TWMS J. App. and Eng. Math. V.15, N.1, 2025, pp. 165-183

A STUDY ON BIPOLAR INTUITIONISTIC FUZZY SOFT GRAPHS AND ITS APPLICATION

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Abstract. The concept of bipolar intuitionistic fuzzy soft sets and graph structure are combined, which is a new concept bipolar intuitionistic fuzzy (BIF) soft graph (SG). In this article, we introduce the notion of a BIF SG and give the concept of regularity on this graph. The concepts of regular, totally regular, perfectly regular, edge-regular, totally edge-regular and perfectly edge-regular BIF graphs are proposed in a soft environment with examples. Also, the concept of perfectly irregular and edge-irregular BIF SGs is introduced and explored a few features. Some related properties of regularity on this graph are investigated. Finally, a real-life application of this graph on a social group has been given to find out the most influenced person.

Keyword: Bipolar intuitionistic fuzzy soft graph; perfectly regular bipolar intuitionistic fuzzy soft graph ; perfectly edge-regular; perfectly edge-irregular.

AMS Subject Classification: 05C72.

1. Introduction

1.1. **Research background and related works.** It is seen in the maximum time that graph theory is found as an essential part of connectivity in some fields of geometry, algebra, topology, number theory, computer science, operations research and optimization. The conception of the fuzzy graph (FG) was first initiated by Rosenfeld [28]. Gani and Radha [7] proposed the conception of the regular FG in 2008. Moderson and Peng [20] propose various operations on FGs. In the field of a mathematical model, incomplete and imprecise data are handled by fuzzy set theory [30]. Atanassov [4] has extended the fuzzy set to an intuitionistic fuzzy set for distinguishing hesitancy more individually.

To deal with vague and imprecise parameters, classical soft sets are not suitable. Maji et al. [17] represented the generalization of the standard soft sets in the form of initialization of the conception of fuzzy soft sets. Many important applications in different fields are handled with fuzzy soft sets. Some characteristics of the said notion are investigated by Maji et al. [16]. Fuzzy number intuitionistic fuzzy soft sets and their properties were

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§ Manuscript received: May 16, 2023; accepted: October 10, 2023.
TWMS Journal of Applied and Engineering Mathematics, Vol.15, No.1;

discussed by Garg et al. [8]. Kalaiarasi et al. [13] studied perfect fuzzy soft tripartite graphs and their complements. Granulation of protein–protein interaction networks in a Pythagorean fuzzy soft environment was discussed by Nawaz and Akram [21]. Also, several researchers have used the idea on various mathematical and algebraic structures.

The main significant difference between soft graphs and fuzzy graphs lies in their approach to representing uncertainty or indeterminacy in graph structures: In a fuzzy graph, each edge and vertex is associated with a degree of membership (between 0 and 1) to indicate the strength of the relationship or presence. This allows for gradual transitions between fully connected and disconnected elements, capturing uncertainty through membership values. Fuzzy graphs are particularly useful when the relationships between vertices and edges are not well-defined or can be graded. Soft graphs, on the other hand, introduce the concept of "soft sets" to graph theory. A soft set is a collection of elements associated with a set of parameters or attributes, where each element may or may not belong to the set. Soft graphs extend this idea to graph structures, where vertices and edges can be associated with soft sets. The inclusion of elements in these soft sets can be partial or vague, reflecting a different form of uncertainty or imprecision compared to fuzzy graphs.

In the year 1998, bipolar fuzzy sets were introduced by Zhang [31]. In a bipolar fuzzy set, positive data reveals the guaranteed possible fact, whereas negative data reveals the forbidden or surely false or impossible fact. The fuzzy sets are generalized by extending the membership value from the range $[0, 1]$ to $[-1, 1]$, which was made known by Lee [15] as a bipolar valued fuzzy set. Membership value belonging to $[-1, 0)$ represents that something that components are satisfying implicit counter property; membership value belonging to $(0, 1]$ stands for the components that are satisfying the correlated property; on the other side, membership value 0 means the elements that are not relevant to correlated property in bipolar fuzzy sets. A recent development by Abdullah et al. [1] and Naz and Shabirh [22] gave an application in the decision-making problem on bipolar fuzzy soft set. Jana and Pal [12] gave an application in decision-making problems on bipolar intuitionistic fuzzy (BIF) soft sets.

Akram explored the concept of bipolar fuzzy graphs in [2], while Akram and Dudek further investigated regularity in this context in [3]. Lakdashti et al. introduced novel ideas related to bipolar fuzzy graphs in their work [14]. Pramanik et al. delved into the realm of bipolar fuzzy planar graphs and their application in image shrinking [25]. Rajamannan and Amsaveni [26] extended the concept to bipolar complex intuitionistic fuzzy graphs. Despite these existing works, there is currently a dearth of research on BIF soft graphs (SGs). To address this gap, we draw upon the foundational principles of intuitionistic fuzzy graphs, bipolar fuzzy graphs, and fuzzy soft theory to formulate BIF SGs. For fundamental terminologies and definition see [23] and for other interesting work on FGs see [5, 9, 10, 11, 19, 27, 29].

1.2. **Motivation of the work.** The main purpose of this article is to discuss the characteristic of any object as bipolar intuitionistic fuzzy sense, i.e., a positive and negative membership and non-membership together work for different parameters in a soft environment. For this combined concept, anyone can analyse the positive and negative membership and non-membership facts in a certain way. This BIF SG model gives more efficient fuzziness results than another fuzzy soft model. For this reason, we are interested in working with this concept. Also, it is very interesting to develop and analyse such combined graphs with examples and related theorems. These definitions and theorems are definitely improving the existing concepts of fuzzy soft graphs and are more reliable for solving any complicated real-life problem.

1.3. **Contribution of the work.** This article introduces and thoroughly discusses specific types of BIF soft graphs, which combine the representation of bipolar intuitionistic fuzzy soft sets with graph structures in a novel manner. The concept of a BIF soft graph is defined, and its properties are explored along with the proof of several theorems related to its aspects. The article proposes and elaborates on the notions of regular, totally regular, perfectly regular, edge-regular, totally edge-regular, and perfectly edge-regular bipolar intuitionistic fuzzy graphs within a soft environment, accompanied by relevant examples. Additionally, various related properties are examined. The article also presents the concept of perfectly irregular and perfectly edge-irregular bipolar intuitionistic fuzzy soft graphs, delving into their distinctive features. Furthermore, the practical application of the BIF soft graph concept is demonstrated in a real-life scenario where it serves as a decision-making tool to identify the most influential individual within a social group.

1.4. **Framework of this study.** The formation of this article is as follows: The novelty and objective of the work are discussed in section 2. Section 3 mentions some useful concepts which are essential for this article. In Section 4, the conception of bipolar intuitionistic fuzzy sets, soft sets and graph structure are combined, which makes a new definition of BIF SG. The concepts of regular (REG), totally REG, perfectly REG and edge REG, totally edge REG, and perfectly edge REG BIF SGs are proposed with suitable examples in this work. Also, some related properties are investigated. In Section 5, the concept of perfectly irregular (IREG) and perfectly edge IREG BIF SGs are formulated and explore a few features. In Section 6, we have discussed a real-life application of this graph on a social group and find out the most influenced person in a social group. Finally, in Section 7, the conclusion of the study has been presented.

Notations and symbols. In Table 1, we have given some notations and abbreviation forms which are used in the whole paper for preparing the theories.

Table 1. The list of abbreviation.

Abbreviation	Meaning	Abbreviation	Meaning
FG	Fuzzy graph	BIF	Bipolar intuitionistic fuzzy
SG	Soft graph	REG	Regular
MS	Membership	IREG	Irregular
NMS	Non membership		

2. Novelty and objective of the work

The novelty of BIF SGs lies in their capacity to combine the concepts of bipolar intuitionistic fuzzy soft sets and graph structures, introducing a new framework for representing and analyzing uncertain or imprecise relationships in graph structures. This integration brings several novel aspects:

• BIF SGs provide a combined representation, incorporating the gradual transition of MS and NMS degrees from intuitionistic fuzzy graphs, positive and negative MS from bipolar concept and the inclusion of uncertain elements through soft sets from soft graphs. This amalgamation allows for a more comprehensive modelling of uncertainty in graph relationships.

