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PRIME - ANTIMAGIC LABELING OF GRAPHS IN POWER MANAGEMENT SYSTEM

M. C. J. ANAND[1*], G. CHITRA[2], N. SWAMY[3], R. SUDHARANI[4]

ABSTRACT. Prime anti-magic labeling techniques play a vital role in various types of real-world applications in sensor networks, smart city management systems, security models, surveillance systems, and so on. A prime anti-magic technique of graph G is considered as finite, simple, and undirected and also it has p number of vertices and q number of edges. If f is an injection from the vertex set of G to the integers $\{1, 2, \ldots, p\}$ such that for each edge uv, the assignment of adjacent vertices of u and v are relatively prime and the edge induced labeling $f(uv) = f(u) + f(v)$ and hence are all distinct. In this research, some special classes of graphs that admit prime anti-magic technique could play a major role in the water tank power management system.

Keywords: Prime, Anti-magic, Star graph, Tripartite graph, Circular ladder graph, Odd prime antimagic.

AMS Subject Classification: 05C90

1. Introduction

This research deals with all the graphs that are finite, connected, and undirected which have p vertices and q edges. Let $|V(G)| = p$ and $|E(G)| = q$. Various type of graph labeling was studied by Gallian's [2]. The idea of the anti-magic graph was introduced by Hartsfield [3]. Each vertex labeling f of a graph $G = (V, E)$ from $\{0, 1, 2, \ldots |E(G)|\}$ induces an edge labeling f^* where $f^*(e)$ is sum the labels of end vertices of an edge e. Labeling f is called anti-magic if and only if all the edge labeling is paired wisely and distinctly. The representation of prime labeling and its techniques originated and hence it

[1] Department of Mathematics, Mount Carmel College (Autonomous), Affiliated to Bengaluru City University, Bengaluru, Karnataka, India, 560052.
e-mail: arjoemi@gmail.com; ORCID: https://orcid.org/0000-0002-1959-7631.
[*] Corresponding author.
[2] Department of Mathematics, Kalasalingam Academy of Research and Education, Krishnankoil, Srivilliputhur, Tamil Nadu, India, 626128.
e-mail: chitraganesan2008@gmail.com; ORCID: https://orcid.org/0000-0003-4082-8550.
[3] Department of Mathematics, KLE Technological University, Dharwad, Karnataka, India, 580031.
e-mail: nswamy@kletech.ac.in; ORCID: https://orcid.org/0000-0002-3393-6361.
[4] Department of Mathematics, Panimalar Engineering College, Chennai, India - 600123.
e-mail: sudhamats@gmail.com; ORCID: https://orcid.org/0000-0002-1267-9343.
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was discussed [8]. The notation of graph labeling and anti-magic labeling was introduced by Harstfield and Ringel [3]. Various graph labeling is studied from [2]. The representation of prime labeling and its techniques were originated by Roger Entringer and hence it was discussed by Tout [8]. Thirugnanasambandam and Chitra [7], introduced the concept of prime anti-magic labeling. Prime labeling of various types of graphs was studied in [5] and [6] and the condition of odd prime labeling was introduced by Maged Zakaria Youssef [4]. The definition of circular ladder graph and its admits prime condition was explained by Batuwita [1]. Weerarthana [9] introduced some special graphs that are admitted prime labeling. Based on the study [9], we have derived some new results in the concept of prime anti-magic labeling. We follow the notation and terminology of [10].

1.1. **Motivation.** Graph Labeling is one of the famous zones in the applications of mathematics. Graph theory as an individual from the discrete mathematics family has an amazing number of applications to software engineering as well as to numerous different sciences (physical, organic, and social), designing, and trade labeled graphs fill in as valuable apparatuses for an expansive scope of applications. The use of graph labeling in many research fields, for example, coding theory problems, X-beam crystallographic investigation, correspondence network structure, ideal circuit design, basic voltage generator, and added substance number theory. Hence this initiative method helps us to find prime anti-magic labeling for various types of special graphs and which could serve a major role in the power management system.

1.2. **Contribution.** Based on the idea of prime labeling and anti-magic labeling, we extend the concept of prime anti-magic labeling to various kinds of special graphs. In this research work, we focused on the applications of prime anti-magic labeling in power management systems. The discussion of prime anti-magic labeling of complete tripartite graphs, stripe blade graphs, and ladder graphs is the primary goal of this study. We have discussed odd prime anti-magic and consecutive prime anti-magic labeling of the scorpion graph.

