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STUDY OF SUBCLASSES OF STARLIKE AND CONVEX FUNCTIONS

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Abstract. In this article, we introduce new subclasses of starlike and convex functions by using a new differential operator defined in the open unit disk $\mathbb{U}$. We study the geometric properties of such subclasses of starlike and convex functions. The vertices, co-vertices, coordinates of foci and equation of directrix are discussed in detail. Finally, we investigate various convolution properties of these subclasses.

Keywords: Analytic functions, Differential operator, Convex functions.

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1. Introduction

Differential operators based on complex valued functions plays an important role in geometric function theory. In recent years, more and more researchers have been interested in studying the differential operators. They have not only introduced new differential operators but also used them for introducing new subclasses of analytic functions. Our motivation by the research works is based on differential operators, (see for example [1, 2, 3, 4, 5, 6, 7]). The articles provides an idea to introduce a new differential operator for forming new subclasses of analytic functions.
Let

$$f(z) = a_1 z + \sum_{n=2}^{\infty} a_n z^n, \tag{1}$$

and

$$f_i(z) = a_{1,i} z + \sum_{n=2}^{\infty} a_{n,i} z^n, g_j(z) = b_{1,j} z + \sum_{n=2}^{\infty} b_{n,j} z^n. \tag{2}$$

The functions analytic and univalent in $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ of the form (1) with $a_1 = 1$ are said to form the class A, and for f_i, g_i we suppose they have the form (1).
The convolution $f * g$ of the functions f and g, given by (1) is defined by $(f * g)(z) = a_1 b_1 z + \sum_{n=2}^{\infty} a_n b_n z^n = (g * f)(z)$.

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We define the differential operator $\Theta^m_{\beta,\gamma}f(z)$ in 2012 (cf. [8]), as follows:

$$\begin{aligned}\Theta^0_{\beta,\gamma}f(z) &= f(z),\\ ((\gamma+1)+\beta)\Theta^1_{\beta,\gamma}f(z) &= (\beta)f(z)+(\gamma+1)zf'(z)\\ &= \Theta_{\beta,\gamma}f(z),\\ \Theta^2_{\beta,\gamma}f(z) &= \Theta_{\beta,\gamma}(\Theta^1_{\beta,\gamma}f(z)),\\ &\vdots\\ \Theta^m_{\beta,\gamma}f(z) &= \Theta_{\beta,\gamma}(\Theta^{m-1}_{\beta,\gamma}f(z)), \qquad m\in\mathbb{N}.\end{aligned} \tag{3}$$

If f belongs to A then by using (3),

$$\Theta^k_{\beta,\gamma}f(z) = z+\sum_{n=2}^{\infty}\left(\frac{\beta+n(\gamma+1)}{\beta+(\gamma+1)}\right)^k a_n z^n, k\in\mathbb{N}_0=\mathbb{N}\cup\{0\}. \tag{4}$$

Consider,

$$f_{\beta,\gamma}(z) = z+\sum_{n=2}^{\infty}\left(\frac{\beta+n(1+\gamma)}{\beta+(1+\gamma)}\right)^k z^n.$$

and define

$$f_{\beta,\gamma}(z)*[f_{\beta,\gamma}(z)]^{-1} = \frac{z}{(1-z)^{\mu}}, \qquad \mu>0,\ z\in\mathbb{U}. \tag{5}$$

After doing calculation and then using (5), we get

$$\Upsilon^{k,\mu}_{\beta,\gamma}f(z) = z+\sum_{n=2}^{\infty}\frac{(\mu)_{n-1}}{(n-1)!}\left(\frac{\beta+(1+\gamma)}{\beta+n(1+\gamma)}\right)^k a_n z^n. \tag{6}$$

For, $0\le\rho<1$, $0\le\delta<1$ and $\eta\ge 0$, let $\ominus^{\mu}_{\beta,\gamma}(k,\rho,\delta,\eta)$ denote the class of analytic functions f given by (1) with $a_1=1$ and satisfying the analytic criterion

$$\Re\left\{\frac{z[\Upsilon^{k,\mu}_{\beta,\gamma}f(z)]'}{(1-\rho)\Upsilon^{k,\mu}_{\beta,\gamma}f(z)+\rho z[\Upsilon^{k,\mu}_{\beta,\gamma}f(z)]'}-\delta\right\} > \eta\left|\frac{z[\Upsilon^{k,\mu}_{\beta,\gamma}f(z)]'}{(1-\rho)\Upsilon^{k,\mu}_{\beta,\gamma}f(z)+\rho z[\Upsilon^{k,\mu}_{\beta,\gamma}f(z)]'}-1\right|. \tag{7}$$

