TWMS J. App. and Eng. Math. V.14, N.3, 2024, pp. 1179-1188

RELATIVELY PRIME INVERSE DOMINATION ON VERTEX SWITCHING OF SOME GRAPHS

C. JAYASEKARAN[1*], L. ROSHINI[1], §

ABSTRACT. Let $G = (V, E)$ be a non-trivial graph. A subset D of the vertex set V of a graph G is called a dominating set of G if every vertex in $V - D$ is adjacent to a vertex in D. The domination number is the lowest cardinality of a dominating set, and it is denoted by $\gamma(G)$. If $V - D$ contains a dominating set S of G, then S is called an inverse dominating set with respect to D. In an inverse dominating set S, every pair of vertices u and v in S such that $(deg(u), deg(v)) = 1$, then S is called relatively prime inverse dominating set. The lowest cardinality of a relatively prime inverse dominating set is called the relatively prime inverse domination number and is denoted by $\gamma_{rp}^{-1}(G)$. In this paper, we find relatively prime inverse domination number on vertex switching of some graphs.

Keywords: Domination, Inverse domination, Relatively Prime Inverse domination, Vertex switching.

AMS Subject Classification: 05C69.

1. INTRODUCTION

When we refer to a graph,we mean a limited number, undirected one having no loops or numerous edges. We cite the book by Chartrand and Lesniak for terms related to graph theory [2]. In this work, it is assumed that every graph is non-trivial. The degree of a vertex v in a graph $G = (V, E)$ is denoted by the symbol $deg(v)$ and is defined as the number of edges that intersect v. The lowest of $\{deg(v) : v \in V(G)\}$ is denoted by $\delta(G)$ and the total of $\{deg(v) : v \in V(G)\}$ is denoted by $\Delta(G)$. The subgraph produced by a set S of vertices of a graph G is denoted by $\langle S\rangle$, with V$(\langle S\rangle) = S$ and $E(\langle S\rangle) = \{uv \in E(G) : u, v \in S\}$. One of the most rapidly expanding fields of graph theory is the study of domination and associated subset problems. A full survey of dominance can be seen in [4]. A set $D \subseteq V(G)$ in a graph G is a dominating set of G if for every vertex v in $V - D$, there exists a vertex u in D such that v is adjacent to u. A dominating set is also called γ-set. The least cardinality of a dominating set of a graph G is called the domination number of G and is represented by $\gamma(G)$. In [8], the concept of inverse domination in graph was introduced by Kulli V. R. et al. Let D be the γ-set of G. If

[1] Department of Mathematics, Pioneer Kumaraswamy College, Nagercoil, 629003, Tamil Nadu, India.
e-mail: jayacpkc@gmail.com; ORCID: https://orcid.org/0000-0001-5731-0980.
[*] Corresponding author.
e-mail: jerryroshini92@gmail.com; ORCID: https://orcid.org/0009-0006-2031-1166.
§ Manuscript received: September 15, 2022; accepted: April 07, 2023.

$V - D$ contains a dominating set S, then S is known as the inverse dominating set of G with respect to D. The inverse domination number $\gamma^{-1}(S)$ is the smallest cardinality of all the least inverse dominating sets of G. A crown graph on $2n$ vertices is an undirected graph with two set of vertices $\{u_1, u_2, ..., u_n\}$ and $\{v_1, v_2, ..., v_n\}$ and with an edge from u_i to v_i, $i \leq i \neq j \leq n$ [1]. The book graph is defind as the cartisian product of star graph S_n and path P_2 [3]. The graph obtained by joining disjoint cycles $u_1u_2...u_nu_1$ and $v_1v_2...v_nv_1$ with an edge u_1v_1 is called the dumbbell graph Db_n[9]. A set $S \subseteq V$ is said to be relatively prime dominating set if it is a dominating set with at least two elements and for every pair of vertices u and v in S such that $(deg(u), deg(v)) = 1$. The lowest cardinality of a relatively prime dominating set of a graph G is known as the relatively prime domination number of G and is represented by $\gamma_{rp}(G)$ [5]. The concept of relatively prime inverse domination of a graph was introduced in [7] and the lowest cardinality of a relatively prime inverse dominating set of a graph G is known as the relatively prime inverse domination number of G and is represented by $\gamma_{rp}^{-1}(G)$. The goal of this paper is to find the parameter $\gamma_{rp}^{-1}(G)$ in graph vertex switching.

Definition 1.1. [7] *Let D be a γ- set of a graph G. If $V - D$ contains a dominating set S of G, then S is called an inverse dominating set with respect to D. If every pair of vertices u and v in S such that (deg(u), deg(v)) = 1, then S is called relatively prime inverse dominating set. The lowest cardinality of a relatively prime inverse dominating set of a graph G is called relatively prime inverse domination number of graph G and is denoted by $\gamma_{rp}^{-1}(G)$. If no such set S exists in $V - D$, then we denote $\gamma_{rp}^{-1}(G) = 0$.*

Definition 1.2. [6] *For a finite undirected graph $G = (V, E)$ and $v \in V$, the graph G^v formed from G by removing all edges incident to v and adding edges that are not adjacent to v is the vertex switching of G by v.*

2. Known Result

Theorem 2.1. [4] If G is a graph with no isolated vertices, then the complement $V - S$ of every minimal dominating set S is also a dominating set.