• By combining the characteristics of BIF graphs and SGs, BIF SGs can capture a wider range of uncertainty types and magnitudes. They can represent a positive and negative membership and non-membership together for different parameters in a soft environment.
• The incorporation of both fuzzy and soft concepts provides a more nuanced approach to addressing complex problems that involve uncertainty. BIF SGs offer a powerful tool for handling situations where traditional crisp graph models fall short.
• BIF SGs contribute to the theoretical exploration of uncertainty in graph theory.

BIF SGs might aim to extend the concepts of intuitionistic fuzzy soft graphs by incorporating bipolarity, which generally refers to the consideration of both positive and negative aspects. Here are some speculative objectives for BIF SGs:
• BIF SGs could aim to capture not only MS and NMS but also represent positive and negative relationships or sentiments between elements.
• By including bipolarity, BIF SGs might strive to provide a more realistic representation of complex real-world relationships, where elements can have mixed or ambivalent sentiments towards each other.
• In situations where there are conflicting pieces of information or opinions, BIF SGs could potentially aid in decision-making processes by considering both positive and negative factors.
• BIF SGs could provide a means to represent multiple perspectives or viewpoints on relationships and interactions, acknowledging that different actors may perceive the same relationship in opposing ways.

3. Preliminaries

Some essential notions recalled here which are needed for the article.

Definition 3.1. [30] *Taking T as a universal set, then a fuzzy set A over T is meant as $A = \{(u, \mu_A(u)) : u \in T\}$, where $\mu_A : T \to I$. Here, $\mu_A(u)$ is the measure of MS of u in A and $I = [0, 1]$.*

In 1965, the notion of fuzzy sets and fuzzy relations was initiated by Zadeh and later, Rosenfeld took fuzzy relations on fuzzy sets and improved the concept of FGs in 1975.

Definition 3.2. [28] *Let $\acute{G} = (\acute{V}, \sigma, \mu)$ be a FG, where $\acute{V}$ is non-empty set (called set of vertices), jointly with two functions $\sigma : \acute{V} \to I$ and $\mu : \acute{V} \times \acute{V} \to I$ such that for all $u, v \in \acute{V}$, $\mu(uv) \leq \min\{\sigma(u), \sigma(v)\}$, μ is a symmetrical fuzzy relation on σ and $I = [0, 1]$.*

In 1986, Atanassov included a new type of uncertainty in fuzzy set, a measure of NMS function and gave a new definition name of intuitionistic fuzzy set.

Definition 3.3. [4] *Taking T as a non-empty set, an intuitionistic fuzzy set $\tilde{A}$ over T is defined by $\tilde{A} = \{(u, \mu_{\tilde{A}}(u),$
$\nu_{\tilde{A}}(u)) : u \in T\}$, where $\mu_{\tilde{A}}(u) \in I$ is the measure of MS of u in $\tilde{A}$ and $\nu_{\tilde{A}}(u) \in I$ is the measure of NMS of u in $\tilde{A}$ with the condition $0 \leq \mu_{\tilde{A}}(u) + \nu_{\tilde{A}}(u) \leq 1$.*

Here, $S_{\tilde{A}}(u) = 1 - (\mu_{\tilde{A}}(u) + \nu_{\tilde{A}}(u))$ is the measure of suspicion of u in $\tilde{A}$, which excludes the measure of MS and NMS.

Definition 3.4. [24] *An IFG is denoted by $\acute{G} = (\acute{V}, \mu, \gamma)$ where $\mu = (\mu_1, \mu_2), \gamma = (\gamma_1, \gamma_2)$ and*

(a) $\mu_1 : \acute{V} \to I$ and $\mu_2 : \acute{V} \to I$ denote the measure of MS and NMS of the vertex $u \in \acute{V}$ respectively and $0 \leq \mu_1(u) + \mu_2(u) \leq 1$ for every $u \in \acute{V}$.

(b) $\gamma_1 : \acute{V} \times \acute{V} \to I$ *and* $\gamma_2 : \acute{V} \times \acute{V} \to I$*, where* $\gamma_1(uv)$ *and* $\gamma_2(uv)$ *denote the measure of MS and NMS value of the edge uv respectively such that* $\gamma_1(uv) \leq \min\{\mu_1(u), \mu_1(v)\}$ *and* $\gamma_2(uv) \leq \max\{\mu_2(u), \mu_2(v)\}$*,* $0 \leq \gamma_1(uv) + \gamma_2(uv) \leq 1$ *for every uv.*

In 1999, Molodtsov proposed soft set theory, a generalization of fuzzy set theory to deal with uncertainty in a parametric manner.

Definition 3.5. [18] *A pair* (ϕ, Q) *is said a soft set over* X*, where* X *and* Q *are universal set and set of parameters respectively and* ϕ *is a mapping of* Q *into power set of* X*.*

In another way, a soft set over X is a family of parametric subset of X. For $e \in Q$, $\phi(e)$ can be taken as the set of e-elements of the soft set (ϕ, Q). An extension of fuzzy sets whose MS degree range is $[-1, 1]$, called bipolar fuzzy set, was given by Zhang [31] in 1998.

Definition 3.6. [31] *A bipolar fuzzy set* $\tilde{A}$ *over* X *is defined by* $\tilde{A} = \{(t, \mu^P(t), \mu^N(t)) : t \in X\}$*, where* $\mu^P : X \to I$ *and* $\mu^N : X \to J$*,* μ^P *represents positive information and* μ^N *represents negative information and* $I = [0, 1], J = [-1, 0]$*.*

In 2015, Ezhilmaran and Sankar [6] initiated the concept of BIF set by combining the concept of intuitionistic fuzzy sets and bipolar fuzzy sets.

Definition 3.7. [6] *The BIF set* $\tilde{A}$ *on* X *is defined by* $\tilde{A} = \{(t, \mu^P(t), \mu^N(t), \gamma^P(t), \gamma^N(t)) : t \in X\}$*, where* $\mu^P : X \to I$*,* $\mu^N : X \to J$*,* $\gamma^P : X \to I$*,* $\gamma^N : X \to J$ *are the functions such that* $0 \leq \mu^P(t) + \gamma^P(t) \leq 1$ *and* $-1 \leq \mu^N(t) + \gamma^N(t) \leq 0$ *and* $\mu^P(t)$*,* $\mu^N(t)$*,* $\gamma^P(t)$ *and* $\gamma^N(t)$ *are respectively present positive MS value, negative MS value, positive NMS value and negative NMS value.*

A combination of BIF set with soft set, called BIF soft set, was given by Jana and Pal [12] in 2018. Let BIF^X be the bipolar intuitionistic fuzzy power set of X.

Definition 3.8. [12] *Let* X *and* Q *be a universal set and parameter set respectively and* $\tilde{A} \subset Q$*. Let us define a mapping* ϕ *from* $\tilde{A}$ *to* BIF^X *as :* $\phi : \tilde{A} \to BIF^X$*. Then* $(\phi, \tilde{A})$ *is said BIF soft set over* X *and is defined by* $\{(t_i, \mu^P_{\tilde{A}}(t_i), \mu^N_{\tilde{A}}(t_i), \gamma^P_{\tilde{A}}(t_i), \gamma^N_{\tilde{A}}(t_i)) : \forall t_i \in X, \forall \alpha_i \in \tilde{A}\}$*, where* $\mu^P_{\tilde{A}} : X \to I$*,* $\mu^N_{\tilde{A}} : X \to J$*,* $\gamma^P_{\tilde{A}} : X \to I$ *and* $\gamma^N_{\tilde{A}} : X \to J$ *are such that* $0 \leq \mu^P_{\tilde{A}}(t) + \gamma^P_{\tilde{A}}(t) \leq 1$ *and* $-1 \leq \mu^N_{\tilde{A}}(t) + \gamma^N_{\tilde{A}}(t) \leq 0$*.*

4. Bipolar intuitionistic fuzzy soft graphs

In this section, we have introduced the idea of BIF SGs, perfectly REG BIF SGs, perfectly edge REG BIF SGs and discussed some of their features.

