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A NOTE ON SOME NEW $\mathcal{P}_\delta$-TRANSFORMS OF ${}_2F_2$ GENERALIZED HYPERGEOMETRIC FUNCTIONS

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Abstract. In this note, we aim to establish $\mathcal{P}_\delta$-transforms of ${}_2F_2$ generalized hypergeometric functions in terms of gamma functions. The results are established from the generalized classical summation theorems due to Gauss's second, Kummer's and Bailey's for the series ${}_2F_1$ obtained earlier by Lavoie *et al.* [2]. Special cases of our main findings are known results derived earlier by Parmar *et al.* [3].

Keywords: Generalized hypergeometric functions, Classical summation theorems, $\mathcal{P}_\delta$-transform, Laplace transform.

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1. Introduction

We define generalized hypergeometric function ${}_pF_q$ with p numerator and q denominator parameters by:

$$ {}_pF_q\left[\begin{matrix}\alpha_1,\cdots,\alpha_p\\ \beta_1,\cdots,\beta_q\end{matrix}\,\middle|\,z\right]=\sum_{k=0}^{\infty}\frac{(\alpha_1)_k\cdots(\alpha_p)_k}{(\beta_1)_k\cdots(\beta_q)_k}\,\frac{z^k}{k!}\,, \tag{1}$$

where, as usual, $(\alpha)_n$ $(\alpha\in\mathbb{C}\setminus\mathbb{Z}_0^-)$ is well known factorial function defined by

$$ (\alpha)_n=\frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}=\begin{cases}1 & n=0\\ \prod\limits_{r=0}^{n-1}(\alpha+r) & n\in\mathbb{N}.\end{cases} \tag{2}$$

For convergence conditions of ${}_pF_q$ and other properties, we refer to [4, 5]. In general, there are four classical summations theorems for the series ${}_2F_1$. However, the three generalized summation theorems, which we shall make use in our present investigations, are given below:
Generalized Gauss's second summation theorem [2]

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$$
{}_2F_1\left(\begin{array}{cc} \alpha, \ \beta & \\ \frac{1}{2}(1+a+\beta+i) & |\frac{1}{2} \end{array}\right) = \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2}\alpha+\frac{1}{2}\beta+\frac{1}{2}i\right)\Gamma\left(\frac{1}{2}+\frac{1}{2}\alpha-\frac{1}{2}\beta-\frac{1}{2}i\right)}{\Gamma\left(\frac{1}{2}+\frac{1}{2}\alpha-\frac{1}{2}\beta-\frac{1}{2}|i|\right)}
$$

$$
\times\left\{\frac{A_i}{\Gamma\left(\frac{1}{2}+\frac{1}{2}\alpha\right)\Gamma\left(\frac{1}{2}+\frac{1}{2}\beta+\frac{1}{2}i-\left[\frac{1}{2}(1+i)\right]\right)}+\frac{B_i}{\Gamma\left(\frac{1}{2}\alpha\right)\Gamma\left(\frac{1}{2}+\frac{1}{2}\beta-\left[\frac{1}{2}i\right]\right)}\right\} \tag{3}
$$

for $i = 0, \pm1, \pm2, \pm3, \pm4, \pm5$.

Generalized Bailey's summation theorem [2]

$$
{}_2F_1\left(\begin{array}{cc} \alpha, \ 1-\alpha+i & \\ \beta & |\frac{1}{2} \end{array}\right) = \frac{\Gamma\left(\frac{1}{2}\right)\Gamma(1-\alpha)\Gamma(\beta)}{2^{\beta-1-i}\Gamma\left(1-\alpha+\frac{1}{2}i+\frac{1}{2}|i|\right)}
$$

$$
\times\left\{\frac{C_i}{\Gamma\left(\frac{1}{2}-\frac{1}{2}\alpha+\frac{1}{2}\beta\right)\Gamma\left(\frac{1}{2}\alpha+\frac{1}{2}\beta-\left[\frac{1}{2}(1+i)\right]\right)}+\frac{D_i}{\Gamma\left(\frac{1}{2}\beta-\frac{1}{2}\alpha\right)\Gamma\left(\frac{1}{2}\alpha+\frac{1}{2}\beta-\frac{1}{2}-\left[\frac{1}{2}i\right]\right)}\right\} \tag{4}
$$

for $i = 0, \pm1, \pm2, \pm3, \pm4, \pm5$.