Definition 1.1. *A graph G is said to have edge anti-magic vertex labeling if the vertices of G by assigned by positive integers while at the same time, all the edge assignments must be distinct.*

Definition 1.2. *A graph is said to have prime labeling if there exists a one-one mapping $f : V \to \{1, 2, \dots, p\}$ such that for all edge $uv \in E(G)$, g.c.d of $f(u)$ and $f(v)$ is 1. The types of tripartite graphs and prime anti-magic graphs and their technique were studied in* [7] *and* [9].

Definition 1.3. *A complete tripartite graph is a graph of the form $K_{p,q,r}$in which it has p number of vertices in one partite set and another partite set contain q vertices, also r vertices in the remaining partite set where $p, q, r \geq 1$ in which each vertex in one partite set is adjacent to all the vertices in the other two partite sets.*

Definition 1.4. *A stripe blade fan graph is a simple undirected finite graph obtained by replacing every edge of a star graph $K_{1,n}$ by the complete tripartite graph $K_{1,m,1}$ for $m = 2, 3, 4$ and 5 which has $3n + 4$ vertices and $6(n - 1)$ edges and is denoted by $F_{1,3,m}$.*

Definition 1.5. *The Cartesian product $P_n \times P_m$ where $n \geq m$ is called a grid graph, here P_n denotes the path on n vertices. If $m = 2$, then the graph is called a ladder graph. A circular ladder graph (Prism graph) denoted by $CL_{n,n}$- Prism graph has $2n$ vertices and $3n$ edges. The notation of graph theory and its terminologies we had to follow* [10].

The following notations are used in this entire research.

$V(G)$	-	Vertex set of G
$E(G)$	-	Edge set of G
p	-	Number of vertices in G
q	-	Number of edges in G
f	-	Vertex function
f^*	-	Edge function
P_n	-	Path on n vertices
$K_{1,n}$	-	Complete bipartite graph (or) Star graph
$K_{1,m,1}$	-	Complete tripartite graph
$K_{1,3,m}$	-	Stripe blade fan graph
$P_n \times P_m$	-	Grid graph
$P_n \times P_2$	-	Grid graph
$CL_{n,n}$	-	Circular ladder graph (or) Prism graph
$S_{(2p,2q,r)}$	-	Scorpion graph

2. Main Results

In this section, prime labeling of special kinds of graphs was given in the following theorems.

Theorem 2.1. *The graph G which is obtained by all edges in a star graph $K_{1,n}$ replaced by the complete tripartite $K_{1,2,1}$ is a prime anti-magic where $n \geq 1$.*

Proof. Here G is attained by replacing every edge of a star graph $K_{1,n}$ by the complete tripartite graph $K_{1,2,1}$ where $n \geq 1$. Let the vertices of $K_{1,n}$ be $u_0, u_1, u_2, \dots u_n$ with u_0 be the center vertex and every edge u_0u_1 of $K_{1,n}$ is replaced by $K_{1,2,1}$ for $1 \leq i \leq n, n \geq 1$. Then, the new set is $V(H)_1 = \{u_0, u_i, u_{i1}, u_{i2}\}$, for $1 \leq i \leq n$ and $E(H)_1 = \{u_0u_i, u_0u_{i1}, u_{i1}u_i, u_0u_{i2}, u_{i2}u_i\}$ for $1 \leq i \leq n$. Hence $|V(H)_1| = 3n + 1$. The function $f : V(H)_1 \to \{1, 2, \dots, 3n + 1\}$ defined by $f(u_0) = 1, f(u_i) = 3i, f(u_{i1}) = 3i - 1, f(u_{i2}) = 3i + 1$ in all above $1 \leq i \leq n$. In all the above vertex assignments, adjacent vertices of the graph vertex are relatively prime and they are distinct. The edge assignment labels are calculated by $f(uv)$ is equal to the sum of $f(u)$ and $f(v)$ and also all the edge labels are distinct. Hence G has admitted prime anti-magic technique. □

Theorem 2.2. *The graph G which is attained by replacing every edge of a star graph $K_{1,n}$ by the complete tripartite graph $K_{1,3,1}$ is a prime anti-magic where $n \geq 1$.*