Definition 4.1. *A BIF SG on a non empty set* $\acute{V}$ *is a tuple* $\Gamma_I = (\phi, \psi, \tilde{E})$ *such that*

1. $\tilde{E}$ *is a non empty parameter set,*
2. $(\phi, \tilde{E})$ *is a BIF soft set on* $\acute{V}$*,*
3. $(\psi, \tilde{E})$ *is a BIF soft set over* $E \subseteq \acute{V} \times \acute{V}$*,*
4. $(\phi(e_i), \psi(e_i))$ *is a connected BIF subgraph,* $\forall e_i \in \tilde{E}$*, i=1,2,3,...,m, that is,*

$$\mu^P_{\psi(e_i)}(tu) \leq min\{\mu^P_{\phi(e_i)}(t), \mu^P_{\phi(e_i)}(u)\}, \mu^N_{\psi(e_i)}(tu) \geq max\{\mu^N_{\phi(e_i)}(t), \mu^N_{\phi(e_i)}(u)\},$$

$$\nu^P_{\psi(e_i)}(tu) \geq max\{\nu^P_{\phi(e_i)}(t), \nu^P_{\phi(e_i)}(u)\}, \nu^N_{\psi(e_i)}(tu) \leq min\{\nu^N_{\phi(e_i)}(t), \nu^N_{\phi(e_i)}(u)\},$$

such that $0 \leq (\mu^P_{\psi(e_i)}(tu)) + (\nu^P_{\psi(e_i)}(tu)) \leq 1$*,* $-1 \leq (\mu^N_{\psi(e_i)}(tu)) + (\nu^N_{\psi(e_i)}(tu)) \leq 0$*,* $\forall e_i \in \tilde{E}$*,* $t, u \in \acute{V}$*. Note that* $(\phi, \tilde{E})$ *is called a BIF soft vertex and* $(\psi, \tilde{E})$ *is called a BIF soft edge. Here,* $\mu^P_{\phi(e_i)}(t)$ *and* $\mu^P_{\psi(e_i)}(tu)$ *denote the degree of positive MS of the vertex* t

and edge tu respectively corresponding to parameter e_i, $\mu^N_{\phi(e_i)}(t)$ and $\mu^N_{\psi(e_i)}(tu)$ denote the degree of negative MS of the vertex t and edge tu respectively corresponding to parameter e_i, $\nu^P_{\phi(e_i)}(t)$ and $\nu^P_{\psi(e_i)}(tu)$ denote the degree of positive NMS of the vertex t and edge tu respectively corresponding to parameter e_i, $\nu^N_{\phi(e_i)}(t)$ and $\nu^N_{\psi(e_i)}(tu)$ denote the degree of negative NMS of the vertex t and edge tu respectively corresponding to parameter e_i. The BIF subgraph $(\phi(e_i), \psi(e_i))$ is denoted by $M(e_i)$.

Definition 4.2. *Suppose Γ_I is a BIF SG over $\acute{V}$. Then Γ_I is said to be REG BIF SG if $M(e_i)$ is a REG BIF graph for all $e_i \in \tilde{E}$. If $M(e_i)$ is a REG BIF graph of degree $(r_i, s_i, \acute{r}_i, \acute{s}_i)$ for all $e_i \in \tilde{E}$, then Γ_I is a REG BIF SG. Here, $(r_i, s_i, \acute{r}_i, \acute{s}_i) = (\sum_{t\neq v} \mu^P_{\psi(e_i)}(tv), \sum_{t\neq v} \mu^N_{\psi(e_i)}(tv), \sum_{t\neq v} \nu^P_{\psi(e_i)}(tv),$ $\sum_{t\neq v} \nu^N_{\psi(e_i)}(tv))$, a degree of a vertex t of Γ_I with respect to parameter e_i, denoted by $deg(t)$.*

Definition 4.3. *Suppose Γ_I is a BIF SG over $\acute{V}$. Then Γ_I is said to be totally REG BIF SG if $M(e_i)$ is a totally REG BIF graph of degree $(l_i, k_i, \acute{l}_i, \acute{k}_i)$ for all $e_i \in \tilde{E}$, then Γ_I is a totally REG BIF SG. Here, $(l_i, k_i, \acute{l}_i, \acute{k}_i) = (\sum_{t\neq v} \mu^P_{\psi(e_i)}(tv) + \mu^P_{\phi(e_i)}(t), \sum_{t\neq v} \mu^N_{\psi(e_i)}(tv) + \mu^N_{\phi(e_i)}(t), \sum_{t\neq v} \nu^P_{\psi(e_i)}(tv) + \nu^P_{\phi(e_i)}(t), \sum_{t\neq v} \nu^N_{\psi(e_i)}(tv) + \nu^N_{\phi(e_i)}(t))$, a total degree of a vertex t of Γ_I with respect to parameter e_i, denoted by $deg[t]$.*

Definition 4.4. *Let Γ_I be a BIF SG over $\acute{V}$. Then Γ_I is called to be perfectly REG BIF SG if $M(e_i)$ is a REG and totally REG BIF graph for all $e_i \in \tilde{E}$.*

Example 1. *Take two nonempty sets $\acute{V} = \{t_1, t_2, t_3\}$ and $E = \{t_1t_2, t_2t_3, t_1t_3\}$. Let $\tilde{E} = \{e_1\}$ and $(\phi, \tilde{E})$ be a BIF soft set on $\acute{V}$ with the function $\phi : \tilde{E} \to P(\acute{V})$ given by $\phi(e_1) = \{(t_1, 0.4, -0.5, 0.5, -0.4), (t_2, 0.4, -0.5, 0.5, -0.4), (t_3, 0.4, -0.5, 0.5, -0.4)\}$. Let $(\psi, \tilde{E})$ be a BIF soft set over E with the function $\psi : \tilde{E} \to P(E)$ defined by $\psi(e_1) = \{(t_1t_2, 0.4, -0.5, 0.5, -0.4), (t_2t_3, 0.4, -0.5, 0.5, -0.4), (t_3t_1, 0.4, -0.5, 0.5, -0.4)\}$.*

It is easy to see through routine calculation that BIF graph $M(e_1) = (\phi(e_1), \psi(e_1))$ is a REG and totally REG as displayed in Figure 1a. Hence Γ_I is a perfectly REG BIF SG.

Proposition 4.1. *For a perfectly REG BIF SG $\Gamma_I = (\phi, \psi, \tilde{E})$, the function ϕ is constant.*

Theorem 4.1. *Suppose Γ_I is a BIF SG. Then Γ_I is perfectly REG iff*

(i) $\sum_{t\neq v} \mu^P_{\psi(e_i)}(tv) = \sum_{u\neq v} \mu^P_{\psi(e_i)}(uv)$, $\sum_{t\neq v} \mu^N_{\psi(e_i)}(tv) = \sum_{u\neq v} \mu^N_{\psi(e_i)}(uv)$, $\sum_{t\neq v} \nu^P_{\psi(e_i)}(tv) = \sum_{u\neq v} \nu^P_{\psi(e_i)}(uv)$, $\sum_{t\neq v} \nu^N_{\psi(e_i)}(tv) = \sum_{u\neq v} \nu^N_{\psi(e_i)}(uv)$ and

(ii) $\mu^P_{\phi(e_i)}(t) = \mu^P_{\phi(e_i)}(u)$, $\mu^N_{\phi(e_i)}(t) = \mu^N_{\phi(e_i)}(u)$, $\nu^P_{\phi(e_i)}(t) = \nu^P_{\phi(e_i)}(u)$, $\nu^N_{\phi(e_i)}(t) = \nu^N_{\phi(e_i)}(u)$, $\forall t, u \in \acute{V}$, $e_i \in \tilde{E}$.

Proof: (i) First assume Γ_I is perfectly REG BIF SG. So Γ_I is REG BIF SG and from the definition of REG BIF SG, we have $deg(\mu^P(t)) = deg(\mu^P(u))$, $deg(\mu^N(t)) = deg(\mu^N(u))$, $deg(\nu^P(t)) = deg(\nu^P(u))$ and $deg(\nu^N(t)) = deg(\nu^N(u))$, $\forall t, u \in \acute{V}$, $e_i \in \tilde{E}$, where $deg(\mu^P(t))$, $deg(\mu^N(t))$ are the degree of positive and negative MS of the vertex t and $deg(\nu^P(t))$, $deg(\nu^N(t))$ are the degree of positive and negative NMS of the vertex t respectively. Then $\sum_{t\neq v} \mu^P_{\psi(e_i)}(tv) = \sum_{u\neq v} \mu^P_{\psi(e_i)}(uv), \sum_{t\neq v} \mu^N_{\psi(e_i)}(tv) = \sum_{u\neq v} \mu^N_{\psi(e_i)}(uv)$, $\sum_{t\neq v} \nu^P_{\psi(e_i)}(tv) = \sum_{u\neq v} \nu^P_{\psi(e_i)}(uv)$, $\sum_{t\neq v} \nu^N_{\psi(e_i)}(tv) = \sum_{u\neq v} \nu^N_{\psi(e_i)}(uv)$, $\forall t, u \in \acute{V}$, $e_i \in \tilde{E}$. Therefore (i) holds. From Proposition 3.1, (ii) is also hold.

