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OPTICAL SOLITON SOLUTIONS OF THE FRACTIONAL PERTURBED NONLINEAR SCHRÖDINGER EQUATION

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Abstract. This paper is interested in a set of conformable fractional derivative for constructing optical soliton solutions to the fractional perturbed nonlinear Schrödinger equation. The powerful Kudryashov method is the integration scheme that has been implemented to retrieve the solitary wave solutions. After converting equation to integer-ordered ordinary differential equations, replacing the suggested form for the solution into the integer-ordered ordinary differential equations, the nonzero coefficients in solutions are detected. Some graphical illustrations of the obtained solutions for the different cases are drawn. Our results prove the correctness and durableness of the method which can be further used for solving such problems appearing in plasma physics, optical fibers, fluid dynamics, nonlinear optics etc.

Keywords: The fractional perturbed nonlinear Schrödinger equation, Kudryashov method, optical solutions, soliton.
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1. Introduction

The theory of nonlinear differential equations (PDEs) has made a significant development during the last decade especially in nonlinear optics, optical fibers, chemical physics, capillary-gravity, fluid dynamics and mechanics, plasmas, condensed matter, electro magnetics and any more. The study of exact solutions of nonlinear PDEs featuring fractional order derivative in space or time variable or both (space-time) is very significant in the understanding of many phenomena in different science and has progressively become one of the most prominent topic both for physicists and mathematicians. As yet, many efficacious and operative method to getting exact solutions of nonlinear PDEs have been presented. There are various schemes have been used to handle like problems besides conventional

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ones for instance the Kudryashov methods (KMs) [1-4] for time fractional KdV-KZK equation, modified extended tanh method for fractional equal width wave equations by Raslan et al. [5], G'/G-expansion method for nonlinear fractional differential equations by Bekir and Guner [6] and for another methods see [7-34].

Schrödinger equations play fundemental roles in characterizing the variable dynamical manners of light pulses. These equations have been studied for more than 40 years and there exist a plethora of results in the literature in regards to these models. It is an influential and plain method and is largely used. The principal goal of this paper is to govern the KM to determine new types of optical soliton solutions to fractional perturbed nonlinear Schrödinger equation defined in terms of conformable fractional derivative [35]. Khalil et al. illustrated this new definition extends the classical limit definition. The conformable fractional derivative of order α is describe as

$$D_t^\alpha f(t) = \lim_{\varepsilon \to 0} \frac{f(t+\varepsilon t^{1-\alpha}) - f(t)}{\varepsilon},$$

for all $t > 0$, $\alpha \in (0,1]$.

The remainder of this study is arranged as follows. In Section 2 we describe Kudryashov method for finding optical soliton solutions of nonlinear fractional evolution equations. In Section 3, we illustrate this method in detail with the fractional perturbed nonlinear Schrödinger equation. Some graphical exemplifications of the obtained solutions are given in Section 4. The last section is a brief conclusion.

2. The properties of the Methodology of Solution

To explain the main mentality of the scheme, take into consideration the following nonlinear fractional differential equation

$$F\left(u,\, D_t^\alpha u,\, D_x^\beta u,\, D_t^{2\alpha} u,\, \ldots\right) = 0, \quad 0 < \alpha, \beta \le 1, \quad t \ge 0. \tag{1}$$

Applying the following transformation

$$u\,(x,\,t) \;= f\,(\xi)\, \exp\left(i\,\varphi(\xi)\right), \qquad \xi = k\,\frac{t^\alpha}{\alpha} - a\,\frac{x^\beta}{\beta}, \quad 0 < \alpha, \beta \le 1,$$

where $f\,(\xi)$ and $\phi\,(\xi)$ are real functions of ξ, k and a are nonzero constants, transforms (1) into an integer order nonlinear ordinary differential equations as follows

$$H(f', f'', f''',\ \ldots\ ) \;= 0, \tag{2}$$

where the derivatives are with respect to ξ. It is supposed that the solutions of (2) is offered as a finite series,

