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TOPOLOGICAL INDICES OF SIERPIŃSKI GASKET AND SIERPIŃSKI GASKET RHOMBUS GRAPHS

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ABSTRACT. Sierpiński graphs $S(n,k)$ were defined originally in 1997 by Sandi Klavžar and Uroš Milutinović. In this paper atom bond connectivity index, fourth atom bond connectivity indices, geometric arithmetic index, fifth geometric arithmetic indices, augmented Zagreb index and sankruti index of Sierpiński Gasket graphs and Sierpiński Gasket Rhombus graphs are determined.

Keywords: Sierpiński graph, sierpiński Gasket graph, Sierpiński Gasket Rhombus graph, topological index, degree, eccentricity.

AMS Subject Classification: 05C90, 05C35, 05C12.

1. INTRODUCTION

A systematic study of topological indices is one of the most striking aspects in many branches of mathematics with its applications and various other fields of science and technology. A topological index is a numeric quantity from the structural graph of a molecule. All the graphs $G = (V, E)$ considered in this paper are simple, undirected and connected graphs. For any vertices $u, v \in V(G)$, the distance $d(u, v)$ is defined as the length of any shortest path connecting u and v in G. For any vertex v_i in G, the degree d_i of v_i is the number of edges incident with v_i and any vertex v_i in G is a pendant vertex if $d_i = 1$. The eccentricity e_i of v_i is the largest distance between v_i and any other vertex of G. The terminology used throughout this paper is based on [5].

Sierpiński Gasket graphs are strongly related to the well known fractal called the Sierpiński Gasket. Sierpiński Gasket graphs appear in different areas of graph theory,

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topology, probability [7, 12], psychology [15]. Sierpiński Gasket graphs S_n are the graphs naturally defined by the finite number of iterations. Grundy, Scorer and Smith introduced this connection in [17]. Teguia and Godbole [19] studied several properties of these graphs.

The Sierpiński Gasket Rhombus graph SR_n is stimulated from the fractal structure given by Marcelo Epstein and Samer M. Adeeb[16]. The graph is obtained by identifying two copies of Sierpiński Gasket graphs along their side edges leading to Rhombus like structure.Topological properties of Sierpiński Gasket Rhombus graph are studied in[21]. In this paper we consider the following topological indices.

Ernesto Estrada et al. [3] proposed a new topological index, named atom-bond connectivity (ABC) index. The main applications of this graph invariant were reported in [2, 3, 11]. The atom bond connectivity(ABC) index of G is defined[3] as

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{(d_u + d_v - 2)}{d_u d_v}}. \tag{1}$$

The fourth atom-bond connectivity index, $ABC_4(G)$ index, was introduced by M. Ghorbani et. al. [8] in 2010 and it is defined as

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{(S_u + S_v - 2)}{S_u S_v}}. \tag{2}$$

where, $S_u = \sum_{uv \in E(G)} d_v$.

In 2009, Vukičević et al.[20] introduced a new topological index geometric arithmetic index $GA(G)$ of a graph G. Further studies on GA index can be found in [1, 22] and it is defined[20] as

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}. \tag{3}$$

The fifth geometric arithmetic index, $GA_5(G)$, was introduced by A. Graovac et. al.[10] in 2011 and it is defined as

$$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v}. \tag{4}$$

where, $S_u = \sum_{uv \in E(G)} d_v$.

In 2009 the augmented Zagreb index (AZI) of G was proposed by Furtula et al.[6] and they showed that AZI is a valuable predictive index in the study of the heat formation in octanes and heptanes, whose prediction power is better than atom-bond connectivity index. The augmented Zagreb index is defined[6] as

$$AZI(G) = \sum_{uv \in E(G)} \left(\frac{d_u d_v}{d_u + d_v - 2} \right)^3. \tag{5}$$

Very recently sankruti index is introduced in[18] and it is defined as

$$\mathcal{S} = \sum_{uv \in E(G)} \left(\frac{S_u S_v}{S_u + S_v - 2} \right)^3. \tag{6}$$

where, S_u is sum of the degrees of all neighbors of vertex u in G.

