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# SOME PROPERTIES OF VAGUE GRAPH STRUCTURES

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ABSTRACT. A graph structure is a generalization of simple graphs. Graph structures are very useful tools for the study of different domains of computational intelligence and computer science. A vague graph structure is a generalization of a vague graph. In this research paper, we present several different types of operations including cartesian product, cross product, lexicographic product, union, and composition on vague graph structures. We also introduce some results of operations.

Keywords: Vague set, vague graph structure, cross product, lexicographic product, composition.

AMS Subject Classification: 05C99, 03E72.

## 1. Introduction

Fuzzy graph models are advantageous mathematical tools for dealing with combinatorial problems of various domains including operations research, optimization, social science, algebra, computer science, and topology. Fuzzy graphical models are obviously better than graphical models due to natural existence of vagueness and ambiguity. Fuzzy set theory [24] is a very strong mathematical tool for solving approximate reasoning related problems. Graph structures, introduced by Sampathkumar (2006), are a generalization of graph which quite useful in studying structures including graphs, signed graphs, and graphs in which every edge is labeled or colored. Gau and Buehrer [7] proposed the concept of vague set in 1993, by replacing the value of an element in a set with a sub-interval of $[0, 1]$. Namely, a true-membership function $t_v(x)$ and a false membership function $f_v(x)$ are used to describe the boundaries of the membership degree. Kauffman defined in [9] a fuzzy graph. Rosenfeld [18] described the structure of fuzzy graph obtaining analogs of several graph theoretical concepts. Bhattacharya [5] gave some remarks on fuzzy graphs. Several concepts on fuzzy graphs were introduced by Mordeson et al. [10]. Dinesh [6] introduced the notion of a fuzzy graph structure and discussed some related properties. Ramakrishna [11] defined the concept of vague graphs and studied some of their properties. Sahoo and Pal [19, 20] studied different types of products on intuitionistic fuzzy graphs.

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Rashmanlou et al. [12, 13, 14, 15, 16, 17] investigated several properties of fuzzy graphs. Ghorai and Pal [8] defined certain types of product bipolar fuzzy graphs. Akram [1] given bipolar fuzzy graphs. Sheikh Hoseini et al. [21] introduced maximal product of graphs under vague environment. Sunitha and Vijayakumar [23] studied some properties of complement on fuzzy graphs. Shahzadi et al. [22] investigated pythagorean fuzzy soft graphs with applications. Borzooei [2, 3, 4] introduced new concepts on vague graphs. In this paper, we present several different types of operations, including cartesian product, cross product, lexicographic product, union, and composition on vague graph structures.

## 2. Preliminaries

A graph structure $G^* = (U, E_1, E_2, \cdots, E_k)$, consists of a non-empty set $U$ together with mutually disjoint, irreflexive, and symmetric relations, $E_1, E_2, \cdots, E_k$ on $U$. If $G_1^*$ and $G_2^*$ are two graph structures given by $(U, E_1, E_2, \cdots, E_k)$ and $(V, E_1', E_2', \cdots, E_k')$ respectively, then cartesian product of $G_1^*$ and $G_2^*$, is denoted by "$G_1^* \times G_2^*$" and given by $G_1 \times G_2^* = (U \times V, E_1 \times E_1', E_2 \times E_2', \cdots, E_k \times E_k')$ where $E_i \times E_i' = \{(u_1v, u_2v)|\ v \in V, u_1u_2 \in E_i\} \cup \{(uv_1, uv_2)|\ u \in U, v_1v_2 \in E_i'\}$, $i = 1, 2, \cdots, k$. Composition of $G_1^*$ and $G_2^*$ is denoted by "$G_1^* \circ G_2^*$" and given by $G_1^* \circ G_2^* = (U \circ V, E_1 \circ E_1', E_2 \circ E_2', \cdots, E_k \circ E_k')$ where $U \circ V = U \times V$ and $E_i \circ E_i' = \{(u_1v, u_2v)|\ v \in V, u_1u_2 \in E_i\} \cup \{(uv_1, uv_2)|\ u \in U, v_1v_2 \in E_i'\} \cup \{(u_1v_1, u_2v_2)|\ u_1u_2 \in E_i, v_1 \neq v_2\}$, $i = 1, 2, \cdots, k$. Union of $G_1^*$ and $G_2^*$ is denoted by "$G_1^* \cup G_2^*$" and given by $G_1^* \cup G_2^* = (U \cup V, E_1 \cup E_1', E_2 \cup E_2', \cdots, E_k \cup E_k')$ and join of $G_1^*$ and $G_2^*$ is given by $G_1^* + G_2^* = (U + V, E_1 + E_1', E_2 + E_2', \cdots, E_k + E_k')$ where $U + V = U \cup V$ and $E_i + E_i' = E_i \cup E_i' \cup E$, for $i = 1, 2, \cdots, k$ such that $E$ is the set consisting of all edges which join vertices of $U$ with vertices of $V$.

**Definition 2.1.** [6] *Let $G^* = (U, E_1, E_2, \cdots, E_k)$ be a graph structure and let $\nu, \rho_1, \rho_2, \cdots, \rho_k$ be the fuzzy subsets of $U, E_1, E_2, \cdots, E_k$ respectively such that:*

$$0 \leq \rho_i(xy) \leq \mu(x) \wedge \mu(y),\ for\ all\ x, y \in V,\ i = 1, 2, \cdots, k.$$

*Then $G = (\nu, \rho_1, \rho_2, \cdots, \rho_k)$ is a fuzzy graph structure of $G^*$.*

**Definition 2.2.** [7] *A vague set $A$ on an ordinary finite non-empty set $X$ is a pair $(t_A, f_A)$ where $t_A : X \to [0, 1]$ and $f_A : X \to [0, 1]$ are true and false membership functions, respectively such that $t_A(x) + f_A(x) \leq 1$, for all $x \in X$. Let $X$ and $Y$ be ordinary finite non-empty sets. Then we call a vague relation to be a vague subset of $X \times Y$, that is an expression $R$ defined by:*

$$R = \{\langle (x, y), t_R(x, y), f_R(x, y)\rangle|\ x \in X, y \in Y\},$$

*where $t_R : X \times Y \to [0, 1]$, $f_R : X \times Y \to [0, 1]$, which satisfies condition $0 \leq t_R(x, y) + f_R(x, y) \leq 1$, for all $(x, y) \in X \times Y$.*

**Definition 2.3.** [11] *A vague graph is a pair of $G = (A, B)$, where $A = (t_A, f_A)$ is a vague set on $V$ and $B = (t_B, f_B)$ is a vague set on $E \subseteq V \times V$ such that $t_B(xy) \leq \min(t_A(x), t_A(y))$ and $f_B(xy) \geq \max(f_A(x), f_A(y))$, for $xy \in E$.*

**Example 2.1.** [11] *Consider a vague graph $G$ such that $V = \{a_1, a_2, a_3\}$ and $E = \{a_1a_2, a_2a_3, a_1a_3\}$. By routin computations, it is easy to show that $G$ is a vague graph.*

## 3. Operations on vague graph structures

**Definition 3.1.** *$\check{G}_v = (A, B_1, B_2, \cdots, B_n)$ is called a vague graph structure (VGS) of a graph structure (GS) $G^* = (U, E_1, E_2, \cdots, E_n)$, if $A = (t_A, f_A)$ is a vague set on $U$ and*

