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A LATTICE STRUCTURE OF Z-SOFT COVERING BASED ROUGH SET AND ITS APPLICATION

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Abstract. The aim of this paper is to construct the lattice structure for Z-soft covering based rough set. First, we define an equivalence relation R' on a universal set to obtain the equivalence classes induced by Z-soft covering-based rough set. Also, we define a relation R_S on the family of Z-soft covering-based rough set (T_S) to show that the relation R_S is a poset on T_S. Second, we define two operations join $\vee$ and meet $\wedge$ on T_S. Using these two operations, we prove that every pair of elements of R_S has a least upper bound and a greatest lower bound and as a result, T_S is a lattice. Finally, we develop a novel Multiple Attribute Group Decision Making (MAGDM) model using Z-soft covering based rough set in medical diagnosis to determine the patients at high risk of chronic kidney disease using the collected data from the UCI Machine Learning Repository.

Keywords: Soft set, Rough set, Soft covering based rough set, Lattice.

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1. Introduction

Zadeh [17] investigated the general theory of uncertainty. In this theory, information was represented as general constraints derived from fuzzy set theory and fuzzy logic, and uncertainty is linked to information through the idea of granular structures. Pawlak [12] proposed the notion of rough set (RS) in 1982. This formal technique was developed in information systems to handle incomplete data. This theory is utilized in data analysis software to find fundamental patterns in data, reduce redundancies and create decision rules. RS is used in a variety of fields, including artificial intelligence, such as pattern recognition, intelligent systems, expert systems, knowledge discovery and others [7, 10]. As an extension of Pawlak's rough sets, covering rough sets (CRS) is an essential research subject for RS. CRS is a valuable technique that allows researchers to look at uncertainty and roughness in a wider sense.

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Molodtsov [11] originally proposed soft set theory in 1999, which is another mathematical technique for dealing with ambiguity. Soft set theory has provided a variety of information descriptions and computing operations. Ali et al. [4] introduced various operations like restricted intersection, union and difference on soft sets. In 2021, Al-Shami introduced a new types of soft compactness on finite spaces and some new types of soft separation axioms called *pt*-soft α regular and *pt*-soft αT_i-spaces (i = 0,1,2,3,4) in [1] and [2], respectively. Likewise, soft somewhat open sets and their behaviours are studied in [3] through some specific topologies. Al-Shami has contributed to the growing literature on soft topology and decision problems through these research works. Ali [5] investigated the interconnections among rough set, fuzzy set and soft set. Roy and Maji [14] proposed a decision-making model by creating a comparison table using fuzzy soft sets. In [6], semiring structures of soft sets are discussed. By combining soft sets with rough sets, Feng et al. proposed soft P-rough set in [8]. Shabir et al. [15] established a modified soft rough set using the concept of a soft P-rough set. In terms of precision, this modified soft rough set exceeds the soft P-rough set. The construction of a soft P-rough set needs additional criteria than the Shabir-soft rough set. Feng et al. [9] defined a multicriteria group decision making algorithm using soft rough set. Yüksel et al. [16] suggested soft covering based rough sets (SCRS) and provided a decision-making algorithm.

Zhan et al. [18] proposed five different types of soft covering based rough sets. They proved that the third type provides a most accurate description of sets than other soft rough sets and soft covering based rough sets. Praba et al. [13] developed a lattice structure for minimal soft rough sets and provided a new decision-making technique based on it. Inspired by these ideas, a lattice structure is constructed for the third type of soft covering based rough set and developed a decision-making algorithm to solve the medical problem of identifying patients at high risk of chronic kidney disease. In this paper, we use the third type of SCRS defined in [18] as Z-soft covering based rough set.

This paper is organized in the following manner: Section 2 provides all the basic definitions for understanding the following sections. Section 3 defines the relation R' on the universal set Ω and prove that the relation R' is an equivalence relation. In Section 4, we define the relation R_S on T_S, where T_S is the family of Z-soft covering based rough sets and prove that T_S is a poset. Two operations join $\vee$ and meet $\wedge$ are defined on T_S and based on these defined operations, T_S is proved to be a lattice. In Section 5, an application of Z-soft covering based rough sets is presented. The conclusion is discussed in Section 6.

2. Preliminaries

In this section, we discuss the basic definitions necessary to understand the following sections. Throughout this paper, Ω represents a finite universal set.

Definition 2.1. [12] *Let R to be an equivalence relation and (Ω, R) be an approximation space. For any $M \subseteq \Omega$, the lower and upper approximation of M with respect to R are given by $\underline{R}(M) = \{v \in \Omega : [v]_R \subseteq M\}$ and $\overline{R}(M) = \{v \in \Omega : [v]_R \cap M \neq \emptyset\}$, respectively and the corresponding rough set is defined as $RS(M) = \left(\underline{R}(M), \overline{R}(M)\right)$.*

Definition 2.2. [11] *Let Ω be a universal set and E be the set of all parameters and $B \subseteq E$. A pair $K = (N, B)$ be a soft set over Ω if N is a mapping defined by $N : B \to P(\Omega)$ where $P(\Omega)$ denote the power set of Ω.*

Definition 2.3. [17] *A fuzzy set B in Ω is a set of ordered pairs:*
$B = \{(v, \mu_B(v) : v \in \Omega)\}$, *where $\mu_B : \Omega \longrightarrow [0, 1] = I$ is a mapping and $\mu_B(v)$ states the grade of belongness of v in B.*

Definition 2.4. [14] *Let I^{Ω} denotes the set of all fuzzy sets on Ω. Let $B \subseteq E$. A pair (N, B) is called a fuzzy soft set over Ω, where N is a mapping defined by $N : B \longrightarrow I^{\Omega}$.*

Definition 2.5. [8] *A soft set $K = (N, B)$ over Ω is called a full soft set if $\bigcup_{b \in B} N(b) = \Omega$.*

Definition 2.6. [8] *A full soft set $K = (N, B)$ over Ω is called a covering soft set denoted as C_K if $N(b) \neq \emptyset \ \forall \ b \in B$.*

Definition 2.7. [16] *Let $K = (N, B)$ be a covering soft set over Ω. The ordered pair $S = (\Omega, C_K)$ represents a soft covering approximation space (SCA).*

3. Z-Soft covering based rough set

In this section, we define a relation R' on Ω and prove that R' is an equivalence relation.