In this paper ABC index, ABC_4 index, GA index, GA_5 index, AZI and sankruti index of Sierpiński Gasket graphs and Sierpiński Gasket Rhombus graphs are determined.

2. Sierpiński Gasket graphs

Sierpiński graphs are $S(n, k)$ defined[14] as follows. For $n \geq 1$ and $k \geq 1$, the vertex set of $S(n, k)$ consists of all n-tuples of integers $1, 2, ..., k$, that is $V(S(n, k)) = \{1, 2, ..., k\}^n$. Two different vertices $u = (u_1, u_2, \ldots, u_n)$ and $v = (v_1, v_2, \ldots, v_n)$ are adjacent if and only if there exists an $h \in \{1, 2, ..., n\}$ such that
(a) $u_t = v_t$ for $t = 1, 2, ..., h - 1$;
(b) $u_h \neq v_h$;
(c) $u_t = v_h$ and $v_t = u_h$ for $t = h + 1, \ldots, n$.

In the rest we will shortly write $u_1 u_2 \cdots u_n$ for $(u_1, u_2, \ldots, u_n)$. The vertex $ii \cdots i$ of $S(n, k)$ is called an extreme vertex.

The Sierpiński Gasket graphs S_n is constructed in[13] in a natural way from the Sierpiński graph $S(n, 3)$ by contracting all of its edges that lie in no triangle K_3. For example the Sierpiński graph $S(3, 3)$ and the Sierpiński Gasket graph S_3 is shown in Fig. 1.

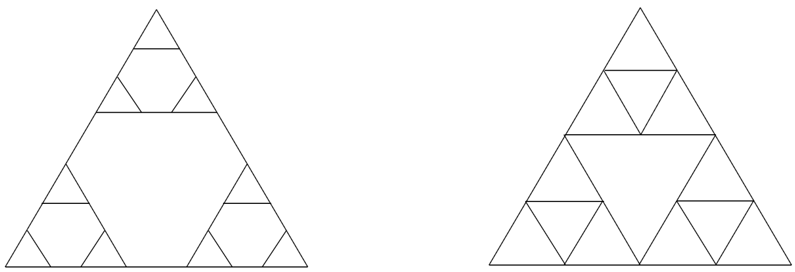

Figure 1: The graphs $S(3, 3)$ and S_3

Theorem 2.1. *If S_n is a Sierpiński Gasket graph, then $ABC(S_n) = 3\sqrt{2} + (3^n - 6)\frac{\sqrt{6}}{4}$.*

Proof. Let $e_{i,j}$ denote the number of edges connecting the vertices of degrees d_i and d_j. Then in graph S_n $e_{2,4} = 6$ and $e_{4,4} = 3^n - 6$. Then,

$$
\begin{aligned}
ABC(S_n) = & \sum_{uv\in E(G)} \sqrt{\frac{(d_u+d_v-2)}{d_ud_v}} \\
=& 6\sqrt{\frac{(2+4-2)}{2\cdot 4}} + (3^n-6)\sqrt{\frac{(4+4-2)}{4\cdot 4}} \\
=& 3\sqrt{2} + (3^n-6)\frac{\sqrt{6}}{4}.
\end{aligned}
$$

□

Theorem 2.2. *If S_n is a Sierpiński Gasket graph, then $GA(S_n) = 3^n + 2(2\sqrt{2}-3)$.*

Proof. In view of proof of Theorem 2.1, we have

$$
\begin{aligned}
GA(S_n) = & \sum_{uv\in E(G)} \frac{2\sqrt{d_ud_v}}{d_u+d_v} \\
=& 6\frac{2\sqrt{2\cdot 4}}{2+4} + (3^n-6)\frac{2\sqrt{4\cdot 4}}{4+4} \\
=& 3^n + 2(2\sqrt{2}-3).
\end{aligned}
$$