Proof. By theorem 2.1, the vertices of a star graph $K_{1,n}$ replaced as $K_{1,3,1}, n \geq 1$. Also $u_0, u_1, u_2, \dots, u_n$ as the vertices of $K_{1,n}$ with u_0 be the center vertex and every edge u_0u_1 of $K_{1,n}$ is replaced by $K_{1,3,1}$ for $1 \leq i \leq n$, where $n \geq 1$. Then, the new vertex set is $V(H)_2 = \{u_0, u_i, u_{i1}, u_{i2}, u_{i3}\}$ for $1 \leq i \leq n$, where $n \geq 1$ and $E(H)_2 = \{u_0u_i, u_0u_{i1}, u_{i1}u_i, u_0u_{i2}, u_{i2}u_i, u_0u_{i3}u_{i3}u_i\}$ for $1 \leq i \leq n, n \geq 1$. Hence $|V(H)_2| = 4n + 1$, where $n \geq 1$.

Vertex Assignment:

$f : V(H)_2 \to \{1, 2, \dots 4n + 1\}$ by $f(u_0) = 1, f(u_i) = 4i - 1, f(u_{i1}) = 4i - 2, f(u_{i2}) = 4i, f(u_{i3}) = 4i + 1$ and also satisfies $1 \leq i \leq n$, where $n \geq 1$. In all the above vertex assignments, it satisfies adjacent vertex in G are relatively prime and they are distinct. The edge numbering is calculated by anti-magic technique and also all edge labels are different. Henceforth G has prime anti-magic labeling and it is proved. □

Theorem 2.3. *Prove that the graph G is constructed by replacing every edge of a star graph $K_{1,n}$ by the complete tripartite graph $K_{1,4,1}$ is a prime anti-magic where $n \geq 1$.*

Proof. Here G is obtained as replacing all the edges of $K_{1,n}$ by $K_{1,4,1}$ where n must be ≥ 1. Take $u_0, u_1, u_2, \ldots u_n$ as the vertices of $K_{1,n}$ with u_0 be the center vertex and every edge u_0u_1 of $K_{1,n}$ is replaced by $K_{1,4,1}$ for $1 \leq i \leq n$, where n should be greater than or equal to 1. Then, the new vertex set is $V(H)_3 = \{u_0, u_i, u_{i1}, u_{i2}, u_{i3}, u_{i4}\}$, for $1 \leq i \leq n$, where $n \geq 1$ and $E(H)_3 = \{u_0u_i, u_0u_{i1}, u_{i1}u_i, u_0u_{i2}, u_{i2}u_i, u_0u_{i3}, u_{i3}u_i, u_0u_{i4}, u_{i4}u_i\}$ for $1 \leq i \leq n, n \geq 1$. Hence $|V(H)_3| = 5n + 1$, where $n \geq 1$. A vertex function $f : V(H)_3 \rightarrow \{1, 2, \ldots 5n + 1\}$ by $f(u_0) = 1$ for all i.

$$f(u_i) = \begin{cases} 5i, & i \equiv 1 \mod (6) \\ 5i-1, & i \equiv 0, 2 \mod (6) \\ 5i-2, & i \equiv 3, 5 \mod (6) \\ 5i-3, & i \equiv 4 \mod (6) \end{cases}$$

$$f(u_{i2}) = \begin{cases} 5i-2, & i \not\equiv 3, 4, 5 \mod (6) \\ 5i-1, & i \equiv 3, 4, 5 \mod (6) \end{cases}$$

$$f(u_{i3}) = \begin{cases} 5i, & i \not\equiv 1 \mod (6) \\ 5i-1, & i \equiv 1 \mod (6) \end{cases}$$

$$f(u_{i1}) = \begin{cases} 5i-3, & i \not\equiv 4 \mod (6) \\ 5i-2, & i \equiv 4 \mod (6) \end{cases}$$

$$f(u_{i4}) = 5i + 1, \text{ for } i \leq i \leq n, \text{ where } n \geq 1.$$

In all the above vertex assignments, they are satisfying the relatively prime condition and they are distinct. To get the edge labels, use the condition of $f(uv) = f(u) + f(v)$ and also all the edge labels are distinct. Hence it is proved as G is a prime anti-magic graph. □