Definition 3.1. [18] *Let $S = (\Omega, C_K)$ be a SCA. For each $v \in \Omega$, the soft adhesion of v are defined as $SA(v) = \{u \in \Omega : \forall \ b \in B(v \in N(b) \leftrightarrow u \in N(b))\}$.*

Definition 3.2. [18] *Let $S = (\Omega, C_K)$ be a SCA. For each subset $M \subseteq \Omega$, the soft covering lower approximation(SCLA) and soft covering upper approximation(SCUA) are respectively defined as $\underline{SC}(M) = \{v \in \Omega : SA(v) \subseteq M\}$ and $\overline{SC}(M) = \{v \in \Omega : SA(v) \cap M \neq \emptyset\}$. If $\underline{SC}(M) \neq \overline{SC}(M)$, then M is called Z-soft covering based rough set; otherwise M is known as Z-soft covering based definable, then the Z-soft covering based rough set is denoted as $SCRS(M)$ is defined by $SCRS(M) = (\underline{SC}(M), \overline{SC}(M))$.*

We define a relation R' on Ω such that $R' = \{(u, v) \in \Omega \times \Omega : SA(u) = SA(v)\}$ by applying soft adhesion.

Lemma 3.1. *R' is an equivalence relation of Ω.*
Proof. The proof is trivial from the statement of the lemma. □

Example 3.1. *Let $\Omega = \{v_1, v_2, v_3, v_4\}$ be a universal set and $B = \{b_1, b_2, b_3\}$ be the set of parameters. Then the soft set over Ω is given by Table 1 where $N(b_1) = \{v_1, v_2, v_3, v_4\}$, $N(b_2) = \{v_2, v_4\}$ and $N(b_3) = \{v_1, v_2, v_3\}$.*
Then, $SA(v_1) = \{v_1, v_3\}$, $SA(v_2) = \{v_2\}$, $SA(v_3) = \{v_1, v_3\}$, $SA(v_4) = \{v_4\}$.

Table 1. Tabular representation of the soft set

	v_1	v_2	v_3	v_4
b_1	1	1	1	1
b_2	0	1	0	1
b_3	1	1	1	0

(i) Let $M = \{v_1, v_4\} \subseteq \Omega$; then $\underline{SC}(M) = \{v_4\}$ and $\overline{SC}(M) = \{v_1, v_3, v_4\}$.
Hence, $SCRS(M) = (\{v_4\}, \{v_1, v_3, v_4\})$.
(ii) Let $M = \{v_2, v_3, v_4\} \subseteq \Omega$; then $\underline{SC}(M) = \{v_2, v_4\}$ and $\overline{SC}(M) = \Omega$.
Hence, $SCRS(M) = (\{v_2, v_4\}, \Omega)$.

The equivalence classes formed by soft adhesion using R' are $[v_1] = \{v_1, v_3\}$, $[v_2] = \{v_2\}$, $[v_4] = \{v_4\}$.

Remark 3.1. *The soft adhesion of v can be obtained directly from the tabular representation of the soft set (see Table 1) , where each row represents an element of the parameter set and each column represents an element of the universal set.*

Elements v_i and v_j belong to the same class if the entries in column v_i and the entries in column v_j are the same, where v_i and $v_j \in \Omega$. If the entries in column v_i are not equal to any other column v_j, then v_i forms a separate class. For example, In Table 1, the entries in v_1 and v_3 are the same. Therefore, they belong to the same class. The entries in v_2 are not equal to any other column entries. Hence, it forms a separate class.

4. Lattice Structure on the family of Z-Soft covering based rough sets

In this section, we show that the family of Z-soft covering based rough set create a lattice. Let $T_S = \{SCRS(M) : M \subseteq \Omega\}$. We define a relation R_S on T_S by $R_S = \{(SCRS(M), SCRS(O)) : SCRS(M) \subseteq SCRS(O)\}$.

Lemma 4.1. *R_S is a poset on T_S.*

Proof. By direct verification.
Now, we define two operations $\vee$ and $\wedge$ on T_S as follows. □

Definition 4.1. *For each two subsets M and O of Ω. $SAW(M) = \{SA(v) : SA(v) \subseteq M\}$. Define the set $M \vee O$ as follows:*
(1) $M \vee O = M \cup O$, if $|SAW(M \cup O)| = |SAW(M)| + |SAW(O)| - |SAW(M \cap O)|$.
(2) If $|SAW(M \cup O)| > |SAW(M)| + |SAW(O)| - |SAW(M \cap O)|$ then there exists $v \in \Omega$ such that $SA(v) \subseteq SAW(M \cup O)$, $SA(v) \nsubseteq M$ and $SA(v) \nsubseteq O$.
(3) Remove v from M (or O).
(4) Name the newly formed set as M (or O).
(5) Redo Step 1 if there is no v such that $SA(v) \nsubseteq M$ and $SA(v) \nsubseteq O$ is found, then $M \vee O = M \cup O$.