□

Theorem 2.3. *If S_n is a Sierpiński Gasket graph, then $AZI(S_n) = \frac{1}{9}(512\times 3^{n-1} - 592)$.*

Proof. In view of proof of Theorem 2.1, we have

$$
\begin{aligned}
AZI(S_n) = & \sum_{uv\in E(G)} \left(\frac{d_ud_v}{d_u+d_v-2}\right)^3 \\
=& 6\left(\frac{2\cdot 4}{2+4-2}\right)^3 + (3^n-6)\left(\frac{4\cdot 4}{4+4-2}\right)^3 \\
=& \frac{1}{9}(512\times 3^{n-1} - 592).
\end{aligned}
$$

□

Theorem 2.4. *If S_n is a Sierpiński Gasket graph, then*

$$
ABC_4(S_n) = \begin{cases} \frac{1}{2}\left(3\sqrt{3} + \sqrt{\frac{11}{2}}\right), & \text{if } n = 2; \\ \\ (3^n-21)\,\frac{\sqrt{30}}{16} + 3\left[\sqrt{\frac{5}{7}} + \frac{\sqrt{26}}{14} + \sqrt{2}\right], & \text{otherwise.} \end{cases}
$$

Proof. Let $e'_{m,n}$ denote the type of edges of Sierpiński Gasket graph with $m = S_u$ and $n = S_v$ where, $uv \in E(S_n)$ and $S_u = \sum_{uv\in E(S_n)} d_v$. We consider the following cases.

Case 1: $n = 2$. The graph S_2 contains 6 number of $e'_{8,12}$ type edges and 3 number of $e'_{12,12}$ type edges. Then,

$$\begin{aligned}ABC_4(S_2) &= \sum_{uv\in E(G)} \sqrt{\frac{(S_u+S_v-2)}{S_uS_v}}\\ &=6\sqrt{\frac{(8+12-2)}{8\cdot 12}}+3\sqrt{\frac{(12+12-2)}{12\cdot 12}}\\ &=\frac{1}{2}\left(3\sqrt{3}+\sqrt{\frac{11}{2}}\right).\end{aligned}$$

Case 2: $n \geq 3$. The graph S_n contains only $e'_{8,14}$, $e'_{14,14}$, $e'_{14,16}$ and $e'_{16,16}$ types of edges. There are 6, 3, 12, and $3^n - 21$ number of edges of $e'_{8,14}$, $e'_{14,14}$, $e'_{14,16}$ and $e'_{16,16}$ types, respectively. Then,

$$\begin{aligned}ABC_4(S_n) &= \sum_{uv\in E(G)} \sqrt{\frac{(S_u+S_v-2)}{S_uS_v}}\\ &=6\sqrt{\frac{(8+14-2)}{8\cdot 14}}+3\sqrt{\frac{(14+14-2)}{14\cdot 14}}+12\sqrt{\frac{(14+16-2)}{14\cdot 16}}\\ &\quad+(3^n-21)\sqrt{\frac{(16+16-2)}{16\cdot 16}}\\ &=(3^n-21)\,\frac{\sqrt{30}}{16}+3\left[\sqrt{\frac{5}{7}}+\frac{\sqrt{26}}{14}+\sqrt{2}\right].\end{aligned}$$

□

Theorem 2.5. *If S_n is a Sierpiński Gasket graph, then $GA_5(S_2) = 3 + \frac{12\sqrt{6}}{5}$ and $GA_5(S_n) \approx 3^n - 0.25415$ for $n \geq 3$.*