Generalized Kummer's summation theorem [2]

$$
{}_2F_1\left(\begin{array}{cc} \alpha, \ \beta & \\ 1+\alpha-\beta+i & |-1 \end{array}\right) = \frac{\Gamma\left(\frac{1}{2}\right)\Gamma(1-\beta)\Gamma(1+\alpha-\beta+i)}{2^{\alpha}\Gamma\left(1-\beta+\frac{1}{2}(|i|+i)\right)}
$$

$$
\times\left\{\frac{E_i}{\Gamma\left(1+\frac{1}{2}\alpha-\beta+\frac{1}{2}i\right)\Gamma\left(\frac{1}{2}+\frac{1}{2}\alpha+\frac{1}{2}i-\left[\frac{1}{2}(1+i)\right]\right)}\right.
$$

$$
\left.+\frac{F_i}{\Gamma\left(\frac{1}{2}+\frac{1}{2}\alpha-\beta+\frac{1}{2}i\right)\Gamma\left(\frac{1}{2}\alpha+\frac{1}{2}i-\left[\frac{1}{2}i\right]\right)}\right\} \tag{5}
$$

for $i = 0, \pm1, \pm2, \pm3, \pm4, \pm5$.

In all these results, $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x and its modulus is denoted by $|x|$. The coefficient are given in Tables 1-3.

For $i = 0$, the result (3), (4) and (5) reduce to classical Gauss second, Bailey and Kummer's summation theorems respectively recorded in [4].

On the other hand the $\mathcal{P}_\delta$-transforms or pathway transforms of the function $f(t)$ $(t \in \mathbb{R})$ is a function $F_{\mathcal{P}}(s)$ of a complex variable s, is defined by (see, *e.g.*, [1])

$$
\mathcal{P}_\delta\{f(t); s\} = F_{\mathcal{P}}(s) = \int_0^\infty [1+(\delta-1)s]^{-\frac{t}{\delta-1}} f(t)\,dt \quad (\delta > 1), \tag{6}
$$

For the sufficient condition for the $\mathcal{P}_\delta$-transform (6) to exist, we refer [1].

Clearly, $\mathcal{P}_\delta$-transforms of the power function $t^{\mu-1}$ is given by [1, p. 7, Eq. (32)]

$$
\mathcal{P}_\delta\{t^{\mu-1}; s\} = \left(\frac{\delta-1}{\ln[1+(\delta-1)s]}\right)^{\mu}\Gamma(\mu) = \frac{\Gamma(\mu)}{[\Delta(\delta; s)]^{\mu}} \quad (\Re(\mu) > 0;\ \delta > 1), \tag{7}
$$

with

$$
\Delta(\delta; s) = \frac{\ln[1+(\delta-1)s]}{(\delta-1)}. \tag{8}
$$

Noting that, upon letting $\delta \mapsto 1$ in the definition (6), the $\mathcal{P}_\delta$-transform reduces to the classical Laplace transform [6]:

$$
L\{f(t); s\} = \int_0^\infty e^{-st} f(t)\,dt \ (\Re(s) > 0). \tag{9}
$$

With the help of the power function formula (7), it is not difficult to derive the $\mathcal{P}_\delta$-transform of the generalized hypergeometric function as [1, p. 8, Eq. (42)]:

$$\int_0^\infty [1+(\delta-1)s]^{-\frac{t}{\delta-1}}\, t^{\mu-1}\, {}_pF_q\left[\begin{matrix} a_1,\cdots,a_p \\ b_1,\cdots,b_q\end{matrix}\Big|\omega t\right] dt = \frac{\Gamma(\mu)}{[\Delta(\delta;s)]^\mu}\; {}_{p+1}F_q\left[\begin{matrix} a_1,\cdots,a_p,\mu \\ b_1,\cdots,b_q\end{matrix}\Big|\frac{\omega}{\Delta(\delta;s)}\right],$$

for $p<q$, $\Re(\mu)>0$, $\Re\left(\frac{\ln[1+(\delta-1)s]}{\delta-1}\right)>0$ and $\delta>1$ or for $p=q$, $\Re(\mu)>0$, $\Re\left(\frac{\ln[1+(\delta-1)s]}{\delta-1}\right)>\Re(\omega)$ and $\delta>1$.

