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KERNEL AND K-KERNEL SYMMETRIC INTUITIONISTIC FUZZY MATRICES

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Abstract. The idea of Kernel and k-Kernel Symmetric (k-KS) Intuitionistic Fuzzy Matrices (IFM) are introduced with an example. We present some basic results of kernel symmetric matrices. We show that k-symmetric implies k-Kernel symmetric but the converse need not be true. The equivalent relations between kernel symmetric, k-kernel symmetric and Moore-Penrose inverse of IFM are explained with numerical results.

Keywords: k-Kernel Symmetric, Intuitionistic Fuzzy Matrices (IFM), k-hermitian, Moore-Penrose inverse, Range symmetric.

AMS Subject Classification: 03E72, 15B15, 15B99.

1. Introduction

The complexity of problems in Economics, Engineering, Environmental Sciences and Social Sciences which cannot be solved by the well-known methods of classical Mathematics pose a great difficulty in today's practical world. To handle this type of situation Zadeh [14] first introduced the notion of fuzzy set to investigate both theoretical and practical applications of our daily activities. This traditional fuzzy set is sometimes may be very difficult to assign the membership value for fuzzy sets. In the current scenario intuitionistic fuzzy set (IFS) initiated by Atanassov [1] is appropriate for such a situation.

It is well known that generalized inverses exist for a complex matrix. However this is the failure for fuzzy matrix, that is for under the max-min fuzzy operations the matrix equation $AXA = A$ need not have a solution X. If A has a generalized inverse ($g-inverse$) then A is said to be regular. The concept of generalized inverse presents a very interesting area of research in matrix theory in the same way a regular matrix as one of which g-inverse exists, lays the foundation for research in fuzzy matrix theory.

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For a fuzzy matrix A, if A^+ exists, then it coincide with A^T, Kim and Roush [6] have studied Generalized fuzzy matrices. A Fuzzy matrix A is range symmetric if $R[A] = R[A^T]$ implies and kernel symmetric $N(A) = N(A^T)$. It is well known that for complex matrices, the concept of range symmetric and kernel symmetric is identical. For Intuitionistic Fuzzy matrix is range symmetric $R[A] = R[A^T]$ implies $N(A) = N(A^T)$ but the converse need not be true. Meenakshi [7] introduced the notion of fuzzy matrix. Let k-be a fixed product of disjoint transpositions in $S_n = 1, 2, ..., n$ and K be the associated permutation matrix. Hill and Waters [5] have introduced on k-real and k-hermitian matrices. Baskett and Katz [3] have studied theorems on products of EPr matrices. Schwerdtfeger [13] has studied the notion of introduction to Linear Algebra and the Theory of matrices. Meenakshi and Jayashri [9] have studied k-Kernel Symmetric Matrices. Riyaz Ahmad Padder and Murugadas [10-12] introduced on idempotent intuitionistic fuzzy Matrices of T-type, reduction of a nilpotent intuitionistic fuzzy matrix using implication operator and determinant theory for intuitionistic fuzzy matrices. Atanassov has studied [2] generalized index matrices. Meenakshi and Krishnamoorthy introduced on $k - EP$ matrices. Ben and Greville [4] developed the concept of range symmetric fuzzy matrix and kernel symmetric fuzzy matrix analogues to that of an EP matrix in the complex field.

1.1 Research gaps

As mentioned in the above introduction section, Meenakshi introduced the concept of Range symmetric and Meenakshi and Jayashri developed the notion of kernel symmetric in fuzzy matrix. Here, we have applied the concept of range symmetric and kernel symmetric in intuitionistic fuzzy matrix (IFM). Both these concepts plays a significant role in hybrid fuzzy structure and we have applied the same in IFM and studied some of the results in detail. First we present equivalent characterizations of a range symmetric matrix and then, derive equivalent conditions for an intuitionistic fuzzy matrix to be kernel symmetric intuitionistic fuzzy matrix and study the relation between range symmetric and kernel symmetric intuitionistic fuzzy matrices. Equivalent condition for varies g-inverses of a kernel symmetric matrix to be kernel symmetric are determined.