Proof. In view of proof of Theorem 2.4, we have,

$$\begin{aligned}GA_5(S_2) &= \sum_{uv\in E(G)} \frac{2\sqrt{S_uS_v}}{S_u+S_v}\\ &=6\left(\frac{2\sqrt{8\cdot 12}}{8+12}\right)+3\left(\frac{2\sqrt{12\cdot 12}}{12+12}\right)\\ &=3+\frac{12\sqrt{6}}{5}.\end{aligned}$$

and

$$
\begin{aligned}
GA_5(S_n) &= \sum_{uv\in E(G)} \frac{2\sqrt{S_uS_v}}{S_u+S_v} \\
&=6\left(\frac{2\sqrt{8\cdot 14}}{8+14}\right)+3\left(\frac{2\sqrt{14\cdot 14}}{14+14}\right)+12\left(\frac{2\sqrt{14\cdot 16}}{14+16}\right) \\
&\quad +(3^n-21)\left(\frac{2\sqrt{16\cdot 16}}{16+16}\right) \\
&=3^n-18+\left(\frac{24}{11}+\frac{48\sqrt{2}}{15}\right)\sqrt{7} \\
&\approx 3^n-0.25415.
\end{aligned}
$$

□

Theorem 2.6. *If S_n is a Sierpiński Gasket graph, then*

$$
\mathcal{S}(S_n)\approx\begin{cases} 1751.5, & \text{if } n=2; \\ 621.38\times 3^n-4566.1, & \text{otherwise.}\end{cases}
$$

Proof. In view of proof of Theorem 2.4, we have,

$$
\begin{aligned}
\mathcal{S}(S_2) &= \sum_{uv\in E(G)}\left(\frac{S_uS_v}{S_u+S_v-2}\right)^3 \\
&=6\left(\frac{8\times 12}{8+12-2}\right)^3+3\left(\frac{12\times 12}{12+12-2}\right)^3 \\
&\approx 1751.5
\end{aligned}
$$

and

$$
\begin{aligned}
\mathcal{S}(S_n) &= \sum_{uv\in E(G)}\left(\frac{S_uS_v}{S_u+S_v-2}\right)^3 \\
&=6\left(\frac{8\times 14}{8+14-2}\right)^3+3\left(\frac{14\times 14}{14+14-2}\right)^3+12\left(\frac{14\times 16}{14+16-2}\right)^3 \\
&\quad +(3^n-21)\left(\frac{16\times 16}{16+16-2}\right)^3 \\
&\approx 621.38\times 3^n-4566.1
\end{aligned}
$$

□

3. Sierpiński Gasket Rhombus graphs

Sierpiński Gasket Rhombus graph is obtained by identifying two copies of Sierpiński Gasket graphs along their side edges leading to Rhombus like structure. A Sierpiński Gasket Rhombus of level n [denoted by SR_n], is obtained by identifying the edges in two Sierpiński Gasket graphs S_n along one of their sides[21]. The Sierpiński Gasket Rhombus graph SR_3 is shown in Fig. 2.

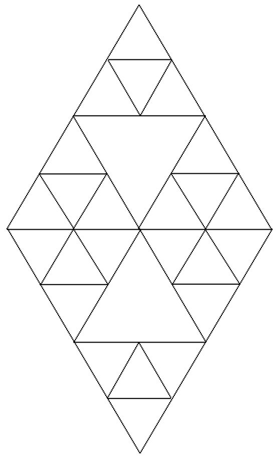

Fig. 2: SR_3

Theorem 3.1. *If SR_n is a Sierpiński Gasket Rhombus graph graph, then $ABC(SR_n) \approx (1.2247)3^n - (0.83779)2^{n-1} + 0.84465$.*