Definition 4.2. *For each subset M and O of Ω, then any element $v \in \Omega$ is called pivot element and $\widehat{P}_{M \cap O} = \{v \in \Omega : SA(v) \cap M \neq \emptyset, SA(v) \cap O \neq \emptyset, SA(v) \nsubseteq M \cap O\}$ is the pivot set for Z-soft covering based rough set.*

Definition 4.3. *For each subset M and O of Ω. The meet of M and O is defined by*
$$M \wedge O = \{v \in \Omega : SA(v) \subseteq M \cap O\} \cup \widehat{P}_{M \cap O}.$$

Example 4.1. *Let $\Omega = \{v_1, v_2, v_3, v_4\}$ be a universal set and $B = \{b_1, b_2, b_3\}$ be the set of parameters. Then the soft set over Ω is given by Table 1 where*
$N(b_1) = \{v_1, v_2, v_3, v_4\}$, $N(b_2) = \{v_2, v_4\}$ and $N(b_3) = \{v_1, v_2, v_3\}$.
Then, $SA(v_1) = \{v_1, v_3\}$, $SA(v_2) = \{v_2\}$, $SA(v_3) = \{v_1, v_3\}$, $SA(v_4) = \{v_4\}$.
Let $M = \{v_2, v_3, v_4\}$ and $O = \{v_1, v_4\}$ are the subsets of Ω.
Then, $M \vee O = \{v_2, v_3, v_4\}$ and $M \wedge O = \{v_1, v_3, v_4\}$.

Our intention is to provide a lattice structure on T_S. To prove this, for each two subsets M and O of Ω, the least upper bound (lub) and greatest lower bound (glb) of $SCRS(M)$ and $SCRS(O)$ are to be found.
In the following, we prove that $SCRS(M \vee O)$ and $SCRS(M \wedge O)$ are the lub and glb of $SCRS(M)$ and $SCRS(O)$ respectively.

Theorem 4.1. *If M and O are any two subsets of Ω then $SCRS(M \vee O)$ is the lub of $SCRS(M)$ and $SCRS(O)$.*

Proof. First, we prove that $SCRS(M) \subseteq SCRS(M \vee O)$. i.e., $\underline{SC}(M) \subseteq \underline{SC}(M \vee O)$ and $\overline{SC}(M) \subseteq \overline{SC}(M \vee O)$.
Let $v \in \underline{SC}(M)$
$\Rightarrow SA(v) \subseteq (M)$

$\Rightarrow SA(v) \subseteq (M \vee O)$
$\Rightarrow v \in \underline{SC}(M \vee O)$.
Hence, $\underline{SC}(M) \subseteq \underline{SC}(M \vee O)$.
Now, to prove that $\overline{SC}(M) \subseteq \overline{SC}(M \vee O)$.
Let $v \in \overline{SC}(M)$
$\Rightarrow SA(v) \cap M \neq \emptyset$. i.e., $SA(v) \cap (M \vee O) \neq \emptyset$
$\Rightarrow v \in \overline{SC}(M \vee O)$
$\therefore \overline{SC}(M) \subseteq \overline{SC}(M \vee O)$.
Similarly, $\underline{SC}(O) \subseteq \underline{SC}(M \vee O)$ and $\overline{SC}(O) \subseteq \overline{SC}(M \vee O)$.
Hence, $SCRS(M) \subseteq SCRS(M \vee O)$ and $SCRS(O) \subseteq SCRS(M \vee O)$.
$\therefore SCRS(M \vee O)$ is an upper bound of $SCRS(M)$ and $SCRS(O)$.
Let $SCRS(Z)$ be any upper bound of $SCRS(M)$ and $SCRS(O)$.
Then, $SCRS(M) \subseteq SCRS(Z)$ and $SCRS(O) \subseteq SCRS(Z)$.
Now, we prove that, $SCRS(M \vee O) \subseteq SCRS(Z)$.
i.e.,$(\underline{SC}(M \vee O), \overline{SC}(M \vee O)) \subseteq (\underline{SC}(Z), \overline{SC}(Z))$.
Let $v \in \underline{SC}(M \vee O)$
$\Rightarrow SA(v) \subseteq (M \vee O)$ then $SA(v) \subseteq M$ or $SA(v) \subseteq O$.
$\Rightarrow v \in \underline{SC}(Z)$
$\therefore \underline{SC}(M \vee O) \subseteq \underline{SC}(Z)$.
Let $v \in \overline{SC}(M \vee O)$
$\Rightarrow SA(v) \cap (M \vee O) \neq \emptyset$
$\Rightarrow SA(v) \cap M \neq \emptyset$ or SA(v) $\cap$ O $\neq \emptyset$
$\Rightarrow SA(v) \cap Z \neq \emptyset$
$\Rightarrow v \in \overline{SC}(Z)$
$\Rightarrow \overline{SC}(M \vee O) \subseteq \overline{SC}(Z)$.
$\therefore SCRS(M \vee O) \subseteq SCRS(Z)$.
Hence, SCRS(M $\vee$ O) is the lub of SCRS(M) and SCRS(O). □

Theorem 4.2. *If M and O are any two subsets of Ω then $SCRS(M \wedge O)$ is the glb of $SCRS(M)$ and $SCRS(O)$.*

