TWMS J. App. and Eng. Math. V.14, N.4, 2024, pp. 1722-1735

MAXWELL-CATTANEO LAW OF HEAT CONDUCTION THROUGH POROUS FERROCONVECTION WITH MAGNETIC FIELD DEPENDENT VISCOSITY

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Abstract. The problem of convective instability in a ferromagnetic fluid saturated porous medium with magnetic field dependent (MFD) viscosity and Maxwell-Cattaneo law is studied using the method of small perturbation. Darcy model is used to describe the fluid motion. The horizontal porous layer is heated from below and cooled from above. Convection is caused by a spatial variation in magnetization which is induced when the magnetization of the ferrofluid is a function of temperature. The non-classical Maxwell-Cattaneo heat flux law involves a wave type of heat transport and does not suffer from the physically unacceptable drawback of infinite heat propagation speed. For a fluid layer contained between magnetically responding and isothermal boundaries, approximate solutions for stationary instability are obtained by using the higher order Galerkin technique. It is shown that the ferromagnetic fluid is distinctly influenced by the effect of magnetic forces and is prone to instability in the presence of second sound and MFD viscosity. It is found that the second sound mechanism works in tandem with the effect of magnetic forces. It is also established that the effects of second sound and MFD viscosity are mutually antagonistic towards influencing the stability of the system and that an increase in MFD viscosity attenuates the threshold of porous ferroconvection.

Keywords: Ferrofluid, MFD Viscosity, Porous Media, Second Sound.

AMS Subject Classification: 47.65.Cb, 83.80. Gv; 47.56. +r; 02.70.Hm;

1. Introduction

Ferromagnetic fluids are formed by suspending submicron sized particles of magnetite in a carrier medium such as kerosene, heptane or water. To prevent the particles from

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§ Manuscript received: September 23, 2022; accepted: July 20, 2023.

agglomerating in the presence of a magnetic field they are surrounded by a surfactant such as oleic acid. The combinations of the short range repulsion due to the surfactant and the thermal agitation yields a material which behaves as a continuum and can experience forces due to magnetic polarization. The magnetic fluids are usually good insulators and forces due to interaction of magnetic fields with currents of free charge, such as found in magnetohydrodynamics, are negligible.

Gupta and Gupta [1] investigated the ferroconvection problem with centrifugal acceleration. It is proved that oscillatory ferroconvection is possible as long as the Prandtl number is less than unity. Bashtovoy et al., [2] developed magnetic fluids for a variety of applications including novel energy conversion devices, levitation devices and rotating seals. Russell et al.,[3] extended the pioneering contribution to deal with large wave number ferroconvection. Aniss et al., [4] investigated the influence of a time-sinusoidal magnetic field on the onset of convection in a horizontal magnetic fluid layer heated from above. In the cases of free-free and rigid-rigid boundaries, the convective threshold is calculated using the Floquet theory. The possibility of a conflict between the harmonic and sub-harmonic modes at the beginning of convection is discussed. When there is a vertical uniform magnetic field, Abraham [5] investigated the RBC problem in a micropolar ferromagnetic fluid layer analytically. It is shown that the micropolar ferromagnetic fluid layer is more stable than the conventional Newtonian ferromagnetic fluid layer when heated from below.

Second sound is a more recent phenomenon involving the propagation of heat as a temperature wave. Second sound is a quantum mechanical phenomenon in which heat transfer occurs by wave-like motion rather than by a more usual mechanism of diffusion. Heat takes the place of pressure in normal sound waves. This leads to a very high thermal conductivity. It is known as "second sound" because the wave motion of heat is similar to the propagation of sound in air. Normal sound waves are fluctuations in the density of molecules in a substance; second sound waves are fluctuations in the density of particle-like thermal excitations. Thermal relaxation effects may be important in biological tissues, in phase changes, in nuclear reactor technology and in surgical procedures and the like.