Proof. Let $e_{i,j}$ denote the number of edges connecting the vertices of degrees d_i and d_j. Then the graph SR_n contains only $e_{2,4} + e_{3,4} + e_{3,6} + e_{6,4} + e_{6,6} + e_{4,4}$ edges. Clearly $e_{2,4} = 4$, $e_{3,4} = 4$, $e_{3,6} = 2$, $e_{6,4} = 4(2^{n-1} - 1)$, $e_{6,6} = 2^{n-1} - 2$, and $e_{4,4} = 2 \cdot 3^n - 3 \cdot 2^n - 4$. Then,

$$
\begin{aligned}
ABC(SR_n) = &\sum_{uv \in E(G)} \sqrt{\frac{(d_u + d_v - 2)}{d_u d_v}} \\
=&4\sqrt{\frac{(2+4-2)}{2\cdot 4}} + 4\sqrt{\frac{(3+4-2)}{3\cdot 4}} + 2\sqrt{\frac{(3+6-2)}{3\cdot 6}} + 4(2^{n-1}-1)\sqrt{\frac{(6+4-2)}{6\cdot 4}} \\
&+ (2^{n-1}-2)\sqrt{\frac{(6+6-2)}{6\cdot 6}} + (2\cdot 3^n - 3\cdot 2^n - 4)\sqrt{\frac{(4+4-2)}{4\cdot 4}} \\
=&\frac{\sqrt{6}}{2}3^n + \left[\frac{4}{\sqrt{3}} + \frac{\sqrt{10}}{6} - \frac{3\sqrt{6}}{2}\right]2^{n-1} + 2\sqrt{2} + 2\sqrt{\frac{5}{3}} + \frac{\sqrt{14}}{3} - \frac{4}{\sqrt{3}} - \frac{\sqrt{10}}{3} - \sqrt{6} \\
\approx&(1.2247)3^n - (0.83779)2^{n-1} + 0.84465.
\end{aligned}
$$

□

Theorem 3.2. *If SR_n is a Sierpiński Gasket Rhombus graph graph, then $GA(SR_n) \approx 2 \cdot 3^n + (1.0808)2^{n-1} - 0.30336$.*

Proof. In view of proof of Theorem 3.1, we have,

$$
\begin{aligned}
GA(SR_n) =& \sum_{uv\in E(G)} \frac{2\sqrt{d_u d_v}}{d_u+d_v}\\
=&4\frac{2\sqrt{2\cdot 4}}{2+4}+4\frac{2\sqrt{3\cdot 4}}{3+4}+2\frac{2\sqrt{3\cdot 6}}{3+6}+4(2^{n-1}-1)\frac{2\sqrt{6\cdot 4}}{6+4}\\
&+(2^{n-1}-2)\frac{2\sqrt{6\cdot 6}}{6+6}+(2\cdot 3^n-3\cdot 2^n-4)\frac{2\sqrt{4\cdot 4}}{4+4}\\
=&2\cdot 3^n+\left(\frac{8\sqrt{6}}{5}-5\right)2^{n-1}+4\sqrt{2}+\frac{16\sqrt{3}}{7}-\frac{8\sqrt{6}}{5}-6\\
\approx&2\cdot 3^n+(1.0808)2^{n-1}-0.30336.
\end{aligned}
$$

□

Theorem 3.3. *If SR_n is a Sierpiński Gasket Rhombus graph graph, then* $AZI(SR_n) \approx 20.439\times 2^n + 37.926\times 3^n - 155.86$

Proof. In view of proof of Theorem 3.1, we have,

$$
\begin{aligned}
AZI(SR_n) =& \sum_{uv\in E(G)} \left(\frac{d_u d_v}{d_u+d_v-2}\right)^3\\
=&4\left(\frac{2\times 4}{2+4-2}\right)^3+4\left(\frac{3\times 4}{3+4-2}\right)^3+2\left(\frac{3\times 6}{3+6-2}\right)^3+4(2^{n-1}-1)\left(\frac{6\times 4}{6+4-2}\right)^3\\
&+(2^{n-1}-2)\left(\frac{6\times 6}{6+6-2}\right)^3+(2\times 3^n-3\times 2^n-4)\left(\frac{4\times 4}{4+4-2}\right)^3\\
\approx&20.439\times 2^n+37.926\times 3^n-155.86.
\end{aligned}
$$