Example 2.1. *Figure 1 shows that the prime anti-magic labeling of the graph which is obtained by replacing every edge of a star graph by $K_{1,3}$ by the complete tripartite graph $K_{1,4,1}$.*

Theorem 2.4. *Prove that every edge of a star graph $K_{1,n}$ is replaced by the complete tripartite graph $K_{1,5,1}$ is a prime anti-magic graph and $n \geq 1$.*

Proof. The proof is the same as in theorem 2.3. Here G is obtained as replacing every edge of $K_{1,n}$ by $K_{1,5,1}$ where $n \geq 1$. Assume $u_0, u_1, u_2, \ldots, u_n$ as the vertices of $K_{1,n}$ with u_0 be the center vertex and every edge u_0u_1 of $K_{1,n}$ is replaced by $K_{1,5,1}$ for $1 \leq i \leq n$, where $n \geq 1$. Then, the new vertex set is $V(H)_4 = \{u_0, u_i, u_{i1}, u_{i2}, u_{i3}, u_{i4}, u_{i5}\}, 1 \leq i \leq n$. Edge set as $E(H)_4 = \{u_0u_i, u_0u_{i1}, u_{i1}u_i, u_0u_{i2}, u_{i2}u_i, u_0u_{i3}, u_{i3}u_i, u_0u_{i4}, u_0u_{i5}, u_{i5}u_i\}, n \geq 1$. Hence $|V(H)_4| = 6n + 1$ and its vertices are assigned by $\{1, 2, \ldots 6n + 1\}$ as $f(u_0) = 1; f(u_i) = 6i-1; f(u_{i1}) = 6i-4; f(u_{i2}) = 6i-3; f(u_{i3}) = 6i-2; f(u_{i4}) = 6i; f(u_{i5}) = 6i+1$ in all the above vertex assignment n should be finite and ≥ 1. □

Theorem 2.5. *The graph G is attaching $K_{1,2}$ at each internal vertex of CL_n is a prime anti-magic graph when $n \geq 3$ and $n \not\equiv 1 \ (mod\, 3)$*

Proof. The internal vertices of the circular graph are denoted as $u_1, u_2, \ldots, u_n$, and its the external vertices of the circular ladder are denoted as $v_1, v_2, \ldots, v_n$. Let v_i, v_{i1} and v_{i2} be the vertices of i^{th} copy of $K_{1,2}$ in which v_i is the central vertex where $1 \leq i \leq n, n \geq 3$ and $n \not\equiv 1(\text{mod}\, 3)$. The vertex set of $G = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n, v_i, v_{i1}, v_{i2}\}$ and edge set of $G = \{u_iu_{i+1}, u_1u_n, v_iv_{i+1}, v_1v_n, v_iv_j\}, 1 \leq i \leq n, n \geq 3$ and $n \not\equiv 1(\text{mod}\ 3), j = 1, 2\}$.