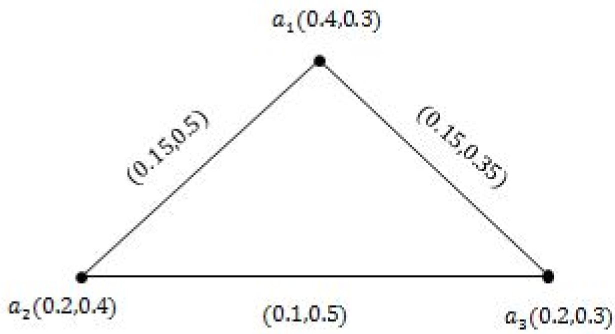


FIGURE 1. Vague graph $G$

*for each* $i = 1, 2, \cdots, n$; $B_i = (t_{B_i}, f_{B_i})$ *is a vague set on* $E_i$ *such that:*

$$t_{B_i}(xy) \leq t_A(x) \wedge t_A(y), \;\; f_{B_i}(xy) \geq f_A(x) \vee f_A(y),$$

$\forall xy \in E_i \subseteq U \times U$. *Note that* $t_{B_i}(xy) = 0 = f_{B_i}(xy)$, *for all* $xy \in U \times U - E_i$ *and* $0 \leq t_{B_i}(xy) \leq 1$, $0 \leq f_{B_i}(xy) \leq 1$, $\forall xy \in E_i$, *where* $U$ *and* $E_i (i = 1, 2, \cdots, n)$ *are called underlying vertex set and underlying i-edge set of* $\check{G}_v$, *respectively.*

**Example 3.1.** *Let* $(U, E_1, E_2)$ *be a graph structure such that* $U = \{a_1, a_2, a_3, a_4\}$, $E_1 = \{a_1a_2, a_2a_3\}$, *and* $E_2 = \{a_3a_4, a_1a_4\}$. *Let* $A$,$B_1$ *and* $B_2$ *be vague subsets of* $U$,$E_1$ *and* $E_2$ *respectively such that:*

$$A = \{(a_1, 0.3, 0.4), (a_2, 0.3, 0.5), (a_3, 0.2, 0.3), (a_4, 0.3, 0.3)\},$$
$$B_1 = \{(a_1a_2, 0.3, 0.5), (a_2a_3, 0.2, 0.5)\}, and\ B_2 = \{(a_3a_4, 0.2, 0.3), (a_1a_4, 0.2, 0.4)\}.$$

*Then* $\check{G}_v = (A, B_1, B_2)$ *is a VGS of* $G^*$ *as shown in Fig. 2.*

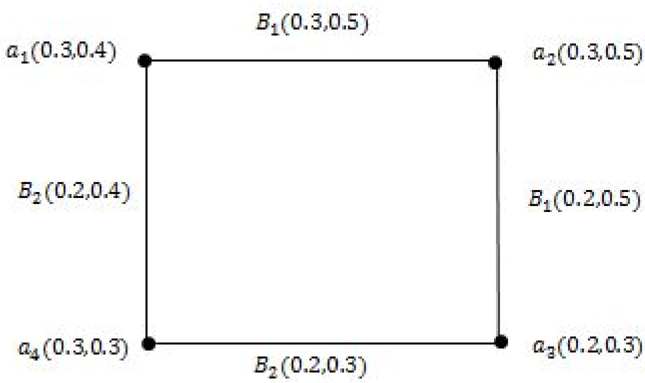


FIGURE 2. VGS $\check{G}_v = (A, B_1, B_2)$

**Definition 3.2.** *Let* $\check{G}_{v1} = (A_1, B_{11}, B_{12}, \cdots, B_{1n})$ *and* $\check{G}_{v2} = (A_2, B_{21}, B_{22}, \cdots, B_{2n})$ *be respective VGSs of GSs* $G_1^* = (U_1, E_{11}, E_{12}, \cdots, E_{1n})$ *and* $G_2^* = (U_2, E_{21}, E_{22}, \cdots, E_{2n})$. *The cartesian product* $\check{G}_{v1} \times \check{G}_{v2}$ *of* $\check{G}_{v1}$ *and* $\check{G}_{v2}$ *is then a VGS of* $G_1^* \times G_2^* = (U_1 \times U_2, E_{11} \times E_{21}, E_{12} \times E_{22}, ...., E_{1n} \times E_{2n})$ *is given by*

$(A_1 \times A_2, B_{11} \times B_{21}, B_{12} \times B_{22}, \cdots, B_{1n} \times B_{2n}) such that$

$$(i) \begin{cases} t_{A_1 \times A_2}(xy) = (t_{A_1} \times t_{A_2})(xy) = t_{A_1}(x) \wedge t_{A_2}(y) \\ f_{A_1 \times A_2}(xy) = (f_{A_1} \times f_{A_2})(xy) = f_{A_1}(x) \vee f_{A_2}(y), \ \forall xy \in U_1 \times U_2 \end{cases}$$

$$(ii) \begin{cases} t_{B_{1i} \times B_{2i}}((xy_1)(xy_2)) = (t_{B_{1i}} \times t_{B_{2i}})((xy_1)(xy_2)) = t_{A_1}(x) \wedge t_{B_{2i}}(y_1y_2) \\ f_{B_{1i} \times B_{2i}}((xy_1)(xy_2)) = (f_{B_{1i}} \times f_{B_{2i}})((xy_1)(xy_2)) = f_{A_1}(x) \vee f_{B_{2i}}(y_1y_2), \\ \forall x \in U_1, y_1y_2 \in E_{2i}, \end{cases}$$

$$(iii) \begin{cases} t_{B_{1i} \times B_{2i}}((x_1y)(x_2y)) = (t_{B_{1i}} \times t_{B_{2i}})((x_1y)(x_2y)) = t_{A_2}(y) \wedge t_{B_{1i}}(x_1x_2) \\ f_{B_{1i} \times B_{2i}}((x_1y)(x_2y)) = (f_{B_{1i}} \times f_{B_{2i}})((x_1y)(x_2y)) = f_{A_2}(y) \vee f_{B_{1i}}(x_1x_2), \\ \forall y \in U_2, x_1x_2 \in E_{1i}, \end{cases}$$

**Example 3.2.** *Let $\check{G}_{v1} = (A_1, B_{11}, B_{12})$ and $\check{G}_{v2} = (A_2, B_{21}, B_{22})$ be respective VGSs of graph structures $G_1^* = (U_1, E_{11}, E_{12})$ and $G_2^* = (U_2, E_{21}, E_{22})$ such that $U_1 = \{a_1, a_2, a_3, a_4\}$, $U_2 = \{b_1, b_2, b_3, b_4\}$, $E_{11} = \{a_1a_2\}$, $E_{12} = \{a_3a_4\}$, $E_{21} = \{b_1b_2\}$, and $E_{22} = \{b_3b_4\}$. $\check{G}_{v1}$ and $\check{G}_{v2}$ are shown in Fig. 3,*