□

Theorem 3.4. *If SR_n is a Sierpiński Gasket Rhombus graph graph, then* $ABC_4(SR_2) = 2\left(\sqrt{\frac{7}{10}}+\frac{\sqrt{28}}{15}\right)+4\left(\sqrt{\frac{9}{70}}+\sqrt{\frac{7}{66}}\right)+\sqrt{\frac{34}{77}}$ *and* $ABC_4(SR_n) \approx (0.68465)3^n - (0.59794)2^{n-1} + 0.59320.$

Proof. Let $e'_{m,n}$ denote the type of edges of Sierpiński Gasket Rhombus graph with $m = S_u$ and $n = S_v$ where, $uv \in E(SR_n)$ and $S_u = \sum_{uv\in E(SR_n)} d_v$. We consider the following cases.

Case 1: $n = 2$. The graph SR_2 contains only $e'_{8,15}$, $e'_{15,15}$, $e'_{14,15}$, $e'_{14,22}$ and $e'_{15,22}$ types of edges. There are 4, 2, 4, 2, and 4, edges of $e'_{8,15}$, $e'_{15,15}$, $e'_{14,15}$, $e'_{14,22}$ and $e'_{15,22}$ types, respectively. Then,

$$
\begin{aligned}
ABC_4(SR_2) &= \sum_{uv\in E(G)} \sqrt{\frac{(S_u+S_v-2)}{S_uS_v}}\\
&=4\sqrt{\frac{(8+15-2)}{8\cdot 15}}+2\sqrt{\frac{(15+15-2)}{15\cdot 15}}+4\sqrt{\frac{(14+15-2)}{14\cdot 15}}\\
&\quad+2\sqrt{\frac{(14+22-2)}{14\cdot 22}}+4\sqrt{\frac{(15+22-2)}{15\cdot 22}}\\
&=2\left(\sqrt{\frac{7}{10}}+\frac{\sqrt{28}}{15}\right)+4\left(\sqrt{\frac{9}{70}}+\sqrt{\frac{7}{66}}\right)+\sqrt{\frac{34}{77}}.
\end{aligned}
$$

Case 2: $n \geq 3$. The graph SR_n contains only $e'_{8,14}$, $e'_{14,14}$, $e'_{14,16}$, $e'_{14,17}$, $e'_{14,25}$, $e'_{16,16}$, $e'_{16,17}$, $e'_{16,20}$, $e'_{17,20}$, $e'_{17,25}$, $e'_{20,25}$, $e'_{20,28}$ and $e'_{25,28}$ types of edges. The number of these edges in SR_n is mentioned in Table 1. Then,

