [4] to codes over the commutative ring $F_p^k + uF_p^k$ where $p$ is a prime, $k \in \mathbb{N}$ and $u^2 = 0$. In [5], it was introduced $(1 + u^2)$-constacyclic codes or cyclic codes over $F_2 + uF_2 + u^2F_2$, $u^3 = 0$ and characterized codes over $F_2$ which are the Gray images of $(1 + u^2)$ -constacyclic or cyclic codes over $F_2 + uF_2 + u^2F_2$. In [2], it was introduced $(1 - u^m)$-constacyclic codes over $F_2 + uF_2 + \ldots + u^mF_2$, $u^{m+1} = 0$ and characterized codes over $F_2$. In 2011, $(1 + v)$-constacyclic codes over $F_2 + uF_2 + vF_2 + uvF_2$ were studied. $(1 + v)$ -constacyclic codes over $F_2 + uF_2 + vF_2 + uvF_2$, $u^2 = v^2 = 0$, $u.v - v.u = 0$ of odd length were characterized with the help of cyclic codes over $F_2 + uF_2 + vF_2 + uvF_2$. A new Gray map was defined. It was shown that the image under the Gray map of $(1 + v)$-constacyclic codes over $F_2 + uF_2 + vF_2 + uvF_2$ are cyclic codes over $F_2 + uF_2$ in [3]. In 2013, X. Xiaofang

¹ Department of Mathematics, Trakya University, Edirne, Turkey.
e-mail: mustafaozkan22@icloud.com, mustafaozkan@trakya.edu.tr.
ORCID: https://orcid.org/0000-0001-7398-8564.

² Department of Mathematics, Ondokuz Mayis University, Samsun, Turkey.
e-mail: abdullah.dertli@gmail.com; ORCID: https://orcid.org/0000-0001-8687-032X.

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studied \((1+v)-\text{constacyclic}\) codes over the ring \(\mathbb{F}_2 + u\mathbb{F}_2 + v\mathbb{F}_2, u^2 = v^2 = 0, u,v = v,u = 0\), \((1+v)-\text{constacyclic}\) codes over \(\mathbb{F}_2 + u\mathbb{F}_2 + v\mathbb{F}_2\) of odd length are characterized with the help of cyclic codes over \(\mathbb{F}_2 + u\mathbb{F}_2 + v\mathbb{F}_2\) in \([7]\).

This paper is organized as follows. In section 2, we give some knowledge about the ring \(R = \mathbb{F}_2 + u_1\mathbb{F}_2 + u_2\mathbb{F}_2\), \(u_1^2 = u_1, u_2^2 = 0, u_1 u_2 = u_2 u_1 = 0\) and the codes over \(R\). In section 3, we have the relationship between cyclic code over and \((1+u_2)-\text{constacyclic}\) code over \(R\). In section 4, the Gray image of \((1+u_2)-\text{constacyclic}\) code over \(R\) of odd length is obtained.

## 2. Preliminaries

The ring \(R = \mathbb{F}_2 + u_1\mathbb{F}_2 + u_2\mathbb{F}_2\) is defined as a characteristic 2 ring subject to the restrictions \(u_1^2 = u_1, u_2^2 = 0, u_1 u_2 = u_2 u_1 = 0\). The isomorphism \(\mathbb{F}_2 + u_1\mathbb{F}_2 + u_2\mathbb{F}_2 \cong \mathbb{F}_2[u_1,u_2]/<u_1^2 = u_1, u_2^2 = 0, u_1 u_2 = u_2 u_1 = 0 >\) is obvious to see. The elements of \(R\) may be written as 0, 1, \(u_1, u_2, 1+u_1, 1+u_2, u_1 + u_2, 1+u_1 + u_2\). Addition and multiplication operations over \(R\) are given in the following tables:

