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CONSTRUCTION OF OPTIMAL REGULATORS FOR MULTIPOINT PROBLEMS OF DYNAMIC SYSTEMS

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ABSTRACT. A multipoint problem of dynamic systems is considering, where the motion is described by a system of ordinary differential equations. Using the LQR theory, a computational algorithm is given for finding a solution to the desired problem, which ensures passage through each of the given points. The results are illustrated using the example of the movement of a quadcopter, for which the proposed algorithm ensures fairly accurate passage of the trajectory through given points.

Keywords: Unmanned aerial vehicle, quadcopter, LQR theory, Riccati equation, motion stabilization.
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1. Introduction

As is known [1-2], when the movement of an object on an infinite time interval $(0, \infty)$ is described by a system of linear ordinary differential equations

$$\dot{x} = Ax + Bu, \quad x(0) = x_0, \tag{1}$$

and it is required to find the regulation law

$$u = Kx, \tag{2}$$

such that problem (11)-(12) was asymptotically stable and the following quadratic functional [16]

$$J = \frac{1}{2} \int_0^\infty (x'Qx + u'Ru)dt, \tag{3}$$

reached minimum value, such a problem is called optimal stabilization by state. Here x is a n -dimensional phase vector, u is the m -dimensional vector of control influence, A and

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B are the $n \times n$ and $n \times m$ -dimensional matrices, respectively, and are a stabilizing pair, $Q \geq 0$, $R \geq 0$ are symmetric matrices of $n \times n$ and $m \times m$ dimension, respectively.

The solution to problem (11)-(13), i.e. the matrix of the feedback circuit K has the form [3-6]:

$$K = -R^{-1}B^{1}S, \tag{4}$$

where $S = S' \geq 0$ is a solution of the algebraic Riccati equation (ARE)

$$SF + F'S - SBR^{-1}B'S + Q = 0. \tag{5}$$

As noted in [7, 8], at a certain time interval, equation (15) has the form

$$\frac{dS}{dt} = SF + F^{1}S - SBR^{-1}BS + Q, \tag{6}$$

where this interval actually describes the transition process, and at point τ is fulfilled [7]

$$\frac{dS}{dt}\,|_{t=\tau} \approx 0, \tag{7}$$

and at $T > \tau$ the system is already in steady state.

Now consider the following optimal control problem on the interval $(0, T)$, where the movement of the object is described by equation (11) with boundary conditions

$$x\,(0) = x_0, \qquad x\,(T) = x_T. \tag{8}$$

It is required to find the control law (12) in such a way that the closed system (11)-(13) satisfies condition (15). Indeed, for a sufficiently large T, equation (11) has the form

$$(x - x_T)' = A\,(x - x_T) + Gu, \tag{9}$$

and feedback (12) ensures convergence $\lim_{t \to T} x\,(t) = x_T$ with a certain sufficient accuracy. Note that equation (19) is equivalent to equation

$$\dot{x} = Ax + Gu - Ax_T. \tag{10}$$

Thus, using a given technique, we determine T , at which $x\,(T)$ is approximately equal to x_T. This technique is applicable to solving multipoint problems of optimization, which is what this article is devoted to. It is assumed that the solution of equation (11) must pass through given points $x_0,\ x_1,\ \ldots, x_l$. To solve such problem, first, for points on the interval $x_0,\ x_1$, the (11)-(13), (18) are solved and the time T_1, is determined, satisfying condition (17). Thus, taking $T_0 = 0$ we find a solution on the time interval (T_0, T_1), the corresponding trajectory of which connects the points x_0 and x_1. Further, this technique is extended to points x_{i-1} and x_i on time intervals (T_{i-1}, T_i) for all $i = 2, \ldots, l$.