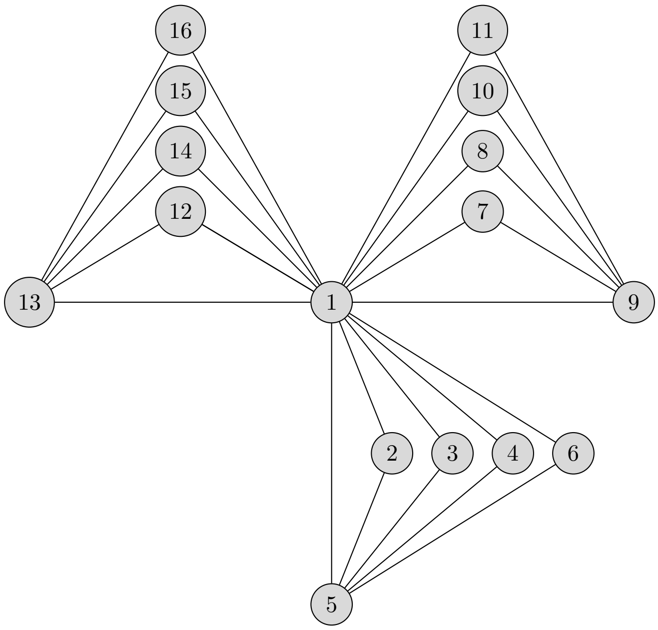

FIGURE 1. Prime anti-magic labeling of the graph

Hence the cardinality of the vertex set G is $4n$ and the size of edge set of G is $5n$. The vertex label is $f : V(G) \to \{1, 2, \dots 4n\}$ such that $f(u_i) = 4i-3; f(v_i) = 4i-1; f(v_{i1}) = 4i-2; f(v_{i2}) = 4i; f(u_{i4}) = 6i; f(u_{i5}) = 6i+1$; and n is greater than or equal to one. Here all the adjacent vertices of u_i's, v_i's and u_iv_i's are relatively prime and hence gcd $(f(u), f(v)) = 1$. Thus G has received prime labeling. The induced edge labeling is identified by the condition that $f(uv) = f(u) + f(v)$ for all u, v in $E(G)$. All the edge assignments must be distinct. Thus G has a prime anti-magic labeling and CL_n is a prime anti-magic graph. □

Example 2.2. *Consider the CL_n graph with $n = 6$. It has 24 vertices and 30 edges. The figure 2 shows the prime anti-magic labeling of CL_6.*

Theorem 2.6. *If n+1 is prime, then scorpion graph $S_{(2p,2q,r)}$ has a prime anti-magic labeling.*

Proof. By using Theorem in 2.1, vertices have prime labeling, which are in the head and body of the scorpion, when $n+1$ is prime. Let the starting vertex in the tail is placed at between n and n+1 vertices, and we claim that the vertex labeling gives prime labeling. □

Theorem 2.7. *If* $2n+1$ *is prime, then the Scorpion graph $S_{(2p,2q,r)}$ has a consecutive cyclic prime anti-magic labeling with the value 1 assigned to vertex V_1, where and $n = p + q$ and $p \geq 1, q \geq 2, r \geq 2$.*

Proof. This proof as in theorem 2.6. □

3. Applications in Water Management System

The prime anti-magic graph could serve as a water power management router for a particular building or city. The center vertex serves as a hub for the total system. All the edges are could serve as a router from the center vertex. All the remaining vertices serve

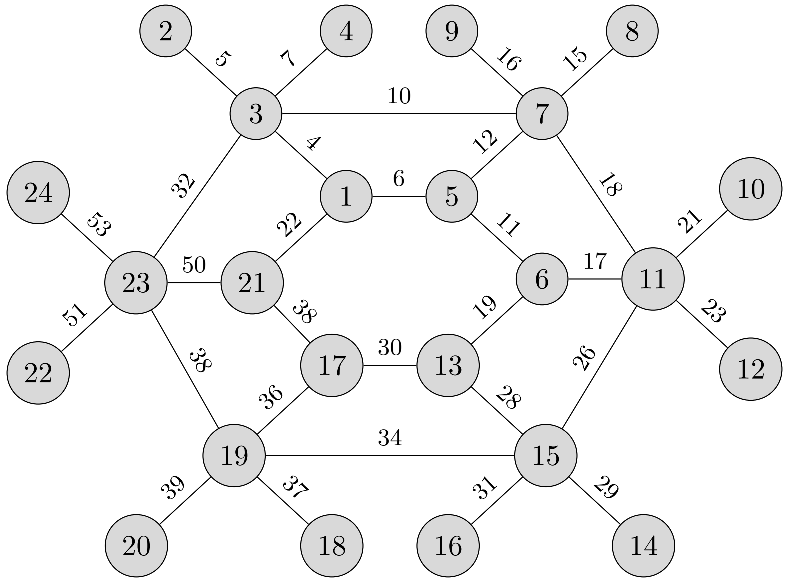

FIGURE 2. Prime anti-magic labeling of the graph CL_6

as a power plant connected to the center of the hub. The condition of prime anti-magic plays a role of control the water level of the building.

3.1. **Odd Prime Anti-Magic Labeling.** Here we have discussed the definition of odd prime anti-magic labeling and the graphs which are satisfying the odd prime anti-magic condition.

Definition 3.1. *A graph with p vertices and q edges is said to have odd prime anti-magic labeling if the vertex function* $f : V \to \{1, 3, 5, \ldots 2p-1\}$ *such that for each edge* $uv \in E(G), (f(u), f(v)) = 1$ *and the induced edge labeling* $f^* : E \to N$ *is defined by* $f^*(uv) = f(u) + f(v)$ *and also all the edge labeling must be distinct.*

Theorem 3.1. *The graph G obtained by replacing every edge of a star graph* $K_{1,n}$ *by the complete tripartite graph* $K_{1,3,1}$ *is odd prime anti-magic where* $n \geq 1$.