Theorem 2.2. [6] *For a path P_n,* $\gamma_{rp}^{-1}(P_n) = \begin{cases} 2 & if\ 3 \leq n \leq 5 \\ 3 & if\ n = 6, 7 \\ 0 & otherwise \end{cases}$

3. Main Results

Theorem 3.1. *Let C_n be the cycle and v be any vertex of C_n. Then for any integer $n \geq 3$,* $\gamma_{rp}^{-1}\ (\bar{C_n}^{v}) = \begin{cases} 2\ if\ n\ is\ odd \\ 0\ if\ n\ is\ even \end{cases}$.

Proof. Let $v_1v_2 \cdots v_n$ be the cycle C_n and $\bar{C_n}$ be the complement graph of C_n. Clearly, $\bar{C_n}^{v_i} \cong \bar{C_n}^{v_j}, 1 \leq i, j \leq n$. Without losing generality, let v be v_i for some $i, 1 \leq i \leq n$. Then in $\bar{C_n}^{v}$, v_i is adjacent with v_{i+1} and v_{i-1}; v_{i+1} and v_{i-1} are adjacent with every vertices except v_{i+2} and v_{i-2}, respectively for $1 \leq i \leq n$ and v_j is adjacent with every vertices except v_{j-1}, v_{j+1} and v_i for $1 \leq j \leq n$ where $j \neq i-1, i, i+1$. Clearly, in $\bar{C_n}^{v}$, $deg(v_i) = 2, deg(v_{i+1}) = deg(v_{i-1}) = n - 2$ and $deg(v_j) = n - 4$. Now we consider the following two cases.

Case 1. n is odd

Let $n = 2k + 1, k \geq 1$. When $k = 1$, $\bar{C_3}^{v} \cong P_3$. Hence, by Theorem 2.2, $\gamma_{rp}^{-1}(\bar{C_3}^{v}) = \gamma_{rp}^{-1}(P_3) = 2$.

Let $k \geq 2$. In $\bar{C_n}^v$, $D = \{v_{i+1}, v_{i+2}\}$ is a γ-set and a corresponding γ^{-1}-set is $S = \{v_{i-1}, v_{i-2}\}$ and $(deg(v_{i-1}), deg(v_{i-2})) = (n-2, n-4) = 1$, since n is odd. Hence, S is a minimum relatively prime inverse dominating set of $\bar{C_n}^v$ and so $\gamma_{rp}^{-1}(\bar{C}_n^v) = 2$.

Case 2. n is even

Let $n = 2k, k \geq 2$. When $k = 2$, $\bar{C_4}^v = K_1 \cup K_3$ which has an isolated vertex given in Figure 1. By Theorem 2.2, $\gamma_{rp}^{-1}(\bar{C_4}^v) = 0$.

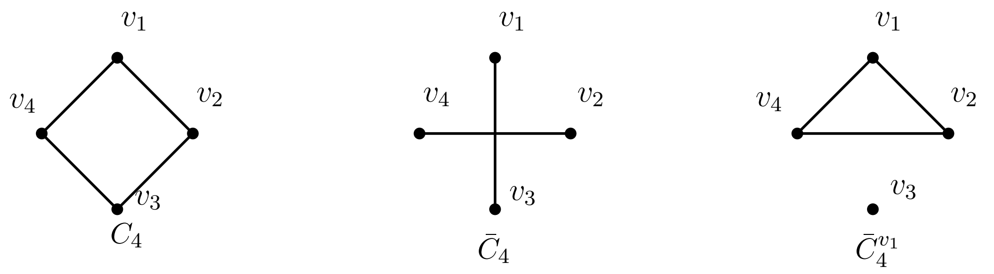


Figure 1

Now consider $\bar{C_{2k}}^v$ where $k \geq 3$. For any given γ-set D of $\bar{C_{2k}}^v$, any corresponding γ^{-1}-set S must contain at least two vertices and each vertex has even degree and so S cannot be a relatively prime inverse dominating set. Hence, $\gamma_{rp}^{-1}(\bar{C_n}^v) = 0$. $\square$

Example 3.1. The graphs C_5, $\bar{C}_5$, and $\bar{C}_5^{v_1}$ are given in the Figure 2. Clearly, $D = \{v_2, v_3\}$ is γ- set of $\bar{C}_5^{v_1}$. In $\bar{C}_5^{v_1}$, $S = \{v_4, v_5\}$ is a corresponding γ^{-1}-set with degree sequence $(1, 3)$. This implies that S is a minimum relatively prime inverse dominating set of $\bar{C}_5^{v_1}$ and so $\gamma_{rp}^{-1}(\bar{C}_5^{v_1}) = 2$.