In case, for $\rho=0$, if $f\in\ominus^{\mu}_{\beta,\gamma}(k,\rho,\delta,\eta)$, then $z[\Upsilon^{k,\mu}_{\beta,\gamma}f(z)]'/\Upsilon^{k,\mu}_{\beta,\gamma}f(z)$ belongs to the region of complex plane i.e. $\{w:\Re(w-\delta)>\eta|w-1|\}$ which contains $w=1$ and is bounded by an ellipse $\frac{\left(u-\frac{\eta^2-\delta}{\eta^2-1}\right)^2}{\frac{\eta^2(1-\delta)^2}{(\eta^2-1)^2}}+\frac{v^2}{\frac{(1-\delta)^2}{\eta^2-1}}=1$ with vertices at the points $\left(\frac{\eta-\delta}{\eta-1},0\right)$, $\left(\frac{\eta+\delta}{\eta+1},0\right)$, $\left(\frac{\eta^2-\delta}{\eta^2-1},\frac{1-\delta}{\sqrt{\eta^2-1}}\right)$ and $\left(\frac{\eta^2-\delta}{\eta^2-1},\frac{\delta-1}{\sqrt{\eta^2-1}}\right)$ respectively. By using vertices and co-vertices of ellipse, one can easily get the coordinates of foci and equation of directrix.

$\oplus^{\mu,k}_{\beta,\gamma}(\rho,\delta,\eta)$ denote the class of analytic functions f given by (1) with $a_1=1$ and satisfying the analytic criterion

$$\Re\left\{\frac{[\Upsilon^{k,\mu}_{\beta,\gamma}f(z)]'+z[\Upsilon^{k,\mu}_{\beta,\gamma}f(z)]''}{[\Upsilon^{k,\mu}_{\beta,\gamma}f(z)]'+\rho z[\Upsilon^{k,\mu}_{\beta,\gamma}f(z)]''}-\delta\right\} > \eta\left|\frac{[\Upsilon^{k,\mu}_{\beta,\gamma}f(z)]'+z[\Upsilon^{k,\mu}_{\beta,\gamma}f(z)]''}{[\Upsilon^{k,\mu}_{\beta,\gamma}f(z)]'+\rho z[\Upsilon^{k,\mu}_{\beta,\gamma}f(z)]''}-1\right|. \tag{8}$$

For $\rho=0$, if $f\in\oplus^{\mu,k}_{\beta,\gamma}(\rho,\delta,\eta)$, then $z[\Upsilon^{k,\mu}_{\beta,\gamma}f(z)]''/[\Upsilon^{k,\mu}_{\beta,\gamma}f(z)]'$ belongs to the region of complex plane i.e. $\{w:\Re(1+w-\delta)>\eta|w|\}$ which contains $w=1$ and is bounded by an ellipse $\frac{\left(u-\frac{1-\delta}{\eta^2-1}\right)^2}{\frac{\eta^2(1-\delta)^2}{(\eta^2-1)^2}}+\frac{v^2}{\frac{(1-\delta)^2}{\eta^2-1}}=1$ with vertices at the points $\left(\frac{1-\delta}{\eta-1},0\right)$, $\left(\frac{\delta-1}{\eta+1},0\right)$, $\left(\frac{1-\delta}{\eta^2-1},\frac{1-\delta}{\sqrt{\eta^2-1}}\right)$

and $\left(\frac{1-\delta}{\eta^2-1}, \frac{\delta-1}{\sqrt{\eta^2-1}}\right)$ respectively. By using vertices and co-vertices of ellipse, one can easily get the coordinates of foci and equation of directrix.
For $\rho = k = 0$, I get the uniformly starlike and convex functions, first studied by Goodman [9, 10] and Ronning [11, 12] respectively. For class of α-uniformly convex function, I refer for study [13]. For special cases of the classes given by (7) and (8), I refer some other papers (for example [14]-[20]). For sake of convenience, we consider $\phi(\beta,\gamma) = \left(\frac{\beta+(\gamma+1)}{\beta+n(\gamma+1)}\right)$.
A function f defined by (1) belongs to class $\ominus^{\mu}_{\beta,\gamma}(k,\rho,\delta,\eta)$ if

$$\sum_{n=2}^{\infty} \frac{(\mu)_{n-1}}{(n-1)!} \left(\phi(\beta,\gamma)\right)^k \left[n(1+\eta) - (\delta+\eta)(1+n\rho-\rho)\right] |a_n| \le (1-\delta). \tag{9}$$