$$
\begin{aligned}
ABC_4(SR_n) &= \sum_{uv\in E(G)} \sqrt{\frac{(S_u+S_v-2)}{S_uS_v}}\\
&=4\sqrt{\frac{(8+14-2)}{8\cdot 14}}+2\sqrt{\frac{(14+14-2)}{14\cdot 14}}+8\sqrt{\frac{(14+16-2)}{14\cdot 16}}\\
&\quad+4\sqrt{\frac{(14+17-2)}{14\cdot 17}}+2\sqrt{\frac{(14+25-2)}{14\cdot 25}}+4\sqrt{\frac{(16+17-2)}{16\cdot 17}}\\
&\quad+(2\cdot 3^n-7\cdot 2^{n-1}-22)\sqrt{\frac{(16+16-2)}{16\cdot 16}}+2(2^{n-1}-2)\sqrt{\frac{(16+20-2)}{16\cdot 20}}\\
&\quad+4\sqrt{\frac{(17+20-2)}{17\cdot 20}}+4\sqrt{\frac{(17+25-2)}{17\cdot 25}}+4\sqrt{\frac{(20+25-2)}{20\cdot 25}}\\
&\quad+4(2^{n-1}-3)\sqrt{\frac{(20+28-2)}{20\cdot 28}}+2\sqrt{\frac{(25+28-2)}{25\cdot 28}}\\
&=\frac{\sqrt{30}}{8}3^n+\left[-\frac{7\sqrt{30}}{16}+\frac{1}{4}\sqrt{\frac{34}{5}}+\sqrt{\frac{46}{35}}\right]2^{n-1}+2\sqrt{\frac{5}{7}}+\frac{\sqrt{26}}{7}+2\sqrt{2}\\
&\quad+4\sqrt{\frac{29}{238}}+\frac{2}{5}\sqrt{\frac{37}{14}}+\sqrt{\frac{31}{17}}-\frac{11\sqrt{30}}{8}-\frac{1}{2}\sqrt{\frac{34}{5}}+2\sqrt{\frac{7}{17}}\\
&\quad+\frac{8}{5}\sqrt{\frac{10}{17}}+\frac{2}{5}\sqrt{\frac{43}{5}}-3\sqrt{\frac{46}{35}}+\frac{1}{5}\sqrt{\frac{51}{7}}\\
&\approx(0.68465)3^n-(0.59794)2^{n-1}+0.59320.
\end{aligned}
$$

□

Table 1

Theorem 3.5. *If SR_n is a Sierpiński Gasket Rhombus graph graph, then*
$GA_5(SR_2)=2\left(1+\frac{\sqrt{77}}{9}\right)+8\left(\frac{2\sqrt{30}}{23}+\frac{\sqrt{210}}{29}++\frac{\sqrt{330}}{37}\right)$ *and*
$GA_5(SR_n)\approx 2\cdot 3^n+(1.0683)2^{n-1}-0.19301.$

Type of edges	Number of edges
$e'_{8,14}$	4
$e'_{14,14}$	2
$e'_{14,16}$	8
$e'_{14,17}$	4
$e'_{14,25}$	2
$e'_{16,16}$	$2 \cdot 3^n - 7 \cdot 2^{n-1} - 22$
$e'_{16,17}$	4
$e'_{16,20}$	$2(2^{n-1} - 2)$
$e'_{17,20}$	4
$e'_{17,25}$	4
$e'_{20,25}$	4
$e'_{20,28}$	$4(2^{n-1} - 3)$
$e'_{25,28}$	2

Proof. In view of proof of Theorem 3.4 and Table 1, we have,

$$\begin{aligned}
GA_5(SR_2) &= \sum_{uv\in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v} \\
&=4\frac{2\sqrt{8\cdot 15}}{8+15} + 2\frac{2\sqrt{15\cdot 15}}{15+15} + 4\frac{2\sqrt{14\cdot 15}}{14+15} + 2\frac{2\sqrt{14\cdot 22}}{14+22} + 4\frac{2\sqrt{15\cdot 22}}{15+22} \\
&=2\left(1+\frac{\sqrt{77}}{9}\right) + 8\left(\frac{2\sqrt{30}}{23} + \frac{\sqrt{210}}{29} + +\frac{\sqrt{330}}{37}\right).
\end{aligned}$$