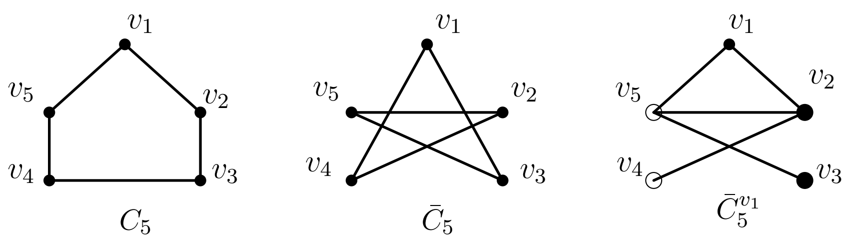


Figure 2

Theorem 3.2. *Let v be any vertex of the the dumbbell graph Db_n. Then*

(1) *for $n = 3$, $\gamma_{rp}^{-1}\ (Db_n^v) = 2$.*

(2) *for $n = 4$,* $\gamma_{rp}^{-1}\ (Db_n^v) = \begin{cases} 3 \ if \ deg_{Db_n}(v) = 2 \\ 4 \ if \ deg_{Db_n}(v) = 3 \end{cases}.$

(3) *for $n = 5$,* $\gamma_{rp}^{-1}\ (Db_n^v) = \begin{cases} 0 \ if \ deg_{Db_n}(v) = 2 \text{ and } v \text{ is adjacent to} \\ \qquad a \text{ vertex of degree } 3 \\ 4 \text{ otherwise} \end{cases}.$

(4) *for $n \geq 6$, $\gamma_{rp}^{-1}\ (Db_n^v) = 0$.*

Proof. The dumbbell graph Db_n is obtained by joining two disjoint cycles $u_1u_2....u_nu_1$ and $v_1v_2....v_nv_1$ with an edge u_1v_1. Then in Db_n, $deg(v) = 2$ if $v \in \{u_i, v_i : 2 \leq i \leq n\}$ and $deg(v) = 3$ if $v \in \{u_1, v_1\}$. Let Db_n^v be the vertex switching graph Db_n with respect

to the vertex v. Let D be a γ- set of Db_n^v and S be a minimum relatively prime inverse dominating set of Db_n^v with respect to D. Now we consider the following five cases.
Case 1. $n = 3$

In Db_3, if $deg(v) = 3$, then v is either u_1 or v_1. Clearly, $Db_3^{v_1} \cong Db_3^{u_1}$. Without losing generality, let v be u_1. The graphs Db_3 and $Db_3^{u_1}$ are given in Figur 3. A γ-set of ${Db_3}^{u_1}$ is $D = \{u_3, v_3\}$ and a corresponding γ^{-1}- set $S = \{u_2, v_2\}$. In $Db_n^{u_1}$, $(deg(u_2), deg(v_2)) = (1, 3) = 1$ and so S is a minimum relatively prime inverse dominating set of $Db_n^{u_1}$. Therefore, $\gamma_{rp}^{-1}\ (Db_3^{u_1}) = 2$.

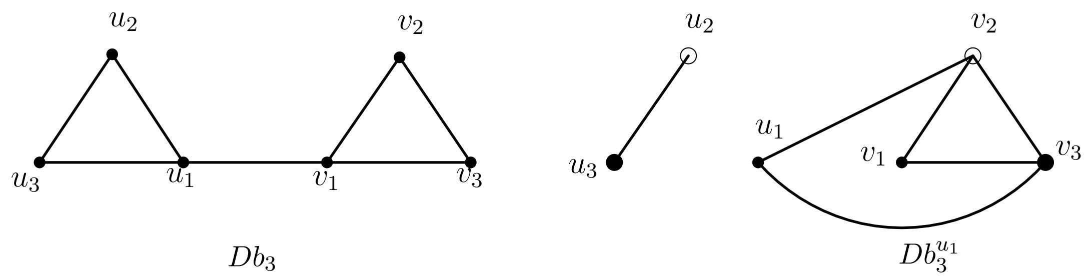


Figure 3

In Db_3, if $deg(v) = 2$, then v is either u_i or v_i where $i = 2, 3$. Clearly, $Db_3^{v_2} \cong Db_3^{v_3} \cong Db_3^{u_2} \cong Db_3^{u_3}$. Without losing generality, let v be u_2. The graph $Db_3^{u_2}$ is given in Figure 4. A γ-set of ${Db_3}^{u_2}$ is $D = \{u_1, v_1\}$ and a corresponding γ^{-1}-set is $S = \{u_3, v_3\}$. In $Db_n^{u_2}$, $(deg(u_3), deg(v_3)) = (1,\ 3) = 1$. Hence, S is a minimum relatively prime inverse dominating set of $Db_n^{u_2}$ and so $\gamma_{rp}^{-1}\ (Db_3^{u_2}) = 2$.

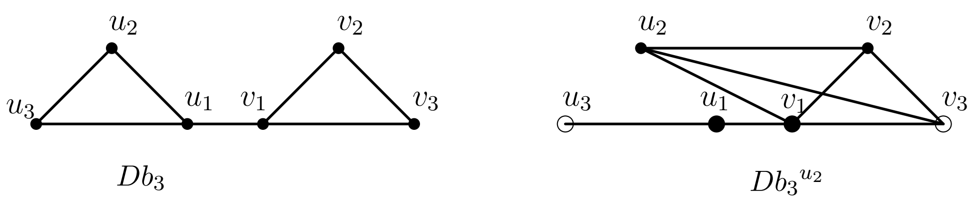


Figure 4

Case 2. $n = 4$

In Db_4, if $deg(v) = 2$, then v is either u_i or v_i, $2 \leq i \leq 4$. Clearly, $Db_4^{v_2} \cong Db_4^{v_4} \cong Db_4^{u_2} \cong Db_4^{u_4}$ and $Db_4^{v_3} \cong Db_4^{u_3}$. The graphs $Db_4^{u_2}$ and $Db_4^{u_3}$ are given in Figure 5.
Subcase 2.1. $v = u_2$

A γ-set of ${Db_4}^{u_2}$ is $D = \{u_2, u_4\}$ and a corresponding γ^{-1}-set is $S = \{u_3, v_1, v_3\}$. In $Db_4^{u_2}$, $(deg(u_3), deg(v_1)) = (1, 4) = 1$, $(deg(v_1),\ deg(v_3)) = (4,\ 3) = 1$, $(deg(v_3), deg(u_3)) = (1,\ 3) = 1$. This implies that S is a minimum relatively prime inverse dominating set of $Db_4^{u_2}$ and so $\gamma_{rp}^{-1}\ (Db_4^{u_2}) = 3$.