The class $\ominus^{\mu}_{\beta,\gamma}(k,\rho,\delta,\eta)$ is nonempty and having the functions of the form

$$f(z) = a_1 z + \sum_{n=2}^{\infty} \frac{(\beta+n(1+\gamma))^k (1-\delta)(n-1)!}{(\mu)_{n-1}(\beta+(1+\gamma))^k \left[n(1+\eta) - (\delta+\eta)(1+n\rho-\rho)\right]} \lambda_n z^n,$$

where $\lambda_n \ge 0$ and $\sum\limits_{n=2}^{\infty} \lambda_n \le 1$; satisfy the inequality given in (9). Similarly, a function f defined by (1) belongs to class $\oplus^{\mu,k}_{\beta,\gamma}(\rho,\delta,\eta)$ if

$$\sum_{n=2}^{\infty} \frac{(\mu)_{n-1}}{(n-1)!} \left(\phi(\beta,\gamma)\right)^k n \left[n(1+\eta) - (\delta+\eta)(1+n\rho-\rho)\right] |a_n| \le (1-\delta). \tag{10}$$

2. Main Results (Convolution Properties)

Theorem 2.1. *Let the functions $f_i \mid_{i=1}^{r}$ and $g_j \mid_{j=1}^{q}$ defined by (2) belong to the class $\ominus^{\mu}_{\beta,\gamma}(k+1,\rho,\delta,\eta)$ and $\ominus^{\mu}_{\beta,\gamma}(k,\rho,\delta,\eta)$ respectively. Then convolution of $f_1 * f_2 * \cdots * f_r * g_1 * g_2 * \cdots * g_q(z)$ belongs to $\ominus^{\mu}_{\beta,\gamma}(r(k+2)+q(k+1)-1,\rho,\delta,\eta)$.*

Proof. For sake of convenience, let convolution of

$$f_1 * f_2 * \cdots * f_r * g_1 * g_2 * \cdots * g_q = G;$$

then obviously,

$$G(z) = \left[\prod_{i=1}^{r} |a_{1,i}|\right]\left[\prod_{j=1}^{q} |b_{1,j}|\right] z + \sum_{n=2}^{\infty}\left[\prod_{i=1}^{r} |a_{n,i}|\right]\left[\prod_{j=1}^{q} |b_{n,j}|\right] z^n.$$

Since $f_i \in \ominus^{\mu}_{\beta,\gamma}(k+1,\rho,\delta,\eta)$, then by using (9), we have

$$\sum_{n=2}^{\infty} \frac{(\mu)_{n-1}}{(n-1)!}(\phi(\beta,\gamma))^{k+1}[n(1+\eta) - (\delta+\eta)(1+n\rho-\rho)]\left|a_{n,i}\right| \le (1-\delta)\left|a_{1,i}\right|.$$

Or

$$\left|a_{n,i}\right|_{i=1}^{r} \le (\phi(\beta,\gamma))^{-k-2}\left|a_{1,i}\right|_{i=1}^{r}. \tag{11}$$

Similarly if $g_j \in \ominus^{\mu}_{\beta,\gamma}(k,\rho,\delta,\eta)$, then

$$\sum_{n=2}^{\infty} \frac{(\mu)_{n-1}}{(n-1)!}(\phi(\beta,\gamma))^{k}[n(1+\eta) - (\delta+\eta)(1+n\rho-\rho)]\left|b_{n,j}\right| \le (1-\delta)\left|b_{1,j}\right|. \tag{12}$$

This implies that

$$\left|b_{n,j}\right|_{i=1}^{q} \le (\phi(\beta,\gamma))^{-k-1}\left|b_{1,j}\right|_{i=1}^{q}. \tag{13}$$

Now we have to do that

$$\sum_{n=2}^{\infty}\frac{(\mu)_{n-1}}{(n-1)!}(\phi(\beta,\gamma))^{t}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]\big[\prod_{i=1}^{r}|a_{n,i}|\big]\big[\prod_{j=1}^{q}|b_{n,j}|\big]$$
$$\leq(1-\delta)\big[\prod_{i=1}^{r}|a_{1,i}|\big]\big[\prod_{j=1}^{q}|b_{1,j}|\big].$$