Proof. First, we prove that $SCRS(M \wedge O)$ is the lower bound of $SCRS(M)$ and $SCRS(O)$.
It is enough to show that $SCRS(M \wedge O) \subseteq SCRS(M)$ and $SCRS(M \wedge O) \subseteq SCRS(O)$.
i.e., $\underline{SC}(M \wedge O) \subseteq \underline{SC}(M)$ and $\overline{SC}(M \wedge O) \subseteq \overline{SC}(M)$,
$\underline{SC}(M \wedge O) \subseteq \underline{SC}(O)$ and $\overline{SC}(M \wedge O) \subseteq \overline{SC}(O)$.
Let $v \in \underline{SC}(M \wedge O)$
$\Rightarrow SA(v) \subseteq (M \wedge O)$
$\Rightarrow SA(v) \subseteq (M \cap O) \subseteq M$
$\Rightarrow v \in \underline{SC}(M)$
$\therefore \underline{SC}(M \wedge O) \subseteq \underline{SC}(M)$.
Similarly, $\underline{SC}(M \wedge O) \subseteq \underline{SC}(O)$.
Now, let $v \in \overline{SC}(M \wedge O)$
$\Rightarrow SA(v) \cap (M \wedge O) \neq \emptyset$
$\Rightarrow SA(v) \cap M \neq \emptyset$ or $SA(v) \cap O \neq \emptyset$
$\Rightarrow v \in \overline{SC}(M)$
$\therefore \overline{SC}(M \wedge O) \subseteq \overline{SC}(M)$.
Similarly, $\overline{SC}(M \wedge O) \subseteq \overline{SC}(O)$.
Hence, $SCRS(M \wedge O) \subseteq SCRS(M)$ and $SCRS(M \wedge O) \subseteq SCRS(O)$.
$\therefore SCRS(M \wedge O)$ is the lower bound of $SCRS(M)$ and $SCRS(O)$.
Let $SCRS(Z)$ be any lower bound of $SCRS(M)$ and $SCRS(O)$.

We have to prove that $SCRS(Z) \subseteq SCRS(M \wedge O)$.
i.e., $\underline{SC}(Z) \subseteq \underline{SC}(M \wedge O)$ and $\overline{SC}(Z) \subseteq \overline{SC}(M \wedge O)$.
Let $v \in \underline{SC}(Z)$
$\Rightarrow SA(v) \subseteq Z$
$\Rightarrow SA(v) \subseteq (M \wedge O)$
$\Rightarrow v \in \underline{SC}(M \wedge O)$
$\therefore \underline{SC}(Z) \subseteq \underline{SC}(M \wedge O)$.
If $v \in \overline{SC}(Z)$
then $SA(v) \cap Z \neq \emptyset$
$\Rightarrow SA(v) \cap M \neq \emptyset$ and $SA(v) \cap$ O $\neq \emptyset$
$\Rightarrow SA(v) \cap (M \wedge O) \neq \emptyset$
$\Rightarrow v \in \overline{SC}(M \wedge O)$
$\therefore \overline{SC}(Z) \subseteq \overline{SC}(M \wedge O)$.
Hence, $SCRS(M \wedge O)$ is the glb of $SCRS(M)$ and $SCRS(O)$. □

Remark 4.1. *If $K = (N, B)$ is a soft set over Ω, then $(T_S, \subseteq)$ is a lattice. $(T_S, \subseteq)$ is known as Z-soft covering based rough lattice.*

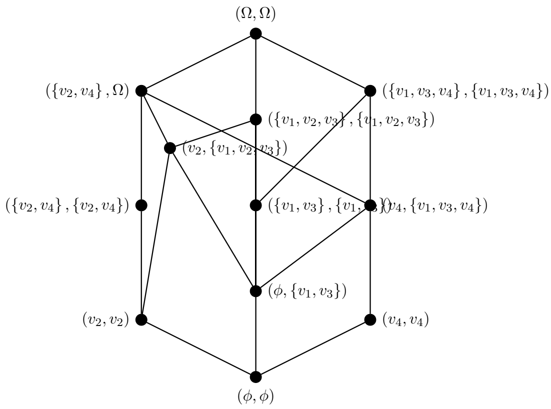
Theorem 4.3. *$(T_S, \subseteq)$ has both minimal and maximal element.*

Proof. It can be easily verified that $SCRS(\emptyset) = (\emptyset,\emptyset)$ is the minimal element and $SCRS(\Omega) = (\Omega, \Omega)$ is the maximal element. □

Example 4.2. *Let $\Omega = \{v_1, v_2, v_3, v_4\}$ be a universal set and $B = \{b_1, b_2, b_3\}$ be the set of parameters. Then the soft set over Ω is given by Table 1 where*
$N(b_1) = \{v_1, v_2, v_3, v_4\}$, $N(b_2) = \{v_2, v_4\}$ *and* $N(b_3) = \{v_1, v_2, v_3\}$.
Then, $SA(v_1) = \{v_1, v_3\}$, $SA(v_2) = \{v_2\}$, $SA(v_3) = \{v_1, v_3\}$, $SA(v_4) = \{v_4\}$.
Let $M = \{v_2, v_3, v_4\}$ *and* $O = \{v_1, v_4\}$ *are the subsets of* Ω.
Then, $SCRS(M) = (\{v_2, v_4\}, \Omega)$ *and* $SCRS(O) = (\{v_4\}, \{v_1, v_3, v_4\})$.
If $M \vee O = \{v_2, v_3, v_4\}$ *and* $M \wedge O = \{v_1, v_3, v_4\}$. *Then,*
$SCRS(M \vee O) = (\{v_2, v_4\}, \Omega)$ *and* $SCRS(M \wedge O) = (\{v_1, v_3, v_4\}, \{v_1, v_3, v_4\})$.
$T_S = \{$ $SCRS(\emptyset)$, $SCRS(v_1)$, $SCRS(v_2)$, $SCRS(v_4)$, $SCRS(\{v_1, v_2\})$, $SCRS(\{v_1, v_3\})$, $SCRS(\{v_1, v_4\})$, $SCRS(\{v_2, v_4\})$, $SCRS(\{v_1, v_2, v_3\})$, $SCRS(\{v_1, v_2, v_4\})$, $SCRS(\{v_1, v_3, v_4\})$, $SCRS(\Omega)$ $\}$.