2. Problem statement

As is known [8-10, 17-19], the movement of a controlled quadcopter (Fig. 1) is described by the following system of second-order ordinary differential equations:

$$m\,\ddot{x} = -u\ sin\theta,$$

$$m\,\ddot{y} = u\ cos\theta\ sin\varphi,$$

$$m\,\ddot{z} = u\ cos\theta\ cos\varphi - mg, \tag{11}$$

$$\ddot{\psi} = \widetilde{\tau}_\psi,$$

$$\ddot{\theta} = \widetilde{\tau}_\theta,$$

$$\ddot{\varphi} = \widetilde{\tau}_\varphi.$$

Figure 1. Quadcopter.

Here m is the quadcopter mass, and control influences are defined as

$$u = f_1 + f_2 + f_3 + f_4,$$

$$\widetilde{\tau}_\psi = [(f_2 + f_4) - (f_1 + f_2)]\, l, \tag{12}$$

$$\widetilde{\tau}_\theta = (f_2 - f_4)l,$$

$$\widetilde{\tau}_\varphi = (f_3 - f_1)l,$$

where l is the distance from the center of the quadcopter to the engines, and f_i is the lifting force of the i- th engine

$$f_i = k_i \omega_i^2 \tag{13}$$

Here ω_i is the angular speed of rotation of the i- th engine (propeller), k_i is an experimentally determined constant. Note that the angular speed of rotation of the motor depends on the electric current I_i supplied to the motor

$$\omega_i = W(I_i) \tag{14}$$

If we divide each side in the first equations of system (11) by m we get

$$\ddot{x} = -\frac{u}{m}\, sin\theta,$$

$$\ddot{y} = \frac{u}{m}\, cos\theta\ sin\varphi, \tag{15}$$

$$\ddot{z} = \frac{u}{m} cos\theta\ cos\varphi - g.$$

Note that here (x, y, z) – are the coordinates of the quadcopter's center of gravity, and (ψ, θ, φ) are Euler angles, i.e. yaw, pitch and roll angles, respectively.

Let's denote

$$x\,(t) = x_1\,(t)\,, \qquad \dot{x}\,(t) = x_2\,(t)\,, \quad y\,(t) = y_1\,(t)\,, \quad \dot{y}\,(t) = y_2\,(t)\,,$$

$$\dot{z}(t) = z_1(t), \qquad \dot{z}(t) = z_2(t), \qquad \theta(t) = \theta_1(t), \qquad \dot{\theta}(t) = \theta_2(t), \tag{16}$$

$$\varphi(t) = \varphi_1(t), \qquad \dot{\varphi}(t) = \varphi_2(t), \qquad \psi(t) = \psi_1(t), \qquad \dot{\psi}(t) = \psi_2(t).$$

Then, taking into account the initial conditions, we obtain the following Cauchy problem

$$\dot{x}_1(t) = x_2(t)\, x_1(0) = x_1^0,$$

$$\dot{x}_2(t) = -\frac{u(t)}{m}\ sin\theta,\ (t)x_2(0) = x_2^0,$$

$$\dot{y}_1(t) = y_2(t)\, y_1(0) = y_1^0,$$

$$\dot{y}_2(t) = \frac{u(t)}{m}cos\theta\ _1(t)\ \ sin\varphi_1(t)y_2(0) = y_2^0,$$

$$\dot{z}_1(t) = z_2(t)\, z_1(0) = z_1^0$$

$$\dot{z}_2(t) = \frac{u(t)}{m}cos\theta\ _1(t) \cdot cos\varphi_1(t) - gz_2(0) = z_2^0, \tag{17}$$

$$\dot{\theta}_1(t) = \theta_2(t) \quad \theta_1(0) = \ \theta_1^0,$$

$$\dot{\theta}_2(t) = \widetilde{\tau}_\theta(t)\ \theta_2(0) = \ \theta_2^0,$$

$$\dot{\varphi}_1(t) = \varphi_2(t)\ \ \varphi_1(0) = \ \varphi_1^0,$$

$$\dot{\varphi}_2(t) = \widetilde{\tau}_\varphi(t)\ \varphi_2(0) = \ \varphi_2^0,$$