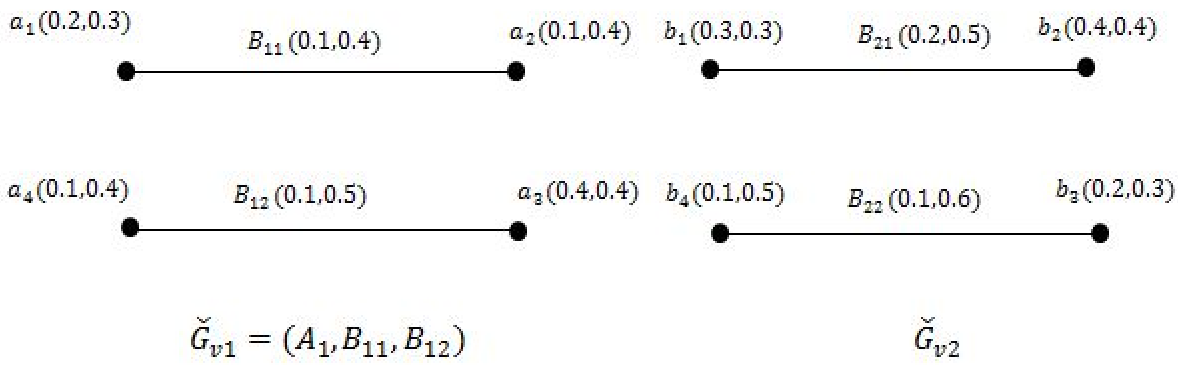


Figure 3. Vague graph structures

*and cartesian product $\check{G}_{v1} \times \check{G}_{v2} = (A_1 \times A_2, B_{11} \times B_{21}, B_{12} \times B_{22})$ is shown in Fig. 4.*

**Theorem 3.1.** *Let $G^* = (U_1 \times U_2, E_{11} \times E_{21}, E_{12} \times E_{22}, \cdots, E_{1n} \times E_{2n})$ be cartesian product of GSs $G_1^* = (U_1, E_{11}, E_{12}, \cdots, E_{1n})$ and $G_2^* = (U_2, E_{21}, E_{22}, \cdots, E_{2n})$. Let $\check{G}_{v1} = (A_1, B_{11}, B_{12}, \cdots, B_{1n})$ and $\check{G}_{v2} = (A_2, B_{21}, B_{22}, \cdots, B_{2n})$ be respective VGSs of $G_1^*$ and $G_2^*$. Then $(A_1 \times A_2, B_{11} \times B_{21}, B_{21} \times B_{22}, \cdots, B_{1n} \times B_{2n})$ is a VGS of $G^*$.*

*Proof.* Case 1. When $u \in U_1$, $b_1b_2 \in E_{2i}$

$$\begin{aligned} t_{B_{1i} \times B_{2i}}\big((ub_1)(ub_2)\big) &= t_{A_1}(u) \wedge t_{B_{2i}}(b_1b_2) \leq t_{A_1}(u) \wedge \big[t_{A_2}(b_1) \wedge t_{A_2}(b_2)\big] \\ &= \big[t_{A_1}(u) \wedge t_{A_2}(b_1)\big] \wedge \big[t_{A_1}(u) \wedge t_{A_2}(b_2)\big] \\ &= t_{A_1 \times A_2}(ub_1) \wedge t_{A_1 \times A_2}(ub_2) \end{aligned}$$

$$\begin{aligned} f_{B_{1i} \times B_{2i}}\big((ub_1)(ub_2)\big) &= f_{A_1}(u) \vee f_{B_{2i}}(b_1b_2) \geq f_{A_1}(u) \vee \big[f_{A_2}(b_1) \vee f_{A_2}(b_2)\big] \\ &= \big[f_{A_1}(u) \vee f_{A_2}(b_1)\big] \vee \big[f_{A_1}(u) \vee f_{A_2}(b_2)\big] \\ &= f_{A_1 \times A_2}(ub_1) \vee f_{A_1 \times A_2}(ub_2) \end{aligned}$$

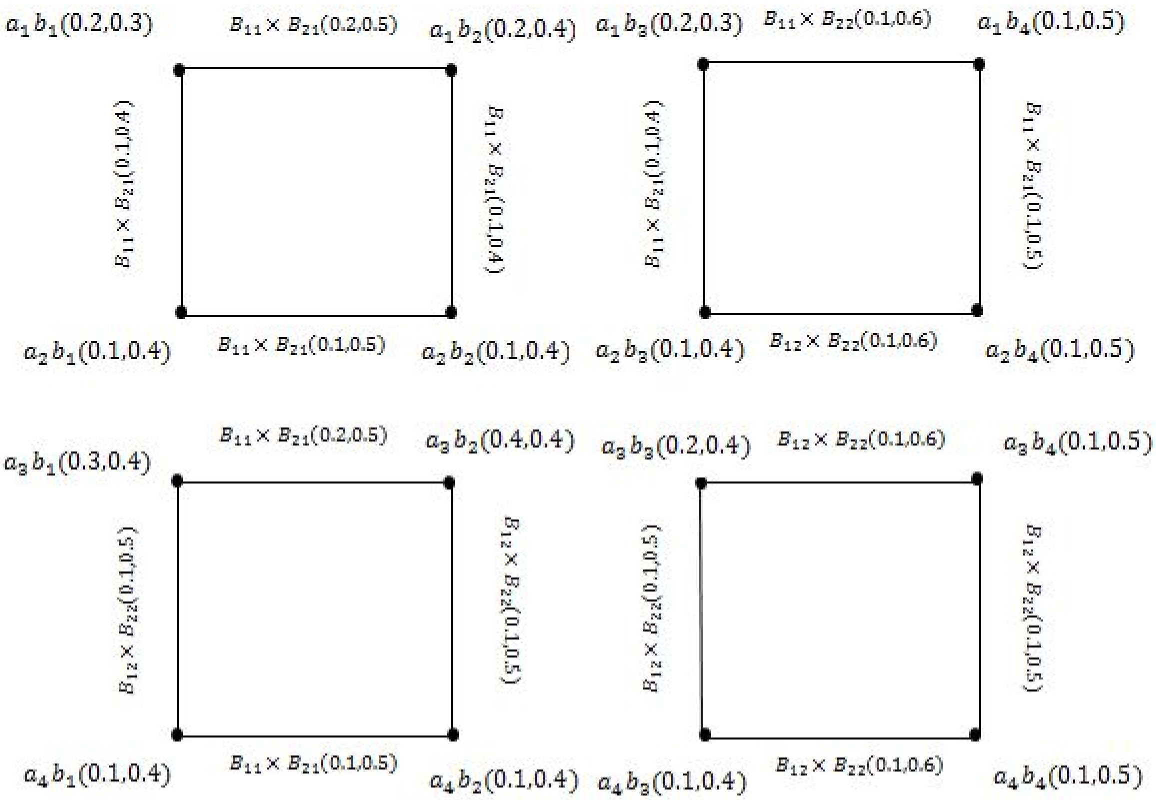


FIGURE 4. Cartesian product of two VGSs

for $ub_1, ub_2 \in U_1 \times U_2$.