Proof. Let G be a graph obtained by replacing every edge of a star graph $K_{1,n}$ by the tripartite graph $K_{1,3,1}$ where $n \geq 1$. Let $u_0, u_1, u_2, \ldots u_n$ be the vertices of $K_{1,n}$ with u_0 be the center vertex and every edge u_0u_1 of $K_{1,n}$ is replaced by $K_{1,3,1}$ for $1 \leq i \leq n$, where $n \geq 1$. Then, the new vertex set is $V(H)_2 = \{u_0, u_i, u_{i1}, u_{i2}, u_{i3}\}$ for $1 \leq i \leq n$, where $n \geq 1$ and $E(H)_2 = \{u_0u_i, u_0u_{i1}, u_{i1}u_i, u_0u_{i2}, u_{i2}u_i, u_0u_{i3}, u_{i3}u_i\}$ for $1 \leq i \leq n, n \geq 1$. Hence $|V(H)_2| = 4n+1$, where $n \geq 1$. Here $f(u_i) = 2i-1$ for $i = 1, 2, \ldots 4n+1$. Note that, in all the vertex assignments are satisfying the condition of prime. Hence

$$gcd(f(u_0), f(u_i)) = gcd(1, f(u_i)) = 1$$
$$gcd(f(u_0), f(u_{i1})) = gcd(1, f(u_{i1})) = 1$$
$$gcd(f(u_0), f(u_{i2})) = gcd(1, f(u_{i2})) = 1$$

Thus, in all the above $i, 1 \leq i \leq n$. Therefore, vertex labels are distinct. Thus, the labeling defined above gives odd prime labeling for the given graph. Also, the distinct edge labeling is satisfying the condition that $f^*(uv) = f(u) + f(v)$. Hence it is proved. □

Example 3.1. *The figure 3 exhibits the odd prime anti-magic labeling of* $K_{1,5}$ *by* $K_{1,2,1}$.

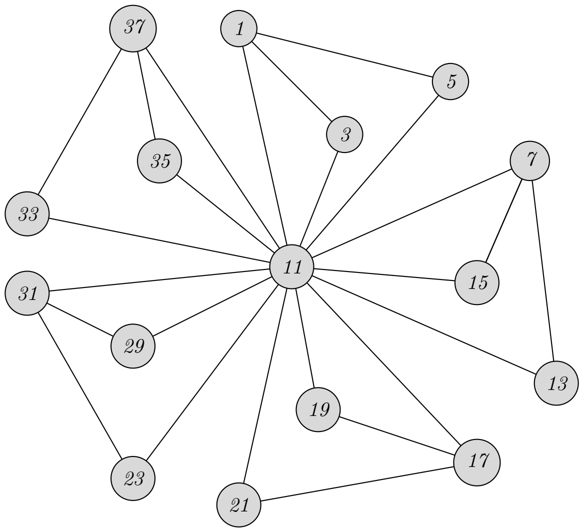

FIGURE 3. Odd prime anti-magic labeling of $K_{1,5}$ by $K_{1,2,1}$

Theorem 3.2. *The graph G obtained by replacing every edge of a star graph $K_{1,n}$ by the complete tripartite graph $K_{1,3,1}$ is an odd prime anti-magic graph, where $n \geq 1$.*

Proof. This proof is similar to the proof of theorem 3.1. However, the function is defined as follows for these types of graphs. Let G be a graph obtained by replacing every edge of a star graph $K_{1,n}$ by $K_{1,3,1}$, where $n \geq 1$. Let the vertices of $K_{1,n}$ be $u_0, u_1, u_2, \ldots u_n$. Then, the new vertex set is $V(H)_2 = \{u_0, u_i, u_{i1}, u_{i2}, u_{i3}\}$, for $1 \leq i \leq n$, where $n \geq 1$ and $E(H)_2 = \{u_0u_i, u_0u_{i1}, u_{i1}u_i, u_0u_{i2}, u_{i2}u_i, u_0u_{i3}, u_{i3}u_i\}$ for $1 \leq i \leq n, n \geq 1$. Hence $|V(H)_2| = 4n + 1$, where $n \geq 1$. Define a function $f : V(H)_2 \to \{1, 3, 5, \ldots 2(4n + 1) - 1\}$ by $f(u_i) = 2i - 1$ for $i = 1, 2, \ldots 4n + 1$. By applying theorem 3.1, we get the proof. □