Considering Bénard and Marangoni problems, Lebon and Cloot [6] investigated the significance of replacing the classical Fourier law on heat conduction with the Maxwell-Cattaneo law. Straughan and Franchi [7] studied the impact of thermal waves on the onset of convective instability in a Newtonian fluid enclosed in a horizontal layer of finite thickness. Boundaries that are stress-free have been considered. It is discovered that the Bénard problem for a Maxwell-Cattaneo fluid is always less stable than the classical one and that overstability only occurs in the heated below case. Pressure and density variations propagate with very small temperature variations in ordinary or first sound; in second sound, temperature variations spread without significantly changing the density or pressure. Recently, it has been realized that this is not just a low temperature phenomenon, but has important applications in such fields as skin burns, phase changes, biological materials and in nanofluids (Straughan [8]). The impact of propagating thermal waves at the commencement of electroconvection in a horizontal layer of dielectric fluid was qualitatively examined by Maruthamanikandan and Smita [9]. The Cattaneo heat flow model is used to implement the linear stability analysis, which is based on the normal mode technique. Instability caused by the Maxwell-Cattaneo heat flux and the internal heat generation/absorption of the non-Newtonian Casson dielectric fluid is studied by Mahanthesh et al., [10].

Lapwood [11] investigated the possibility of convective flow in a layer of fluid subjected to a vertical temperature gradient in a porous medium. Wooding [12] investigated the conditions for the occurrence of Rayleigh instability of a thermal boundary layer. The external magnetic field hinders the free rotation of the magnetic particles and thus increases the viscosity of magnetic fluid is known as magnetorheological effect.The contemporary applications of the magnetorheological effect include dampers, brakes, pumps, clutches, valves, robotic control systems etc. Ramanathan and Suresh [13] examined the impact of magnetic field-dependent viscosity on the onset of ferroconvection in an anisotropic, densely packed porous medium. The distribution is assumed to be anisotropic along the vertical axis and isotropic along the horizontal axis. The stability criterion, which includes the critical centrifugal Rayleigh number, the critical wave number and the flow characteristics at the thresholds was studied by Saravanan and Yamaguchi [14].

To understand the control over convection, Saravanan [15] studied how the magnetic field affects the onset of centrifugal convection in an anisotropic porous medium saturated with magnetic fluid. Sekar and Murugan [16] used the Darcy model to investigate the Soret-driven thermoconvective instability of a ferromagnetic fluid layer heated from below and salted from above while rotating a densely packed anisotropic porous medium with magnetic field dependent (MFD) viscosity. Ramachandramurthy et al.,[17] investigated convective instability and heat transfer in a temperature-sensitive rotating Newtonian liquid with a volumetric heat source and sink, as well as linear and weak nonlinear stability. When a gravitational field is present, Vidya Shree et al.,[18] investigated the impact of MFD viscosity on the ferroconvective instability of a fluid-saturated porous medium in the presence of a varying gravitational field.

Rudresha et al.,[19, 20, 21] studied the effect of electric field modulation in a dielectric fluid saturating porous medium using a regular perturbation approach. The stability of the system characterized by a correction Rayleigh number is computed as a function of thermal, electric, and porous parameters, and the frequency of electric field modulation. It is found that the onset of electroconvection can be delayed or advanced by the presence of these parameters. The effect of various parameters is found to be significant for moderate values of the frequency of electric field modulation. The system stability is strongly influenced by the couple stress parameter, and the Prandtl number diminishes the stabilizing impact. The stability of a horizontal sparsely packed porous layer of a ferromagnetic fluid heated from below is examined by Balaji et al.,[22, 23] when the fluid layer is subjected to time-dependent magnetic field modulation. The effects of the oscillating magnetic field are treated by a perturbation expansion in powers of the amplitude of the applied magnetic field. The onset criterion is derived under the assumption that the principle of exchange of stabilities holds true and couple stress parameter has a stabilizing effect on the system. To improve the heat transfer connected with the flow system, the flow of numerous fluids of various types has been addressed ([24, 25, 26, 27, 28, 29, 30, 31]).

The purpose of this paper is to investigate the qualitative effect of the Maxwell-Cattaneo law in a ferrofluid layer with magnetic field-dependent viscosity in the presence of a uniform vertical magnetic field and porous medium. The understanding of the control of ferroconvection by means of variable viscosity is useful in many heat transfer problems, particularly in materials science processing. The resulting eigenvalue problem is numerically solved for magnetically responding isothermal boundary conditions using the Galerkin method.