**Table 1**

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The units of \(R\) can be found to be following \(R^* = \{1,1+u_2\}\). It can be easily find all the ideals of \(R\) to be listed as,

\[
\{0\} = I_0 \subset I_{u_1} \subset I_{u_1+u_2} \subset R = I_{1+u_2}
\]

\[
\{0\} = I_0 \subset I_{u_2} \subset I_{1+u_1} = I_{1+u_1+u_2} \subset R = I_{1+u_2}
\]

\(R\) is not a finite chain ring. It has got two maximal ideals, \(I_{u_1+u_2}\) and \(I_{u_1}\). It is semi local ring. Moreover, \(R\) is principal ring. We take \(R\) to be a natural extension of the ring
\( R_2 = \mathbb{F}_2 + u_2 \mathbb{F}_2, u_2^2 = 0 \). The elements of \( R_2 \) may be written as 0, 1, \( u_2 \), 1 + \( u_2 \) where 1 and 1 + \( u_2 \) are only units in \( R_2 \). \( R_2 \) has three ideals \((0), (1) \) and \((u_2)\).

A linear code \( C \) over \( R \) of length \( n \) is a \( R \) submodule of \( R^n \). A linear code \( C \) over \( \mathbb{F}_2 \) of length \( n \) is a \( \mathbb{F}_2 \) subvector space \( \mathbb{F}_2^n \). An element of \( C \) is called a codeword. Each codeword \( c \) such a code \( C \) is an \( n \)-tuple of the form \( c = (c_0, c_1, \ldots, c_{n-1}) \in R^n \) or \( R_2^n \) and can be represented by

\[
c = (c_0, c_1, \ldots, c_{n-1}) \iff c(x) = \sum_{i=0}^{n-1} c_i x^i \in R[x] \text{ (or } R_2[x] , \mathbb{F}_2[x] \).
\]

The Gray map \( \Phi_1 \) on \( R \) is given by

\[
\Phi_1 : R \rightarrow R_2^2
\]

\[
a + u_1 b + u_2 c \mapsto \Phi_1(a + u_1 b + u_2 c) = \Phi_1(r + u_1 q) = (u_2 r, q)
\]

where \( r = a + u_2 c \) and \( q = b + u_2 c \). We will extend \( \Phi_1 \) to \( R^n \) naturally as follows

\[
\Phi_1(c_0, c_1, \ldots, c_{n-1}) = (u_2 r_0, u_2 r_1, \ldots, u_2 r_{n-1}, q_0, q_1, \ldots, q_{n-1}) \text{ where } c_i = r_i + u_2 q_i \text{ for all } i = 0, 1, \ldots, n - 1.
\]

The Gray map \( \Phi_2 \) on \( R_2 \) is given by

\[
\Phi_2 : R_2 \rightarrow \mathbb{F}_2^n
\]

\[
s + u_2 t \mapsto (s, t)
\]

where \( s, t \in \mathbb{F}_2 \). We will extend \( \Phi_2 \) to \( R_2^n \) naturally as follows

\[
\Phi_2 : R_2^n \rightarrow \mathbb{F}_2^{2n}
\]

\[
(c_0, \ldots, c_{n-1}) \mapsto (s_0, \ldots, s_{n-1}, t_0, \ldots, t_{n-1})
\]

where \( c_i = s_i + u_2 t_i \), \( s_i, t_i \in \mathbb{F}_2 \) for all \( i = 0, 1, \ldots, n - 1 \).

The weight \( w_1(r) \) of \( r \in R \) is given by

\[
w_1(r) = \begin{cases} 
0 & r = 0 \\
1 & r = 1, u_1, u_2 \\
2 & r = 1 + u_1, 1 + u_2, u_1 + u_2 \\
3 & r = 1 + u_1 + u_2
\end{cases}
\]

This extends to a weight function in \( R^n \). If \( r = (r_0, r_1, \ldots, r_{n-1}) \in R^n \) then \( w_1(r) = \sum_{i=0}^{n-1} w_1(r_i) \). The distance \( d_1(x, y) \) between any distinct vectors \( x, y \in R^n \) is defined to be \( w_1(x - y) \). The \( d_1 \) minimum distance of \( C \) is defined as \( d_1(C) = \min\{d_1(x, y)\} \) for any \( x, y \in C, x \neq y \).