Theorem 3.3. *The graph G obtained by replacing every edge of a star graph $K_{1,n}$ by the complete tripartite graph $K_{1,4,1}$ is an odd prime anti-magic graph, where $n \geq 1$.*

Proof. Let G be a graph obtained by replacing every edge of a star graph $K_{1,n}$ by the complete tripartite graph $K_{1,4,1}$, where $n \geq 1$. Let $u_0, u_1, u_2, \ldots, u_n$ be the vertices of $K_{1,n}$ with u_0 be the center vertex and every edge u_0u_1 of $K_{1,n}$ is replaced by $K_{1,4,1}$ for $1 \leq i \leq n$, where $n \geq 1$. Then, the new vertex set is $V(H)_3 = \{u_0, u_i, u_{i1}, u_{i2}, u_{i3}, u_{i4}\}$, for $1 \leq i \leq n$, where $n \geq 1$ and $E(H)_3 = \{u_0u_i, u_0u_{i1}, u_{i1}u_i, u_0u_{i2}, u_{i2}u_i, u_0u_{i3}, u_{i3}u_i, u_0u_{i4}, u_{i4}u_i\}$ for $1 \leq i \leq n, n \geq 1$. Hence $|V(H)_3| = 5n + 1$, where $n \geq 1$. Define a function $f : V(H)_3 \to \{1, 3, 5, \ldots 2(5n + 1) - 1\}$ by $f(u_i) = 2i - 1$ for $i = 1, 2, \ldots 5n + 1$. By applying theorem 3.1, we get the proof. □

Theorem 3.4. *The graph G obtained by replacing every edge of a star graph $K_{1,n}$ by the complete tripartite graph $K_{1,5,1}$ is an odd prime anti-magic graph, where $n \geq 1$.*

Proof. By theorem 2.4 and 3.1, we have received its vertex and edge labeling. Thus proved. □

Theorem 3.5. *The graph G obtained by attaching $K_{1,2}$ at each internal vertex of CL_n is odd prime anti-magic graph when $n \geq 3$ and $n \not\equiv 1\,(mod\,3)$.*

Proof. Here the internal vertices of the circular graph are denoted as $u_1, u_2, \ldots, u_n$ and the external vertices of the circular ladder are denoted as $v_1, v_2, \ldots v_n$. Let v_i, v_{i1} and v_{i2} be

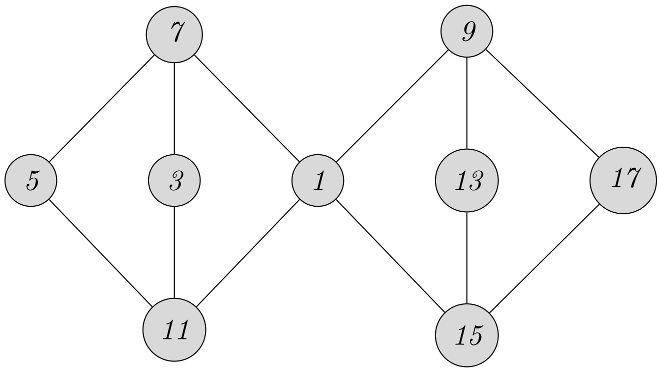

FIGURE 4. Odd Prime anti - magic labeling of $G(2,2)$

the vertices of i^{th} copy of $K_{1,2}$ in which v_i is the central vertex where $1 \leq i \leq n$, $n \geq 3$ and $n \not\equiv 1(\mod 3)$. The vertex set of $G = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n, v_i, v_{i1}, v_{i2}\}$ and edge set of $G = \{u_iu_{i+1}, u_1u_n, v_iv_{i+1}, v_1v_n, v_iv_j, 1 \leq i \leq n, n \geq 3 \text{ and } n \not\equiv 1 \pmod 3), j = 1, 2\}$. Hence the cardinality of vertex set G is $4n$ and the size of edge set of G is $5n$. The vertex assignment is defined by $f : V(G) \to \{1, 3, 5 \ldots 2(4n) - 1\}$ such that $f(u_i) = 2i - 1$ for $i = 1, 2, \ldots 4n$. Hence by theorem 3.1, we have received its edge labeling. □