Case 2. When $u \in U_2$, $b_1b_2 \in E_{1i}$

$$\begin{aligned} t_{B_{1i}\times B_{2i}}\big(b_1u)(b_2u)\big) &= t_{A_2}(u) \wedge t_{B_{1i}}(b_1b_2) \leq t_{A_2}(u) \wedge \big[t_{A_1}(b_1) \wedge t_{A_1}(b_2)\big] \\ &= \big[t_{A_2}(u) \wedge t_{A_1}(b_1)\big] \wedge \big[t_{A_2}(u) \wedge t_{A_1}(b_2)\big] \\ &= t_{A_1\times A_2}(b_1u) \wedge t_{A_1\times A_2}(b_2u) \end{aligned}$$

$$\begin{aligned} f_{B_{1i}\times B_{2i}}\big(b_1u)(b_2u)\big) &= f_{A_2}(u) \vee f_{B_{1i}}(b_1b_2) \geq f_{A_2}(u) \vee \big[f_{A_1}(b_1) \vee f_{A_1}(b_2)\big] \\ &= \big[f_{A_2}(u) \vee f_{A_1}(b_1)\big] \vee \big[f_{A_2}(u) \vee f_{A_1}(b_2)\big] \\ &= f_{A_1\times A_2}(b_1u) \vee f_{A_1\times A_2}(b_2u) \end{aligned}$$

for $b_1u, b_2u \in U_1 \times U_2$. Both cases hold for $i = 1, 2, \cdots, n$. This completes the proof. □

**Definition 3.3.** *Let $\check{G}_{v1} = (A_1, B_{11}, B_{12}, \cdots, B_{1n})$ and $\check{G}_{v2} = (A_2, B_{21}, B_{22}, \cdots, B_{2n})$ be respective VGSs of GSs $G_1^* = (U_1, E_{11}, E_{12}, \cdots, E_{1n})$ and $G_2^* = (U_2, E_{21}, E_{22}, \cdots, E_{2n})$. The cross product $\check{G}_{v1} * \check{G}_{v2}$ of $\check{G}_{v1}$ and $\check{G}_{v2}$ is a VGS of $G_1^* * G_2^* = (U_1 * U_2, E_{11} * E_{21}, E_{12} * E_{22}, \cdots, E_{1n} * E_{2n})$ is given by $(A_1 * A_2, B_{11} * B_{21}, B_{12} * B_{22}, \cdots, B_{1n} * B_{2n})$, such that*

$$(i)\begin{cases} t_{A_1*A_2}(xy) = (t_{A_1} * t_{A_2})(xy) = t_{A_1}(x) \wedge t_{A_2}(y) \\ f_{A_1*A_2}(xy) = (f_{A_1} * f_{A_2})(xy) = f_{A_1}(x) \vee f_{A_2}(y), \ \forall xy \in U_1 \times U_2, \end{cases}$$

$$(ii)\begin{cases} t_{B_{1i}*B_{2i}}\big((x_1y_1)(x_2y_2)\big) = (t_{B_{1i}} * t_{B_{2i}})\big((x_1y_1)(x_2y_2)\big) = t_{B_{2i}}(y_1y_2) \wedge t_{B_{1i}}(x_1x_2), \\ f_{B_{1i}*B_{2i}}\big((x_1y_1)(x_2y_2)\big) = (f_{B_{1i}} * f_{B_{2i}})\big((x_1y_1)(x_2y_2)\big) = f_{B_{2i}}(y_1y_2) \vee f_{B_{1i}}(x_1x_2), \end{cases}$$

$\forall y_1y_2 \in E_{2i},\ x_1x_2 \in E_{1i}$.

**Example 3.3.** *Let $\check{G}_{v1}$ and $\check{G}_{v2}$be VGSs as shown in Figure 3 and cross products $\check{G}_{v1} * \check{G}_{v2} = (A_1 * A_2, B_{11} * B_{21}, B_{12} * B_{22})$ is shown in Figure 5.*

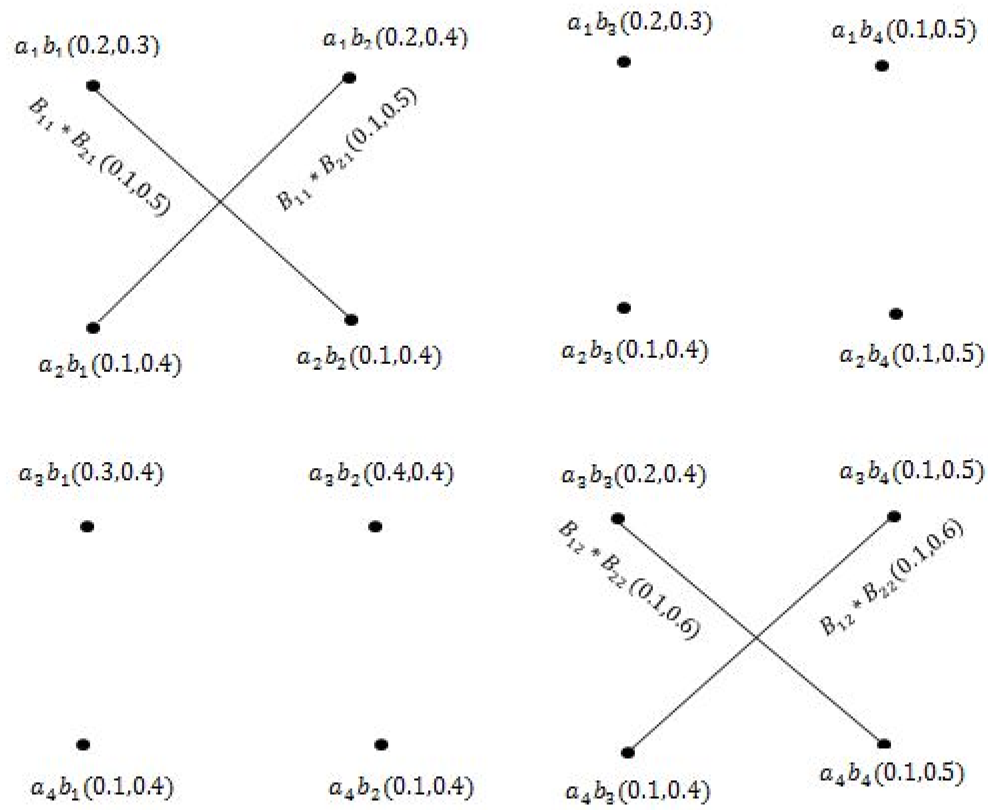


FIGURE 5. Cross product of two VGSs

**Theorem 3.2.** *Let $G^* = (U_1 * U_2, E_{11} * E_{21}, E_{12} * E_{22}, \cdots, E_{1n} * E_{2n})$ be cross product of GSs $G_1^* = (U_1, E_{11}, E_{12}, \cdots, E_{1n})$ and $G_2^* = (U_2, E_{21}, E_{22}, \cdots, E_{2n})$. Let $\check{G}_{v1} = (A_1, B_{11}, B_{12}, \cdots, B_{1n})$ and $\check{G}_{v2} = (A_2, B_{21}, B_{22}, \cdots, B_{2n})$ be respective VGSs of GSs $G_1^*$ and $G_2^*$. Then $(A_1 * A_2, B_{11} * B_{21}, B_{12} * B_{22}, \cdots, B_{1n} * B_{2n})$ is a GVS of $G^*$.*