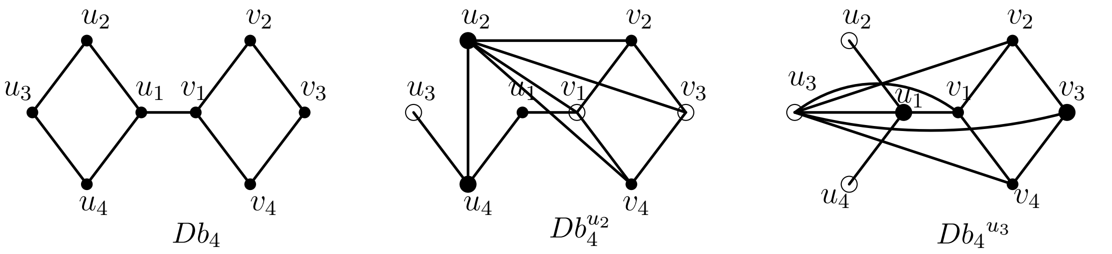


Figure 5

Subcase 2.2. $v = u_3$

A γ-set of $Db_4{}^{u_3}$ is $D = \{u_1, v_3\}$ and a corresponding γ^{-1}-set is $S = \{u_2, u_3, u_4\}$. In $Db_4^{u_3}$, $deg(u_2) = deg(u_4) = 1$ and $deg(u_3) = 5$, and thereby S is a minimum relatively prime inverse dominating set of $Db_4^{u_3}$. Therefore, $\gamma_{rp}^{-1}\ (Db_4^{u_3}) = 3$.

In Db_4, if $deg(v) = 3$, then v is either u_1 or v_1. Clearly, $Db_4^{u_1} \cong Db_4^{v_1}$. Without losing generality, let v be u_1. The graph $Db_4^{u_1}$ is given in Figure 6. A γ-set of $Db_4{}^{u_1}$ is $D = \{u_1, u_3, v_4\}$ and a corresponding γ^{-1}-set is $S = \{u_2, u_4, v_1, v_2\}$. In $Db_4^{u_1}$, $deg(u_2) = deg(u_4) = 1$ $deg(v_1) = 2$ and $deg(v_2) = 3$. Hence, S is a minimum relatively prime inverse dominating set of $Db_4^{u_1}$ and so $\gamma_{rp}^{-1}\ (Db_4^{u_1}) = 4$.