2. PRELIMINARIES AND NOTATIONS

Let the function be defined as $k(x) = (x_{k[1]}, x_{k[2]}, x_{k[3]}, \cdots, x_{k[n]}) \in F_{n\times 1}$ for $x = x_1, x_2, ..., x_n \in F_{[1\times n]}$, where K is involuntary, the following conditions are satisfied. The associated permutation matrix, where K is a permutation matrix, $KK^T = K^T K = I_n$ then $K^T = K$.

(P_1) $K = K^T$, $K^2 = I$, and $k(x) = Kx$ for all $A \in (IF)_n$,
(P_2) $N(A) = N(AK) = N(KA)$,
(P_3) $(AK)^+ = KA^+$ and $(KA)^+ = A^+K$ exists, if A^+ exists,
(P_4) A^T is a g-inverse of A iff A^+ exist.

Notations. For IFM of $A \in (IF)_n$,
A^T: transpose of A, $R(A)$: Row space of A, $C(A)$: Column space of A, $N(A)$: Null Space of A.
A^+: Moore-Penrose inverse of A, $(IF)_n$: Square Intutionistic Fuzzy Matrix. $F_{[1\times n]}$: The matrix one row n columns. $F_{[n\times 1]}$: The matrix n rows one column.

3. DEFINITIONS AND THEOREMS

Definition 3.1. *Let A be a IFM, if $R[A] = R[A^T]$ then A is called as range symmetric.*

Example 3.1.

$$A = \begin{bmatrix} < 0.2, 0.5 > & < 0, 0 > & < 0.7, 0.2 > \\ < 0, 0 > & < 0, 0 > & < 0, 0 > \\ < 0.7, 0.2 > & < 0, 0 > & < 0.3, 0.2 > \end{bmatrix}.$$

The following matrices are not range symmetric

$$A = \begin{bmatrix} < 1, 0 > & < 1, 0 > & < 0, 0 > \\ < 0, 0 > & < 1, 0 > & < 1, 0 > \\ < 0, 0 > & < 0, 0 > & < 1, 0 > \end{bmatrix},$$

$$A^T = \begin{bmatrix} < 1, 0 > & < 0, 0 > & < 0, 0 > \\ < 1, 0 > & < 1, 0 > & < 0, 0 > \\ < 0, 0 > & < 1, 0 > & < 1, 0 > \end{bmatrix},$$

$$(< 1, 0 > < 1, 0 > < 0, 0 >) \in R(A), \qquad (< 1, 0 > < 1, 0 > < 0, 0 >) \in R(A^T)$$

$$(< 0, 0 > < 1, 0 > < 1, 0 >) \in R(A), \qquad (< 0, 0 > < 1, 0 > < 1, 0 >) \in R(A^T)$$

$$(< 0, 0 > < 0, 0 > < 1, 0 >) \in R(A), \qquad (< 0, 0 > < 0, 0 > < 1, 0 >) \notin R(A^T)$$

$R(A) \notin R(A^T)$.

Definition 3.2. *Let $A \in F_n$ be an Intuitionistic fuzzy matrix, if $N(A) = N(A^T)$ then A is called kernel symmetric IFM, where $N(A) = \{x/xA =< 0, 0 > \text{ and } x \in F_{1\times n}\}$.*

Example 3.2.

$$A = \begin{bmatrix} < 0.4, 0.5 > & < 0, 0 > & < 0.6, 0.4 > \\ < 0, 0 > & < 0, 0 > & < 0, 0 > \\ < 0.4, 0.5 > & < 0, 0 > & < 0.4, 0.3 > \end{bmatrix},$$

$$N(A) = N(A^T) = \{(< 0, 0 >, < a_{ij\alpha}, a_{ij\beta} >, < 0, 0 >)/a_{ij\alpha}, a_{ij\beta} \in F\}.$$

Definition 3.3. *Let IFM be represented in the form of $A = [y_{ij}, \langle a_{ij\alpha}, a_{ij\beta}\rangle]$, where $a_{ij\alpha}$ and $a_{ij\beta}$ are called the degree of membership and also the non-membership of y_{ij} in A, with the condition $0 \leq a_{ij\alpha} + a_{ij\beta} \leq 1$.*