*Proof.* For all $b_1u_1, b_2u_2 \in U_1 * U_2$;

$$\begin{aligned}
t_{B_{1i}\times B_{2i}}\big((b_1u_1)(b_2u_2)\big) &= t_{B_{2i}}(u_1u_2) \wedge t_{B_{1i}}(b_1b_2)\\
&\leq \big[t_{A_2}(u_1) \wedge t_{A_2}(u_2)\big] \wedge \big[t_{A_1}(b_1) \wedge t_{A_1}(b_2)\big]\\
&= \big[t_{A_2}(u_1) \wedge t_{A_1}(b_1)\big] \wedge \big[t_{A_2}(u_2) \wedge t_{A_1}(b_2)\big]\\
&= t_{A_1*A_2}(b_1u_1) \wedge t_{A_1*A_2}(b_2u_2)
\end{aligned}$$

$$\begin{aligned}
f_{B_{1i}\times B_{2i}}\big((b_1u_1)(b_2u_2)\big) &= f_{B_{2i}}(u_1u_2) \vee f_{B_{1i}}(b_1b_2)\\
&\geq \big[f_{A_2}(u_1) \vee f_{A_2}(u_2)\big] \vee \big[f_{A_1}(b_1) \vee f_{A_1}(b_2)\big]\\
&= \big[f_{A_2}(u_1) \vee f_{A_1}(b_1)\big] \vee \big[f_{A_2}(u_2) \vee f_{A_1}(b_2)\big]\\
&= f_{A_1*A_2}(b_1u_1) \vee f_{A_1*A_2}(b_2u_2)
\end{aligned}$$

for $i = 1, 2, \cdots, n$. This complete the proof. □

**Definition 3.4.** *Let $\check{G}_{v1} = (A_1, B_{11}, B_{12}, \cdots, B_{1n})$ and $\check{G}_{v2} = (A_2, B_{21}, B_{22}, \cdots, B_{2n})$ be respective VGSs of GSs $G_1^* = (U_1, E_{11}, E_{12}, \cdots, E_{1n})$ and $G_2^* = (U_2, E_{21}, E_{22}, \cdots, E_{2n})$. The lexicographic product $\check{G}_{v1} \bullet \check{G}_{v2}$ of $\check{G}_{v1}$ and $\check{G}_{v2}$ is a VGS of $G_1^* \bullet G_2^* = (U_1 \bullet U_2, E_{11} \bullet E_{21}, E_{12} \bullet E_{22}, \cdots, E_{1n} \bullet E_{2n})$ is given by $(A_1 \bullet A_2, B_{11} \bullet B_{21}, B_{12} \bullet B_{22}, \cdots, B_{1n} \bullet B_{2n})$ such that*

$$(i) \begin{cases} t_{A_1 \bullet A_2}(xy) = (t_{A_1} \bullet t_{A_2})(xy) = t_{A_1}(x) \wedge t_{A_2}(y) \\ f_{A_1 \bullet A_2}(xy) = (f_{A_1} \bullet f_{A_2})(xy) = f_{A_1}(x) \vee f_{A_2}(y), \ \forall xy \in U_1 \times U_2, \end{cases}$$

$$(ii) \begin{cases} t_{B_{1i} \bullet B_{2i}}\big((xy_1)(xy_2)\big) = (t_{B_{1i}} \bullet t_{B_{2i}})\big((xy_1)(xy_2)\big) = t_{A_1}(x) \wedge t_{B_{2i}}(y_1y_2) \\ f_{B_{1i} \bullet B_{2i}}\big((xy_1)(xy_2)\big) = (f_{B_{1i}} \bullet f_{B_{2i}})\big((xy_1)(xy_2)\big) = f_{A_1}(x) \vee f_{B_{2i}}(y_1y_2), \\ \forall x \in U_1, y_1y_2 \in E_{2i}, \end{cases}$$

$$(iii) \begin{cases} t_{B_{1i} \bullet B_{2i}}\big((x_1y_1)(x_2y_2)\big) = (t_{B_{1i}} \bullet t_{B_{2i}})\big((x_1y_1)(x_2y_2)\big) = t_{B_{2i}}(y_1y_2) \wedge t_{B_{1i}}(x_1x_2) \\ f_{B_{1i} \bullet B_{2i}}\big((x_1y_1)(x_2y_2)\big) = (f_{B_{1i}} \bullet f_{B_{2i}})\big((x_1y_1)(x_2y_2)\big) = f_{B_{2i}}(y_1y_2) \vee f_{B_{1i}}(x_1x_2), \\ \forall y_1y_2 \in E_{2i}, x_1x_2 \in E_{1i}. \end{cases}$$

**Example 3.4.** *Let $\check{G}_{v1}$ and $\check{G}_{v2}$ be VGSs shown in Figure 3 and lexicographic product $\check{G}_{v1} \bullet \check{G}_{v2} = (A_1 \bullet A_2, B_{11} \bullet B_{21}, B_{12} \bullet B_{22})$ is as shown in Figure 6.*

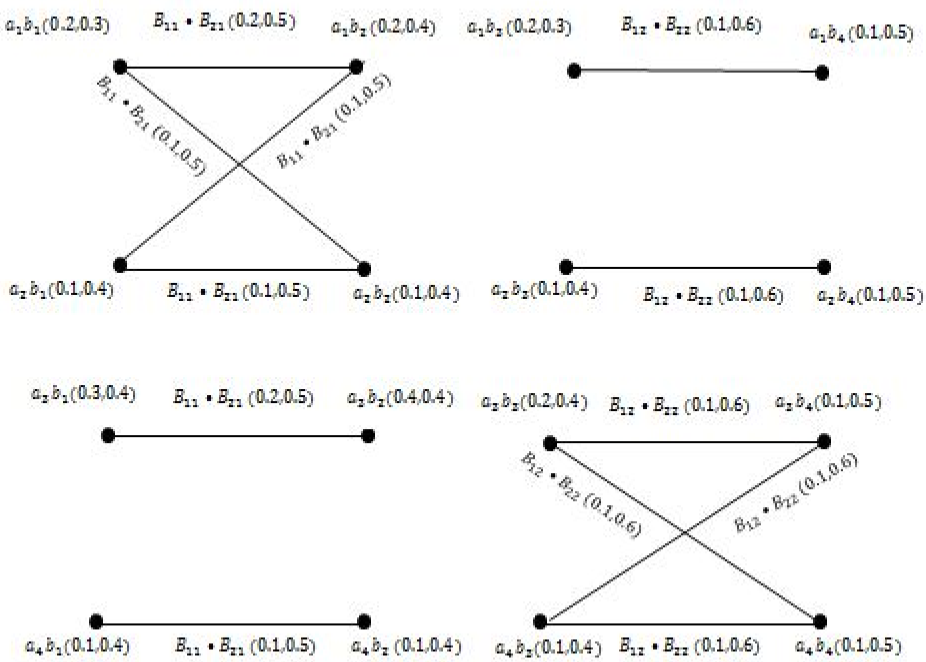


FIGURE 6. Lexicographic product of two VGSs

**Theorem 3.3.** *Let $G^* = (U_1 \bullet U_2, E_{11} \bullet E_{21}, E_{12} \bullet E_{22}, \cdots, E_{1n} \bullet E_{2n})$ be lexicographic product of GSs $G_1^* = (U_1, E_{11}, E_{12}, \cdots, E_{1n})$ and $G_2^* = (U_2, E_{21}, E_{22}, \cdots, E_{2n})$. Let $\check{G}_{v1} = (A_1, B_{11}, B_{12}, \cdots, B_{1n})$ and $\check{G}_{v2} = (A_2, B_{21}, B_{22}, \cdots, B_{2n})$ be respective VGSs of $G_1^*$ and $G_2^*$. Then $(A_1 \bullet A_2, B_{11} \bullet B_{21}, B_{12} \bullet B_{22}, \cdots, B_{1n} \bullet B_{2n})$ is a VGS of $G^*$.*