Conversely, suppose Γ_I is a BIF SG so that it satisfies (i) and (ii). By (i), we get $\sum_{t\neq v}\mu^P_{\psi(e_i)}(tv) = \sum_{u\neq v}\mu^P_{\psi(e_i)}(uv), \sum_{t\neq v}\mu^N_{\psi(e_i)}(tv) = \sum_{u\neq v}\mu^N_{\psi(e_i)}(uv)$,
$\sum_{t\neq v}\nu^P_{\psi(e_i)}(tv) = \sum_{u\neq v}\nu^P_{\psi(e_i)}(uv), \sum_{t\neq v}\nu^N_{\psi(e_i)}(tv) = \sum_{u\neq v}\nu^N_{\psi(e_i)}(uv)$
$\Rightarrow deg(\mu^P(t)) = deg(\mu^P(u))$
$= k_i$, $deg(\mu^N(t)) = deg(\mu^N(u)) = l_i$, $deg(\nu^P(t)) = deg(\nu^P(u)) = \acute{k}_i$ and $deg(\nu^N(t)) = deg(\nu^N(u)) = \acute{l}_i$, $\forall t,u \in \acute{V}$, $e_i \in \tilde{E}$. This implies that $M(e_i)$ is a REG BIF graph. Hence Γ_I is a REG BIF SG.

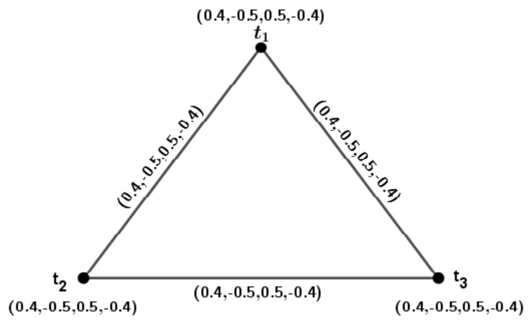
Now from (ii), $\mu^P_{\phi(e_i)}(t) = \mu^P_{\phi(e_i)}(u) = r_i$, $\mu^N_{\phi(e_i)}(t) = \mu^N_{\phi(e_i)}(u) = s_i$, $\nu^P_{\phi(e_i)}(t) = \nu^P_{\phi(e_i)}(u) = \acute{r}_i$, $\nu^N_{\phi(e_i)}(t) = \nu^N_{\phi(e_i)}(u) = \acute{s}_i$, $\forall t,u \in \acute{V}$, $e_i \in \tilde{E}$. Thus ϕ is a constant function. So, $deg(\mu^P[t]) = deg(\mu^P(t)) + \mu^P_{\phi(e_i)}(t) = k_i + r_i, deg(\mu^N[t]) = deg(\mu^N(t)) + \mu^N_{\phi(e_i)}(t) = l_i + s_i$, $deg(\nu^P[t]) = deg(\nu^P(t)) + \nu^P_{\phi(e_i)}(t) = \acute{k}_i + \acute{r}_i, deg(\nu^N[t]) = deg(\nu^N(t)) + \nu^N_{\phi(e_i)}(t) = \acute{l}_i + \acute{s}_i$.

Similarly, $deg(\mu^P[u]) = k_i + r_i, deg(\mu^N[u]) = l_i + s_i, deg(\nu^P[u]) = \acute{k}_i + \acute{r}_i$,
$deg(\nu^N[u]) = \acute{l}_i + \acute{s}_i$, $\forall t,u \in \acute{V}$, $e_i \in \tilde{E}$, where $deg(\mu^P[t])$, $deg(\mu^N[t])$ are the total degree of positive and negative MS of the vertex t and $deg(\nu^P[t])$, $deg(\nu^N[t])$ are the total degree of positive and negative NMS of the vertex t respectively. Therefore, $deg(\mu^P[t]) = deg(\mu^P[u]) = f_i(say)$, $deg(\mu^N[t]) = deg(\mu^N[u]) = g_i(say)$, $deg(\nu^P[t]) = deg(\nu^P[u]) = \acute{f}_i(say)$ and $deg(\nu^N[t]) = deg(\nu^N[u]) = \acute{g}_i(say)$, i.e, $deg[t] = deg[u] = (f_i, g_i, \acute{f}_i, \acute{g}_i)$, $\forall t,u \in \acute{V}$, $e_i \in \tilde{E}$, where f_i, g_i, $\acute{f}_i$ and $\acute{g}_i$ are the total degree of positive MS, negative MS, positive NMS and negative NMS of each vertex of Γ_I with respect to parameter e_i respectively. So, $M(e_i)$ is a $(f_i, g_i, \acute{f}_i, \acute{g}_i)$-totally REG BIF graph. Hence Γ_I is totally REG BIF SG. This implies Γ_I is a perfectly REG BIF SG.

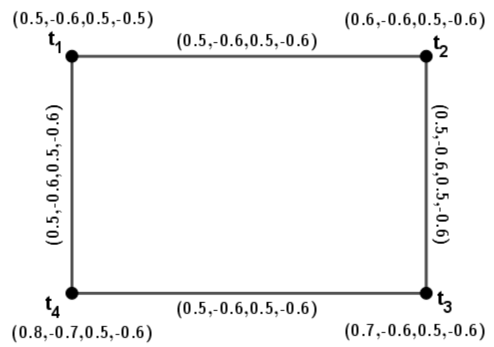
Theorem 4.2. *Let $\Gamma_I = (\phi, \psi, \tilde{E})$ be a perfectly REG BIF SG. Then size of $M(e_i)$ is $S(M(e_i)) = \frac{|\acute{V}|}{2}(k_i, l_i, \acute{k}_i, \acute{l}_i)$, where $(k_i, l_i, \acute{k}_i, \acute{l}_i)$ is the degree of a vertex in $M(e_i)$, $\forall e_i \in \tilde{E}$.*

The converse part of the Theorem 4.2 need not be accurate, as shown in the following example.

Example 2. *Consider the BIF SG Γ_I in Figure 1b. Here $|\acute{V}| = 4$ and $deg(t_1) = deg(t_2) =$*



(A) Perfectly REG BIF soft graph $\Gamma_I = \{M(e_1)\}$.



(B) A connected BIF soft graph $\Gamma_I = \{M(e_1)\}$.

FIGURE 1

$deg(t_3) = deg(t_4) = (1, -1.2, 1, -1.2)$. Size of $M(e_1)$ is $S(M(e_1)) = (2, -2.4, 2, -2.4)$. From Theorem 4.2, we have $S(M(e_i)) = \frac{|\acute{V}|}{2}(k_i, l_i, \acute{k}_i, \acute{l}_i) = \frac{|\acute{V}|}{2}(k_1, l_1, \acute{k}_1, \acute{l}_1) = \frac{4}{2}(1, -1.2, 1, -1.2) = (2, -2.4, 2, -2.4)$. Now, $deg[t_1] = (1.5, -1.8, 1.5, -1.7)$ and $deg[t_2] = (1.6, -1.8,$

$1.5, -1.8)$. *So,* $deg[t_1] \neq deg[t_2]$. *Therefore,* Γ_I *is not perfectly REG BIF SG. Hence, the converse part of the Theorem 4.2 is not true.*

Definition 4.5. *Let* Γ_I *be a BIF SG over* $\acute{V}$. *Then* Γ_I *is called to be edge REG BIF SG if* $M(e_i)$ *is an edge REG BIF graph for all* $e_i \in \tilde{E}$. *If* $M(e_i)$ *is an edge REG BIF graph of degree* $(r_i, s_i, \acute{r}_i, \acute{s}_i)$ *for all* $e_i \in \tilde{E}$, *then* Γ_I *is an edge REG BIF SG.*

Definition 4.6. *Let* Γ_I *be a BIF SG over* $\acute{V}$. *Then* Γ_I *is called to be totally edge REG BIF SG if* $M(e_i)$ *is a totally edge REG BIF graph for all* $e_i \in \tilde{E}$. *If* $M(e_i)$ *is a totally edge REG BIF graph of degree* $(r_i, s_i, \acute{r}_i, \acute{s}_i)$ *for all* $e_i \in \tilde{E}$, *then* Γ_I *is a totally edge REG BIF SG.*