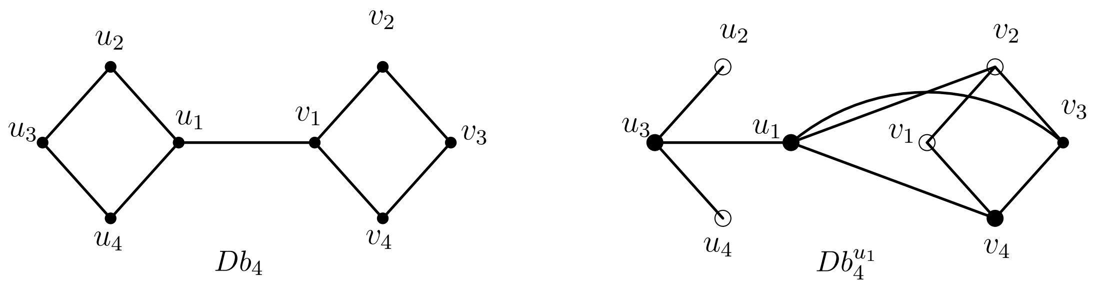


Figure 6

Case 3. $n = 5$

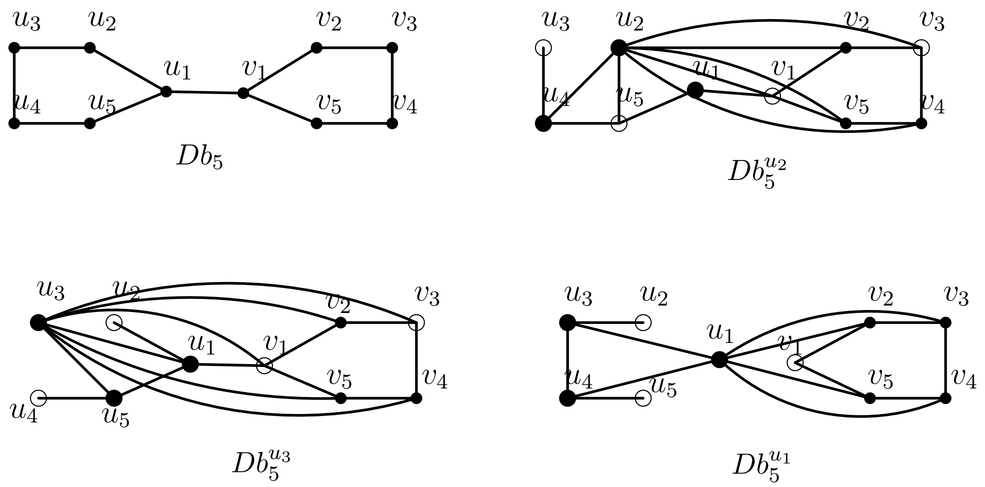
In Db_5, if $deg(v) = 2$, then v is either u_i or v_i, $2 \leq i \leq 4$. Clearly, $Db_5^{v_2} \cong Db_5^{v_5} \cong Db_5^{u_2} \cong Db_5^{u_5}$ and $Db_5^{v_3} \cong Db_5^{u_3} \cong Db_5^{v_4} \cong Db_5^{u_4}$. The graphs Db_5, $Db_5^{u_1}$, $Db_5^{u_2}$ and $Db_5^{u_3}$ are given in Figure 7.

Subcase 3.1. $v = u_2$

The vertex u_2 is adjacent to only u_1 and u_3 in Db_5 and so u_2 is adjacent to all vertices except u_1 and u_3 in $Db_5^{u_2}$. Also the vertices u_1 and u_3 are non-adjacent in $Db_5^{u_2}$. Hence, a γ-set of $Db_5^{u_2}$ must contain exactly three vertices. Since the degree sequence of vertices of $Db_5^{u_2}$ is (7, 4, 3, 3, 3, 3, 3, 3, 2, 1), any corresponding γ^{-1}-set must contain at least two vertices of degree 3 and thereby there does not exist any relatively prime inverse dominating set. Therefore, $\gamma_{rp}^{-1}(Db_5^{u_2}) = 0$.

Subcase 3.2. $v = u_3$

A γ-set of $Db_5{}^{u_3}$ is $D = \{u_1, u_3, u_5\}$ and a corresponding $\gamma{-}1$-set is $S = \{u_2, u_4, v_1, v_3\}$. Since the degree sequence of vertices is S is (4, 3, 1, 1), S is a minimum relatively prime inverse dominating set of $Db_5^{u_3}$ and so γ_{rp}^{-1} $(Db_5^{u_3}) = 4$.



Fgure 7

In Db_5, if $deg(v) = 3$, then v is either u_1 or v_1. Clearly, $Db_4^{v_1} \cong Db_5^{u_1}$. Without losing generality, let v be u_1. A γ-set of $Db_5{}^{u_1}$ is $D = \{u_1, u_3, u_4, v_2\}$ and a corresponding γ^{-1}-set is $S = \{u_2, u_5, v_1, v_4\}$. Since the degree sequence of vertices of S in $Db_5^{u_1}$ is $(3, 2, 1, 1)$, S is a minimum relatively prime inverse dominating set of $Db_5^{u_1}$ and so $\gamma_{rp}^{-1}(Db_5^{u_1}) = 4$.
Case 4. $n \geq 6$

In Db_n, if $deg(v) = 3$, then the degree sequence of Db_n^v is $(n-4, 3, 3, ..., 3, 2, 1, 1)$. Any γ-set D of Db_n^v contains four vertices and any γ^{-1}-set S must contain at least two vertices of degree 3 and so Db_n^v has no relatively prime inverse dominating set. This implies that $\gamma_{rp}^{-1}(Db_n^v) = 0$.

In Db_n, if $deg(v) = 2$, then the degree sequence of Db_n^v is $(n-3, 4, 3, 3, ..., 3, 2, 1)$. Clearly, any γ-set D of Db_n^v contains three vertices and any γ^{-1}-set S must contain at least two vertices of degree 3 and so Db_n^v has no relatively prime inverse dominating set. This implies that $\gamma_{rp}^{-1}(Db_n^v) = 0$.

The theorem follows from the above four cases. $\square$

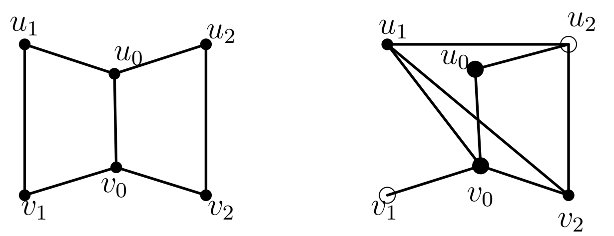
Theorem 3.3. *Let v be any vertex of the book graph B_n. Then*

$$\gamma_{rp}^{-1}(B_n^v) = \begin{cases} 2 & if\ n = 2\ and\ deg_{B_n}(v) = 2 \\ 3 & if\ n \geq 3,\ deg_{B_n}(v) = 2\ and\ 2n \not\equiv 1 (mod\ 3) \\ n & if\ deg_{B_n}(v) = n+1 \\ 0 & otherwise \end{cases}.$$

Proof. Let $v_0, v_1, ..., v_n$ and $u_0, u_1, ..., u_n$ be the vertices of two copies of star $K_{1,n}$ with central vertices v_0 and u_0 respectively. Join u_i with v_i for all i, $1 \leq i \leq n$. The resultant graph is book graph B_n with vertex set $V(B_n) = \{v_0, u_0, v_i, u_i : 1 \leq i \leq n\}$ and edge set $E(B_n) = \{v_0u_0, v_iu_i, v_0v_i, u_0u_i : 1 \leq i \leq n\}$. Then B_n has $2n + 2$ vertices and $3n + 1$ edges. Also $deg_{B_n}(v) = n$ for $v \in \{u_0, v_0\}$ and $deg_{B_n}(v) = 2$ otherwise. Let D be a γ-set of B_n and S be a corresponding minimum relatively prime inverse dominating set of B_n. Now we consider the following three cases.