If $p=q=2$, we get the formula :

$$\int_0^\infty [1+(\delta-1)s]^{-\frac{t}{\delta-1}}\, t^{b_2-1}\, {}_2F_2\left[\begin{matrix} a_1,a_2 \\ b_1,b_2\end{matrix}\Big|\omega t\right] dt = \frac{\Gamma(b_2)}{[\Delta(\delta;s)]^{b_2}}\; {}_2F_1\left[\begin{matrix} a_1,a_2, \\ b_1\end{matrix}\Big|\frac{\omega}{\Delta(\delta;s)}\right],$$

for $\Re(b_1)>0$, $\Re(b_2)>0$, $\Re\left(\frac{\ln[1+(\delta-1)s]}{\delta-1}\right)>\Re(\omega)$ and $\delta>1$.
In this note, an attempt has been made to obtain $\mathcal{P}_\delta$-transforms of ${}_2F_2$ generalized hypergeometric functions by employing the results (3) to (5).

2. $\mathcal{P}_\delta$-TRANSFORMS OF ${}_2F_2(z)$

This section deals with the general $\mathcal{P}_\delta$-transforms of ${}_2F_2$ generalized hypergeometric functions asserted in the following theorems:

Theorem 2.1. *If $\Re(b)>0$, $\Re\left(\frac{\ln[1+(\delta-1)s]}{\delta-1}\right)>0$ and $\delta>1$, then for $i=0,\pm1,\pm2,\pm3,\pm4,\pm5$, the following general result holds true:*

$$\int_0^\infty [1+(\delta-1)s]^{-\frac{t}{\delta-1}}\, t^{d-1}\, {}_2F_2\left[\begin{matrix} a,b \\ d,\frac{1}{2}(a+b+i+1)\end{matrix}\Big|\frac{t\,\Delta(\delta;s)}{2}\right] dt = \frac{\Gamma(d)}{[\Delta(\delta;s)]^d}\frac{\Gamma(\frac{1}{2})\Gamma(\frac{1}{2}a+\frac{1}{2}b+\frac{1}{2}i+\frac{1}{2})\Gamma(\frac{1}{2}a-\frac{1}{2}b-\frac{1}{2}i+\frac{1}{2})}{\Gamma(\frac{1}{2}a-\frac{1}{2}b+\frac{1}{2}|i|+\frac{1}{2})} \times\left\{\frac{A_i'}{\Gamma(\frac{1}{2}a+\frac{1}{2})\Gamma(\frac{1}{2}b+\frac{1}{2}i+\frac{1}{2}-\lfloor\frac{1+i}{2}\rfloor)} + \frac{B_i'}{\Gamma(\frac{1}{2}a)\Gamma(\frac{1}{2}b+\frac{1}{2}i-\lfloor\frac{i}{2}\rfloor)}\right\}.$$

Theorem 2.2. *For* $\Re(1-a+i)>0\,(i=0,\pm1,\pm2,\pm3,\pm4,\pm5)$, $\Re\left(\frac{\ln[1+(\delta-1)s]}{\delta-1}\right)>0$ *and* $\delta>1$, *the following general result holds true:*

$$\begin{aligned}\int_0^\infty &[1+(\delta-1)s]^{-\frac{t}{\delta-1}}\,t^{d-1}\,{}_2F_2\left[\begin{matrix}a,1-a+i\\ d,b\end{matrix}\;\middle|\frac{t\,\Delta(\delta;s)}{2}\right]\mathrm{d}t\\ &=\frac{\Gamma(d)}{[\Delta(\delta;s)]^d}\,\frac{\Gamma(\frac{1}{2})\Gamma(b)\Gamma(1-a)}{2^{b-i-1}\Gamma(1-a+\frac{1}{2}i+\frac{1}{2}|i|)}\\ &\times\left\{\frac{C_i'}{\Gamma(\frac{1}{2}b-\frac{1}{2}a+\frac{1}{2})+\Gamma(\frac{1}{2}b+\frac{1}{2}a-\lfloor\frac{1+i}{2}\rfloor)}\right.\\ &\left.+\frac{D_i'}{\Gamma(\frac{1}{2}b-\frac{1}{2}a)\Gamma(\frac{1}{2}b+\frac{1}{2}a-\frac{1}{2}-\lfloor\frac{i}{2}\rfloor)}\right\}.\end{aligned}$$