$$\dot{\psi}_1(t) = \psi_2(t)\ \ \psi_1(0) = \ \psi_1^0,$$

$$\dot{\psi}_2(t) = \widetilde{\tau}_\psi(t)\psi_2(0) = \ \psi_2^0.$$

Let's assume that $cos\theta \cdot cos\varphi \neq 0$. Next, assuming that the angles θ, φ are sufficiently small, taking into account (15) we obtain from system (11)

$$\ddot{x} = -g\theta,$$

$$\ddot{\theta} = \widetilde{\tau}_\varphi,$$

$$\ddot{y} = g\varphi,$$

$$\ddot{\varphi} = \widetilde{\tau}_\varphi, \tag{18}$$

$$\ddot{z} = \frac{\tilde{u}}{m} - g = u_z,$$

$$\ddot{\psi} = \widetilde{\tau}_\psi.$$

Here $\tilde{u} = z_1 + mg = u\ cos\theta\ cos\varphi$.

In this case, the system of equations (17) can be written in the following form:

$$
\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= -g\theta_1, \\
\dot{y}_1 &= y_2, \\
\dot{y}_2 &= g\ \varphi_1, \\
\dot{z}_1 &= z_2, \\
\dot{z}_2 = \frac{\tilde{u}}{m} - g &= \frac{z_1}{m} = u_z, \qquad (19) \\
\dot{\theta}_1 &= \theta_2, \\
\theta_2 &= \widetilde{\tau}_\theta, \\
\dot{\varphi}_1 &= \varphi_2, \\
\dot{\varphi}_2 &= \widetilde{\tau}_\varphi, \\
\dot{\psi}_1 &= \psi_2, \\
\dot{\psi}_2 &= \widetilde{\tau}_\theta.
\end{aligned}
$$

Initial conditions remain unchanged.
Next, if we denote

$$
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -g & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & g & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
$$

$$P=\begin{bmatrix} x_1\\ x_2\\ y_1\\ y_2\\ z_1\\ z_2\\ \theta_1\\ \theta_2\\ \varphi_1\\ \varphi_2\\ \psi_1\\ \psi_2 \end{bmatrix},\quad q=\begin{bmatrix} 0\\ 0\\ 0\\ 0\\ 0\\ \frac{\tilde{u}}{m}-g\\ 0\\ \widetilde{\tau}_\theta\\ 0\\ \widetilde{\tau}_\varphi\\ 0\\ \widetilde{\tau}_\psi \end{bmatrix},\quad P^0=\begin{bmatrix} x_1^0\\ x_2^0\\ y_1^0\\ y_2^0\\ z_1^0\\ z_2^0\\ \theta_1^0\\ \theta_2^0\\ \varphi_1^0\\ \varphi_2^0\\ \psi_1^0\\ \psi_2^0 \end{bmatrix}, \tag{20}$$

then system (19) can be written in the following compact form:

$$\dot{P}=AP+q,\quad P(0)=P^0. \tag{21}$$

3. Construction of the regulator

Now let's consider the following problem. We will look for such a vector q, more precisely, such functions $\tilde{u},\widetilde{\tau}_\theta,\widetilde{\tau}_\varphi,\widetilde{\tau}_\psi$, so that the equality $P(T)=P^T$ holds. To begin with, let's assume that $P^T=0$. In equation (21), let's the vector q represent as

$$q=Bu, \tag{22}$$

where

$$B=\begin{bmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 1&0&0&0\\ 0&0&0&0\\ 0&1&0&0\\ 0&0&0&0\\ 0&0&1&0\\ 0&0&0&0\\ 0&0&0&1 \end{bmatrix},\qquad u=\begin{bmatrix} u_z\\ \widetilde{\tau}_\theta\\ \widetilde{\tau}_\varphi\\ \widetilde{\tau}_\psi \end{bmatrix}. \tag{23}$$

Then (21) takes the following form [20]

$$\dot{P}=AP+Bu,\qquad P(0)=P^0. \tag{24}$$

To synthesize a linear optimal controller of problem (24), we use the following functional as a quality criterion:

$$J=\int_0^\infty \left(P'QP+u'Ru\right)dt, \tag{25}$$

where $Q = Q^1 \geq 0$ is the matrix of dimension 12×12, and $R = R^1 > 0$ is the 4×4 -dimensional matrix of the following form:

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad Q = \begin{bmatrix} Q_1^{4,4} & & 0^{4,8} & \\ 0^{4,4} & Q_2^{4,4} & 0^{4,4} & \\ 0^{2,4} & 0^{2,4} & Q_3^{2,2} & 0^{2,2} \\ 0^{2,10} & & Q_4^{2,2} & \end{bmatrix},$$

$$Q_1^{4,4} = \begin{bmatrix} 1 & -2 & -4 & 6 \\ -2 & 4 & 8 & -12 \\ -4 & 8 & 16 \cdot 10^4 & -24 \\ 6 & -12 & -24 & 1 \end{bmatrix} \quad Q_3^{2,2} = Q_4^{2,2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$Q_2^{4,4} = \begin{bmatrix} 1 & -2 & -4 & 6 \\ -2 & 4 & 8 & -12 \\ -4 & 8 & 16 & -24 \\ 6 & -12 & -24 & 36 \end{bmatrix}. \tag{26}$$

Here $0^{k,n}$ is the $k \times n$-dimensional matrix.

Now, using control action

$$u(t) = K\ P(t), \tag{27}$$

we minimize the functional (25) in such a way that the closed-loop system

$$\dot{P} = (A + BK)\, P\,(t)\,, \qquad P\,(0) = P^0 \tag{28}$$

was asymptotically stable, i.e. the inequality

$$Re\lambda(P + AK) < 0 \tag{29}$$

was fulfilled.

Thus, the problem is reduced to finding the matrix K. To find it, first solve the Riccati equation

$$A'S + SA - SBR^{\,-1}\ \ B'S + Q = 0, \tag{30}$$

and $S \geq 0$ is determined. Further according to the formula

$$K = R^{\,-1}\ \ B'S, \tag{31}$$

K is determined. Then, using this matrix, the Cauchy problem (28) is solved and thereby the trajectory $P\,(t)$ of the optimal control problem (24)-(25) is found, and the optimal control $u(t)$ is determined by formula (27).

Since closed system (28) is asymptotically stable, it is obvious

$$\lim_{t \to \infty} P\,(t) = 0\ \ , \tag{32}$$

i.e. $P\,(t) \overset{t\to\infty}{\longrightarrow} P^T$.

Now let's assume that $P^T \neq 0$. For problem (24)-(25), we find a control $u(t)$, such that the solution $P\,(t)$ of the Cauchy problem (24) satisfies $P\,(t) \overset{t\to\infty}{\longrightarrow} P^T$. For this purpose, we introduce the following notation:

$$\overline{P}(t) = P(t) -\ P^T.$$

Then the initial condition passes to $\overline{P}(0) = P(0) - P^T = P^0 - P^T = \overline{P}^0$. Then, similarly to (28), we obtain the following Cauchy problem in the form of a closed system

$$\dot{\overline{P}}(t) = (A + BK)\,\overline{P}(t), \quad \overline{P}(0) = \overline{P}^0, \tag{33}$$

for which, due to asymptotically stability, we have $\overline{P}(t) \overset{t\to\infty}{\longrightarrow} 0$, and the control influence has the form $u = K\overline{P}(t) = K(P(t) - P^T)$. And this means that $P(t) \overset{t\to\infty}{\longrightarrow} P^T$. From (33) we easily obtain

$$\dot{P}(t) = (A + BK)\left(P(t) - P^T\right), \quad P(0) = P^0. \tag{34}$$

In other words, if equation (33) is replaced by equation

$$\dot{P}(t) = (A + BK)\; P(t) - \; (A + BK)\, P^T\;, \tag{35}$$

then its solution will satisfy the condition $P(t) \to P^T$. Thus, solving equation (35) with the corresponding initial condition we obtain a solution that satisfies the condition