Case 1. $n = 2$ and $deg_{B_n}(v) = 2$

Here v is either u_i or v_i, $1 \leq i \leq 2$. Clearly, $B_n^{u_1} \cong B_n^{u_2} \cong B_n^{v_1} \cong B_n^{v_2}$. Without losing generality, let v be u_1. The graphs B_2 and $B_2^{u_1}$ are given in Figure 8. A γ-set of $B_n^{u_1}$ is $D = \{u_0, v_0\}$ and a corresponding γ^{-1}- set is $S = \{u_2, v_1\}$. In $B_n^{u_1}$, $deg(u_2) = 3$, $deg(v_1) = 1$ and $(deg(u_n), deg(v_1)) = (3, 1) = 1$. Hence, S is a minimum relatively prime inverse dominating set of $B_n^{u_1}$ and so $\gamma_{rp}^{-1}(B_n^{u_1}) = 2$.

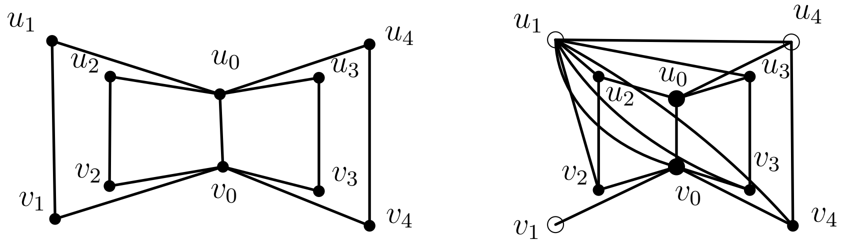


B_2 $\qquad$ $B_2^{u_1}$

Figure 8

Case 2. $n \geq 3$ and $deg_{B_n}(v) = 2$

Here v is either u_i or v_i, $1 \leq i \leq n$. Clearly, $B_n^{u_1} \cong B_n^{u_2} \cong ... \cong B_n^{u_n} \cong B_n^{v_1} \cong B_n^{v_2} \cong ... \cong B_n^{v_n}$. Without losing generality, let v be u_1. The graphs B_4 and $B_4^{u_1}$ are given in Figure 9. Consider the graph ${B_n}^{u_1}$. In ${B_n}^{u_1}$, $deg(u_1) = 2n-1$, $deg(u_i) = deg(v_i) = 3$, $2 \leq i \leq n$, $deg(v_1) = 1$, $deg(u_0) = n - 1$ and $deg(v_0) = n + 1$. A γ-set of ${B_n}^{u_1}$ is $D = \{u_0, v_0\}$ and a corresponding γ^{-1}-set is $S = \{u_1, u_n, v_1\}$. In ${B_n}^{u_1}$, $(deg(u_1),\ deg(v_1)) = (2n-3, 1) = 1$, $(deg(u_n), deg(v_1)) = (3, 1) = 1$ and $(deg(u_1), deg(u_n)) = (2n - 1, 3)$. Hence, S is a minimum relatively prime inverse dominating set if and only if $(2n - 1, 3) = 1$ if and only if $2n - 1 \not\equiv 0(mod\ 3)$ if and only if $2n \not\equiv 1(mod\ 3)$.



B_4 $\qquad$ $B_4^{u_1}$

Figure. 9

Case 3. $deg_{B_n}(v) = n + 1$

Here v is either u_0 or v_0. Without losing generality, let v be u_0 since $B_n^{u_0} \cong B_n^{v_0}$. The graphs B_5 and $B_5^{u_0}$ are given in Figure 10. In $B_n^{u_0}, deg(u_0) = deg(v_0) = n$, $deg(u_i) = 1$ and $deg(v_i) = 3, 1 \leq i \leq n$. A γ-set of $B_n^{u_0}$ is $D = \{u_1, v_2, v_3, ..., v_n\}$ and a corresponding γ^{-1}-set is $S = \{v_1, u_2, u_3, ..., u_n\}$. Also in $B_n^{u_0}$, $(deg(v_1), deg(u_j)) = (3, 1) = 1, 2 \leq j \leq n$ and $(deg(u_l), deg(u_m)) = (1, 1) = 1$ for $2 \leq l \neq m \leq n$. Hence, S is a minimum relatively prime inverse dominating set of $B_n^{u_0}$ and so $\gamma_{rp}^{-1}(B_n^{u_0}) = n$.