Definition 3.4. *Suppose a and b are two IFM elements $a = \langle a_{ij\alpha}, a_{ij\beta}\rangle$, $b = \langle b_{ij\alpha}, b_{ij\beta}\rangle$, the component wise addition and multiplication are described as below,*

$a + b = \langle \min\{a_{ij\alpha}, b_{ij\alpha}\}, \max\{a_{ij\beta}, b_{ij\beta}\}\rangle$ *and*

$a.b = \langle \max\{a_{ij\alpha}, b_{ij\alpha}\}, \min\ of\{a_{ij\beta}, b_{ij\beta}\}\rangle$

Note:1. For Intuitionistic fuzzy matrix $A \in F_n$ with $\det A >< 0, 0 >$, has non-zero rows and non-columns, hereafter $N(A) =< 0, 0 >= N(A^T)$. Furthermore, a symmetric matrix $A = A^T$, that is $N(A) = N(A^T)$.

Theorem 3.1. *For $A, B \in (IF)_n$ and K be an Intuitionistic fuzzy permutation matrix, $N(A) = N(B) \Leftrightarrow N(KAK^T) = N(KBK^T)$.*

Proof. Let $w \in N(KAK^T)$

$\Rightarrow w(KAK^T) =< 0, 0 >$

$\Rightarrow zK^T =< 0, 0 >$ where $z = wKA$

$\Rightarrow z \in N(K^T)$.

Since, $\det K = \det K^T > < 0, 0 >$ (By Note 1) Therefore, $N(K^T) =< 0, 0 >$.

Hence, $z =< 0, 0 >$.

$\Rightarrow wKA =< 0, 0 >$

$\Rightarrow wK \in N(A) = N(B)$
$\Rightarrow wKBK^T = <0,0>$
$\Rightarrow w \in N(KBK^T)$
$N(KAK^T) \subseteq N(KBK^T)$.
Similarly, $N(KBK^T) \subseteq N(KAK^T)$.
Therefore, $N(KAK^T) = N(KBK^T)$.
Conversely, if $N(KAK^T) = N(KBK^T)$, then by the above proof, $N(A) = N(B)$. □

Example 3.3.

$$A = \begin{bmatrix} <0,0.5> & <0,0> & <0.3,0> \\ <0,0> & <0,0> & <0,0> \\ <0.7,0> & <0,0> & <0.3,0.2> \end{bmatrix}, \quad B = \begin{bmatrix} <0.3,0.4> & <0,0> & <0.4,0.2> \\ <0,0> & <0,0> & <0,0> \\ <0.5,0.3> & <0,0> & <0.5,0.3> \end{bmatrix},$$

$$K = \begin{bmatrix} <1,0> & <0,1> & <0,1> \\ <0,1> & <1,0> & <0,1> \\ <0,1> & <0,1> & <1,0> \end{bmatrix},$$

$$N(KAK^T) = \{(<0,0>, <a_{ij\alpha}., a_{ij\beta}>, <0,0>)/a_{ij\alpha}, a_{ij\beta} \in F\},$$

$$w = \{(<0,0>, <a_{ij\alpha}., a_{ij\beta}>, <0,0>)/ \text{ for all } a_{ij\alpha}, a_{ij\beta} \in F\},$$

$$KAK^T = \begin{bmatrix} <1,0> & <0,1> & <0,1> \\ <0,1> & <1,0> & <0,1> \\ <0,1> & <0,1> & <1,0> \end{bmatrix} \begin{bmatrix} <0,0.5> & <0,0> & <0.3,0> \\ <0,0> & <0,0> & <0,0> \\ <0.7,0> & <0,0> & <0.3,0.2> \end{bmatrix}$$

$$\begin{bmatrix} <1,0> & <0,1> & <0,1> \\ <0,1> & <1,0> & <0,1> \\ <0,1> & <0,1> & <1,0> \end{bmatrix}$$

$$KAK^T = \begin{bmatrix} <0,0.5> & <0,0> & <0.3,0> \\ <0,0> & <0,0> & <0,0> \\ <0.7,0> & <0,0> & <0.3,0.2> \end{bmatrix}$$

$$w(KAK^T) = \begin{bmatrix} <0,0> & <a_{ij\alpha}, a_{ij\beta}> & <0,0> \end{bmatrix}$$

$$\begin{bmatrix} <0,0.5> & <0,0> & <0.3,0> \\ <0,0> & <0,0> & <0,0> \\ <0.7,0> & <0,0> & <0.3,0.2> \end{bmatrix} = <0,0>$$