The Hasse diagram of Z-soft covering based rough lattice on T_S is shown in FIGURE 1.

FIGURE 1. Lattice structure for Z-soft covering based rough set



5. A novel approach to MAGDM using Z-SCRS

In this section, a novel decision-making method is created to select the best object from a list of possible objects Ω.

5.1. **Description and process.** Let $\Omega = \{v_1, v_2, ..., v_j\}$ be j alternatives and let B be the parameter set. Assume that, we have an expert group $D = \{D_1, D_2, ..., D_m\}$ consisting of m specialists to evaluate the alternatives in Ω. Each specialist must examine all objects in Ω and is only allowed to recommend "the best alternatives" as a result of their evaluation. As a result, the primary evaluation result of each specialist is a subset of Ω. We assume that the evaluations of these specialists in D are equally important. The primary assessment result of expert group D is referred to as the assessment soft set $K_1 = (T, D)$ over Ω, where $T : D \to P(\Omega)$ is given by $T(D_m) = M_m$ $(m = 1, 2, ..., p)$.
From the soft set $K_1 = (T, D)$, we get only the initial evaluation dataset. But, soft rough approximations help us to gain more useful information. We consider the soft rough approximations of the specialist D_m's primary evaluation result M_m with respect to the soft approximation space $P = (\Omega, K)$. The soft covering lower approximation $\underline{T}(D_m)$ can be regarded as the group of objects that are definitely the best candidates according to specialist D_m's opinion. For instance, if $v_2 \in \underline{T}(D_2)$ we can say that the specialist D_2 thinks with high confidence that v_2 is an optimal alternative. Similarly, the soft covering upper approximation $\overline{T}(D_m)$ can be regarded as the group of objects that are possibly the best candidates according to specialist D_m's opinion.
Using soft rough approximations, we finally obtain two other soft sets $\underline{K}_1 = (\underline{T}, D)$ and $\overline{K}_1 = (\overline{T}, D)$ over Ω where,

$\underline{T} : D \to P(\Omega)$,
$\underline{T}(D_m) = \underline{SC}(T(D_m))$, m = 1,2,...,p.
$\overline{T} : D \to P(\Omega)$,
$\overline{T}(D_m) = \overline{SC}(T(D_m)$, m = 1,2,...,p.

As mentioned above, the soft set $\underline{K}_1$ represents the evaluation result of the whole expert group D with high confidence, while $\overline{K}_1$ represents the evaluation result of the whole expert group D with low confidence. Furthermore, the primary assessment, namely the soft set K_1 is considered as the whole group evaluation result with middle confidence.
It is important to note that fuzzy sets can also be used to express the evaluation result of the entire expert group D. Let $M \subseteq \Omega$, the characteristic function of M is denoted by χ_M. Based on the soft set $K_1 = (T, D)$, we define the fuzzy set μ_{K_1} in Ω by

$\mu_{K_1} : \Omega \to [0, 1]$,

$$\mu_{K_1}(v_i) = \frac{1}{p} \sum_{m=1}^{p} \chi_{T(D_m)}.$$

Similarly, the fuzzy sets $\mu_{\underline{K}_1}$ and $\mu_{\overline{K}_1}$ can be formulated as follows,

$\mu_{\underline{K}_1} : \Omega \to [0, 1]$,

$$\mu_{\underline{K}_1}(v_i) = \frac{1}{p} \sum_{m=1}^{p} \chi_{\underline{T}(D_m)}.$$

$\mu_{\overline{K}_1} : \Omega \to [0, 1]$,

$$\mu_{\overline{K}_1}(v_i) = \frac{1}{p} \sum_{m=1}^{p} \chi_{\overline{T}(D_m)}.$$

where $\underline{T}(D_m) = \underline{SC}(T(D_m))$ and $\overline{T}(D_m) = \overline{SC}(T(D_m))$ where $i = 1, ..., j$.
From $\underline{K}_1 \subseteq K_1 \subseteq \overline{K}_1$, we can say that $\mu_{\underline{K}_1} \subseteq \mu_{K_1} \subseteq \mu_{\overline{K}_1}$. The risky factors of the patients can be classified as fuzzy sets $\mu_{\underline{K}_1}$, μ_{K_1} and $\mu_{\overline{K}_1}$ respectively with the ambiguous concept