The theorem follows from cases 1, 2 and 3. $\square$

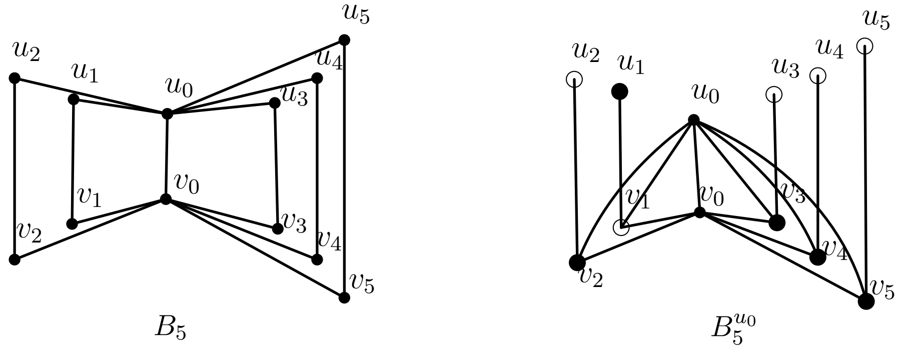


Figure 10

Theorem 3.4. *Let v be any vertex of the Barbell graph $B_{n,n}$. Then*

$$\gamma_{rp}^{-1}\ (B_{n,n}^{v}) = \begin{cases} 0 \ if\ deg_{B_{n,n}}(v) = n \ and\ n\ is\ even. \\ 2\ otherwise \end{cases}$$

Proof. Consider two copies of K_n with the vertex sets $\{u_1, u_2, ..., u_n\}$ and $\{v_1, v_2, ..., v_n\}$. Join u_1 and v_1, we get the barbell graph $G = B_{n,n}$ with the vertex set $V(B_{n,n}) = \{u_i, v_i : 1 \leq i \leq n\}$ and edge set $E(B_{n,n}) = \{u_1v_1, u_iu_j, v_iv_j : 1 \leq i < j \leq n\}$. Also in $B_{n,n}$, $deg(u_1) = deg(v_1) = n$, $deg(u_i) = deg(v_i) = n-1$, $2 \leq i \leq n$. Let $B_{n,n}^{v}$ be the vertex switching of $B_{n,n}$ with respect to the vertex v. Let D be a γ-set of $B_{n,n}^{v}$ and S be a corresponding γ^{-1}-set. We consider the following two cases.

Case 1. $deg_{B_{n,n}}(v) = n-1$

Here v is either u_i or v_i, $2 \leq i \leq n$. Clearly, $B_{n,n}^{u_i} \cong B_{n,n}^{v_i} \cong B_{n,n}^{u_j} \cong B_{n,n}^{u_j}$, $2 \leq i \neq j \leq n$. Without losing generality, v be u_2. In $B_{n,n}^{v}$, v is adjacent to v_i, $1 \leq i \leq n$, and a γ-set of $B_{n,n}^{v}$ is $D = \{v, u_i\}$, for some i, $2 \leq i \leq n$ and a corresponding γ^{-1}- set is $S = \{v_i, u_1\}$, for some i, $2 \leq i \leq n$. Also in $B_{n,n}^{v}$, $(deg(v_i), deg(u_1)) = (n, n-1) = 1$. Hence, S is a minimum relatively prime inverse dominating set of $B_{n,n}^{v}$ and so $\gamma_{rp}^{-1}\ (B_{n,n}^{v}) = 2$.

Case 2. $deg_{B_{n,n}}(v) = n$

Here v is either u_1 or v_1. Let v be u_1. In $B_{n,n}^{v}$, v is adjacent to v_i, $1 \leq i \leq n$, $d_{B_{n,n}^{v}}(v) = n$. A γ-set of $B_{n,n}^{v}$ is $D = \{u_i, v_i\}$, $1 \leq i \leq n$. and a corresponding γ^{-1}-set is $S = \{u_j, v_j\}$, $2 \leq i \leq n$, $i \neq j$. Also in $B_{n,n}^{v}$, $(deg(v_j), deg(u_j)) = (n, n-2)$. Hence, S is a minimum relatively prime inverse dominating set of $B_{n,n}^{v}$ if and only if $(n, n-2) = 1$ if and only if n is odd. This implies that $\gamma_{rp}^{-1}\ (B_{n,n}^{v}) = 2$ if and only if n is odd.

Thus the theorem follows from the above two cases. □

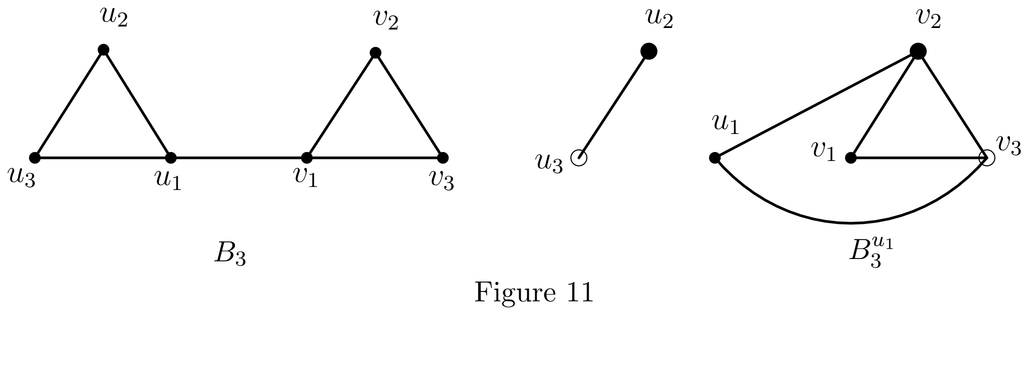


Figure 11

Theorem 3.5. *Let $H_{n,n}$ be a crown graph and v be any vertex of $H_{n,n}$. Then*

$$\gamma_{rp}^{-1}(H_{n,n}^{v}) = \begin{cases} 2 & if\ n\ is\ odd \\ 0 & otherwise \end{cases}$$