$\Rightarrow zK^T = <0,0>$ *where* $z = wKA$.
Hence, $z = <0,0>$.

$$KA = \begin{bmatrix} <0,0.5> & <0,0> & <0.3,0> \\ <0,0> & <0,0> & <0,0> \\ <0.7,0> & <0,0> & <0.3,0.2> \end{bmatrix}$$

$$w(KA) = \begin{bmatrix} <0,0> & <a_{ij\alpha}, a_{ij\beta}> & <0,0> \end{bmatrix} \begin{bmatrix} <0,0.5> & <0,0> & <0.3,0> \\ <0,0> & <0,0> & <0,0> \\ <0.7,0> & <0,0> & <0.3,0.2> \end{bmatrix} = <0,0>$$

$\Rightarrow wKA = <0,0>$
$\Rightarrow wK \in N(A) = N(B)$
$\Rightarrow wKBK^T = <0,0>$
$\Rightarrow w \in N(KBK^T)$
$N(KAK^T) \subseteq N(KBK^T)$.
Similarly, $N(KBK^T) \subseteq N(KAK^T)$.

Theorefore, $N(KAK^T) = N(KBK^T)$.
Conversely, if $N(KAK^T) = N(KBK^T)$*, then by the above proof,* $N(A) = N(B)$.

Theorem 3.2. *For* $A \in (IF)_n$ *is kernel symmetric intuitionistic fuzzy matrix and* K *being a permutation matrix if and only if* $N(KAK^T) = N(KA^TK^T)$.

Proof. Let $x \in N(KAK^T)$
$\Rightarrow x(KAK^T) =< 0,0 >$
$\Rightarrow yK^T =< 0,0 >$ where $y = xKA$
$\Rightarrow y \in N(K^T)$
Since, $\det K = \det K^T > < 0,0 >$.
Therefore, $N(K^T) =< 0,0 >$.
Hence, $y =< 0,0 >$.
$\Rightarrow xKA =< 0,0 >$
$\Rightarrow xK \in N(A) = N(A^T)$
$\Rightarrow xKA^TK^T =< 0,0 >$
$\Rightarrow x \in N(KA^TK^T)$
$N(KAK^T) \subseteq N(KA^TK^T)$.
Similarly, $N(KA^tK^T) \subseteq N(KAK^T)$.
Therefore, $N(KAK^T) = N(KA^TK^T)$.
Conversely, if $N(KAK^T) = N(KA^TK^T)$, then by the above proof, $N(A) = N(A^T)$. □

Example 3.4.

$$A = \begin{bmatrix} < 0,0.5 > & < 0,0 > & < 0.3,0 > \\ < 0,0 > & < 0,0 > & < 0,0 > \\ < 0.7,0 > & < 0,0 > & < 0.3,0.2 > \end{bmatrix}, \quad K = \begin{bmatrix} < 1,0 > & < 0,1 > & < 0,1 > \\ < 0,1 > & < 1,0 > & < 0,1 > \\ < 0,1 > & < 0,1 > & < 1,0 > \end{bmatrix}$$

$N(A) = N(A^T) = \{(< 0.0 >, < a_{ij\alpha}, a_{ij\beta} >, < 0,0 >)/a_{ij\alpha}, a_{ij\beta} \in F\}$.

Theorem 3.3. *For* $A \in (IF)_n$ *is kernel symmetric intuitionistic fuzzy matrix, then* $N(AA^T) = N(A) = N(A^TA)$.

Proof. Let, $x \in N(A)$
$\Leftrightarrow xA =< 0,0 >$
$\Leftrightarrow xAA^T =< 0,0 >$
$\Leftrightarrow x \in N(AA^T)$
$\Leftrightarrow N(A) \subseteq N(AA^T)$
Similarly, $N(AA^T) \subseteq N(A)$.
Therefore, $N(A) = N(AA^T)$.
Similarly, $N(A) = N(A^TA)$.
Therefore, $N(AA^T) = N(A) = N(A^TA)$. □

Example 3.5.

$$A = \begin{bmatrix} < 0.4,0.5 > & < 0,0 > & < 0.6,0.4 > \\ < 0,0 > & < 0,0 > & < 0,0 > \\ < 0.4,0.5 > & < 0,0 > & < 0.4,0.3 > \end{bmatrix},$$

$N(AA^T) = N(A) = N(A^TA) = \{(< 0.0 >, < a_{ij\alpha}, a_{ij\beta} >, < 0,0 >)/a_{ij\alpha}, a_{ij\beta} \in F\}$.