$$f\,(\xi) = \sum_{n=0}^{N} a_n\, Q^n\,(\xi)\,, \tag{3}$$

where a_n, $n = 0,\, 1,\, 2,\, \ldots\ldots, N(a_N \neq 0)$ are constants can be calculated and $Q\,(\xi)$ is the following function:

$$Q\,(\xi) = \frac{1}{1 + d\,\exp\,(\xi)},$$

which supplies the following first-order equation

$$Q'\,(\xi) + Q\,(\xi) = Q^2\,(\xi)\,.$$

It can be referred that the value of N is usually determined by balancing the linear and nonlinear terms of highest orders in (1). Substituting Eq. (3) and its necessary derivatives, for example

$$f' = \sum_{n=1}^{N} a_n n\, Q^n (Q-1),$$
$$f'' = \sum_{n=1}^{N} a_n n\; Q^n (Q-1)\ \left((1+n)\, Q - n\right),$$

into (3) gives

$$P(Q(\xi)) = 0, \tag{4}$$

where $P(Q(\xi))$ is a polynomial in $Q(\xi)$. By equating the coefficient of each power of $Q(\xi)$ in (4) to zero, a system of algebraic equations can be acquired whose solution yields the exact solutions of (1).

3. Construction and Implementation of the method

Now, the exact solutions of the perturbed nonlinear Schrödinger equation have been built using Kudryashov method.

3.1. The fractional perturbed nonlinear Schrödinger equation. Consider the fractional perturbed nonlinear Schrödinger equation in the form [34],

$$\begin{aligned} &iD_x^{\alpha}u(x,t) + \frac{1}{2}D_{tt}^{2\alpha}u(x,t) + \gamma_1|u(x,t)|^2u(x,t) + \gamma_2|u(x,t)|^4u(x,t) + \\ &pu(x,t) - \tau_R u(x,t)D_t^{\alpha}|u(x,t)|^2 + isD_t^{\alpha}|u(x,t)|^2u(x,t) = 0, \end{aligned} \tag{5}$$

where $u(x,t)$ is the normalized slowly varying amplitude, x and t stands for the normalized dispersion distance variable and the normalized time variable. γ_1, γ_2, p, τ_R, and s are the Kerr law (cubic) nonlinear coefficient, the saturation of the nonlinear refractive index (quintic) coefficient, the debasing parameter [36], the Raman effect coefficient, and the self-steepening coefficient, respectively [37]. We use the following fractional complex transformation

$$u(x,t) = f(\xi)\ \exp(i\,\varphi(\xi)), \qquad \xi = k\,\frac{t^{\alpha}}{\alpha} - a\,\frac{x^{\beta}}{\beta}. \tag{6}$$

Substituting the above ansatz (6) into (5), we converts (5) into an integer order nonlinear ordinary differential equation as the following

$$a\,f\ \phi' + \frac{1}{2}k^2 f'' - \frac{1}{2}k^2 f\ \ \phi'^2 + \gamma_1 f^3 + \gamma_2 f^5 + p\,f - 2\tau_R\,k\,f^2 f' - s\,k\,f^3\phi' = 0, \tag{7}$$

$$-a\,f' + k^2 f'\,\phi' + \frac{1}{2}k^2 f\ \phi'' + 3s\,k\,f^2 f' = 0. \tag{8}$$

Assuming that

$$\phi' = b + c\,f^2, \tag{9}$$

where b and c are real constants. Adding the above ansatz (9) into (8), to do the right-hand side of (8) zero, we must take $b = \frac{a}{k^2}$ and $c = \frac{-3\,s}{2k}$, then the statement of $\phi(\xi)$ is got

$$\phi(\xi) = \int \left[\frac{a}{k^2} - \frac{3\,s}{2k}f^2(\xi)\right]\, d\xi. \tag{10}$$