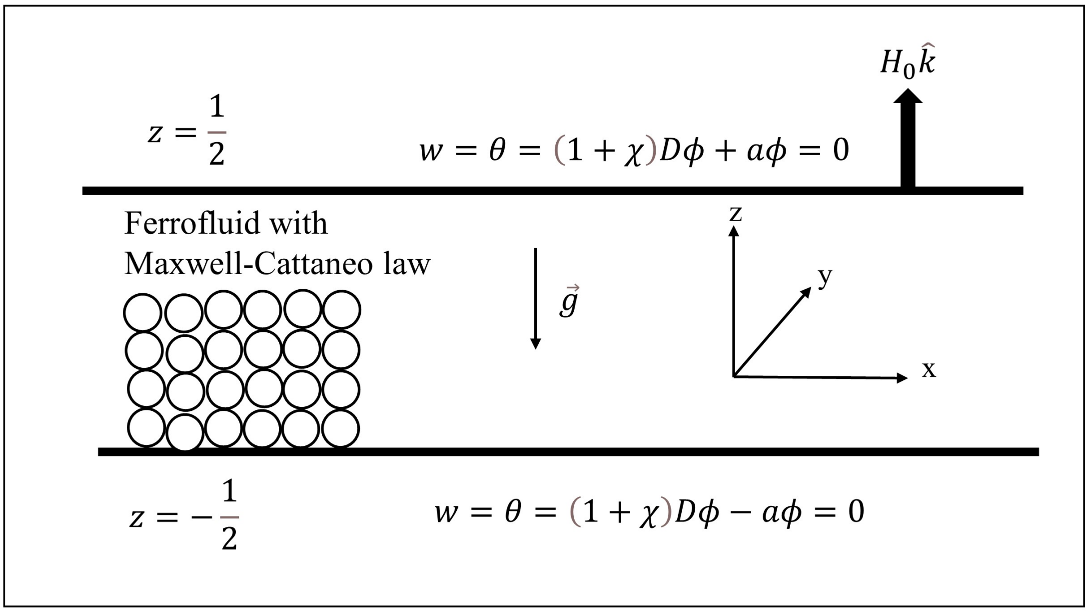

Figure 1. Schematic of the Problem.

2. Mathematical formulation

We consider a horizontal layer of Boussinesq ferromagnetic fluid saturated a porous layer of depth d, which is heated from below and cooled from above and confined between two parallel planes $z = -\frac{d}{2}$ and $z = \frac{d}{2}$ maintained at different uniform temperatures with a temperature gradient ΔT across the fluid layer. A Cartesian frame of reference is chosen with x and y axes at the lower boundary plane and z-axis acting vertically upwards. The solid temperature equation is modified to allow heat transfer via the Cattaneo heat flux theory, while the usual Fourier heat transfer law is used with regard to the heat transfer in ferrofluids. An inverse linear relationship is considered for the viscosity variation when the magnetic field is dependent on the viscosity of the magnetic fluid. It should be remarked that the use of realistic flow boundary conditions does not qualitatively, but quantitatively change the critical values (Chandrasekhar [32]). Similarly the use of realistic boundary conditions on the magnetic potential is of only very limited impact on the stability of the system.

The basic equations governing the flow of an incompressible ferrofluid saturating a layer of Darcy porous medium with Cattaneo effects in the solid are as follows:

The general form of the continuity equation is

$$\frac{D\rho}{Dt} + \rho\left(\nabla \cdot \vec{q}\right) = 0. \tag{1}$$

Equation (1), for a fluid with Boussinesq approximation, reduces to

$$\nabla \cdot \vec{q} = 0. \tag{2}$$

The momentum equation for a ferromagnetic fluid under the Boussinesq approximation with variable viscosity and the Darcy law is

$$\rho_o \left[\frac{1}{\varepsilon} \frac{\partial \vec{q}}{\partial t} + \frac{1}{\varepsilon^2} (\vec{q} \cdot \nabla) \vec{q} \right] = -\nabla p + \rho \vec{g} - \frac{\mu_f}{k} \vec{q} + \nabla \cdot \left(\vec{H} \vec{B} \right) \tag{3}$$

where $\vec{q} = (u, v, w)$ is the fluid velocity, ρ is a reference density, ε is the porosity of the porous medium, p is the pressure, $\vec{H}$ is the magnetic field, $\vec{B}$ is the magnetic induction, μ_f is the dynamic viscosity, k is the permeability of the porous medium. The left side of equation (3) represents the rate of change of momentum per unit volume. The four terms on the right side represent, respectively, the pressure force due to normal stress, body force due to gravity, Darcy resistance due to porous medium and a pondermotive force arising due to the magnetization of the ferromagnetic fluid.