The weight \( w_2(t) \) of \( t \in R_2 \) is given by

\[
w_2(t) = \begin{cases} 
0 & t = 0 \\
1 & t = 1, u_1 \\
2 & t = 1 + u_1 \\
3 & t = 1 + u_1 + u_2
\end{cases}
\]

This extends to a weight function in \( R_2^n \). If \( t = (t_0, t_1, \ldots, t_{n-1}) \in R_2^n \) then \( w_2(t) = \sum_{i=0}^{n-1} w_2(t_i) \). The distance \( d_2(x, y) \) between any distinct vectors \( x, y \in R_2^n \) is defined to be \( w_2(x - y) \). The \( d_2 \) minimum distance of \( C \) is defined as \( d_2(C) = \min\{d_2(x, y)\} \) for any \( x, y \in C, x \neq y \).

Let \( C \) be a code over \( \mathbb{F}_2 \) of length \( n \) and let \( c = (c_0, c_1, \ldots, c_{n-1}) \) be a codeword of \( C \). The Hamming weight of \( C \) is defined as

\[
w_H(c) = \sum_{i=0}^{n-1} w_H(c_i)
\]
where \( w_H(c_i) = 1 \) if \( c_i = 1 \) and \( w_H(c_i) = 0 \) if \( c_i = 0 \). The minimum Hamming distance of \( C \) is defined as \( d_H = \min \{ d_H(c, c') \} \) for any \( c, c' \in C, c \neq c' \).

\( \Phi_1 \) and \( \Phi_2 \) are distance preserving map from \((R^n, d_1)\) to \((R_2^{2n}, d_2)\) and \((R_2^{2n}, d_2)\) to \((\mathbb{F}_2^{2n}, d_H)\), respectively.

Expressing elements of \( R \) as \( a + u_1 b + u_2 c = r + u_1 q \) where \( r = a + u_2 c \) and \( q = b + u_2 c \) are both in \( R_2 \), we see that

\[
w_1(a + u_1 b + u_2 c) = w_1(r + u_1 q) = w_2(u_2 r, q) = w_H(0, b, a, c)
\]

A cyclic shift on \( R^n \) is a permutation \( \sigma \) such that

\[
\sigma(c_0, c_1, \ldots, c_{n-1}) = (c_{n-1}, c_0, \ldots, c_{n-2})
\]

A linear code \( C \) over \( R \) of length \( n \) is said to be cyclic code if it is invariant under the cyclic shift \( \sigma(C) = C \).

A \((1 + u_2)\)-constacyclic shift \( \gamma \) act on \( R^n \) as

\[
\gamma(c_0, c_1, \ldots, c_{n-1}) = ((1 + u_2)c_{n-1}, c_0, \ldots, c_{n-2})
\]

A linear code \( C \) over \( R \) of length \( n \) is said to be \((1 + u_2)\)-constacyclic code if it is invariant under the \((1 + u_2)\)-constacyclic shift \( \gamma(C) = C \).

Let \( C \) be a code of length \( n \) over \( R \) and \( P(C) \) be its polynomial representation,

\[
P(C) = \left\{ \sum_{i=0}^{n-1} r_i x^i : (r_0, r_1, \ldots, r_{n-1}) \in C \right\}
\]

A code \( C \) of length \( n \) over \( R \) is cyclic if and only if \( P(C) \) is an ideal of \( R[x]/ \langle x^n - 1 \rangle \).

A code \( C \) of length \( n \) over \( R \) is \((1 + u_2)\)-constacyclic code if and only if \( P(C) \) is an ideal of \( R[x]/ \langle x^n - (1 + u_2) \rangle \).