Definition 4.7. *A perfect edge REG BIF SG is a BIF SG that is both edge REG and totally edge REG BIF SG.*

Example 3. *Take two non empty sets* $\acute{V} = \{t_1, t_2, t_3, t_4\}$ *and* $E = \{t_1t_2, t_2t_3, t_3t_4, t_1t_4\}$. *Let* $\tilde{E} = \{e_1, e_2\}$ *and* $(\phi, \tilde{E})$ *be a BIF soft set on* $\acute{V}$ *with the function* $\phi : \tilde{E} \to P(\acute{V})$ *defined by*

$\phi(e_1) = \{(t_1, 0.6, -0.6, 0.6, -0.6), (t_2, 0.6, -0.6, 0.6, -0.6), (t_3, 0.6, -0.6, 0.6, -0.6), (t_4, 0.6, -0.6, 0.6, -0.6)\}$, $\phi(e_2) = \{(t_1, 0.7, -0.7, 0.6, -0.6), (t_2, 0.7, -0.7, 0.6, -0.6), (t_3, 0.7, -0.7, 0.6, -0.6), (t_4, 0.7, -0.7, 0.6, -0.6)\}$. *Let* $(\psi, \tilde{E})$ *be a BIF soft set on* E *with the function* $\psi : \tilde{E} \to P(E)$ *defined by* $\psi(e_1) = \{(t_1t_2, 0.5, -0.5, 0.6, -0.6), (t_2t_3, 0.5, -0.5, 0.6, -0.6), (t_3t_4, 0.5, -0.5, 0.6, -0.6), (t_1t_4, 0.5, -0.5, 0.6, -0.6)\}$, $\psi(e_2) = \{(t_1t_2, 0.6, -0.6, 0.6, -0.6), (t_2t_3, 0.6, -0.6, 0.6, -0.6), (t_3t_4, 0.6, -0.6, 0.6, -0.6), (t_1t_4, 0.6, -0.6, 0.6, -0.6), (t_1t_3, 0.6, -0.6, 0.6, -0.6), (t_2t_4, 0.6, -0.6, 0.6, -0.6)\}$.

It is easy to see through routine calculation that BIF graphs $M(e_1) = (\phi(e_1), \psi(e_1))$, $M(e_2) = (\phi(e_2), \psi(e_2))$ *are edge REG and totally edge REG BIF graphs as displayed in Figure 2. Since* $deg(t_1t_2) = deg(t_2t_3) = deg(t_3t_4) = deg(t_1t_4) = (1.2, -1.2, 1.2, -1.2)$

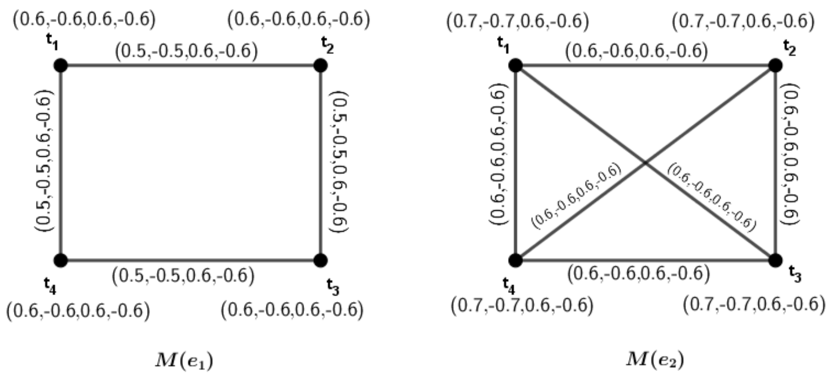


FIGURE 2. Perfectly edge REG BIF soft graph $\Gamma_I = \{M(e_1), M(e_2)\}$.

and $deg[t_1t_2] = deg[t_2t_3] = deg[t_3t_4] = deg[t_1t_4] = (1.7, -1.7, 1.8, -1.8)$ *in* $M(e_1)$ *and* $deg(t_1t_2) = deg(t_2t_3) = deg(t_3t_4) = deg(t_1t_4) = deg(t_1t_3) = deg(t_2t_4) = (1.4, -1.4, 1.2, -1.2)$ *and* $deg[t_1t_2] = deg[t_2t_3] = deg[t_3t_4] = deg[t_1t_4] = deg[t_1t_3] = deg[t_2t_4] = (2, -2, 1.8, -1.8)$ *in* $M(e_2)$. *Hence* $\Gamma_I = (M(e_1), M(e_2))$ *is a perfectly edge REG BIF SG.*

Proposition 4.2. *For a perfectly edge REG BIF SG* $\Gamma_I = (\phi, \psi, \tilde{E})$, *the function* ψ *is constant.*

The reverse part of the above Proposition need not be true, as discussed in the following example.

Example 4. *From the BIF SG Γ_I as shown in Figure 3a, we see that* $\psi(e_1)(t_1t_2) = \psi(e_1)(t_2t_3) = \psi(e_1)(t_3t_4) = \psi(e_1)(t_4t_1) = \psi(e_1)(t_4t_2) = (0.6, -0.6, 0.6, -0.6)$.

So ψ is a constant function. But $deg(t_1t_4) = (1.3, -1.4, 1.2, -1.2)$ and $deg(t_1t_2) = (1.4, -1.3, 1.2, -1.2)$, so $deg(t_1t_4) \neq deg(t_1t_2)$. This implies $M(e_1)$ is not perfectly REG BIF graph. Hence Γ_I is not perfectly REG BIF SG.

Theorem 4.3. *Let Γ_I be a BIF SG. Then Γ_I is perfectly edge REG BIF SG if and only if*

(i) $\sum_{t\neq v} \mu^P_{\psi(e_i)}(tv)+\sum_{u\neq v} \mu^P_{\psi(e_i)}(uv)-2\mu^P_{\psi(e_i)}(tu) = \sum_{x\neq z} \mu^P_{\psi(e_i)}(xz)+\sum_{y\neq z} \mu^P_{\psi(e_i)}(yz)-2\mu^P_{\psi(e_i)}(xy)$,

$\sum_{t\neq v} \mu^N_{\psi(e_i)}(tv) + \sum_{u\neq v} \mu^N_{\psi(e_i)}(uv) - 2\mu^N_{\psi(e_i)}(tu) = \sum_{x\neq z} \mu^N_{\psi(e_i)}(xz) + \sum_{y\neq z} \mu^N_{\psi(e_i)}(yz) - 2\mu^N_{\psi(e_i)}(xy)$,

$\sum_{t\neq v} \nu^P_{\psi(e_i)}(tv) + \sum_{u\neq v} \nu^P_{\psi(e_i)}(uv) - 2\nu^P_{\psi(e_i)}(tu) = \sum_{x\neq z} \nu^P_{\psi(e_i)}(xz) + \sum_{y\neq z} \nu^P_{\psi(e_i)}(yz) - 2\nu^P_{\psi(e_i)}(xy)$,

$\sum_{t\neq v} \nu^N_{\psi(e_i)}(tv) + \sum_{u\neq v} \nu^N_{\psi(e_i)}(uv) - 2\nu^N_{\psi(e_i)}(tu) = \sum_{x\neq z} \nu^N_{\psi(e_i)}(xz) + \sum_{y\neq z} \nu^N_{\psi(e_i)}(yz) - 2\nu^N_{\psi(e_i)}(xy)$,

(ii) $\mu^P_{\psi(e_i)}(tu) = \mu^P_{\psi(e_i)}(xy), \mu^N_{\psi(e_i)}(tu) = \mu^N_{\psi(e_i)}(xy), \nu^P_{\psi(e_i)}(tu) = \nu^P_{\psi(e_i)}(xy)$, $\nu^N_{\psi(e_i)}(tu) = \nu^N_{\psi(e_i)}(xy)$, $\forall\ tu, xy \in E$, $e_i \in \tilde{E}$.