Theorem 2.3. *Assuming that* $\Re(b)>0$, $\Re\left(\frac{\ln[1+(\delta-1)s]}{\delta-1}\right)>0$ *and* $\delta>1$, *thereupon for* $i=0,\pm1,\pm2,\pm3,\pm4,\pm5$, *the following general result holds true:*

$$\begin{aligned}\int_0^\infty &[1+(\delta-1)s]^{-\frac{t}{\delta-1}}\,t^{d-1}\,{}_2F_2\left[\begin{matrix}a,b\\ d,1+a-b+i\end{matrix}\;\middle|-t\,\Delta(\delta;s)\right]\mathrm{d}t=\\ &=\frac{\Gamma(d)}{[\Delta(\delta;s)]^d}\,\frac{\Gamma(\frac{1}{2})\Gamma(1-b)\Gamma(1+a-b+i)}{\Gamma(1-b+\frac{1}{2}i+\frac{1}{2}|i|)}\\ &\times\left\{\frac{E_i'}{\Gamma(\frac{1}{2}a-b+\frac{1}{2}i+1)\Gamma(\frac{1}{2}a+\frac{1}{2}i+\frac{1}{2}-\lfloor\frac{1+i}{2}\rfloor)}\right.\\ &\left.+\frac{F_i'}{\Gamma(\frac{1}{2}a-b+\frac{1}{2}i+\frac{1}{2})\Gamma(\frac{1}{2}a+\frac{1}{2}i-\lfloor\frac{i}{2}\rfloor)}\right\}.\end{aligned}$$

Proofs. The proofs of the theorems are quite straight forward. In order to prove Theorem 2.1, setting $\omega=\frac{\Delta(\delta;s)}{2}$, $b_1=\frac{1}{2}(a+b+i+1)$ and $b_2=d$ for $i=0,\pm1,\pm2,\pm3,\pm4,\pm5$ in (10), we have

$$\begin{aligned}\int_0^\infty &[1+(\delta-1)s]^{-\frac{t}{\delta-1}}\,t^{d-1}\,{}_2F_2\left[\begin{matrix}a,b\\ d,\frac{1}{2}(a+b+i+1)\end{matrix}\;\middle|\frac{t\,\Delta(\delta;s)}{2}\right]\mathrm{d}t\\ &=\frac{\Gamma(d)}{[\Delta(\delta;s)]^d}\;{}_2F_1\left[\begin{matrix}a,b\\ \frac{1}{2}(a+b+i+1)\end{matrix}\;\middle|\frac{1}{2}\right].\end{aligned}$$

We, now observe that the ${}_2F_1$ appearing on the right-hand side of (10) can be evaluated with the help of summation theorem (3) by replacing $\alpha=a$ and $\beta=b$. A_i' and B_i' can be evaluated from Table-1 by replacing $\alpha=a$ and $\beta=b$. This yields the desired formula (10).

The results in Theorem 2.2 and Theorem 2.3 can also be proven in a similar fashion by applying summation theorems (4) and (5), respectively. So we prefer to omit the details.

3. Special Cases

For $d=b$, the Theorems 2.1 to 2.3, reduce the following corollaries respectively:

Corollary 3.1. *If* $\Re(b) > 0$, $\Re\left(\frac{\ln[1+(\delta-1)s]}{\delta-1}\right) > 0$ *and* $\delta > 1$, *for* $i = 0, \pm1, \pm2, \pm3, \pm4, \pm5$, *the following general result holds true:*

$$\int_0^\infty [1+(\delta-1)s]^{-\frac{t}{\delta-1}}\, t^{b-1}\, {}_1F_1\left[\begin{matrix} a \\ \frac{1}{2}(a+b+i+1)\end{matrix} \,\Big|\, \frac{t\,\Lambda(\delta;s)}{2}\right] \mathrm{d}t$$
$$= \frac{\Gamma(b)}{[\Lambda(\delta;s)]^b}\,\frac{\Gamma(\frac{1}{2})\Gamma(\frac{1}{2}a+\frac{1}{2}b+\frac{1}{2}i+\frac{1}{2})\Gamma(\frac{1}{2}a-\frac{1}{2}b-\frac{1}{2}i+\frac{1}{2})}{\Gamma(\frac{1}{2}a-\frac{1}{2}b+\frac{1}{2}|i|+\frac{1}{2})}$$
$$\times\left\{\frac{A_i'}{\Gamma(\frac{1}{2}a+\frac{1}{2})\Gamma(\frac{1}{2}b+\frac{1}{2}i+\frac{1}{2}-\lfloor\frac{1+i}{2}\rfloor)} + \frac{B_i'}{\Gamma(\frac{1}{2}a)\Gamma(\frac{1}{2}b+\frac{1}{2}i-\lfloor\frac{i}{2}\rfloor)}\right\}.$$