like the patients under "high level of risk", the patients under "average level of risk" and the patients under "low level of risk" respectively.
Now, we use the concept of fuzzy soft sets to combine the above soft or fuzzy evaluation results. Let $Q = \{L, A, H\}$ be the parameters. Let L be the low level of risk, A be the average level of risk and H be the high level of risk. We define a fuzzy soft set $K_F = (X, Q)$ over Ω where $X : Q \to I^{\Omega}$ is given by $X(L) = \mu_{\overline{K}_1}$, $X(A) = \mu_{K_1}$, and $X(H) = \mu_{\underline{K}_1}$. Since I^{Ω} denotes the set of all fuzzy sets on Ω.
Let the weighting vector $R = (r_L, r_A, r_H)$, so that $r_L + r_A + r_H = 1$.
$w(v_i) = r_L \cdot X(L)(v_i) + r_A \cdot X(A)(v_i) + r_H \cdot X(H)(v_i)$.
is the weighted assessment values of the alternatives $v_i \in \Omega$, i = 1,...,j.
Finally, we select the object v_i such that $w(v_i) = max\,\{w(v_i) : i = 1, ..., j\}$ as the best alternative.
The decision-making method is summarized as follows:
Step 1: Consider the original soft set $K = (N, B)$.
Step 2: Formulate the soft set $K_1 = (T, D)$ by using the first assessment results of the specialist group D.
Step 3: Calculate SCLA and SCUA and get the soft set $\underline{K}_1 = (\underline{T}, D)$ and $\overline{K}_1 = (\overline{T}, D)$.
Step 4: Calculate the fuzzy sets μ_{k_1}, $\mu_{\underline{k}_1}$ and $\mu_{\overline{k}_1}$ of the sets $K_1 = (T, D)$, $\underline{K}_1 = (\underline{T}, D)$ and $\overline{K}_1 = (\overline{T}, D)$.
Step 5: Determine the fuzzy soft set $K_F = (X, Q)$ using the fuzzy sets $\mu_{\underline{k}_1}$, μ_{k_1} and $\mu_{\overline{k}_1}$.
Step 6: Considering the weighting vector R, calculate the weighted assessment values $w(v_i)$ of any alternatives $v_i \in \Omega$. Using $w(v_i)$, we rank the alternatives to select the elements with the highest weighted evaluation values.

5.2. **Illustrative example.** In this work, we use soft adhesion to find SCLA and SCUA. Our intention is to assist the doctors in determining the patients with a high risk of chronic kidney disease for kidney transplant using the parameters-blood urea level(b_1), diabetes(b_2), coronary artery disease(b_3), blood pressure level(b_4) and bacteria in urine(b_5).

We select 40 patients from the UCI Machine Learning Repository with chronic kidney disease as the data mentioned in the Table 2.
Step 1: Let $\Omega = \{v_i : v_1 = 1, v_2 = 2, v_3 = 3, \cdots, v_{39} = 39, v_{40} = 40\}$ be the universal set and let B = {blood urea level(b_1), diabetes(b_2), coronary artery disease(b_3),blood pressure level(b_4), bacteria(b_5) } be the set of parameters. The patients whose blood urea level is 75 and greater than 75, patients with diabetes, patients with coronary artery disease, patients with blood pressure level is 80 and greater than 80, patients with bacteria in urine are chosen. We create the soft set $K = (N, B)$ which is mainly based on the parameters over Ω is given in Table 3. Let $S = (\Omega, C_K)$ be the SCA. Let
$N(b_1) = \{2, 4, 5, 6, 7, 9, 10, 11, 12, 13, 16, 17, 18, 19, 20, 21, 24, 32, 34, 35, 37, 39, 40\}$,
$N(b_2) = \{2, 3, 4, 5, 7, 8, 9, 10, 12, 16, 17, 18, 19, 24, 25, 26, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40\}$,
$N(b_3) = \{4, 5, 7, 9, 14, 16, 17, 34, 40\}$,
$N(b_4) = \{2, 4, 5, 6, 9, 11, 12, 14, 17, 19, 20, 21, 22, 24, 26, 28, 32, 33, 34, 36, 37, 38\}$,
$N(b_5) = \{4, 12, 14, 16, 20, 28, 30, 31, 36, 37, 38, 40\}$.
Step 2: Let $D = \{D_1, D_2, D_3, D_4\}$ be the expert doctors where they assess the patients with the help of parameters. We create a soft set $K_1 = (T, D)$ over Ω by using the first assessment values of expert doctors D. Each expert evaluate all the elements in Ω and will be pointing out "the best alternatives" as their assessment result. Therefore, each experts primary assessment values are subsets of Ω. We consider the assessments of these experts in $D = \{D_1, D_2, D_3, D_4\}$ are with the equal importance.
$T(D_1) = \{1, 2, 4, 5, 7, 9, 10, 12, 14, 16, 17, 18, 19, 20, 22, 24, 27, 28, 30, 32, 34, 35, 37, 39, 40\}$,

TABLE 2. Tabular representation of parameter values of some patients

Ω	b_1	b_2	b_3	b_4	b_5
v_1	56	No	No	70	Not Present
v_5	148	Yes	Yes	80	Not Present
v_{11}	107	No	No	90	Not Present
v_{16}	82	Yes	Yes	70	Present
v_{19}	166	Yes	No	90	Not Present
v_{24}	235	Yes	No	90	Not Present
v_{28}	40	No	No	90	Present
v_{31}	67	No	No	60	Present
v_{35}	150	Yes	No	70	Not Present
v_{40}	96	Yes	Yes	70	Present

$T(D_2) = \{1, 2, 3, 6, 8, 10, 11, 13, 15, 16, 18, 19, 20, 21, 23, 24, 25, 26, 27, 29, 32, 33, 35, 37, 39\}$,
$T(D_3) = \{2, 3, 4, 5, 8, 9, 12, 14, 16, 17, 20, 22, 25, 26, 28, 30, 32, 33, 34, 36, 37, 38, 40\}$,
$T(D_4) = \{1, 3, 5, 6, 8, 9, 10, 11, 15, 17, 18, 19, 21, 23, 25, 27, 29, 31, 34, 35, 36, 38, 39\}$.