Proof: (i) Suppose Γ_I is perfectly edge REG BIF SG. So Γ_I is edge REG BIF SG and from the definition of edge REG BIF SG, we get $deg(\mu^P(tu)) = deg(\mu^P(xy)), deg(\mu^N(tu)) = deg(\mu^N(xy)), deg(\nu^P(tu)) = deg(\nu^P(xy)), deg(\nu^N(tu)) = deg(\nu^N(xy))$, $\forall\ tu, xy \in E$. Then

$\sum_{t\neq v} \mu^P_{\psi(e_i)}(tv) + \sum_{u\neq v} \mu^P_{\psi(e_i)}(uv) - 2\mu^P_{\psi(e_i)}(tu) = \sum_{x\neq z} \mu^P_{\psi(e_i)}(xz) + \sum_{y\neq z} \mu^P_{\psi(e_i)}(yz) - 2\mu^P_{\psi(e_i)}(xy)$,

$\sum_{t\neq v} \mu^N_{\psi(e_i)}(tv) + \sum_{u\neq v} \mu^N_{\psi(e_i)}(uv) - 2\mu^N_{\psi(e_i)}(tu) = \sum_{x\neq z} \mu^N_{\psi(e_i)}(xz) + \sum_{y\neq z} \mu^N_{\psi(e_i)}(yz) - 2\mu^N_{\psi(e_i)}(xy)$,

$\sum_{t\neq v} \nu^P_{\psi(e_i)}(tv) + \sum_{u\neq v} \nu^P_{\psi(e_i)}(uv) - 2\nu^P_{\psi(e_i)}(tu) = \sum_{x\neq z} \nu^P_{\psi(e_i)}(xz) + \sum_{y\neq z} \nu^P_{\psi(e_i)}(yz) - 2\nu^P_{\psi(e_i)}(xy)$,

$\sum_{t\neq v} \nu^N_{\psi(e_i)}(tv) + \sum_{u\neq v} \nu^N_{\psi(e_i)}(uv) - 2\nu^N_{\psi(e_i)}(tu) = \sum_{x\neq z} \nu^N_{\psi(e_i)}(xz) + \sum_{y\neq z} \nu^N_{\psi(e_i)}(yz) - 2\nu^N_{\psi(e_i)}(xy)$.

Thus (i) holds. From Proposition 4.2, (ii) is also hold.

Conversely, let Γ_I be a BIF SG and (i) and (ii) hold. From (i), we have $deg(\mu^P(tu)) = deg(\mu^P(xy)) = k_i$, $deg(\mu^N(tu)) = deg(\mu^N(xy)) = l_i$, $deg(\nu^P(tu)) = deg(\nu^P(xy)) = \acute{k}_i$, $deg(\nu^N(tu))$
$= deg(\nu^N(xy)) = \acute{l}_i$, $\forall\ tu, xy \in E$, $e_i \in \tilde{E}$. This implies $M(e_i)$ is $(k_i, l_i, \acute{k}_i, \acute{l}_i)$ edge REG BIF graph. So Γ_I is edge REG BIF SG.

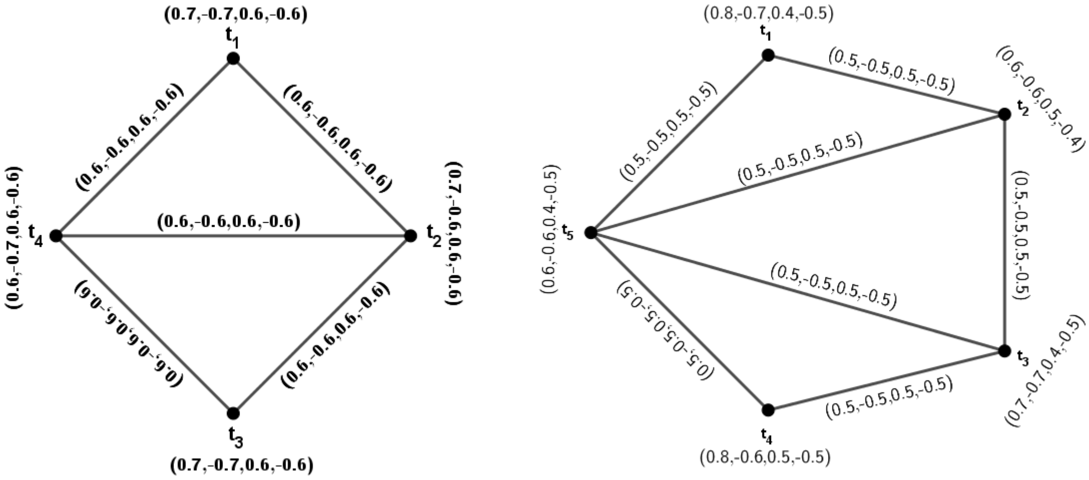
Now from (ii), we have $\mu^P_{\psi(e_i)}(tu) = \mu^P_{\psi(e_i)}(xy) = f_i$, $\mu^N_{\psi(e_i)}(tu) = \mu^N_{\psi(e_i)}(xy) = g_i$, $\nu^P_{\psi(e_i)}(tu) = \nu^P_{\psi(e_i)}(xy) = \acute{f}_i$, $\nu^N_{\psi(e_i)}(tu) = \nu^N_{\psi(e_i)}(xy) = \acute{g}_i$, $\forall\ tu, xy \in E$, $e_i \in \tilde{E}$. Thus ψ is a constant function. So, $deg(\mu^P[tu]) = deg(\mu^P(tu)) + \mu^P_{\psi(e_i)}(tu) = k_i + f_i$, $deg(\mu^N[tu]) = deg(\mu^N(tu)) + \mu^N_{\psi(e_i)}(tu) = l_i + g_i$, $deg(\nu^P[tu]) = deg(\nu^P(tu)) + \nu^P_{\psi(e_i)}(tu) = \acute{k}_i + \acute{f}_i$, $deg(\nu^N[tu]) = deg(\nu^N(tu)) + \nu^N_{\psi(e_i)}(tu) = \acute{l}_i + \acute{g}_i$. Similarly, $deg(\mu^P[xy]) = k_i + f_i$, $deg(\mu^N[xy]) = l_i+g_i$, $deg(\nu^P[xy]) = \acute{k}_i+\acute{f}_i$, $deg(\nu^N[xy]) = \acute{l}_i+\acute{g}_i$. Therefore, $deg(\mu^P[tu]) = deg(\mu^P[xy]) = t_i(say)$, $deg(\mu^N[tu]) = deg(\mu^N[xy]) = u_i(say)$, $deg(\nu^P[tu]) = deg(\nu^P[xy]) =$

$\acute{t}_i(say)$, $deg(\nu^N[tu]) = deg(\nu^N[xy]) = \acute{u}_i(say)$. So $M(e_i)$ is totally edge REG BIF graph. Hence Γ_I is totally edge REG BIF SG. Therefore Γ_I is a perfectly edge REG BIF SG.

Corollary 4.1. *Let Γ_I be a perfectly edge REG BIF SG. Then size of $M(e_i)$ is $S(M(e_i)) = |E|(k_i, l_i, \acute{k}_i, \acute{l}_i)$, where $(k_i, l_i, \acute{k}_i, \acute{l}_i) = (\mu^P_{\psi(e_i)}(tu), \mu^N_{\psi(e_i)}(tu), \nu^P_{\psi(e_i)}(tu), \nu^N_{\psi(e_i)}(tu))$, $\forall\ tu \in E$, $e_i \in \tilde{E}$.*

The reverse part of Corollary 4.1 need not be true, as it is shown in the following example.

Example 5. *Take the BIF SG Γ_I as shown in Figure 3b. Here $|E| = 7$ and size of $M(e_1)$ is*



(A) BIF soft graph $\Gamma_I = \{M(e_1)\}$. (B) BIF soft graph $\Gamma_I = \{M(e_1)\}$.

FIGURE 3

(3.5,-3.5,3.5,-3.5). Now by Corollary 4.1, size of $M(e_1)$ is $S(M(e_1)) = |E|(k_i, l_i, \acute{k}_i, \acute{l}_i) = 7(0.5, -0.5, 0.5, -0.5) = (3.5, -3.5, 3.5, -3.5)$. But $deg(\mu^P(t_1t_2)) = 1.4$, $deg(\mu^N(t_1t_2)) = -1.3$, $deg(\nu^P(t_1t_2)) = 0.9$, $deg(\nu^N(t_1t_2)) = -0.9$, $deg(\mu^P(t_2t_3)) = 1.3$, $deg(\mu^N(t_2t_3)) = -1.3$, $deg(\nu^P(t_2t_3)) = 0.9$, $deg(\nu^N(t_2t_3)) = -0.9$. So $deg(t_1t_2) \neq deg(t_2t_3)$. Therefore, $M(e_1)$ is not perfectly edge REG BIF SG. This shows that Corollary 4.1 is not accurate.