$$\begin{aligned}
GA_5(SR_n) &= \sum_{uv\in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v} \\
&=4\frac{2\sqrt{8\cdot 14}}{8+14} + 2\frac{2\sqrt{14\cdot 14}}{14+14} + 8\frac{2\sqrt{14\cdot 16}}{14+16} + 4\frac{2\sqrt{14\cdot 17}}{14+17} + 2\frac{2\sqrt{14\cdot 25}}{14+25} \\
&\quad + (2\cdot 3^n - 7\cdot 2^{n-1} - 22)\frac{2\sqrt{16\cdot 16}}{16+16} + 4\frac{2\sqrt{16\cdot 17}}{16+17} + 2(2^{n-1}-2)\frac{2\sqrt{16\cdot 20}}{16+20} \\
&\quad + 4\frac{2\sqrt{17\cdot 20}}{17+20} + 4\frac{2\sqrt{17\cdot 25}}{17+25} + 4\frac{2\sqrt{20\cdot 25}}{20+25} + 4(2^{n-1}-3)\frac{2\sqrt{20\cdot 28}}{20+28} + 2\frac{2\sqrt{25\cdot 28}}{25+28} \\
&=2\cdot 3^n + \left[-7 + \frac{8\sqrt{5}}{9} + \frac{2\sqrt{35}}{3}\right] 2^{n-1} + \frac{16\sqrt{7}}{11} + \frac{32\sqrt{14}}{15} + \frac{8\sqrt{238}}{31} \\
&\quad + \frac{20\sqrt{14}}{39} + \frac{32\sqrt{17}}{33} + \frac{16\sqrt{85}}{37} + \frac{20\sqrt{17}}{21} + \frac{40\sqrt{7}}{53} - 2\sqrt{35} - 20 \\
&\approx 2\cdot 3^n + (1.0683)2^{n-1} - 0.19301.
\end{aligned}$$

□

Theorem 3.6. *If SR_n is a Sierpiński Gasket Rhombus graph graph, then*

$$\mathcal{S}(SR_n) \approx \begin{cases} 8505.7, & \text{if } n = 2; \\ 2267.3 \times 2^n + 1242.8 \times 3^n - 6469.8, & \text{otherwise.} \end{cases}$$

Proof. In view of proof of Theorem 3.4 and Table 1, we have,

$$\begin{aligned}\mathcal{S}(SR_2) =& \sum_{uv\in E(G)} \left(\frac{S_uS_v}{S_u+S_v-2}\right)^3\\ =&4\left(\frac{8\times 15}{8+15-2}\right)^3 + 2\left(\frac{15\times 15}{15+15-2}\right)^3 + 4\left(\frac{14\times 15}{14+15-2}\right)^3\\ &+ 2\left(\frac{14\times 22}{14+22-2}\right)^3 + 4\left(\frac{15\times 22}{15+22-2}\right)^3\\ \approx& 8505.7\end{aligned}$$

$$\begin{aligned}\mathcal{S}(SR_n) =& \sum_{uv\in E(G)} \left(\frac{S_uS_v}{S_u+S_v-2}\right)^3\\ =&4\left(\frac{8\times 14}{8+14-2}\right)^3 + 2\left(\frac{14\times 14}{14+14-2}\right)^3 + 8\left(\frac{14\times 16}{14+16-2}\right)^3\\ &+ 4\left(\frac{14\times 17}{14+17-2}\right)^3 + 2\left(\frac{14\times 25}{14+25-2}\right)^3 + (2\times 3^n - 7\times 2^{n-1} - 22)\left(\frac{16\times 16}{16+16-2}\right)^3\\ &+ 4\left(\frac{16\times 17}{16+17-2}\right)^3 + 2(2^{n-1}-2)\left(\frac{16\times 20}{16+20-2}\right)^3 + 4\left(\frac{17\times 20}{17+20-2}\right)^3\\ &+ 4\left(\frac{17\times 25}{17+25-2}\right)^3 + 4\left(\frac{20\times 25}{20+25-2}\right)^3 + 4(2^{n-1}-3)\left(\frac{20\times 28}{20+28-2}\right)^3 + 2\left(\frac{25\times 28}{25+28-2}\right)^3\\ \approx& 2267.3\times 2^n + 1242.8\times 3^n - 6469.8.\end{aligned}$$

□

Conclusion

In this paper, we computed the atom bond connectivity index, fourth atom bond connectivity indices, geometric arithmetic index, fifth geometric arithmetic indices, augmented Zagreb index and sankruti index of Sierpiński Gasket graphs and Sierpiński Gasket Rhombus graphs with the help of graph theory and mathematical derivation.

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