Substituting (10) into Eq.(7), we have

$$k^2 f'' + \mu_0\, f^2 f' + \lambda_0 f + \delta_0 f^3 + q_0 f^5 = 0, \tag{11}$$

where

$$\mu_0 = -4k\tau_R, \lambda_0 = \frac{a^2}{k^2} + 2p,$$

$$\delta_0 = 2\gamma_1 - \frac{2s\,a}{k}, q_0 = 2\gamma_2 + \frac{3s^2}{4}.$$

Balancing f'' and f^5 in (11) results $N+2=5N$, and so $N=\frac{1}{2}$, but we know that N must be positive integer number, so we choose the transformation function $f(\xi) = g^{1/2}(\xi)$ and substituting into (11), we get

$$-k^2 g'^2 + 2k^2 g\, g'' + 2\mu_0\, g^2 g' + 4\lambda_0 g^2 + 4\delta_0 g^3 + 4q_0 g^4 = 0. \tag{12}$$

Now, we make balancing $g\,g''$ and g^4 in (12) results $N+N+2=4N$, and so $N=1$. This presents a truncated series as the following form

$$g(\xi) = a_0 + a_1\, Q(\xi)\,. \tag{13}$$

By substituting (13) into (12) and equating the coefficient of each power of $Q(\xi)$ to zero. A system of algebraic equations are generated as follows

$$\begin{aligned}
&\tfrac{a^2 a_0{}^2}{k^2} + 2p\, a_0{}^2 - \tfrac{2a\ s\ a_0{}^3}{k} + \tfrac{3}{4}s^2\ a_0{}^4 + 2\,\gamma_1 a_0{}^3 + 2\,\gamma_2 a_0{}^4 = 0,\\
&\tfrac{2a^2\, a_0\ a_1}{k^2} + \tfrac{1}{2}k^2 a_0\, a_1 + 4p\, a_0\, a_1 - \tfrac{(6a\ s\ a_0{}^2\ a_1)}{k} + 3s^2 a_0{}^3 a_1 + 6\,\gamma_1 a_0{}^2 a_1\\
&\quad + 8\,\gamma_2 a_0{}^3 a_1 + 2k a_0{}^2 a_1\, \tau_R = 0,\\
&-\tfrac{3}{2}k^2 a_0\, a_1 + \tfrac{a^2\ a_1{}^2}{k^2} + \tfrac{1}{4}k^2 a_1{}^2 + 2p\, a_1{}^2 - \tfrac{(6a\ s\ a_1{}^2\ a_0)}{k} + \tfrac{9}{2}s^2 a_0{}^2 a_1{}^2 + 6\,\gamma_1 a_1{}^2 a_0\\
&\quad + 12\gamma_2 a_0{}^2 a_1{}^2 - 2k\, a_0{}^2 a_1\, \tau_R + 4k\, a_1{}^2\, a_0 \tau_R = 0,\\
&k^2 a_0\ a_1 - k^2 a_1{}^2 - \tfrac{(2a\ s\ a_1{}^3\,)}{k} + 3s^2 a_0 a_1{}^3 + 2\gamma_1 a_1{}^3 + 8\,\gamma_2 a_1{}^3 a_0 - 4k\, a_1{}^2 a_0\, \tau_R\\
&\quad + 2k\, a_1{}^3\, \tau_R = 0,\\
&\tfrac{3}{2}k^2 a_1{}^2 + \tfrac{3}{4}s^2 a_1{}^4 + 2\gamma_2 a_1{}^4 - 2k\, a_1{}^3\, \tau_R = 0.
\end{aligned}$$

Solving the above system, following cases are obtained:

Case 1. When we take

$$a_0 = -\frac{3k}{4\,\tau_R},\ \ a_1 = 0,$$

$$p = \frac{-27\,k^4 s^2 - 72\,k^4\gamma_2 - 96ak^2 s\ \tau_R + 96k^3\,\gamma_1\,\tau_R - 64\,a^2\,\tau_R{}^2}{128\,k^2\,\tau_R{}^2},$$

and substituting them into (13) we have,

$$g(\xi) = -\frac{3\,k}{4\tau_R}.$$

Since$f(\xi) = g^{1/2}(\xi)$, then the corresponding solution to (11) is

$$f(\xi) = \sqrt{-\frac{3\,k}{4\,\tau_R}}. \tag{14}$$

Using (14) and (10), we get

$$\phi(\xi) = \int \left[\frac{a}{k^2} + \frac{9\,s\,k}{8\,k\,\tau_R}\right] d\xi = \left(\frac{a}{k^2} - \frac{9\,s\,k}{8\,k\,\tau_R}\right)\ \xi\ + C_1, \tag{15}$$

where C_1 is a constant of integration. Substituting (14) and (15) into (6), then the solution of (5) is formed as:

$$u_1(x,\,t)\ = \sqrt{-\frac{3\,k}{4\,\tau_R}}\ \exp\left(i\ \left[\left(\frac{a}{k^2} + \frac{9\,s\,k}{8\,k\,\tau_R}\right)\ \xi\ + C_1\right]\right)\,,\qquad \xi = k\,\frac{t^\alpha}{\alpha} - a\,\frac{x^\beta}{\beta}.$$

Case 2. When we choose

$$\begin{aligned}
&a_1 = 0,\ \ p = \tfrac{1}{8}\left(\tfrac{-4\,a^2}{k^2} + 2\,k^2 + 3s^2 a_0{}^2 + 8a_0{}^2\gamma_2 + 8k\,a_0\,\tau_R\right),\\
&\gamma_1 = \frac{-k^2 + \frac{4a\ s\ a_0}{k} - 3\,s^2 a_0{}^2 - 8a_0{}^2\gamma_2 - 4\,k\,a_0\ \tau_R}{4\,a_0},
\end{aligned}$$

and substituting them into (13) we have,

$$g(\xi) = a_0 .$$

Since$f(\xi) = g^{1/2}(\xi)$, then the corresponding solution to equation (11) is

$$f(\xi) = \sqrt{a_0}. \tag{16}$$

Using (16) and (10), we get

$$\phi(\xi) = \int \left[\frac{a}{k^2} - \frac{3\,s\,a_0}{2\,k}\right] d\xi = \left(\frac{a}{k^2} - \frac{3\,s\,a_0}{2\,k}\right)\,\xi\,+C_2, \tag{17}$$

where C_2 is a constant of integration.

Substituting (16) and (17) into (6), hence, the solution of (5) is composed as:

$$u_2(x,\,t) \;=\sqrt{a_0}\,\exp\left(i\,\left[\left(\frac{a}{k^2} - \frac{3\,s\,a_0}{2\,k}\right)\,\xi\,+C_2\right]\right), \qquad \xi = k\,\frac{t^\alpha}{\alpha} - a\,\frac{x^\beta}{\beta}.$$

Case 3. If we take

$$p = \frac{-4a^2 - k^4}{8\,k^2}, a_0 = 0, \quad \gamma_1 = \frac{1}{2}\left(\frac{2\,a\,s}{k} + \frac{k^2}{a_1} - 2\,k\,\tau_R\right),$$

$$\gamma_2 = \frac{-3k^2 - 3\,s^2\,a_1{}^2 + 8k\;a_1\,\tau_R}{8\,a_1{}^2},$$

and substituting them into (13) we have,

$$g(\xi) = a_1\;Q(\xi) = \frac{a_1}{1 + d\;\exp(\xi)}.$$

Since $f(\xi) = g^{1/2}(\xi)$, then the corresponding solution to (11) is

$$f(\xi) = \sqrt{\frac{a_1}{1 + d\;\exp(\xi)}}. \tag{18}$$

Using (18) and (10), we get

$$\phi(\xi) = \int \left[\frac{a}{k^2} - \frac{3\,s\;a_1}{2\,k\;(1 + d\;\exp(\xi))}\right] d\xi = \frac{a}{k^2}\xi - \frac{3\,s\;a_1}{2\,k}\;[\xi - \log(1 + d\;\exp(\xi))] + C_3, \tag{19}$$

where C_3 is a constant of integration.