The heat transport equation for the considered ferromagnetic fluid which obeys modified Fourier law is

$$\begin{aligned} \varepsilon \left[\rho_o C_{V,H} - \mu_o \vec{H} \cdot \left(\frac{\partial \vec{M}}{\partial T} \right)_{V,H} \right] \left[\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T \right] + (1 - \varepsilon) (\rho_o C)_s \frac{\partial T}{\partial t} \\ + \mu_o T \left(\frac{\partial \vec{M}}{\partial T} \right)_{V,H} \cdot \left[\frac{\partial \vec{H}}{\partial t} + (\vec{q} \cdot \nabla) \vec{H} \right] = -\nabla \cdot \vec{Q} \end{aligned} \tag{4}$$

where μ_o is the magnetic permeability, T is the temperature, $\vec{M}$ is the magnetization, C is the specific heat, $C_{V,H}$ is the specific heat at constant volume and constant magnetic field. Here the subscript s represents the solid. The Maxwell-Cattaneo heat flux equation is

$$\tau \left[\frac{\partial \vec{Q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{Q} + \vec{\omega} \times \vec{Q} \right] = -\vec{Q} - k_1 \nabla T \tag{5}$$

where k_1 is the thermal conductivity, τ is the constant relaxation time, $\vec{Q}$ is the heat flux vector, $\vec{\omega} = \frac{1}{2} (\nabla \times \vec{q})$.
The density is a linear function of temperature and the same is given by

$$\rho = \rho_o [1 - \alpha (T - T_a)] \tag{6}$$

where α is the coefficient of thermal expansion and $T_a = \frac{T_0 + T_1}{2}$ is the average temperature.

The ferromagnetic fluid considered typically of a suspension of submicron sized particles of magnetite in a nonmagnetic liquid carrier. In addition, the ferromagnetic fluid obeys Maxwell's equations. In writing Maxwell's equations, one has to keep in mind that the conductivity of ferromagnetic fluid is very small. Therefore, we assume that the fluid is electrically nonconducting with the current density zero and hence the magnetic field equations, neglecting the displacement current, are [33, 34, 35, 36, 37].

$$\nabla \cdot \vec{B} = 0 \tag{7}$$

$$\nabla \times \vec{H} = \vec{0}. \tag{8}$$

Additionally, $\vec{B}$, $\vec{M}$ and $\vec{H}$ are connected through the relationship

$$\vec{B} = \mu_o\left(\vec{H} + \vec{M}\right). \tag{9}$$

We assume that the magnetization is aligned with the magnetic field, but allow a dependence on the magnitude of magnetic field as well as the temperature in the form

$$\vec{M} = \frac{\vec{H}}{H} M\left(H, T\right). \tag{10}$$

The magnetic equation of state is linearized about H_0 and T_a to become

$$M = M_o + \chi\left(H - H_o\right) - K\left(T - T_a\right) \tag{11}$$

where $\chi = \left(\frac{\partial M}{\partial H}\right)_{H_0, T_a}$ is the magnetic susceptibility, $K = -\left(\frac{\partial M}{\partial T}\right)_{H_0, T_a}$ is the pyromagnetic coefficient and $M_0 = M\left(H_0, T_a\right)$.

2.1. **Basic State.** The quiescent basic state is represented as

$$\begin{gathered}\frac{\partial}{\partial t} = 0;\ \vec{q} = \vec{q}_b\left(z\right) = 0;\ T = T_b\left(z\right);\ p = p_b\left(z\right);\ \rho = \rho_b\left(z\right);\\ \vec{H} = \left(0, 0, H_b\left(z\right)\right);\ \vec{M} = \left(0, 0, M_b\left(z\right)\right);\ \vec{B} = \left(0, 0, B_b(z)\right);\ \mu_f = \mu_{f_b}(z).\end{gathered} \tag{12}$$

In the basic state, the temperature, density, magnetic induction and magnetization equations are as follows [18, 39]

$$\begin{gathered}T_b = T_a - \beta z,\ \rho_b = \rho_0\left[1 + \alpha\beta z\right],\ H_b = H_0 - \frac{K\beta z}{1+\chi},\\ M_b = M_0 + \frac{K\beta z}{1+\chi},\ \mu_{f_b}\left(H\right) = \frac{\mu_1}{1 + \frac{\delta K\beta z}{1+\chi}}\end{gathered} \tag{13}$$

where $\beta = \frac{\Delta T}{d}$ is the temperature gradient and the subscript b denotes the basic state. It may be noted that the fluid and solid phases have the same temperatures at the bounding surfaces of the porous layer.