$$P(t) \overset{t\to\infty}{\longrightarrow} P^T, \tag{36}$$

and the control action is determined by the formula

$$u(t) = K(P(t) - P^T). \tag{37}$$

From (36) it follows that for a sufficiently small $\varepsilon > 0$ one can find such T, for which $\left|P(T) - P^T\right| < \varepsilon$. Thus, we can state $P(T) \approx P^T$ with sufficient accuracy. Note that T can be found from conditions (17). In this case, it can be argued that the solution to equation (35) will satisfy the boundary condition (18)

$$P(T) \approx P^T.$$

4. Regulator for multi-point problem

Now let's consider the problem of constructing a regulator for controlling the motion of a quadcopter [13-15], the trajectory of which should pass through the points $P^0,\ P^1,\ \ldots, P^l$. Let's first consider the problem

$$\dot{P}_1(t) = (A + BK)\left(P_1(t) - P^1\right), \quad P_1(T_0) = P^0. \tag{38}$$

$$J_1 = \int_{T_0}^{\infty} \left({P_1}'QP_1 + {u_1}'Ru_1\right) dt, \tag{39}$$

where $A,\ B, Q,\ R$ and K are determined by the formulas (20), (23), (26) and (31). Solving this synthesis problem as in the previous paragraph and using (17) we can determine a $T_1 > T_0$, for which $P_1(T_1) \approx P^1$. Thus, we have shown that the solution of problem (38)-(39) at $u_1(t) = K\left(P_1(t) - P^1\right)$ satisfies the boundary conditions $P_1(T_0) = P^0$ and $P_1(T_1) = P^1$. Next, we move on to pairs of points (P^1, P^2),..., (P^{i-1}, P^i),..., (P^{l-1}, P^l). Then for $i = 2, 3, \ldots, l$ we consider the problem

$$\dot{P}_i(t) = (A + BK)\left(P_i(t) - P^i\right), \quad P_i(T_{i-1}) = P^{i-1}, \tag{40}$$

$$J_i = \int_{T_{i-1}}^{\infty} \left(P_i{}' Q P_i + u_i{}' R u_i \right) dt. \tag{41}$$

Solving the synthesis problem (40), (41) for each i we can determine such $T_i > T_{i-1}$, for which $P_i(T_i) \approx P^i$. The control action on the interval (T_{i-1}, T_i) is defined as $u_i(t) = K\left(P_i(t) - P^i\right)$.

5. Example

Let $l = 4$ and points $P^0,\ P^1, P^2, P^3, P^4$ are given as follows:

$$P^0 = \begin{bmatrix} 20 \\ 0 \\ -10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad P^1 = \begin{bmatrix} 20 \\ 0 \\ -10 \\ 0 \\ 200 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad P^2 = \begin{bmatrix} 150 \\ 0 \\ -10 \\ 0 \\ 200 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad P^3 = \begin{bmatrix} 150 \\ 0 \\ 100 \\ 0 \\ 200 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad P^4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

Using the proposed methodology, we have proposed a regulator synthesis algorithm, with the help of which the trajectory of the center of gravity of the controlled quadcopter is found (Fig. 2).

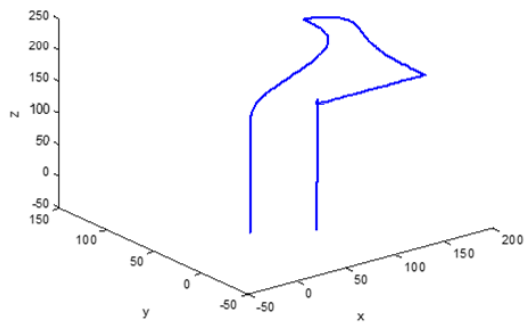

Figure 1. Trajectory of the center of gravity of a controlled quadcopter.

Here area 1 means vertical movement, 2 means lateral movement, 3 means horizontal movement, and 4 means landing the quadcopter.

6. Conclusions

We consider the problem of controlling the movement of a quadcopter, the trajectory of which must pass through each of the given points. To solve this problem using the LQR theory, a computational algorithm is proposed, which ensures fairly accurate passage of the trajectory through given points.

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