Theorem 4.4. *If Γ_I is a REG BIF SG and ψ is a constant function, then Γ_I is a perfectly edge REG BIF SG.*

Proof: Let Γ_I be a REG BIF SG and $\psi = (\mu^P_{\psi(e_i)}(tu), \mu^N_{\psi(e_i)}(tu), \nu^P_{\psi(e_i)}(tu), \nu^N_{\psi(e_i)}(tu)) = (k_i, l_i, \acute{k}_i, \acute{l}_i)$, $\forall\, tu \in E$ be a constant function. Now, $deg(\mu^P(tu)) = deg(\mu^P(t)) + deg(\mu^P(u)) - 2\mu^P_{\psi(e_i)}(tu) = f_i + f_i - 2k_i = 2(f_i - k_i)$, $deg(\mu^N(tu)) = deg(\mu^N(t)) + deg(\mu^N(u)) - 2\mu^N_{\psi(e_i)}(tu) = f_i + f_i - 2k_i = 2(f_i - k_i)$, $deg(\nu^P(tu)) = deg(\nu^P(t)) + deg(\nu^P(u)) - 2\nu^P_{\psi(e_i)}(tu) = f_i + f_i - 2k_i = 2(f_i - k_i)$, $deg(\nu^N(tu)) = deg(\nu^N(t)) + deg(\nu^N(u)) - 2\nu^N_{\psi(e_i)}(tu) = f_i + f_i - 2k_i = 2(f_i - k_i)$, $\forall\ tu \in E$, $e_i \in \tilde{E}$. This implies that $M(e_i)$ is an edge REG BIF graph. Hence Γ_I is an edge REG BIF SG.

Now, $deg(\mu^P[tu]) = deg(\mu^P(tu)) + \mu^P_{\psi(e_i)}(tu) = 2(f_i - k_i) + k_i = 2f_i - k_i$. Similarly, $deg(\mu^N[tu]) = 2g_i - l_i$, $deg(\nu^P[tu]) = 2\acute{f}_i - \acute{k}_i$, $deg(\nu^N[tu]) = 2\acute{g}_i - \acute{l}_i$. Thus $deg[tu] = (2f_i - k_i, 2g_i - l_i, 2\acute{f}_i - \acute{k}_i, 2\acute{g}_i - \acute{l}_i)$, $\forall\, tu \in E$, $e_i \in \tilde{E}$. Thus $M(e_i)$ is totally edge REG BIF SG. Hence Γ_I is perfectly edge REG BIF SG.

If Γ_I is totally edge REG BIF SG and ψ is a constant function, then Γ_I may not be perfectly edge REG BIF SG as discussed below.

Example 6. *Consider the BIF SG Γ_I, which is shown in Figure 5a. Here $\psi = (\mu^P_{\psi(e_i)}(tu), \mu^N_{\psi(e_i)}(tu), \nu^P_{\psi(e_i)}(tu), \nu^N_{\psi(e_i)}(tu)) = (0.3, -0.2, 0.4, -0.1)$, $\forall\ tu \in E$. Now, $deg[t_1] = deg[t_2] = deg[t_3] = deg[t_4] = (1.3, -0.9, 1.2, -0.3)$. So, $M(e_1)$ is totally REG BIF graph. Hence Γ_I is totally REG BIF SG. Now $deg(t_1t_2) = (1.1, -0.8, 0.4, -0.1)$, $deg(t_1t_3) = (0.8, -0.6, 0.0, 0.0)$ but $deg(t_1t_2) \neq deg(t_1t_3)$. Hence Γ_I is not perfectly edge REG BIF SG.*

Theorem 4.5. *If Γ_I is perfectly REG and complete BIF SG, then Γ_I is perfectly edge REG BIF SG.*

Proof: Since Γ_I is perfectly REG BIF SG, so by Proposition 4.1, we have $\mu^P_{\phi(e_i)}(t) = \mu^P_{\phi(e_i)}(u)$, $\mu^N_{\phi(e_i)}(t) = \mu^N_{\phi(e_i)}(u)$, $\nu^P_{\phi(e_i)}(t) = \nu^P_{\phi(e_i)}(u)$, $\nu^N_{\phi(e_i)}(t) = \nu^N_{\phi(e_i)}(u)$, $\forall t, u \in \acute{V}$, $e_i \in \tilde{E}$...... (1).

As Γ_I is complete BIF SG, so $\mu^P_{\psi(e_i)}(tu) = min\{\mu^P_{\phi(e_i)}(t), \mu^P_{\phi(e_i)}(u)\}$, $\mu^N_{\psi(e_i)}(tu) = max\{\mu^N_{\phi(e_i)}(t), \mu^N_{\phi(e_i)}(u)\}$, $\nu^P_{\psi(e_i)}(tu) = max\{\nu^P_{\phi(e_i)}(t), \nu^P_{\phi(e_i)}(u)\}$, $\nu^N_{\psi(e_i)}(tu) = min\{\nu^N_{\phi(e_i)}(t), \nu^N_{\phi(e_i)}(u)\}$, $\forall\ tu \in E$, $e_i \in \tilde{E}$...... (2).

Combining (1) and (2), we say that ψ is a constant function. Since Γ_I is a perfectly REG BIF SG, Γ_I is a REG BIF SG and ψ is constant. Thus by Theorem 4.4, Γ_I is a perfectly edge REG BIF SG.

5. Perfectly IREG and perfectly edge IREG BIF soft graph

This section has discussed the perfectly IREG and the edge IREG BIF SG.

Definition 5.1. *A BIF SG $\Gamma_I = (\phi, \psi, \tilde{E})$ is called a neighborly IREG BIF SG if $M(e_i)$ is neighborly IREG BIF graph $\forall\ e_i \in \tilde{E}$, i.e, if the degree of each pair of adjacent vertices of $M(e_i)$ are different, $\forall\ e_i \in \tilde{E}$.*

Definition 5.2. *A BIF SG $\Gamma_I = (\phi, \psi, \tilde{E})$ is said to be totally neighborly IREG BIF SG if $M(e_i)$ is totally neighborly IREG BIF graph $\forall\ e_i \in \tilde{E}$, i.e, if the total degree of each pair of adjacent vertices of $M(e_i)$ are different, $\forall\ e_i \in \tilde{E}$.*

Example 7. *Take two non-empty sets $\acute{V} = \{t_1, t_2, t_3, t_4\}$ and $E = \{t_1t_2, t_2t_3, t_3t_4\}$. Let $\tilde{E} = \{e_1, e_2\}$ and $(\phi, \tilde{E})$ be a BIF soft set on $\acute{V}$ with approximate function $\phi : \tilde{E} \to P(\acute{V})$ defined by $\phi(e_1) = \{(t_1, 0.5, -0.3, 0.4, -0.6), (t_2, 0.4, -0.5, 0.3, -0.5), (t_3, 0.2, -0.1, 0.6, -0.4), (t_4, 0.3, -0.4, 0.6, -0.7)\}$, $\phi(e_2) = \{(t_1, 0.4, -0.4, 0.5, -0.3), (t_2, 0.5, -0.3, 0.4, -0.4), (t_3, 0.7, -0.6, 0.3, -0.5), (t_4, 0.6, -0.5, 0.5, -0.7)\}$. Let $(\psi, \tilde{E})$ be a BIF soft set on E with approximate function $\psi : \tilde{E} \to P(E)$ defined by $\psi(e_1) = \{(t_1t_2, 0.4, -0.3, 0.4, -0.6), (t_1t_4, 0.3, -0.3, 0.6, -0.7), (t_2t_3, 0.1, -0.1, 0.7, -0.5)\}$, $\psi(e_2) = \{(t_1t_2, 0.4, -0.3, 0.4, -0.6), (t_1t_4, 0.3, -0.3, 0.6, -0.7), (t_2t_3, 0.1, -0.1, 0.7, -0.5)\}$.*

It is easy to see through routine calculation that BIF graphs $M(e_1) = (\phi(e_1), \psi(e_1))$, $M(e_2) = (\phi(e_2), \psi(e_2))$, are neighborly IREG and totally neighborly IREG BIF graphs as displayed in Figure 4. Hence Γ_I is neighborly IREG and totally neighborly IREG BIF SG.