Theorem 3.4. *Let A,B be the IFM and K IFPM,* $R(A) = R(B) \Leftrightarrow R(KAK^T) = R(KBK^T)$.

Proof. Let $R(A) = R(B)$.
Then,

$$R(AK^T) = R(A)K^T$$
$$= R(B)K^T$$
$$= R(BK^T)$$

Let $z \in R(KAK^T)$
$z = w(KAK^T)$ for some $w \in V^n$
$z = rAK^T,\ r = wK$
$z \in R\ AK^T = R\ B\ K^T$
$z = uBK^T$ for some $u \in V^n$
$z = uK^T\ KBK^T$
$z = vKBK^T$ for some $v \in V^n$
$z \in R(KBK^T)$.

Therefore, $R(KAK^T) \subseteq R(KBK^T)$.
Similarly, $R(KBK^T) \subseteq R(KAK^T)$.
Therefore, $R(KAK^T) = R(KBK^T)$.
Conversely, let $R(KAK^T) = R(KBK^T)$. Then by above proof

$$R(A) = R[K^T(KAK^T)K]$$
$$= R[K^T(KBK^T)K]$$
$$R(A) = R(B)$$

□

Example 3.6.

$$A = \begin{bmatrix} <0.2,0.5> & <0,0> & <0.7,0.2> \\ <0,0> & <0,0> & <0,0> \\ <0.7,0.2> & <0,0> & <0.3,0.2> \end{bmatrix}, \quad B = \begin{bmatrix} <0.7,0.2> & <0,0> & <0.3,0.2> \\ <0,0> & <0,0> & <0,0> \\ <0.2,0.5> & <0,0> & <0.7,0.2> \end{bmatrix},$$

$$K = \begin{bmatrix} <1,0> & <0,1> & <0,1> \\ <0,1> & <1,0> & <0,1> \\ <0,1> & <0,1> & <1,0> \end{bmatrix},$$

$R(A) = R(B) \Leftrightarrow R(KAK^T) = R(KBK^T)$.

Theorem 3.5. *For $A \in (IF)_n$ be the IFM and K IFPM, $R(A) = R(A^T) \Leftrightarrow R(KAK^T) = R(KA^TK^T)$.*

Example 3.7.

$$A = \begin{bmatrix} <0.4,0.5> & <0,0> & <0.3,0.5> \\ <0,0> & <0,0> & <0,0> \\ <0.3,0.5> & <0,0> & <0.3,0.2> \end{bmatrix}, \quad K = \begin{bmatrix} <1,0> & <0,1> & <0,1> \\ <0,1> & <1,0> & <0,1> \\ <0,1> & <0,1> & <1,0> \end{bmatrix}.$$

Theorem 3.6. *Let A,B be the Intuitionistic fuzzy matrix and K being a permutation matrix, $C(A) = C(B) \Leftrightarrow C(KAK^T) = C(KBK^T)$.*

Example 3.8.

$$A = \begin{bmatrix} <0.2,0.5> & <0,0> & <0.7,0.2> \\ <0,0> & <0,0> & <0,0> \\ <0.7,0.2> & <0,0> & <0.3,0.2> \end{bmatrix}, \quad B = \begin{bmatrix} <0.7,0.2> & <0,0> & <0.2,0.5> \\ <0,0> & <0,0> & <0,0> \\ <0.3,0.2> & <0,0> & <0.7,0.2> \end{bmatrix},$$

$C(A) = C(B) \Leftrightarrow C(KAK^T) = C(KBK^T)$.