Theorem 3.6. *If $n+1$ is prime, then scorpion graph $S_{(2p,2q,r)}$ has odd prime anti-magic labeling.*

Proof. By using Theorem in 2.6, vertices have odd prime labeling, which is in the head and body of the scorpion, when $n+1$ is prime. Let the starting vertex in the tail is placed at between n and $n+1$ vertices, and we claim that the vertex labeling gives odd prime labeling. □

Theorem 3.7. *If $2n+1$ is prime, then the Scorpion graph $S_{(2p,2q,r)}$ has a consecutive cyclic prime anti-magic labeling with the value 1 assigned to vertex V_1, where and $n = p+q$ and $p \geq 1, q \geq 2, r \geq 2$.*

Proof. This proof as in theorem 3.6. □

Definition 3.2. *For $n \geq 2$, let G_n be the graph obtained from a wheel graph of order $2n+1$ by deleting n spokes where no two of the spokes are consecutive. Such a graph is called a gear graph. The core of the wheel is also called the core of the graph.*

Theorem 3.8. *$G(n_1, n_2)$ admits odd prime anti-magic labeling where $n_1 = n_2$.*

Proof. Let $G = G(n_1, n_2)$. It has $2n_1 + 1$ vertices. The vertex assignment is defined by $f : V(G) \to \{1, 3, 5 \ldots 2(2n_1 + 1) - 1\}$ such that $f(u_i) = 2i - 1$ for $i = 1, 2, \ldots (2n_1 + 1)$. Therefore, vertex labels are distinct. Thus, the labeling defined above gives odd prime labeling for the given graph. Also the distinct edge labeling is satisfying the condition that $f^*(uv) = f(u) + f(v)$. Hence it is proved. □

Example 3.2. *The figure 4 is the odd prime anti-magic labeling of $G(2,2)$.*

4. CONCLUSIONS

Based on the idea of prime labeling and anti-magic labeling, we extended the concept of prime anti-magic labeling to various kinds of special graphs. The prime anti-magic labeling of complete tripartite graphs, stripe blade graphs, and ladder graphs are the new findings of this present study and also we have discussed odd prime anti-magic and consecutive prime anti-magic labeling of the scorpion graph. Mainly we focused on the applications of prime anti-magic labeling in power management systems.

5. Future Directions

We can extend this prime anti-magic labeling technique in the applications of sensor networks and on these effective anti-magic labeling methods for use with sensor network applications. We also extend this idea to different kinds of graphs.

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Dr. M. Clement Joe Anand is working as an Assistant Professor in the Department of Mathematics, Mount Carmel College (Autonomous), Bengaluru, Karnataka, India. Previously, he was an Associate Professor at MVJ College of Engineering (Autonomous), Bengaluru, and Hindustan Institute of Technology and Science (Deemed to be University), Chennai; and a Lecturer at Loyola College, Chennai. He is a Member of the Indian Mathematical Society, the Ramanujan Mathematical Society, and IAENG.

Dr. G. Chitra is currently an Assistant Professor in the Department of Mathematics, Kalasalingam Academy of Research and Education, Krishnankoil. She is specialized in the area of Graph Theory. She received her Ph.D. degree from Thiruvalluvar University, Vellore. Her current research focuses on the application of Graph Theory in Sensor Networks.

Dr. Narayan Swamy is working as an Assistant Professor in the Department of Mathematics, KLE's Technological University, Hubballi. He obtained his Master of Science in Mathematics (2007) and Doctor of Philosophy (2014) degrees from Karnatak University. Previously, he worked in B.V. Bhoomaraddi College of Engineering. His research areas are golden graphs, eigenvalues of a graph, Zagreb indices, cluster graphs, degree, adjacency polynomials of a graph, and cyber security threats using artificial intelligence.

Ms. Sudharani R is working as an Assistant Professor in the Department of Mathematics at Panimalar Engineering College, Chennai, Tamilnadu. She completed her Master's degree in Mathematics at Stella Maris College, Chennai, and a degree awarded from the University of Madras, Chennai, India. She has a Master of Philosophy from Alagappa University, Karaikudi. She has completed a Bachelor of Education from N.K.T. National College of Education for Women, Chennai, Tamilnadu. Her area of interest includes Fuzzy Sets and Logic.