2.2. **Linear Stability Theory.** To investigate the conditions under which the quiescent solution is stable against small disturbances, we consider a perturbed state in the form

$$\begin{gathered}\vec{q} = \vec{q}_b + \vec{q}',\ p = p_b + p',\ T = T_b + T',\ \mu_f = \mu_{f_b} + {\mu'}_f,\\ \vec{M} = \vec{M}_b + \vec{M}',\ \phi = \phi_b + \phi',\ \vec{H} = \vec{H}_b + \vec{H}',\ \rho = \rho_b + \rho'\end{gathered} \tag{14}$$

Substituting (14) into equations (2) through (11), linearizing, eliminating the pressure term by taking curl twice, the following equations are obtained

$$\frac{\rho_0}{\varepsilon}\frac{\partial}{\partial t}\left(\nabla^2 W'\right) = \alpha\rho_0 g\nabla_1^2 T' - \frac{\mu_{fb}}{k}\left(\nabla^2 W'\right) + \frac{\mu_0 K^2\beta}{1+\chi}\nabla_1^2 T' - \mu_0 K\beta\frac{\partial}{\partial z}\nabla_1^2 T' \tag{15}$$

$$\left(1 + \tau\frac{\partial}{\partial t}\right)\begin{bmatrix}(\rho_0 c)_1\dfrac{\partial T'}{\partial t} - \mu_0 K T_a\dfrac{\partial}{\partial t}\left(\dfrac{\partial\phi'}{\partial z}\right)\\ -\left\{(\rho_0 c)_2 - \dfrac{\mu_0 K^2 T_a}{1+\chi}\right\}\beta w'\end{bmatrix} = k_1\nabla^2 T' - \frac{\tau k_1\beta}{2}\nabla^2 W' \tag{16}$$

$$\left(1 + \frac{M_0}{H_0}\right)\nabla_1^2\phi' + \left(1+\chi\right)\frac{\partial^2\phi'}{\partial z^2} - K\frac{\partial T'}{\partial z}. \tag{17}$$

The normal mode solution is adopted and same takes the form

$$\begin{bmatrix} W' \\ T' \\ \phi' \end{bmatrix} = \begin{bmatrix} W(z) \\ \theta(z) \\ \phi(z) \end{bmatrix} e^{i(lx+my)+\sigma t} \tag{18}$$

Substitution of (18) into equations (15) through (17) leads to

$$\begin{aligned} \frac{\rho}{\varepsilon}\sigma\left(D^2-k_h^2\right)W = -\alpha\rho_0 g k_h^2\theta - \frac{\mu_1}{k\left(1+\frac{\delta K\beta z}{1+\chi}\right)}\left(D^2-k_h^2\right)W \\ -\frac{\mu_0 K^2\beta}{1+\chi}k_h^2\theta + \mu_0 K\beta k_h^2 D\phi \end{aligned} \tag{19}$$

$$(1+\tau\sigma)\begin{bmatrix} (\rho_0 C)_1\sigma\theta - \mu_0 K T_a\sigma D\phi \\ -\left\{(\rho_0 C)_2 - \frac{\mu_0 K^2 T_a}{1+\chi}\right\}\beta W \end{bmatrix} = k_1\left(D^2-k_h^2\right)\theta - \frac{\tau k_1\beta}{2}\left(D^2-k_h^2\right)W \tag{20}$$

$$(1+\chi)D^2\phi - \left(1+\frac{M_0}{H_0}\right)k_h^2\phi - KD\theta = 0 \tag{21}$$

where $D=\frac{d}{dz}$ and $k_h^2=l^2+m^2$ is the overall horizontal wavenumber. Non-dimensionalizing equations (19), (20) and (21) using the scaling

$$W^*=\frac{Wd}{\kappa};\ \ \theta^*=\frac{\theta}{\beta d};\ \ \phi^*=\frac{\phi(1+\chi)}{K\beta d^2};\ \ \sigma^*=\frac{\sigma d^2}{\kappa};\ \ z^*=\left(\frac{z}{d}\right);\ \ a=k_h d \tag{22}$$

we obtain

$$\frac{\sigma}{Va}\left(D^2-a^2\right)W = -(R+N)a^2\theta - g(z)\left(D^2-a^2\right)W + Na^2D\phi \tag{23}$$

$$(1+2C\sigma)(\lambda\sigma\theta - W) + C\left(D^2-a^2\right)W - \left(D^2-a^2\right)\theta = 0 \tag{24}$$