Theorem 3.7. *For IFM $A \in (IF)_n$, the following statements are equivalent*

(i) $N(A) = N(A^T)$.
(ii) $N(KAK^T) = N(KA^TK^T)$, *for some permutation IFM K.*

(*iii*) *Intutionistic permutation fuzzy matrices K such that*

$$KAK^T = \begin{bmatrix} D & <0,0> \\ <0,0> & <0,0> \end{bmatrix}$$

with $\det D > <0,0>$.

Proof. (*i*) iff (*ii*). This equivalence follows from the theorem (3.2).
(*i*) iff (*iii*): Let $N(A) = N(A^T)$.
If $\det A >< 0,0 >$ then A has no zero row and columns.

Hence (*iii*) holds by taking $K = I$ and $D = A$ itself.
Suppose $\det A =< 0,0 >$ then $N(A) = N(A^T)$ then $N(A) = N(A^T) \neq < 0,0 >$.

For $x \neq v < 0,0 >$, $x \in N(A)$ corresponding to each non zero co efficient x_i of x, the fuzzy sums $\sum x_i a_{ik} =< 0,0 >$ and $\sum x_i a_{ki} =< 0,0 >$, for all k.

Hence the i^{th} column and i^{th} row of A are full of zeros.

Now by permutating the rows and columns suitably.
We can move all the zero rows to the bottom and all the zero columns to the right.

Thus A is of the form

$$KAK^T = \begin{bmatrix} D & <0,0> \\ <0,0> & <0,0> \end{bmatrix},$$

where D - square matrix, D has non-zero rows and non-zero columns.
Therefore, $\det D > < 0,0 >$.
Thus (*iii*) holds.
(*iii*) implies (*ii*) : If $\det A > < 0,0 >$ by remark, D is kernel symmetric,

$$\begin{bmatrix} D & <0,0> \\ <0,0> & <0,0> \end{bmatrix}$$

is also kernel symmetric (*ii*) holds. □

Example 3.9. *Consider IFM,*

$$A = \begin{bmatrix} <0,0.5> & <0,0> & <0.3,0> \\ <0,0> & <0,0> & <0,0> \\ <0.7,0> & <0,0> & <0.3,0.2> \end{bmatrix},$$

$$K = \begin{bmatrix} <1,0> & <0,1> & <0,1> \\ <0,1> & <0,1> & <1,0> \\ <0,1> & <1,0> & <0,1> \end{bmatrix},$$

$\det(A) =< 0,0 >,\ N(A) = N(A^T) = \{(< 0.0 >, < a_{ij\alpha}, a_{ij\beta} >, < 0,0 >)/a_{ij\alpha}, a_{ij\beta} \in F\}$,

$$KAK^T = \begin{bmatrix} D & <0,0> \\ <0,0> & <0,0> \end{bmatrix},$$

where

$$D = \begin{bmatrix} <0,0.5> & <0.3,0> \\ <0.7,0> & <0.3,0.2> \end{bmatrix},$$

determined of $D > < 0,0 >$.

4. k-KERNEL SYMMETRIC IFM

Definition 4.1. *Let A be an intuitionistic fuzzy matrix. A belongs to F_n is called k-Kernel symmetric intuitionistic fuzzy if $N(A) = N(KA^TK)$.*

Note:3. Let A is k-Symmetric IFM implies it is k-kernel symmetric IFM, for $A = K(A^T)K$ spontaneously implies $N(A) = N(KA^TK)$. Example 4.1 shows that the converse need not be true.

Example 4.1. *Consider IFM,*

$$A = \begin{bmatrix} <0,0> & <0,0> & <0.3,0.4> \\ <0.5,0.4> & <0.1,0.4> & <0,0> \\ <0.4,0.5> & <0.3,0.4> & <0,0> \end{bmatrix}, \quad K = \begin{bmatrix} <0,1> & <0,1> & <1,0> \\ <0,1> & <1,0> & <0,1> \\ <1,0> & <0,1> & <0,1> \end{bmatrix},$$

$$KA^TK = \begin{bmatrix} <0,0> & <0,0> & <0.3,0.4> \\ <0.3,0.4> & <0.1,0.4> & <0,0> \\ <0.4,0.5> & <0.5,0.4> & <0,0> \end{bmatrix}.$$

Therfore, $A \neq KA^TK$.
But, $N(A) = N(KA^TK) = <0,0>$.