Proof. Consider the complete bipartite graph $K_{n,n}$ with vertex set $V(K_{n,n}) = \{u_i, v_i : 1 \leq i \leq n\}$. By removing the edges u_iv_i, $1 \leq i \leq n$, we get the crown graph with vertex set $V(H_{n,n}) = \{u_i, v_i : 1 \leq i \leq n\}$ and edge set $E(H_{n,n}) = \{u_iv_j : 1 \leq i \neq j \leq n\}$. In $H_{n,n}$, $deg(u_i) = deg(v_i) = n-1$, $1 \leq i \leq n$. Let D be a γ-set of $H_{n,n}^{v}$ and S be a minimum relatively prime inverse dominating set with respect to D.

Clearly, $H_{n,n}^{u_1} \cong H_{n,n}^{u_2} \cong ... \cong H_{n,n}^{u_n} \cong H_{n,n}^{v_1} \cong H_{n,n}^{v_2} \cong ... \cong H_{n,n}^{v_n}$. Let v be u_1. The graphs $H_{3,3}$ and $H_{3,3}^{u_1}$ are given in Figure 12. In $H_{n,n}^{u_1}$, $deg(u_i) = n$, $1 \leq i \leq n$, $deg(v_1) = n$, $deg(v_i) = n-2$, $2 \leq i \leq n$ and hence the degree sequence of $H_{n,n}^{u_1}$ contains only the numbers $n-2$ and n. A γ- set of $H_{n,n}^{u_1}$ is $D = \{u_n, v_n\}$ and a corresponding γ^{-1}-set is $S = \{u_{n-1}, v_{n-1}\}$. In $H_{n,n}^{u_1}$, $(deg(u_{n-1}), deg(v_{n-1})) = (n, n-2)$. Hence, S is a minimum relatively prime inverse dominating set if and only if $(n, n-2) = 1$ if and only if n is odd. This implies that $\gamma_{rp}^{-1}(H_{n,n}^{u_1}) = 2$ if and only if n is odd. □

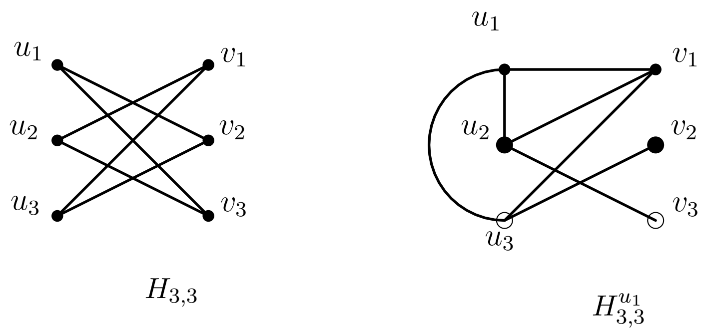


Figure 12

4. CONCLUSION

In this paper, we have found the relatively prime inverse domination number on vertex switching of some standard graphs like dumbbell graph, book graph, barbell graph, and crown graph.

REFERENCES

[1] Agarwal P. K., Alon N., Aronov B., Suri S., (1994), Can visibility graphs be represented compactly, Discrete and Computational Geometry, 12(1), pp. 347-365.
[2] Chartrand G., Lesniak L., (2005), Graphs and Digraphs, Fourth ed., CRC press, BoCa Raton.
[3] Gallian J., (2018), Dynamic Survey of Graph Labelling, Elec. J. Combin, DS6, 21.
[4] Haynes W., Hedetniemi S. T., Slater P. J., (1998), Domination in Graphs : Advanced Topics, Marcel Dekker, New York.
[5] Jayasekaran C., Jancy Vini A., (2017), Results on relatively prime dominating sets in graphs, Annals of Pure and Applied Mathematics, 14(3), pp. 359-369.
[6] Jayasekaran C., Jancy Vini A., (2020), Results on relatively prime domination number of vertex switching of complement graphs, Advances in Mathematics: Scientific Journal, 9(4), pp. 1601-1609.
[7] Jayasekaran C., Roshini L., (2020), Relatively prime inverse dominating sets in graphs, Malaya Journal of Matematik, 8(4), pp. 2292-2295.
[8] Kulli V. R., Sigarkant S. C., (1991), Inverse domination in graphs, Nat. Acad Sci. Letters, 14, pp. 473-475.

[9] Wang J. F., Belardo F., Li Marzi E. M., (2009), A note on the spectral characterization of dumbbell graphs, Linear Algebra Appl, 431, pp. 1707-1714.

Dr. C. Jayasekaran, is working as an Associate Professor of Mathematics, Pioneer Kumaraswamy College, Nagercoil, Tamilnadu, India. Under his guidance, 12 scholars have been awarded Ph.D. degree and 5 have been submitted their doctoral thesis. He is a referee for 5 International journals. He has published 86 research papers in foreign journals. His current research focuses on switching and domination in graph theory.

L. Roshini is a Research Scholar in the Department of Mathematics, Pioneer Kumaraswamy College, Nagercoil, Tamil Nadu, India.