Let \( a \in R_2^{2n} \) with \( a = (a_0, a_1, \ldots, a_{2n-1}) = (a^{(0)}|a^{(1)}) \), \( a^{(i)} \in R_2^n \) for all \( i = 0, 1 \). Let \( \sigma^{\oplus 2} \) be the map from \( R_2^{2n} \) to \( R_2^{2n} \) given by

\[
\sigma^{\oplus 2}(a) = \left( \sigma(a^{(0)}), \sigma(a^{(1)}) \right)
\]

where \( \sigma \) is the usual cyclic shift. A code \( \hat{C} \) of length \( 2n \) over \( R_2 \) is said to be quasi-cyclic code of index \( 2 \) of \( \sigma^{\oplus 2}(\hat{C}) = \hat{C} \).

Let \( a \in \mathbb{F}_2^{2n} \) with \( a = (a_0, a_1, \ldots, a_{4n-1}) = (a^{(0)}|a^{(1)}|a^{(2)}|a^{(3)}) \), \( a^{(i)} \in \mathbb{F}_2^n \) for all \( i = 0, 1, 2, 3 \). Let \( \sigma^{\oplus 4} \) be the map from \( \mathbb{F}_2^{2n} \) to \( \mathbb{F}_2^{2n} \) given by

\[
\sigma^{\oplus 4}(a) = \left( \sigma(a^{(0)}), \sigma(a^{(1)}), \sigma(a^{(2)}), \sigma(a^{(3)}) \right)
\]

where \( \sigma \) is the usual cyclic shift. A code \( \hat{C} \) of length \( 4n \) over \( \mathbb{F}_2 \) is said to be quasi-cyclic code of index \( 4 \) of \( \sigma^{\oplus 4}(\hat{C}) = \hat{C} \).

\section{The relationship between Cyclic Codes Over \( R \) and \((1 + u_2)\)-Constacyclic Codes Over \( R \)}

Suppose \( n \) is odd. Let

\[
\mu : R[x]/ \langle x^n - 1 \rangle \to R[x]/ \langle x^n - (1 + u_2) \rangle
\]

\[
r(x) \mapsto r((1 + u_2)x)
\]

The \( \mu \) is a ring isomorphism. So \( I \) is an ideal of \( R[x]/ \langle x^n - 1 \rangle \) if and only if \( \mu(I) \) is an ideal of \( R[x]/ \langle x^n - (1 + u_2) \rangle \).
If $\overline{\mu}$ is given as follows,
\[
\overline{\mu} : R^n \rightarrow R^n
\]
\[
r = (r_0, ..., r_{n-1}) = (r_0, (1 + u_2)r_1, ..., (1 + u_2)^{n-1}r_{n-1})
\]
then we have,

**Proposition 3.1.** A code $C$ of length $n$ over $R$ is cyclic code if and only if $\overline{\mu}(C)$ is linear $(1 + u_2)$-constacyclic code.

### 4. $(1 + u_2)$-Constacyclic Codes Over $R$ of Odd Length and Their Images

Firstly, we obtained even length quasi-cyclic codes of index 2 over $\Phi 2$ over $R_2$ as the $\Phi 1$ Gray images of $(1 + u_2)$-constacyclic codes over $R$, later we obtained the $\Phi 2$ Gray image of quasi-cyclic code of index 2 over $R_2$ with length even.

**Proposition 4.1.** $\sigma \otimes^2 \Phi_1 = 1 \Phi_1$

**Proof.** Let $c = (c_0, c_1, ..., c_{n-1}) \in R^n$ where $c_i = r_i + u_1q_i$ for $i = 0, 1, ..., n-1$. If
\[
\Phi_1(c_0, c_1, ..., c_{n-1}) = \Phi_1(r_0 + u_1q_0, r_1 + u_1q_1, ..., r_{n-1} + u_1q_{n-1}) = (u_2r_0, u_2r_1, ..., u_2r_{n-1}, q_0, ..., q_{n-1})
\]
then $\sigma \otimes^2 \Phi_1(c) = (u_2r_0, u_2r_1, ..., u_2r_{n-1}, q_0, ..., q_{n-2}, q_{n-1}).$