Applying simultaneously (11), (12) and (13) for $|_{i=1}^{r}$, $j=q$ and $|_{j=1}^{q-1}$ respectively. Consider $[t=r(k+2)+q(k+1)-1)]$

$$\sum_{n=2}^{\infty}\frac{(\mu)_{n-1}}{(n-1)!}(\phi(\beta,\gamma))^{t}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]\big[\prod_{i=1}^{r}|a_{n,i}|\big]\big[\prod_{j=1}^{q}|b_{n,j}|\big]$$
$$\leq\sum_{n=2}^{\infty}\frac{(\mu)_{n-1}}{(n-1)!}(\phi(\beta,\gamma))^{t}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]\big[(\phi(\beta,\gamma))^{-r(k+2)}$$
$$\times(\phi(\beta,\gamma))^{-(q-1)(k+1)}|b_{n,q}|\prod_{i=1}^{r}|a_{1,i}|\big]\big[\prod_{j=1}^{q-1}|b_{1,j}|\big]$$
$$=\sum_{n=2}^{\infty}\frac{(\mu)_{n-1}}{(n-1)!}(\phi(\beta,\gamma))^{k}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]|b_{n,q}|\big[\prod_{i=1}^{r}|a_{1,i}|\prod_{j=1}^{q-1}|b_{1,j}|\big]$$
$$\leq(1-\delta)\big[\prod_{i=1}^{r}|a_{1,i}|\big]\big[\prod_{j=1}^{q}|b_{1,j}|\big]\Rightarrow G\in\ominus_{\beta,\gamma}^{\mu}(r(k+2)+q(k+1)-1,\rho,\delta,\eta).$$

This completes the proof of Theorem 2.1. □

Corollary 2.1. *Let the functions* $f_i\ |_{i=1}^{r}$ *and* $g_j\ |_{j=1}^{q}$ *defined by (2) belong to the class* $\ominus_{\beta,\gamma}^{\mu}(k+1,\rho,\delta,\eta)$ *and* $\ominus_{\beta,\gamma}^{\mu}(k-1,\rho,\delta,\eta)$ *respectively. Then convolution of* $f_1*f_2*\cdots*f_r*g_1*g_2*\cdots*g_q(z)$ *belongs to* $\ominus_{\beta,\gamma}^{\mu}(r(k+2)+qk-1,\rho,\delta,\eta)$.

Corollary 2.2. *Let the functions* $f_i\ |_{i=1}^{r}$ *and* $g_j\ |_{j=1}^{q}$ *defined by (2) belong to the class* $\ominus_{\beta,\gamma}^{\mu}(k,\rho,\delta,\eta)$ *and* $\ominus_{\beta,\gamma}^{\mu}(k,\rho,\delta,\eta)$ *respectively. Then convolution of* $f_1*f_2*\cdots*f_r*g_1*g_2*\cdots*g_q(z)$ *belongs to* $\ominus_{\beta,\gamma}^{\mu}((r+q)(k+1)-1,\rho,\delta,\eta)$.

Theorem 2.2. *Let the function* $f_i\ |_{i=1}^{r}$ *defined by (2) belongs to the class* $\ominus_{\beta,\gamma}^{\mu}(k+1,\rho,\delta,\eta)$. *Then convolution of* $f_1*f_2*\cdots*f_r$ *belongs to* $\ominus_{\beta,\gamma}^{\mu}(r(k+2)-1,\rho,\delta,\eta)$.

Proof. We consider $f_1*f_2*\cdots*f_r=G$. Since $f_i\in\ominus_{\beta,\gamma}^{\mu}(k+1,\rho,\delta,\eta)$, this implies that

$$\sum_{n=2}^{\infty}\frac{(\mu)_{n-1}}{(n-1)!}(\phi(\beta,\gamma))^{k+1}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]\,|a_{n,i}|\leq(1-\delta)\,|a_{1,i}|\,. \quad (14)$$

Or

$$|a_{n,i}|_{i=1}^{r}\leq(\phi(\beta,\gamma))^{-k-2}\,|a_{1,i}|_{i=1}^{r}\,. \quad (15)$$