*Proof.* Case 1. When $u \in U_1$, $b_1b_2 \in E_{2i}$

$$\begin{aligned} t_{B_{1i}\bullet B_{2i}}\big((ub_1)(ub_2)\big) &= t_{A_1}(u) \wedge t_{B_{2i}}(b_1b_2) \leq t_{A_1}(u) \wedge \big[t_{A_2}(b_1) \wedge t_{A_2}(b_2)\big] \\ &= \big[t_{A_1}(u) \wedge t_{A_2}(b_1)\big] \wedge \big[t_{A_1}(u) \wedge t_{A_2}(b_2)\big] \\ &= t_{A_1\bullet A_2}(ub_1) \wedge t_{A_1\bullet A_2}(ub_2) \end{aligned}$$

$$\begin{aligned} f_{B_{1i}\bullet B_{2i}}\big((ub_1)(ub_2)\big) &= f_{A_1}(u) \vee f_{B_{2i}}(b_1b_2) \geq f_{A_1}(u) \vee \big[f_{A_2}(b_1) \vee f_{A_2}(b_2)\big] \\ &= \big[f_{A_1}(u) \vee f_{A_2}(b_1)\big] \vee \big[f_{A_1}(u) \vee f_{A_2}(b_2)\big] \\ &= f_{A_1\bullet A_2}(ub_1) \vee f_{A_1\bullet A_2}(ub_2) \end{aligned}$$

for $ub_1, ub_2 \in U_1 \bullet U_2$.
Case 2. When $u_1u_2 \in E_{2i}$, $b_1b_2 \in E_{1i}$

$$\begin{aligned} t_{B_{1i}\bullet B_{2i}}\big((b_1u_1)(b_2u_2)\big) &= t_{B_{2i}}(u_1u_2) \wedge t_{B_{1i}}(b_1b_2) \\ &\leq \big[t_{A_2}(u_1) \wedge t_{A_2}(u_2)\big] \wedge \big[t_{A_1}(b_1) \wedge t_{A_1}(b_2)\big] \\ &= \big[t_{A_2}(u_1) \wedge t_{A_1}(b_1)\big] \wedge \big[t_{A_2}(u_2) \wedge t_{A_1}(b_2)\big] \\ &= t_{A_1\bullet A_2}(b_1u_1) \wedge t_{A_1\bullet A_2}(b_2u_2), \end{aligned}$$

$$\begin{aligned} f_{B_{1i}\bullet B_{2i}}\big((b_1u_1)(b_2u_2)\big) &= f_{B_{2i}}(u_1u_2) \vee f_{B_{1i}}(b_1b_2) \\ &\geq \big[f_{A_2}(u_1) \vee f_{A_2}(u_2)\big] \vee \big[f_{A_1}(b_1) \vee f_{A_1}(b_2)\big] \\ &= \big[f_{A_2}(u_1) \vee f_{A_1}(b_1)\big] \vee \big[f_{A_2}(u_2) \vee f_{A_1}(b_2)\big] \\ &= f_{A_1\bullet A_2}(b_1u_1) \vee f_{A_1\bullet A_2}(b_2u_2), \end{aligned}$$

for $b_1u_1, b_2u_2 \in U_1 \bullet U_2$. Both cases hold for $i = 1, 2, \cdots, n$. This completes the proof. □

**Definition 3.5.** *Let $\check{G}_{v1} = (A_1, B_{11}, B_{12}, \cdots, B_{1n})$ and $\check{G}_{v2} = (A_2, B_{21}, B_{22}, \cdots, B_{2n})$ be respective VGSs of GSs $G_1^* = (U_1, E_{11}, E_{12}, \cdots, E_{1n})$ and $G_2^* = (U_2, E_{21}, E_{22}, \cdots, E_{2n})$.*

*The composition $\check{G}_{v1} \circ \check{G}_{v2}$ of $\check{G}_{v1}$ and $\check{G}_{v2}$ is then a VGS of $G_1^* \circ G_2^* = (U_1 \circ U_2, E_{11} \circ E_{21}, E_{12} \circ E_{22}, \cdots, E_{1n} \circ E_{2n})$ is given by $(A_1 \circ A_2, B_{11} \circ B_{21}, B_{12} \circ B_{22}, \cdots, B_{1n} \circ B_{2n})$ such that*

$$(i) \begin{cases} t_{A_1\circ A_2}(xy) = (t_{A_1} \circ t_{A_2})(xy) = t_{A_1}(x) \wedge t_{A_2}(y) \\ f_{A_1\circ A_2}(xy) = (f_{A_1} \circ f_{A_2})(xy) = f_{A_1}(x) \vee f_{A_2}(y), \ \forall xy \in U_1 \times U_2, \end{cases}$$

$$(ii) \begin{cases} t_{B_{1i}\circ B_{2i}}\big((xy_1)(xy_2)\big) = (t_{B_{1i}} \circ t_{B_{2i}})\big((xy_1)(xy_2)\big) = t_{A_1}(x) \wedge t_{B_{2i}}(y_1y_2) \\ f_{B_{1i}\circ B_{2i}}\big((xy_1)(xy_2)\big) = (f_{B_{1i}} \circ f_{B_{2i}})\big((xy_1)(xy_2)\big) = f_{A_1}(x) \vee f_{B_{2i}}(y_1y_2), \\ \forall x \in U_1, y_1y_2 \in E_{2i}, \end{cases}$$

$$(iii) \begin{cases} t_{B_{1i}\circ B_{2i}}\big((x_1y)(x_2y)\big) = (t_{B_{1i}} \circ t_{B_{2i}})\big((x_1y)(x_2y)\big) = t_{A_2}(y) \wedge t_{B_{1i}}(x_1x_2) \\ f_{B_{1i}\circ B_{2i}}\big((x_1y)(x_2y)\big) = (f_{B_{1i}} \circ f_{B_{2i}})\big((x_1y)(x_2y)\big) = f_{A_2}(y) \vee f_{B_{1i}}(x_1x_2), \\ \forall y \in U_2, x_1x_2 \in E_{1i}, \end{cases}$$

$$(iv) \begin{cases} t_{B_{1i}\circ B_{2i}}\big((x_1y_1)(x_2y_2)\big) = (t_{B_{1i}} \circ t_{B_{2i}})\big((x_1y_1)(x_2y_2)\big) = t_{A_2}(y_1) \wedge t_{A_2}(y_2) \wedge t_{B_{1i}}(x_1x_2) \\ f_{B_{1i}\circ B_{2i}}\big((x_1y_1)(x_2y_2)\big) = (f_{B_{1i}} \circ f_{B_{2i}})\big((x_1y_1)(x_2y_2)\big) = f_{A_2}(y_1) \vee f_{A_2}(y_2) \vee f_{B_{1i}}(x_1x_2), \\ \forall y_1, y_2 \in U_2, x_1x_2 \in E_{1i}, \end{cases}$$