Theorem 4.1. *For Intuitionistic fuzzy matrix $A \in (IF)_n$, the given statements are equivalent:*

(1) $N(A) = N(KA^TK)$,
(2) $N(KA) = N((KA)^T)$,
(3) $N(AK) = N((AK)^T)$,
(4) $N(A^T) = N(KA)$,
(5) $N(A) = N((AK)^T)$,
(6) A^+ *is k-KSIFM,*
(7) $N(A) = N(A^+K)$,
(8) $KA^+A = AA^+K$,
(9) $A^+AK = KAA^+$.

Proof. (1) implies (2)

$$\Leftrightarrow N(A) = N(KA^TK)$$
$$\Leftrightarrow N(KA) = N(A^TK) \qquad (\text{By } P_2)(K^2 = I)$$
$$\Leftrightarrow N(KA) = N((KA)^T) \qquad (\text{Because, } (KA)^T = A^TK^T = A^TK)$$
$$\Leftrightarrow KA \text{ is Kernel symmetric.}$$

Theorefore, (2) holds.
(1) implies (3)

$$\Leftrightarrow N(A) = N(KA^TK)$$
$$\Leftrightarrow N(AK) = N(KA^T) \qquad (\text{By } P_2)(K^2 = I)$$
$$\Leftrightarrow N(AK) = N((AK)^T) \qquad (\text{Because, } (AK)^T = K^TA^T = KA^T)$$
$$\Leftrightarrow AK \text{ is Kernel symmetric.}$$

Theorefore, (3) holds.
(2) implies (4)
$$\Leftrightarrow N(KA) = N(KA)^T = N(A^TK)$$
$$\Leftrightarrow N(KA) = N(A^T) \qquad (\text{By } P_2).$$

Theorefore, (4) holds.
(3) implies (5)

$\Leftrightarrow N(A.K) = N((A.K)^T)$
$\Leftrightarrow N(A) = N((AK)^T)$ (By P_2).

(2) implies (6)

$\Leftrightarrow N(KA) = N(KA)^T$
$\Leftrightarrow N(KA) = N(A^TK)$ (By P_2)
$\Leftrightarrow N(KA) = N(A^+K)$ (Since $N(KA^+K) = N(A^+K)$)
$\Leftrightarrow N(KA) = N(A^+)$
$\Leftrightarrow A^+$ is k-Kernel symmetric IFM.

(1) implies (7)

$\Leftrightarrow N(A) = N(KA^TK)$
$\Leftrightarrow N(A) = N(KA^TK) = N(A^TK)$
$\Leftrightarrow N(A) = N(KA)^T$
$\Leftrightarrow N(A) = N(A^+K)$ (By P_2).

(2) implies (8)
AK is Kernel symmetric IFM.

$\Leftrightarrow (AK)(AK)^+ = (AK)^+(AK)$
$\Leftrightarrow (AK)(KA^+) = (KA^+)(AK)$
$\Leftrightarrow (AA^+) = KA^+AK$
$\Leftrightarrow AA^+K = KA^+A.$

Thus equivalence of (3) and (8) is proved.
(8) $\Leftrightarrow$ (9): Since, by the property (P_1), $K^2 = I$, this uniformity follows by pre- and post multiplying by K.

$\Leftrightarrow KA^+A = AA^+K$
$\Leftrightarrow K^2A^+AK = KAA^+K^2$
$\Leftrightarrow A^+AK = KAA^+.$

□

Example 4.2. *Consider IFM,*

$$A = \begin{bmatrix} < 0.8, 0.2 > & < 0.6, 0.3 > \\ < 0.6, 0.3 > & < 0.7, 0.1 > \end{bmatrix}, \quad K = \begin{bmatrix} < 0, 1 > & < 1, 0 > \\ < 1, 0 > & < 0, 1 > \end{bmatrix}.$$

5. Conclusions

We introduced the concept of Kernel and k-Kernel Symmetric Intuitionistic Fuzzy Matrices with suitable examples. In addition, we have investigated some results of k - kernel symmetric Intuitionistic Fuzzy Matrices with examples. In future, we shall prove some related properties of $g-$inverse of $k-$Kernel Symmetric Intuitionistic Fuzzy Matrices .

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