$$\left(D^2-M_3a^2\right)\phi - D\phi = 0 \tag{25}$$

where $Va=\frac{\varepsilon\nu d^2}{\kappa k}$ is the Vadasz number, $R=\frac{\alpha g\beta k d^2}{\nu\kappa}$ is the Darcy Rayleigh number, $N=\frac{\mu_0 K^2\beta^2 d^2 k}{\mu_1\kappa(1+\chi)}$ is the Darcy-Magnetic Rayleigh number, $C=\frac{\tau\kappa}{2d^2}$ is the Cattaneo number, $M_2=\frac{\mu_0 K^2 T_a}{(1+\chi)(\rho_0 C)_2}$ is the magnetization parameter, $M_3=\frac{M_0+H_0}{(1+\chi)H_0}$ is the nonlinearity of Magnetization, $V=\frac{\delta K\beta d}{1+\chi}$ variable viscosity parameter and $g(z)=(1-Vz)^{-1}$.

The relevant boundary conditions are [33, 40, 45]

$$\begin{aligned} W = \theta = 0 \quad &\text{at } z = \pm\frac{1}{2} \\ D\phi - \frac{a\phi}{1+\chi} = 0 \quad &\text{at } z = -\frac{1}{2} \\ D\phi + \frac{a\phi}{1+\chi} = 0 \quad &\text{at } z = +\frac{1}{2}. \end{aligned} \tag{26}$$

2.3. **Stationary Instability.** The stationary instability related simultaneous differential equations are found to be (with $\sigma = 0$)

$$g(z)\left(D^2 - a^2\right)W - (R+N)\,a^2\theta + Na^2 D\phi = 0 \tag{27}$$

$$W - C\left(D^2 - a^2\right)W + \left(D^2 - a^2\right)\theta = 0 \tag{28}$$

$$\left(D^2 - M_3 a^2\right)\phi - D\phi = 0 \tag{29}$$

2.4. **Numerical solution.** Equations (27)-(29) together with the corresponding boundary conditions (26) constitute an eigenvalue problem with R as the eigenvalue. The eigenvalue problem is solved numerically using the Galerkin technique. The Galerkin method is used to solve the eigenvalue problem as explained in the book by Finlayson [38]. In this method, the test (weighted) functions are the same as the base (trial) functions. Accordingly, W, θ and ϕ are written as

$$W_i = \left(z^2 - \frac{1}{4}\right)^i, \; \theta_i = \left(z^2 - \frac{1}{4}\right)^i \text{ and } \phi_i = z^{2i-1} \tag{30}$$

The trial functions W_i, θ_i and ϕ_i are usually chosen to satisfy the corresponding boundary conditions, but not the differential equations.

3. Results and discussion

In this paper we study the effect of Maxwell-Cattaneo ferroconvection in a densely packed porous medium with variable viscosity. The dynamic viscosity is taken to be a function of the strength of the magnetic field. The results are obtained exactly for magnetically responding, isothermal boundaries. Critical Rayleigh numbers and the corresponding wave numbers are obtained using the higher order Galerkin technique. The Galerkin method yields an eigenvalue which is stationary to small changes in the trial functions because the Galerkin method is equivalent to an adjoint variational principle. The mathematical application package MATHEMATICA is used to determine the eigenvalue expressions and the associated critical numbers and the results are displayed graphically. The role of various magnetic and non-magnetic properties and their mutual interplay for the instability is examined. The values of the parameters arising in the study are fairly standard and are experimentally relevant ([39-46]). The range of the Cattaneo number C adopted in the study at hand is such that the stationary mode of porous ferroconvection is the preferred mode.

The simultaneous change in the critical thermal Rayleigh number R_c with the magnetic Rayleigh number N is displayed in Figures 2 through 5. The magnetic Rayleigh number N signifies the ratio of release of energy due to magnetic stress to energy dissipation caused by viscosity and temperature fluctuations. It is observed that magnetic mechanism has the stabilising effect in the presence of second sound and MFD viscosity as there is a monotonic drop in R_c with an increase in the magnetic Rayleigh number N. The spatial variation resulting from the magnetization due to the application of both temperature and external magnetic field is largely responsible for inducing ferroconvection (Finlayson [33]).