TABLE 3. Tabular representation of the soft set $K = (N, B)$

Ω	b_1	b_2	b_3	b_4	b_5
v_1	0	0	0	0	0
v_5	1	1	1	1	0
v_{11}	1	0	0	1	0
v_{16}	1	1	1	0	1
v_{19}	1	1	0	1	0
v_{24}	1	1	0	1	0
v_{28}	0	0	0	1	1
v_{31}	0	0	0	0	1
v_{35}	1	1	0	0	0
v_{40}	1	1	1	0	1

Step 3: Now we use SCLA and SCUA in this decision making problem. Let $S = (\Omega, C_K)$ be a SCA. By using this, we get two soft sets $\underline{K}_1 = (\underline{T}, D)$ and $\overline{K}_1 = (\overline{T}, D)$ over Ω where,

$\underline{T} : D \to P(\Omega)$,
$\underline{T}(D_m) = \underline{SC}(T(D_m))$, m = 1,2,3,4.
$\overline{T} : D \to P(\Omega)$,
$\overline{T}(D_m) = \overline{SC}(T(D_m))$, m = 1,2,3,4.

The soft sets $\overline{K}_1$ and $\underline{K}_1$ are the assessment values of the experts group D with less confidence and more confidence respectively. We get the SCLA and SCUA of first assessment values of experts group D to obtain the soft sets $\underline{K}_1$ and $\overline{K}_1$. Consider,
$\underline{T}(D_1) = \{2, 4, 5, 7, 9, 10, 12, 14, 16, 17, 18, 19, 20, 22, 24, 28, 30, 32, 34, 35, 37, 39, 40\}$,
$\underline{T}(D_2) = \{1, 2, 3, 6, 8, 10, 11, 13, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 29, 32, 33, 35, 39\}$,
$\underline{T}(D_3) = \{3, 4, 5, 8, 9, 12, 14, 16, 17, 20, 22, 25, 26, 28, 30, 33, 34, 36, 37, 38, 40\}$,
$\underline{T}(D_4) = \{1, 3, 5, 6, 8, 9, 10, 11, 15, 17, 18, 21, 23, 25, 27, 29, 31, 34, 35, 36, 38, 39\}$.
$\overline{T}(D_1) = \{1, 2, 4, 5, 7, 9, 10, 12, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 27, 28, 29, 30, 32, 34, 35, 37, 39, 40\}$,
$\overline{T}(D_2) = \{1, 2, 3, 6, 8, 10, 11, 12, 13, 15, 16, 18, 19, 20, 21, 23, 24, 25, 26, 27, 29, 32, 33, 35, 37, 39, 40\}$,

$\overline{T}(D_3) = \{2, 3, 4, 5, 8, 9, 12, 14, 16, 17, 19, 20, 22, 24, 25, 26, 28, 30, 32, 33, 34, 36, 37, 38, 40\}$,
$\overline{T}(D_4) = \{1, 2, 3, 5, 6, 8, 9, 10, 11, 15, 17, 18, 19, 21, 23, 24, 25, 27, 29, 31, 32, 34, 35, 36, 38, 39\}$.
Step 4: The outcomes of expert doctors assessment can be formulated into fuzzy sets. Let $M \subseteq \Omega$, the characteristic function of M is denoted by χ_M. Based on the soft set $K_1 = (T, D)$, we define the fuzzy set μ_{K_1} in Ω by

$\mu_{K_1} : \Omega \to [0, 1]$,

$\mu_{K_1}(v_i) = \frac{1}{4} \sum\limits_{m=1}^{4} \chi_{T(D_m)}$.

Similarly, the fuzzy sets $\mu_{\underline{K}_1}$ and $\mu_{\overline{K}_1}$ can be formulated as follows,

$\mu_{\underline{K}_1} : \Omega \to [0, 1]$,

$\mu_{\underline{K}_1}(v_i) = \frac{1}{4} \sum\limits_{m=1}^{4} \chi_{\underline{T}(D_m)}$.

$\mu_{\overline{K}_1} : \Omega \to [0, 1]$,

$\mu_{\overline{K}_1}(v_i) = \frac{1}{4} \sum\limits_{m=1}^{4} \chi_{\overline{T}(D_m)}$.

where $\underline{T}(D_i) = \underline{SC}(T(D_m))$ and $\overline{T}(D_m) = \overline{SC}(T(D_m))$ where $i = 1, ..., 40$.
From $\underline{K}_1 \subseteq K_1 \subseteq \overline{K}_1$, we can say that $\mu_{\underline{K}_1} \subseteq \mu_{K_1} \subseteq \mu_{\overline{K}_1}$. The risky factors of the patients can be classified as fuzzy sets $\mu_{\underline{K}_1}$, μ_{K_1} and $\mu_{\overline{K}_1}$ respectively with the ambiguous concept like the patients under "high level of risk", the patients under "average level of risk" and the patients under "low level of risk" respectively. In this way, we obtain the fuzzy sets μ_{k_1}, $\mu_{\underline{k}_1}$ and $\mu_{\overline{k}_1}$ by the memberships obtained above. The membership values of some patients are given in Table 4. For example, we get the membership values of fuzzy sets for the first patient:
$\mu_{\overline{K}_1}(v_1) = 3/4$, $\mu_{K_1}(v_1) = 3/4$, and $\mu_{\underline{K}_1}(v_1) = 2/4$.