*such that $y_1 \neq y_2$.*

**Theorem 3.4.** *Let $G^* = (U_1 \circ U_2, E_{11} \circ E_{21}, E_{12} \circ E_{22}, \cdots, E_{1n} \circ E_{2n})$ be the composition of GSs $G_1^* = (U_1, E_{11}, E_{12}, \cdots, E_{1n})$ and $G_2^* = (U_2, E_{21}, E_{22}, \cdots, E_{2n})$. Let $\check{G}_{v1} = (A_1, B_{11}, B_{12}, \cdots, B_{1n})$ and $\check{G}_{v2} = (A_2, B_{21}, B_{22}, \cdots, B_{2n})$ be respective VGSs of*

*$G_1^*$ and $G_2^*$. Then $\check{G}_{v1} \circ \check{G}_{v2}=(A_1 \circ A_2, B_{11} \circ B_{21}, B_{12} \circ B_{22}, \cdots, B_{1n} \circ B_{2n})$ is a VGS of $G^*$.*

*Proof.* Case 1. When $u \in U_1$, $b_1b_2 \in E_{2i}$,

$$\begin{aligned} t_{B_{1i}\circ B_{2i}}\big((ub_1)(ub_2)\big) &= t_{A_1}(u) \wedge t_{B_{2i}}(b_1b_2) \leq t_{A_1}(u) \wedge \big[t_{A_2}(b_1) \wedge t_{A_2}(b_2)\big] \\ &= \big[t_{A_1}(u) \wedge t_{A_2}(b_1)\big] \wedge \big[t_{A_1}(u) \wedge t_{A_2}(b_2)\big] \\ &= t_{A_1\circ A_2}(ub_1) \wedge t_{A_1\circ A_2}(ub_2), \end{aligned}$$

$$\begin{aligned} f_{B_{1i}\circ B_{2i}}\big((ub_1)(ub_2)\big) &= f_{A_1}(u) \vee f_{B_{2i}}(b_1b_2) \geq f_{A_1}(u) \vee \big[f_{A_2}(b_1) \vee f_{A_2}(b_2)\big] \\ &= \big[f_{A_1}(u) \vee f_{A_2}(b_1)\big] \vee \big[f_{A_1}(u) \vee f_{A_2}(b_2)\big] \\ &= f_{A_1\circ A_2}(ub_1) \vee f_{A_1\circ A_2}(ub_2), \end{aligned}$$

for $ub_1, ub_2 \in U_1 \circ U_2$.
Case 2. When $u \in U_2$, $b_1b_2 \in E_{1i}$

$$\begin{aligned} t_{B_{1i}\circ B_{2i}}\big((b_1u)(b_2u)\big) &= t_{A_2}(u) \wedge t_{B_{1i}}(b_1b_2) \leq t_{A_2}(u) \wedge \big[t_{A_1}(b_1) \wedge t_{A_1}(b_2)\big] \\ &= \big[t_{A_2}(u) \wedge t_{A_1}(b_1)\big] \wedge \big[t_{A_2}(u) \wedge t_{A_1}(b_2)\big] \\ &= t_{A_1\circ A_2}(b_1u) \wedge t_{A_1\circ A_2}(b_2u), \end{aligned}$$

$$\begin{aligned} f_{B_{1i}\circ B_{2i}}\big((b_1u)(b_2u)\big) &= f_{A_2}(u) \vee f_{B_{1i}}(b_1b_2) \geq f_{A_2}(u) \vee \big[f_{A_1}(b_1) \vee f_{A_1}(b_2)\big] \\ &= \big[f_{A_2}(u) \vee f_{A_1}(b_1)\big] \vee \big[f_{A_2}(u) \vee f_{A_1}(b_2)\big] \\ &= f_{A_1\circ A_2}(b_1u) \vee f_{A_1\circ A_2}(b_2u), \end{aligned}$$

for $b_1u, b_2u \in U_1 \circ U_2$.
Case 3. When $b_1b_2 \in E_{1i}$, $u_1, u_2 \in U_2$ such that $u_1 \neq u_2$,

$$\begin{aligned} t_{B_{1i}\circ B_{2i}}\big((b_1u_1)(b_2u_2)\big) &= t_{A_2}(u_1) \wedge t_{A_2}(u_2) \wedge t_{B_{1i}}(b_1b_2) \\ &\leq t_{A_2}(u_1) \wedge t_{A_2}(u_2) \wedge \big[t_{A_1}(b_1) \wedge t_{A_1}(b_2)\big] \\ &= \big[t_{A_2}(u_1) \wedge t_{A_1}(b_1)\big] \wedge \big[t_{A_2}(u_2) \wedge t_{A_1}(b_2)\big] \\ &= t_{A_1\circ A_2}(b_1u_1) \wedge t_{A_1\circ A_2}(b_2u_2), \end{aligned}$$

$$\begin{aligned} f_{B_{1i}\circ B_{2i}}\big((b_1u_1)(b_2u_2)\big) &= f_{A_2}(u_1) \vee f_{A_2}(u_2) \vee f_{B_{1i}}(b_1b_2) \\ &\geq f_{A_2}(u_1) \vee f_{A_2}(u_2) \vee \big[f_{A_1}(b_1) \vee f_{A_1}(b_2)\big] \\ &= \big[f_{A_2}(u_1) \vee f_{A_1}(b_1)\big] \vee \big[f_{A_2}(u_2) \vee f_{A_1}(b_2)\big] \\ &= f_{A_1\circ A_2}(b_1u_1) \vee f_{A_1\circ A_2}(b_2u_2), \end{aligned}$$

for $b_1u_1, b_2u_2 \in U_1 \circ U_2$. All three cases hold for $i = 1, 2, \cdots, n$. This completes the proof. □

**Definition 3.6.** *Let $\check{G}_{v1} = (A_1, B_{11}, B_{12}, \cdots, B_{1n})$ and $\check{G}_{v2} = (A_2, B_{21}, B_{22}, \cdots, B_{2n})$ be respective VGSs of GSs $G_1^* = (U_1, E_{11}, E_{12}, \cdots, E_{1n})$ and $G_2^* = (U_2, E_{21}, E_{22}, \cdots, E_{2n})$ and let $U_1 \cap U_2 = \emptyset$. The union $\check{G}_{v1} \cup \check{G}_{v2}$ of $\check{G}_{v1}$ and $\check{G}_{v2}$ is then a VGS $G_1^* \cup G_2^* = (U_1 \cup U_2, E_{11} \cup E_{21}, E_{12} \cup E_{22}, \cdots, E_{1n} \cup E_{2n})$ is given by $(A_1 \cup A_2, B_{11} \cup B_{21}, B_{12} \cup B_{22}, \cdots, B_{1n} \cup B_{2n})$ such that $A_1 \cup A_2$ is defined by*

$$t_{A_1\cup A_2}(x) = (t_{A_1} \cup t_{A_2})(x) = t_{A_1}(x) \vee t_{A_2}(x)$$
$$f_{A_1\cup A_2}(x) = (f_{A_1} \cup f_{A_2})(x) = f_{A_1}(x) \wedge f_{A_2}(x), \ \forall x \in U_1 \cup U_2,$$

*(assuming $t_{A_j}(x) = 0$, $f_{A_j}(x) = 0$, if $x \in U_j$, $j = 1, 2$)*

*and* $B_{1i} \cup B_{2i}$, *for* $i = 1, 2, \cdots, n$ *is defined by*

$$t_{B_{1i} \cup B_{2i}}(xy) = (t_{B_{1i}} \cup t_{B_{2i}})(xy) = t_{B_{1i}}(xy) \vee t_{B_{2i}}(xy)$$
$$f_{B_{1i} \cup B_{2i}}(xy) = (f_{B_{1i}} \cup f_{B_{2i}})(xy) = f_{B_{1i}}(xy) \wedge f_{B_{2i}}(xy), \ \forall xy \in E_{1i} \cup E_{2i},$$