Substituting (18) and (19) into (6), hence, the solution of (5) is composed as:

$$u_3(x,\,t) \;= \sqrt{\tfrac{a_1}{1+d\;\exp(\xi)}}\;\exp\left(i\;\left(\tfrac{a}{k^2}\xi - \tfrac{3\,s\;a_1}{2\,k}\;[\xi - \log(1 + d\;\exp(\xi))] + C_3\right)\;\right),$$

$$\xi = k\,\tfrac{t^\alpha}{\alpha} - a\,\tfrac{x^\beta}{\beta}.$$

Case 4. If we choose

$$a_1 = 0, \qquad a_0 = \tfrac{-3\,k}{4\,\tau_R},\;\; p = \tfrac{1}{128}\left(\tfrac{-64\,a^2}{k^2} - 64\,k^2 + \tfrac{27\;k^2\,s^2}{\tau_R{}^2} + \tfrac{72\;k^2\,\gamma_2}{\tau_R{}^2}\right),$$
$$\gamma_1 = \tfrac{1}{12}\left(\tfrac{12\,a\,s}{k} + \tfrac{27\;k\;s^2}{4\tau_R} + \tfrac{18\;k^2\gamma_2}{\tau_R} - 8\,k\,\tau_R\right),$$

and substituting into (13) we have,

$$g(\xi) = a_0\;.$$

Since$f(\xi) = g^{1/2}(\xi)$, then the corresponding solution to (11) is

$$f(\xi) = \sqrt{a_0}. \tag{20}$$

Using (16) and (10), we get

$$\phi(\xi) = \int \left[\frac{a}{k^2} - \frac{3\,s\,a_0}{2\,k}\right] d\xi = \left(\frac{a}{k^2} - \frac{3\,s\,a_0}{2\,k}\right)\xi + C_4, \tag{21}$$

where C_4 is a constant of integration.

Substituting (20) and (21) into (6), so, the solution of (5) is composed as:

$$u_4(x,\,t) = \sqrt{a_0}\,\exp\left(i\left(\left(\frac{a}{k^2} - \frac{3\,s\,a_0}{2\,k}\right)\xi + C_4\right)\right), \qquad \xi = k\,\frac{t^\alpha}{\alpha} - a\,\frac{x^\beta}{\beta}.$$

Case 5. When we take into considerstion

$$a_1 = -a_0, \qquad \gamma_1 = \frac{1}{2}\left(\frac{2\,a\,s}{k} + \frac{k^2}{a_0} + 2\,k\,\tau_R\right), \qquad p = \frac{-4\,a^2 - k^4}{8\,k^2}$$
$$\gamma_2 = \frac{-3\,k^2 - 3\,s^2 a_0{}^2 - 8\,k\,a_0\,\tau_R}{8\,a_0{}^2},$$

and substituting into (13) we have,

$$g(\xi) = -a_1 + a_1\,Q(\xi) = -a_1 + \frac{a_1}{1 + d\,\exp(\xi)}.$$

Since $f(\xi) = g^{1/2}(\xi)$, then the corresponding solution to (11) is

$$f(\xi) = \sqrt{-a_1 + \frac{a_1}{1 + d\,\exp(\xi)}}. \tag{22}$$

Using (22) and (10), we get

$$\begin{aligned} \varphi(\xi) &= \int \left[\frac{a}{k^2} - \frac{3\,s}{2\,k}\left(-a_1 + \frac{a_1}{1+d\,\exp(\xi)}\right)\right] d\xi = \\ &\frac{a}{k^2}\xi - \frac{3\,s}{2\,k}\left[-a_1\,\xi + a_1\,(\xi - \log(1 + d\,\exp(\xi)))\right] + C_5, \end{aligned} \tag{23}$$

where C_5 is a constant of integration.