TABLE 4. Tabular representation of the membership of some patients

Ω	$\mu_{\overline{K}_1}$	μ_{K_1}	$\mu_{\underline{K}_1}$
v_1	3/4	3/4	2/4
v_5	3/4	3/4	3/4
v_{11}	2/4	2/4	2/4
v_{16}	3/4	3/4	2/4
v_{19}	1	3/4	2/4
v_{24}	1	2/4	2/4
v_{28}	2/4	2/4	2/4
v_{31}	1/4	1/4	1/4
v_{35}	3/4	3/4	3/4
v_{40}	3/4	2/4	2/4

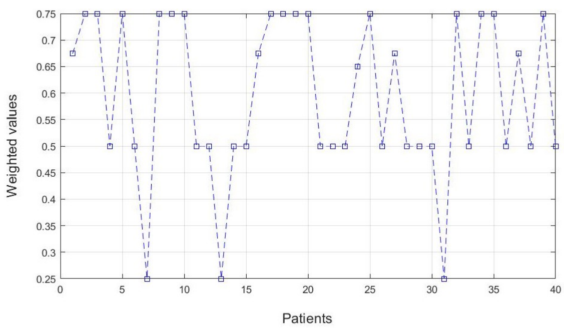
Step 5: Let $Q = \{L, A, H\}$ be the parameters. Let L be the low level of risk, A be the average level of risk and H be the high level of risk. We define a fuzzy soft set $K_F = (X, Q)$ over Ω where $X : Q \to I^{\Omega}$ is given by $X(L) = \mu_{\overline{K}_1}$, $X(A) = \mu_{K_1}$, and $X(H) = \mu_{\underline{K}_1}$. Since I^{Ω} denotes the set of all fuzzy sets on Ω.
Step 6: Let the weighting vector $R = (0.3, 0.4, 0.3)$.
The weighted assessment values of the alternatives $v_i \in \Omega$ is given by
$w(v_i) = 0.3 \cdot X(L)(v_i) + 0.4 \cdot X(A)(v_i) + 0.3 \cdot X(H)(v_i)$.
Tabular representation of fuzzy soft set $K_F = (X, Q)$ with the weighted assessment values

of some patients are given in Table 5. The ranking of the alternatives according to their weighted values is as follows:
$2 \approx 3 \approx 5 \approx 8 \approx 9 \approx 10 \approx 17 \approx 18 \approx 19 \approx 20 \approx 25 \approx 32 \approx 34 \approx 35 \approx 39 = 0.75 > 1 \approx 16 \approx 24 \approx 27 \approx 37 = 0.675 > 4 \approx 6 \approx 11 \approx 12 \approx 14 \approx 15 \approx 21 \approx 22 \approx 23 \approx 26 \approx 28 \approx 29 \approx 30 \approx 33 \approx 36 \approx 38 \approx 40 = 0.5 > 7 \approx 13 \approx 31 = 0.25$.

Table 5. Tabular representation of fuzzy soft set $K_F = (X, Q)$ with the weighted assessment values of some patients

Ω	L	A	H	$w(v_j)$
v_1	3/4	3/4	2/4	0.675
v_5	3/4	3/4	3/4	0.75
v_{11}	2/4	2/4	2/4	0.5
v_{16}	3/4	3/4	2/4	0.675
v_{19}	1	3/4	2/4	0.75
v_{24}	1	2/4	2/4	0.65
v_{28}	2/4	2/4	2/4	0.5
v_{31}	1/4	1/4	1/4	0.25
v_{35}	3/4	3/4	3/4	0.75
v_{40}	3/4	2/4	2/4	0.5

Figure 2. Graphical representation of the weighted assessment values of patients



In step 6, we found the weighted assessment values $\{0.75, 0.675, 0.5, 0.25\}$ for every patient. According to these values, we set the rules as follows:
Rule 1: If a patient's weighted assessment value is 0.75, the patient is at high risk for chronic kidney disease.
Rule 2: If a patient's weighted assessment value is 0.675, the patient is at an average risk for chronic kidney disease.
Rule 3: If a patient's weighted assessment value is 0.5, the patient is at low risk for chronic kidney disease.
Rule 4: If a patient's weighted assessment value is 0.25, the patient is at a very low level of risk for chronic kidney disease.
From the above, the rule sets are given by,
$R_1 = \{2, 3, 5, 8, 9, 10, 17, 18, 19, 20, 25, 32, 34, 35, 39\}$
$R_2 = \{1, 16, 24, 27, 37\}$

$R_3 = \{4, 6, 11, 12, 14, 15, 21, 22, 23, 26, 28, 29, 30, 33, 36, 38, 40\}$
$R_4 = \{7, 13, 31\}$
We conclude from this method that kidney transplantation should be used on patients who are at high risk of chronic kidney disease. The kidney transplant is not necessary for patients at an average level of risk of chronic kidney disease, but the patients must follow the doctor's advice. Patients at low and very low risk of chronic kidney disease do not require a kidney transplant or doctor's care. Therefore, kidney transplantation is needed for the patients in set R_1. Using MATLAB, the weighted assessment values of patients are plotted in the graph shown in the Fig. 2 for better understanding.

6. Conclusion

Rough set theory and soft set theory are two different mathematical tools for discussing uncertainty. A combination of these theories is a recently developing concept. In this paper, a relation R_S on the family of Z-soft covering based rough set (T_S) is defined and proved that R_S is a partially ordered set in T_S. Join ($\vee$) and meet ($\wedge$) are the two operations defined on T_S to prove that every pair of elements of R_S has a least upper bound and a greatest lower bound showing that T_S is a lattice. Furthermore, Z-soft covering based rough set is applied to a concrete example of selecting the right patient for a kidney transplant to demonstrate its practical application. For this process, 40 patients are selected from the UCI Machine Learning Repository dataset and the proposed decision-making algorithm is applied. As a result, we obtained that 15 out of 40 patients are at high risk of chronic kidney disease. We plan to extend our study in the following areas: (1) Semiring on Z-soft covering based rough set (2) Characterization of Z-soft covering based rough semiring.

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Conflict of interest The authors declare that they have no conflict of interest.

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