By using (14) and (15) for $i=r$ and $i=1,2,\cdots,r-1$ respectively, we get

$$\sum_{n=2}^{\infty}\frac{(\mu)_{n-1}}{(n-1)!}(\phi(\beta,\gamma))^{r(k+2)-1}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]\left[\prod_{i=1}^{r}|a_{n,i}|\right]$$
$$\leq\sum_{n=2}^{\infty}\frac{(\mu)_{n-1}}{(n-1)!}(\phi(\beta,\gamma))^{r(k+2)-1}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]\,|a_{n,r}|\times$$

$$\left[(\phi(\beta,\gamma))^{-(r-1)(k+2)}\prod_{i=1}^{r-1}|a_{1,i}|\right]$$

$$=\sum_{n=2}^{\infty}\frac{(\mu)_{n-1}}{(n-1)!}(\phi(\beta,\gamma))^{k+1}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]\,|a_{n,r}|\left[\prod_{i=1}^{r-1}|a_{1,i}|\right]$$

$$\leq(1-\delta)\left[\prod_{i=1}^{r}|a_{1,i}|\right].$$

Hence the proof is complete. □

Theorem 2.3. *If the function $g_i\mid_{i=1}^{q}$ defined by (2) belongs to the class $\ominus^{\mu}_{\beta,\gamma}(k,\rho,\delta,\eta)$. Then convolution of $g_1*g_2*\cdots*g_q$ belongs to the class $\ominus^{\mu}_{\beta,\gamma}(q(k+1)-1,\rho,\delta,\eta)$.*

Proof. Let $g_i\in\ominus^{\mu}_{\beta,\gamma}(k,\rho,\delta,\eta)$, this implies that

$$\sum_{n=2}^{\infty}\frac{(\mu)_{n-1}}{(n-1)!}(\phi(\beta,\gamma))^{k}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]\,|b_{n,i}|\leq(1-\delta)\,|b_{1,i}|\,, \tag{16}$$

and

$$\frac{(\mu)_{n-1}}{(n-1)!}(\phi(\beta,\gamma))^{k}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]\,|b_{n,i}|_{i=1}^{q}\leq(1-\delta)\,|b_{1,i}|_{i=1}^{q}\,,$$

or

$$|b_{n,i}|_{i=1}^{q}\leq(\phi(\beta,\gamma))^{-k-1}\,|b_{1,i}|_{i=1}^{q}\,. \tag{17}$$

Now we need to do that

$$\sum_{n=2}^{\infty}\frac{(\mu)_{n-1}}{(n-1)!}(\phi(\beta,\gamma))^{q(k+1)-1}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]\left[\prod_{i=1}^{q}|b_{n,i}|\right]$$

$$\leq(1-\delta)\left[\prod_{i=1}^{q}|b_{1,i}|\right].$$

Using (16) and (17) for $i=r$ and $i=1,2,\cdots,q-1$, we have

$$\sum_{n=2}^{\infty}\frac{(\mu)_{n-1}}{(n-1)!}(\phi(\beta,\gamma))^{q(k+1)-1}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]\left[\prod_{i=1}^{q}|b_{n,i}|\right]$$

$$\leq\sum_{n=2}^{\infty}\frac{(\mu)_{n-1}}{(n-1)!}(\phi(\beta,\gamma))^{q(k+1)-1}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]\,|b_{n,q}|\times$$

$$\left[(\phi(\beta,\gamma))^{-(q-1)(k+1)}\prod_{i=1}^{q-1}|b_{1,i}|\right]$$

$$\leq(1-\delta)\left[\prod_{i=1}^{q}|b_{1,i}|\right].$$

This completes proof of Theorem 2.3. □

Similarly, we can prove the following Theorems.

Theorem 2.4. *Let the functions $f_i\mid_{i=1}^{r}$ and $g_j\mid_{j=1}^{q}$ defined by (2) belong to the class $\ominus^{\mu}_{\beta,\gamma}(k-1,\rho,\delta,\eta)$ and $\ominus^{\mu}_{\beta,\gamma}(k-1,\rho,\delta,\eta)$. Then Hadamard product of $f_1*f_2*\cdots*f_r*g_1*g_2*\cdots*g_q(z)$ belongs to the class $R^{\mu}_{\beta,\gamma}((r+q)k-1,\rho,\delta,\eta)$.*

Theorem 2.5. *Let the functions* $f_i\ |_{i=1}^{r}$ *and* $g_j\ |_{j=1}^{q}$ *defined by (2) belong to the class* $\ominus_{\beta,\gamma}^{\mu}(k-1,\rho,\delta,\eta)$ *and* $\ominus_{\beta,\gamma}^{\mu}(0,\rho,\delta,\eta)$. *Then Hadamard product of* $f_1 * f_2 * \cdots * f_r * g_1 * g_2 * \cdots * g_q(z)$ *belongs to the class* $R_{\beta,\gamma}^{\mu}(rk+q-1,\rho,\delta,\eta)$.

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