Corollary 3.2. *For* $\Re(1-a+i) > 0\,(i = 0, \pm1, \pm2, \pm3, \pm4, \pm5)$, $\Re\left(\frac{\ln[1+(\delta-1)s]}{\delta-1}\right) > 0$ *and* $\delta > 1$, *the following general result holds true:*

$$\int_0^\infty [1+(\delta-1)s]^{-\frac{t}{\delta-1}}\, t^{-a+i}\, {}_1F_1\left[\begin{matrix} a \\ b\end{matrix} \,\Big|\, \frac{t\,\Lambda(\delta;s)}{2}\right] \mathrm{d}t$$
$$= \frac{\Gamma(1-a+i)}{[\Lambda(\delta;s)]^{1-a+i}}\,\frac{\Gamma(\frac{1}{2})\Gamma(b)\Gamma(1-a)}{2^{b-i-1}\Gamma(1-a+\frac{1}{2}i+\frac{1}{2}|i|)}$$
$$\times\left\{\frac{C_i'}{\Gamma(\frac{1}{2}b-\frac{1}{2}a+\frac{1}{2})+\Gamma(\frac{1}{2}b+\frac{1}{2}a-\lfloor\frac{1+i}{2}\rfloor)} + \frac{D_i'}{\Gamma(\frac{1}{2}b-\frac{1}{2}a)\Gamma(\frac{1}{2}b+\frac{1}{2}a-\frac{1}{2}-\lfloor\frac{i}{2}\rfloor)}\right\}.$$

Corollary 3.3. *Suppose* $\Re(b) > 0$, $\Re\left(\frac{\ln[1+(\delta-1)s]}{\delta-1}\right) > 0$ *and* $\delta > 1$, *then the following general result holds true:*

$$\int_0^\infty [1+(\delta-1)s]^{-\frac{t}{\delta-1}}\, t^{b-1}\, {}_1F_1\left[\begin{matrix} a \\ 1+a-b+i\end{matrix} \,\Big|\, -t\,\Lambda(\delta;s)\right] \mathrm{d}t =$$
$$= \frac{\Gamma(b)}{[\Lambda(\delta;s)]^b}\,\frac{\Gamma(\frac{1}{2})\Gamma(1-b)\Gamma(1+a-b+i)}{\Gamma(1-b+\frac{1}{2}i+\frac{1}{2}|i|)}$$
$$\times\left\{\frac{E_i'}{\Gamma(\frac{1}{2}a-b+\frac{1}{2}i+1)\Gamma(\frac{1}{2}a+\frac{1}{2}i+\frac{1}{2}-\lfloor\frac{1+i}{2}\rfloor)} + \frac{F_i'}{\Gamma(\frac{1}{2}a-b+\frac{1}{2}i+\frac{1}{2})\Gamma(\frac{1}{2}a+\frac{1}{2}i-\lfloor\frac{i}{2}\rfloor)}\right\},$$

where $i = 0, \pm1, \pm2, \pm3, \pm4, \pm5$.

Remark 3.1. *The results* (10) *to* (10) *have recently been obtained by Parmar et al.* [3].

4. Concluding Remarks

In this note an attempt has been made to obtain $\mathcal{P}_\delta$-transforms of ${}_2F_2$ generalized hypergeometric functions in terms of gamma functions by employing generalizations of the classical summation theorems due to Gauss second, Bailey and Kummer for $i = \pm1, \pm2, \pm3, \pm4, \pm5$. We conclude this note by remarking that the new and interesting

$\mathcal{P}_\delta$-transforms by employing generalizations of classical summation theorems for the sere is ${}_3F_2$ such as those of Watson, Dixon and Whipple are further research findings and will form a subsequent paper in this direction.