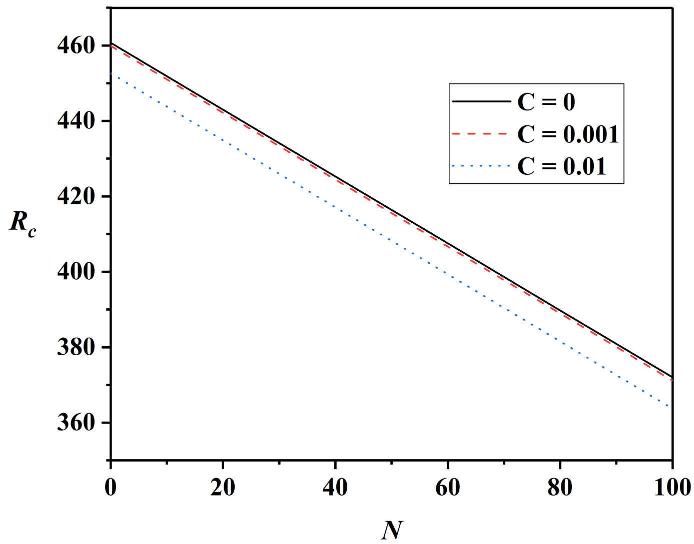

FIGURE 2. Variation of N with respect to R_c and C for fixed values of $M_3 = 5$, $V = 0.5$ and $\chi = 3$.

In Figure 2, we have plotted the critical thermal Rayleigh number R_c versus the magnetic Rayleigh number N for different values of C and for fixed values of V, M_3 and χ. It is found that R_c decreases monotonically with an increase in C indicating that the effect of second sound phenomenon causes ferroconvection to occur at lower values of R_c. The Cattaneo number accelerates the onset of ferroconvection because it characterizes the scaled relaxation time. The treatment of equation of energy as an equation of hyperbolic type, thereby encompassing a damped equation of wave, is responsible for the augmenting effect of second sound (Straughan and Franchi [7]).

On the other hand, we see from Figure 3 that the variable viscosity parameter V designating the MFD viscosity effect is to delay the threshold of porous ferroconvection. The stabilizing effect of V is heightened when the variable viscosity parameter V is large. Technological and biomedical applications of magnetic liquids indicate that the effective viscosity of a ferromagnetic liquid is enhanced by the application of a magnetic field. This reversible effect, known as magnetorheological effect, is a consequence of the fact that the particles magnetize in the presence of a magnetic field and form chain-like clusters that align with the applied field. These chain-like alignments of the dispersed solid particles impede the motion of the liquid thereby increasing the viscous characteristics of the suspension (Maruthamanikandan [39]).

Figure 4 is a plot of R_c versus the magnetic Rayleigh number N for different values of the magnetization parameter M_3 and fixed values of C, V and χ. The parameter

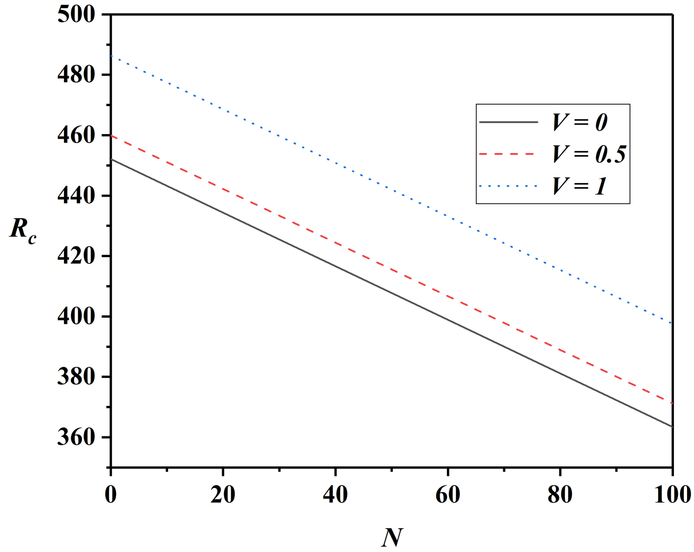

FIGURE 3. Variation of N with respect to R_c and V for fixed values of $C = 0.001$, $M_3 = 5$ and $\chi = 3$.