Substituting (22) and (23) into (6), thus, the solution of (5) is composed as:

$$u_5(x,\,t) = \sqrt{-a_1 + \frac{a_1}{1+d\,\exp(\xi)}}$$
$$\exp\left(i\left(\frac{a}{k^2}\xi - \frac{3\,s}{2\,k}\left[-a_1\,\xi + a_1\,(\xi - \log(1 + d\,\exp(\xi)))\right] + C_5\right)\right),$$

$$\xi = k\,\frac{t^\alpha}{\alpha} - a\,\frac{x^\beta}{\beta}.$$

4. Some graphical illustrations

We depict in this section some graphical illustrations of the obtained solutions for the perturbed nonlinear Schrodinger equation. To reveal the clear picture of the obtained solutions, both the two and three dimensional plots for the solutions are given.

5. Conclusion

In this study, optical soliton solutions for the fractional perturbed nonlinear Schrödinger equation is obtained by Kudryashov method. For this purpose we use the fractional complex transformation. After reducing equation to integer-ordered ordinary differential equations, substituting the proposed form for the solution into the integer-ordered ordinary differential equations, the nonzero coefficients in solutions are determined. This paper shows that the proposed method is effective and can be used for many other nonlinear PDEs in mathematical physics and nonlinear sciences.

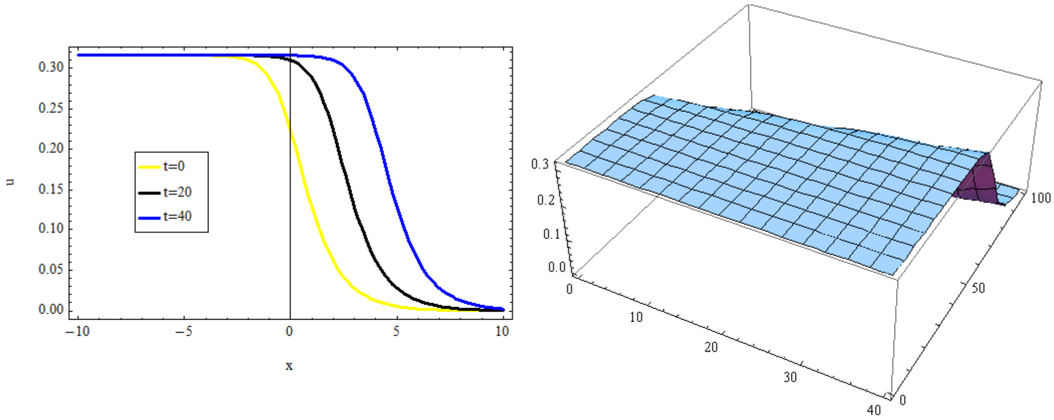

FIGURE 1. Graph of case (1) of the perturbed nonlinear Schrodinger equation using Kudryashov method.

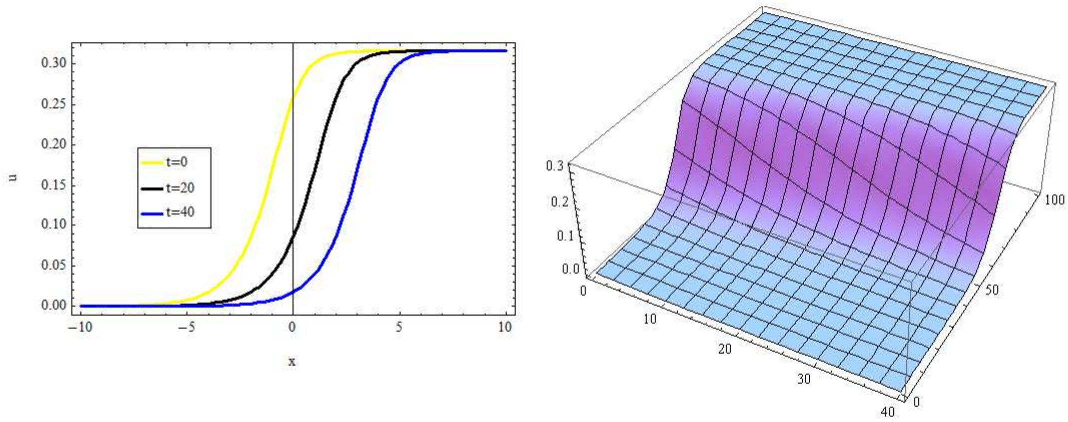

FIGURE 2. Graph of case (2) of the perturbed nonlinear Schrodinger equation using Kudryashov method.