*(assuming* $t_{B_{ji}}(xy) = 0, \ f_{B_{ji}}(xy) = 0, \ if \ xy \notin E_{ji}, \ j = 1, 2$*).*

**Example 3.5.** *Let* $\check{G}_{v1}$ *and* $\check{G}_{v2}$ *be VGSs as shown in Figure 3. Their union represented by* $\check{G}_{v1} \cup \check{G}_{v2} = (A_1 \cup A_2, B_{11} \cup B_{21}, B_{21} \cup B_{22})$ *is shown in Figure 7.*

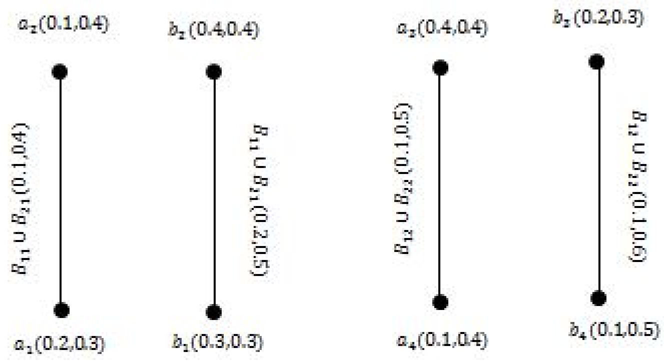


FIGURE 7. Union of two VGSs

**Theorem 3.5.** *Let* $G^* = (U_1 \cup U_2, E_{11} \cup E_{21}, E_{12} \cup E_{22}, \cdots, E_{1n} \cup E_{2n})$ *be the union of GSs* $G_1^* = (U_1, E_{11}, E_{12}, \cdots, E_{1n})$ *and* $G_2^* = (U_2, E_{21}, E_{22}, \cdots, E_{2n})$*. Let* $\check{G}_{v1} = (A_1, B_{11}, B_{12}, \cdots, B_{1n})$ *and* $\check{G}_{v2} = (A_2, B_{21}, B_{22}, \cdots, B_{2n})$ *be respective VGSs of* $G_1^*$ *and* $G_2^*$*. Then* $(A_1 \cup A_2, B_{11} \cup B_{21}, B_{12} \cup B_{22}, \cdots, B_{1n} \cup B_{2n})$ *is a VGS of* $G^*$*.*

*Proof.* Let $u_1u_2 \in E_{1i} \cup E_{2i}$.
Case 1. When $u_1, u_2 \in U_1$, then by Definition 3.6

$$t_{A_2}(u_1) = t_{A_2}(u_2) = t_{B_{2i}}(u_1u_2) = 0, \ f_{A_2}(u_1) = f_{A_2}(u_2) = f_{B_{2i}}(u_1u_2) = 0.$$

So,we have

$$\begin{aligned} t_{B_{1i} \cup B_{2i}}(u_1u_2) &= t_{B_{1i}}(u_1u_2) \vee t_{B_{2i}}(u_1u_2) = t_{B_{1i}}(u_1u_2) \vee 0 \\ &\leq \big[t_{A_1}(u_1) \wedge t_{A_1}(u_2)\big] \vee 0 \\ &= \big[t_{A_1}(u_1) \vee 0\big] \wedge \big[t_{A_1}(u_2) \vee 0\big] \\ &= \big[t_{A_1}(u_1) \vee t_{A_2}(u_1)\big] \wedge \big[t_{A_1}(u_2) \vee t_{A_2}(u_2)\big] \\ &= t_{A_1 \cup A_2}(u_1) \wedge t_{A_1 \cup A_2}(u_2), \end{aligned}$$

$$\begin{aligned} f_{B_{1i} \cup B_{2i}}(u_1u_2) &= f_{B_{1i}}(u_1u_2) \wedge f_{B_{2i}}(u_1u_2) = f_{B_{1i}}(u_1u_2) \wedge 0 \\ &\geq \big[f_{A_1}(u_1) \vee f_{A_1}(u_2)\big] \wedge 0 \\ &= \big[f_{A_1}(u_1) \wedge 0\big] \vee \big[f_{A_1}(u_2) \wedge 0\big] \\ &= \big[f_{A_1}(u_1) \wedge f_{A_2}(u_1)\big] \vee \big[f_{A_1}(u_2) \wedge f_{A_2}(u_2)\big] \\ &= f_{A_1 \cup A_2}(u_1) \vee f_{A_1 \cup A_2}(u_2), \end{aligned}$$

for $u_1, u_2 \in U_1 \cup U_2$.
Case 2. When $u_1, u_2 \in U_2$, then by Definition 3.6,

$$t_{A_1}(u_1) = t_{A_1}(u_2) = t_{B_{1i}}(u_1u_2) = 0, \ f_{A_1}(u_1) = f_{A_1}(u_2) = f_{B_{1i}}(u_1u_2) = 0,$$

so, we have

$$\begin{aligned}
t_{B_{1i}\cup B_{2i}}(u_1u_2) &= t_{B_{1i}}(u_1u_2) \vee t_{B_{2i}}(u_1u_2) = 0 \vee t_{B_{2i}}(u_1u_2)\\
&\le 0 \vee \big[t_{A_2}(u_1) \wedge t_{A_2}(u_2)\big]\\
&= \big[0 \vee t_{A_2}(u_1)\big] \wedge \big[0 \vee t_{A_2}(u_2)\big]\\
&= \big[t_{A_1}(u_1) \vee t_{A_2}(u_1)\big] \wedge \big[t_{A_1}(u_2) \vee t_{A_2}(u_2)\big]\\
&= t_{A_1\cup A_2}(u_1) \wedge t_{A_1\cup A_2}(u_2),
\end{aligned}$$

$$\begin{aligned}
f_{B_{1i}\cup B_{2i}}(u_1u_2) &= f_{B_{1i}}(u_1u_2) \wedge f_{B_{2i}}(u_1u_2) = 0 \wedge f_{B_{2i}}(u_1u_2)\\
&\ge 0 \wedge \big[f_{A_2}(u_1) \vee f_{A_2}(u_2)\big]\\
&= \big[0 \wedge f_{A_2}(u_1)\big] \vee \big[0 \wedge f_{A_2}(u_2)\big]\\
&= \big[f_{A_1}(u_1) \wedge f_{A_2}(u_1)\big] \vee \big[f_{A_1}(u_2) \wedge f_{A_2}(u_2)\big]\\
&= f_{A_1\cup A_2}(u_1) \vee f_{A_1\cup A_2}(u_2),
\end{aligned}$$

for $u_1, u_2 \in U_1 \cup U_2$. Both cases hold for $i = 1, 2, \cdots, n$. This completes the proof. □

## 4. Conclusion

It is well known that graphs are among the most ubiquitous models of both natural and humman-made structures. They can be used to model many types of relations and process dynamics in computer science, biological, social systems and physical. So we have applied the concept of vague sets to graph structures. We have discussed some operations on vague graph structures. In our future work, we will define vague soft graph structures, cubic vague graph structures, bondage number and non-bondage number of vague graph structures, and give some applications that will be useful in our daily life.

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