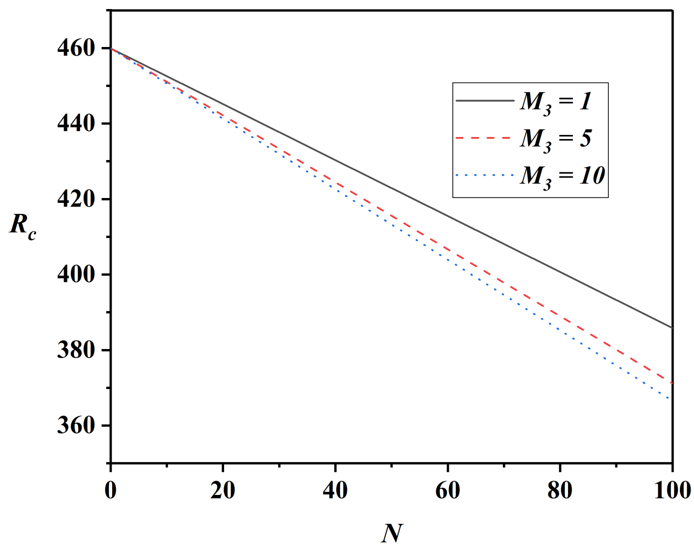

FIGURE 4. Variation of N with respect to R_c and M_3 for fixed values of $C = 0.001$, $V = 0.5$ and $\chi = 3$.

M_3 represents the departure of the magnetic equation of state from linearity. We see that the effect of increasing M_3 is to decrease R_c monotonically. Thus the threshold of ferroconvection in a porous layer with second sound is hastened as the magnetic equation of state becomes more and more nonlinear. It should be mentioned that the destabilizing effect of M_3 is almost insignificant when the magnetic Rayleigh number N is small.

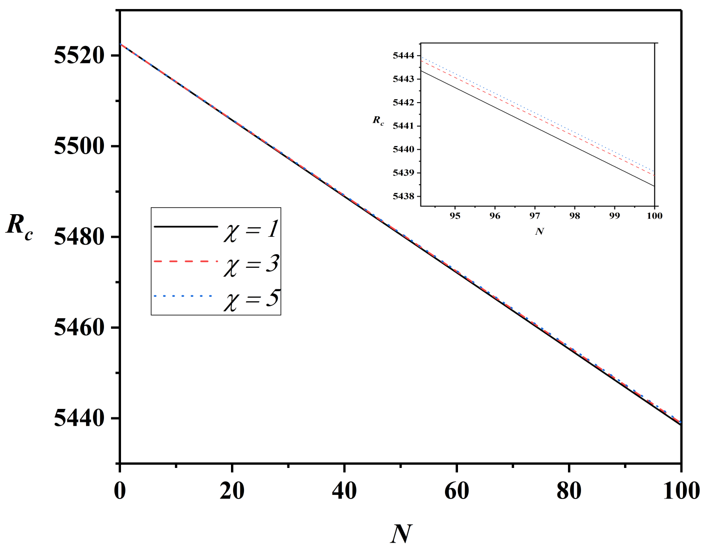

FIGURE 5. Variation of N with respect to R_c and χ for fixed values of $C = 0.001$, $M_3 = 5$ and $V = 0.5$.

Figure 5 shows that the variation of critical Rayleigh number R_c with magnetic Rayleigh number N for different values of the magnetic susceptibility χ and for fixed values of C, V and M_3. The range of values of the magnetic susceptibility χ is 1 to 5 for most ferromagnetic fluids (Finlayson [33]). It is observed that the critical Rayleigh number R_c increases with an increase in the magnetic susceptibility χ.As with the existing works, the stabilizing influence of the magnetic susceptibility χ is inconsequential as far as the stability of the system is concerned.Computations also reveal that convection cell size is more sensitive with the parameters N, C and V compared to that with M_3 and χ.

4. Conclusions

The effect of non-classical heat conduction on the onset of Rayleigh-Benard instability in a horizontal layer of densely packed porous medium saturated with a Boussinesq-Cattaneo-ferromagnetic fluid subjected to the simultaneous action of a vertical magnetic field and vertical temperature gradient is investigated analytically by the method of small perturbation. The stability criteria associated with the stationary instability are delineated in terms of the critical Rayleigh number, wave number, Cattaneo number, variable viscosity parameter, magnetic and porous parameters. It is shown that the effect of magnetic force and second sound is to destabilize the system and the MFD viscosity tends to diminish the threshold of porous ferroconvection. The onset of porous ferroconvection is enhanced as the magnetic equation of state becomes more and more nonlinear and the system is only stabilized slightly due to an increase in the magnetic susceptibility. The outcome of the study may serve as a tool for engineering and industrial applications such as electronic devices, computer storage devices, rotating machinery and the like.

Acknowledgement. The authors are grateful to the anonymous reviewers for their valuable comments towards the improvement of this paper.

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