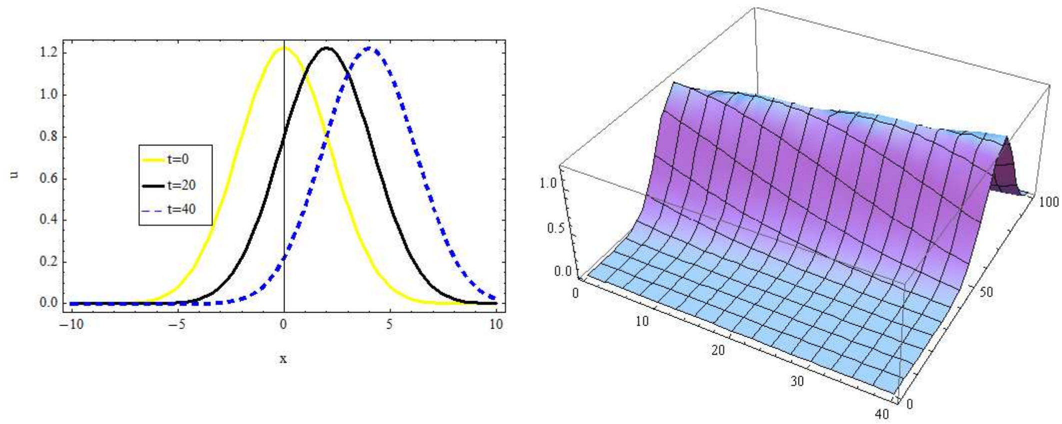

FIGURE 3. Graph of case (3) of the perturbed nonlinear Schrodinger equation using Kudryashov method.

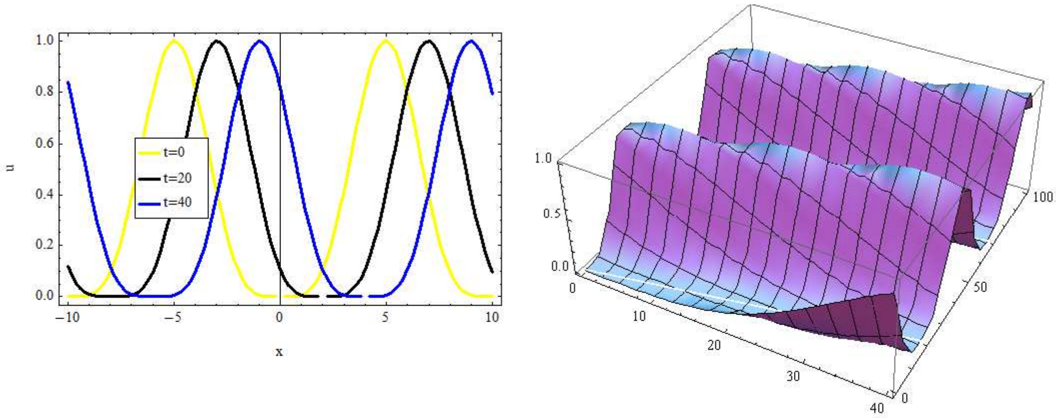

FIGURE 4. Graph of case (4) of the perturbed nonlinear Schrodinger equation using Kudryashov method.

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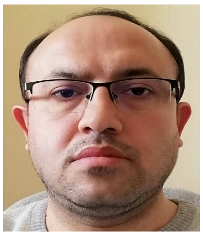

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