Table 1

i	A'_i	B'_i
5	$-(\beta+\alpha+6)^2+\frac{1}{2}(\beta-\alpha+6)(\beta+\alpha+6)+\frac{1}{4}(\beta-\alpha+6)^2+11(\beta+\alpha+6)-\frac{13}{2}(\beta-\alpha+6)-20$	$(\beta+\alpha+6)^2+\frac{1}{2}(\beta-\alpha+6)(\beta+\alpha+6)-\frac{1}{4}(\beta-\alpha+6)^2-17(\beta+\alpha+6)-\frac{1}{2}(\beta-\alpha+6)+62$
4	$-\frac{1}{4}(\beta+\alpha+1)(\beta+\alpha-3)-\frac{1}{4}(\beta-\alpha+3)(\beta-\alpha-3)$	$-2(\beta+\alpha-1)$
3	$-\frac{1}{2}(\beta+3\alpha-2)$	$\frac{1}{2}(3\beta+\alpha-2)$
2	$\frac{1}{2}(\beta+\alpha-1)$	-2
1	-1	1
0	1	0
-1	1	1
-2	$\frac{1}{2}(\beta+\alpha-1)$	2
-3	$\frac{1}{2}(3\alpha+\beta-2)$	$\frac{1}{2}(3\beta+\alpha-2)$
-4	$\frac{1}{2}(\beta+\alpha-3)(\beta+\alpha+1)-\frac{1}{4}(\beta-\alpha-3)(\beta-\alpha+3)$	$2(\beta+\alpha-1)$
-5	$(\beta+\alpha-4)^2-\frac{1}{2}(\beta-\alpha-4)(\beta+\alpha-4)-\frac{1}{4}(\beta-\alpha-4)^2+4(\beta+\alpha-4)-\frac{7}{2}(\beta-\alpha-4)$	$(\beta+\alpha-4)^2+\frac{1}{2}(\beta-\alpha-4)(\beta+\alpha-4)-\frac{1}{4}(\beta-\alpha-4)^2+8(\beta+\alpha-4)-\frac{1}{2}(\beta-\alpha-4)+12$

Table 2

i	C'_i	D'_i
5	$-4\beta^2+2\alpha\beta+\alpha^2+22\beta-13\alpha-20$	$4\beta^2+2\alpha\beta-\alpha^2-34\beta-\alpha+62$
4	$2(\beta-2)(\beta-4)-(\alpha-1)(\alpha-4)$	$12-4\beta$
3	$\alpha-2\beta+3$	$a+2\beta-7$
2	$\beta-2$	-2
1	-1	1
0	1	0
-1	1	1
-2	β	2
-3	$2\beta-\alpha$	$2\beta+\alpha+2$
-4	$2\beta(\beta+2)-\alpha(\alpha+3)$	$4(\beta+1)$
-5	$4\beta^2-2\alpha\beta-\alpha^2+8\beta-7\alpha$	$4\beta^2+2a\beta-\alpha^2+16\beta-\alpha+12$

Table 3

i	E'_i	F'_i
5	$-4(6+\alpha-\beta)^2+2\beta(\alpha-\beta+6)+\beta^2$ $+22(\alpha-\beta+6)-13b-20$	$4(6+\alpha-\beta)^2+2\beta(\alpha-\beta+6)-\beta^2$ $-34(\alpha-\beta+6)-\beta+62$
4	$2(3+\alpha-b)(1+\alpha-\beta)-(\beta-1)(\beta-4)$	$-4(2+\alpha-\beta)$
3	$3\beta-2\alpha-5$	$2\alpha-\beta+1$
2	$\alpha-\beta+1$	-2
1	-1	1
0	1	0
-1	1	1
-2	$\alpha-\beta-1$	2
-3	$2\alpha-3\beta-1$	$2\alpha-\beta-1$
-4	$2(\alpha-\beta-3)(\alpha-\beta-1)-\beta(\beta+3)$	$4(\alpha-b-2)$
-5	$4(\alpha-\beta-4)^2-2\beta(\alpha-\beta-4)-\beta^2$ $+8(\alpha-\beta-4)-7\beta$	$4(\alpha-\beta-4)^2+2\beta(\alpha-\beta-4)-\beta^2$ $+16(\alpha-\beta-4)-\beta+12$

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