On the other hand $\gamma(c_0, ..., c_{n-1}) = ((1 + u_2)c_{n-1}, c_0, ..., c_{n-2})$ where $(1 + u_2)c_{n-1} = r_{n-1} + u_2r_{n-1} + u_1q_{n-1}$. Then $\Phi_1(\gamma(c)) = \Phi_1((r_{n-1} + u_2r_{n-1} + u_1q_{n-1}, r_0 + u_1q_0, ..., r_{n-2} + u_1q_{n-2}, q_{n-1}, q_0, ..., q_{n-2}) = (u_2r_0, u_2r_1, ..., u_2r_{n-1}, q_0, ..., q_{n-2}).$

**Theorem 4.1.** A code $C$ of length $n$ over $R$ is $(1 + u_2)$-constacyclic code if and only if $\Phi_1(C)$ is quasi-cyclic code of index 2 and length $2n$ over $R_2$.

**Proof.** Suppose $C$ is $(1 + u_2)$-constacyclic code, then $\gamma(C) = C$. By applying $\Phi_1$, we have $\Phi_1(\gamma(C)) = \Phi_1(C)$. By using the Proposition 4.1, we have $\sigma \otimes^2(\Phi_1(C)) = \Phi_1(\gamma(C)) = \Phi_1(C)$. So $\Phi_1(C)$ is quasi-cyclic code of index 2. Conversely, if $\Phi_1(C)$ is quasi-cyclic code of index 2, then $\sigma \otimes^2(\Phi_1(C)) = \Phi_1(C)$. By using the Proposition 4.1, we have $\sigma \otimes^2(\Phi_1(C)) = \Phi_1(\gamma(C)) = \Phi_1(C)$. Since $\Phi_1$ is injective it follows that $\gamma(C) = C$.

Now, we will obtain the $\Phi_2$ Gray image of even length quasi-cyclic code of index 2 over $R_2$.

** Proposition 4.2.** $\sigma \otimes^4 \Phi_2 = \Phi_2 \sigma \otimes^2$

**Proof.** It is proved as in the proof of the Proposition 4.1.

**Theorem 4.2.** A code $B$ length $2n$ over $R_2$ is quasi-cyclic code of index 2 if and only if $\Phi_2(B)$ is quasi-cyclic code of index 4 over $F_2$ with length $4n$.

**Proof.** It is proved as in the proof of the Theorem 4.1.

**Corollary**

A code $C$ odd length $n$ over $R$ is $(1 + u_2)$-constacyclic if and only if $\Phi_2(\Phi_1(C))$ is quasi-cyclic code of index 4 and length $4n$ over $F_2$.

### 5. CONCLUSION

It is introduced that the finite ring $F_2 + u_1F_2 + u_2F_2, u_1^2 = u_1, u_2^2 = 0, u_1u_2 = u_2u_1 = 0$. Also, it is obtained that the Gray image of linear $(1 + u_2)$-constacyclic code over $R$ of odd length $n$. 
References


Mustafa Özkan is a doctor in the Department of Mathematics at Trakya University in Turkey. He received his PhD degree in 2016 from mathematics at Trakya University. He has given talks at international conferences. His research interest is algebraic coding theory.

Abdullah Dertli is a research assistant doctor in the Department of Mathematics, Ondokuz Mays University, Samsun, Turkey. He received his B.Sc. and M.Sc. in Mathematics from Trakya University of Turkey. He received his Ph.D. in mathematics from Ondokuz Mays University of Turkey. His research interests include algebra and coding theory.

Yasemin Cengellenmis got MSc degree in mathematics from Trakya University, Turkey. She received her PhD degree in mathematics from Trakya University. Currently, she is associate professor in the Department of Mathematics at Trakya University. Her research area is coding theory.