Definition 5.3. *A BIF SG Γ_I is said to be perfectly IREG if $M(e_i)$ is perfectly IREG BIF graph $\forall\ e_i \in \tilde{E}$, i.e, 1. all vertex degrees of $M(e_i)$ are different, 2. all vertex total degrees of $M(e_i)$ are different.*

Theorem 5.1. *If Γ_I is perfectly IREG BIF SG, then Γ_I is necessarily neighbourly IREG, totally neighbourly IREG and highly IREG BIF SG.*

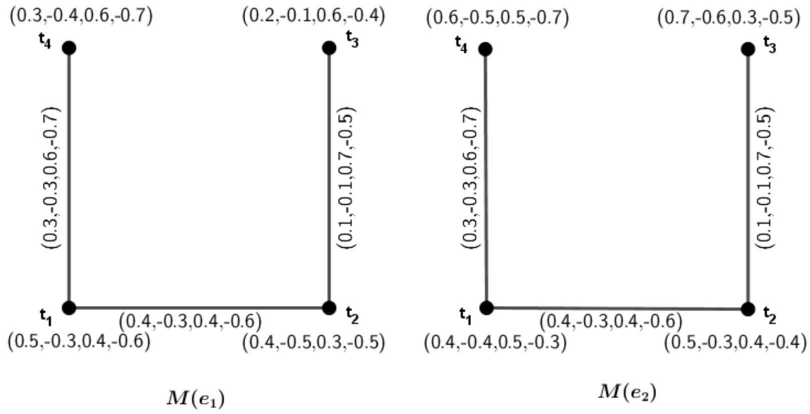
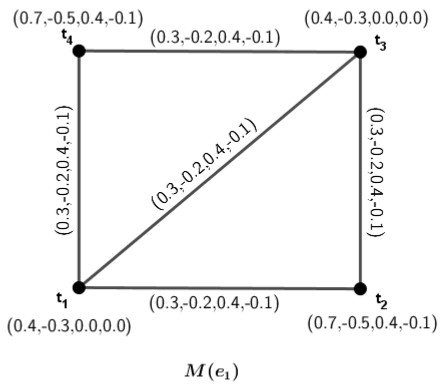


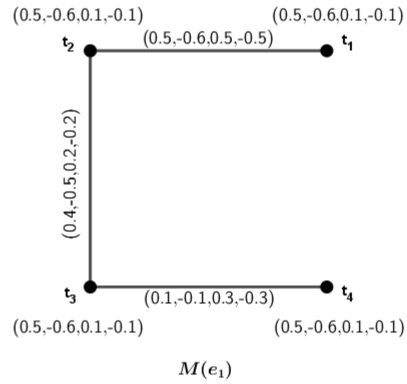
FIGURE 4. Neighborly IREG and totally neighborly IREG BIF soft graph $\Gamma_I = \{M(e_1), M(e_2)\}$.

Proof: Let Γ_I be a perfectly IREG BIF SG. So each vertex of $M(e_i)$ has a different degree. Then every two adjacent vertices of $M(e_i)$ are of distinct degrees. Therefore $M(e_i)$ is neighbourly IREG BIF graph. Hence Γ_I is a neighbourly IREG BIF SG. Since Γ_I is perfectly IREG BIF SG, the total degrees of each vertex of $M(e_i)$ are different. Then every two adjacent vertices of $M(e_i)$ are of distinct degrees. Therefore $M(e_i)$ is a totally neighbourly IREG BIF graph. Hence, Γ_I is a totally neighbourly IREG BIF SG. Since Γ_I is perfectly IREG BIF SG, the degrees of each vertex of $M(e_i)$ are distinct. Thus the degrees of the adjacent vertices of each vertex of $M(e_i)$ are different. Therefore, $M(e_i)$ is a highly IREG BIF graph. Hence, Γ_I is a highly IREG BIF SG.

Example 8. *Consider a BIF SG* $\Gamma_I = (\phi, \psi, \tilde{E})$ *as shown in Figure 5b.*



(A) BIF soft graph $\Gamma_I = \{M(e_1)\}$.



(B) BIF soft graph $\Gamma_I = \{M(e_1)\}$.

FIGURE 5

Now, $deg(t_1) = (0.5, -0.6, 0.5, -0.5)$, $deg(t_2) = (0.9, -1.1, 0.7, -0.7)$, $deg(t_3) = (0.5, -0.6, 0.5, -0.5)$, $deg(t_4) = (0.1, -0.1, 0.3, -0.3)$. *Thus,* Γ_I *is neighborly IREG BIF SG. Also,* $deg[t_1] = (1, -1.2, 0.6, -0.6)$, $deg[t_2] = (1.4, -1.7, 0.8, -0.8)$, $deg[t_3] = (1.1, -1.2, 0.6, -0.6)$, $deg[t_4] = (0.6, -0.7, 0.4, -0.4)$. *Thus,* Γ_I *is totally neighborly IREG BIF SG. But* $deg(t_1) = deg(t_3)$, *so* Γ_I *is not perfectly IREG BIF SG.*

Similarly, we can show a BIF SG Γ_I, a highly IREG BIF SG but not a perfectly IREG BIF SG.

Theorem 5.2. *The sufficient condition of a neighbourly IREG and totally neighbourly IREG BIF SG to be perfectly IREG BIF SG is that there exists an edge between every pair of vertices of $M(e_i)$, $\forall\ e_i \in \tilde{E}$.*

Proof: Let Γ_I be a neighborly IREG and totally neighborly IREG BIF SG and there exists an edge between every pair of vertices of $M(e_i)$, $\forall\ e_i \in \tilde{E}$. Since Γ_I is neighborly IREG BIF SG, so $deg(t) \neq deg(u)$ for all adjacent vertices $\forall\ t, u \in \acute{V}$, $e_i \in \tilde{E}$.....(1). But between each pair of vertices of there is an edge in $M(e_i)$. This means each pair of vertices of $M(e_i)$ are adjacent, i.e, $\forall\ tu \in E$, $e_i \in \tilde{E}$.....(2). Using (1) and (2), we have $deg(t) \neq deg(u)$, $\forall\ t, u \in \acute{V}$, $e_i \in \tilde{E}$. Similarly, it can be shown that $deg[t] \neq deg[u]$, $\forall\ t, u \in \acute{V}$, $e_i \in \tilde{E}$. Therefore the degree and total degree of each vertices of $M(e_i)$ are different. Hence $M(e_i)$ is perfectly IREG BIF graph, $\forall\ e_i \in \tilde{E}$. So Γ_I is perfectly IREG BIF SG.

Corollary 5.1. *For a perfectly IREG BIF SG, ϕ need not be constant.*

Theorem 5.3. *If in a BIF SG Γ_I, $deg(t_j) \neq deg(t_k)$, $\mu^P_{\phi(e_i)}(t_j) = \mu^P_{\phi(e_i)}(t_k)$, $\mu^N_{\phi(e_i)}(t_j) = \mu^N_{\phi(e_i)}(t_k)$, $\nu^P_{\phi(e_i)}(t_j) = \nu^P_{\phi(e_i)}(t_k)$ and $\nu^N_{\phi(e_i)}(t_j) = \nu^N_{\phi(e_i)}(t_k)$, $j \neq k$, $\forall\ t_j, t_k \in \acute{V}$. Then Γ_I is perfectly IREG BIF SG.*

Proof: Let $\mu^P_{\phi(e_i)}(t_j) = \mu^P_{\phi(e_i)}(t_k) = f_i$, $\mu^N_{\phi(e_i)}(t_j) = \mu^N_{\phi(e_i)}(t_k) = g_i$, $\nu^P_{\phi(e_i)}(t_j) = \nu^P_{\phi(e_i)}(t_k) = \acute{f}_i$, $\nu^N_{\phi(e_i)}(t_j) = \nu^N_{\phi(e_i)}(t_k) = \acute{g}_i$, $j \neq k$, $\forall\ t_j, t_k \in \acute{V}$. Now $deg(t_j) \neq deg(t_k)$ implies that $deg(\mu^P(t_j)) \neq deg(\mu^P(t_k))$, $deg(\mu^N(t_j)) \neq deg(\mu^N(t_k))$, $deg(\nu^P(t_j)) \neq deg(\nu^P(t_k))$ and $deg(\nu^N(t_j)) \neq deg(\nu^N(t_k))$, $\forall\ t_j, t_k \in \acute{V}$. So, $deg(\mu^P(t_j)) + f_i \neq deg(\mu^P(t_k)) + f_i$ and $deg(\mu^N(t_j)) + g_i \neq deg(\mu^N(t_k)) + g_i$, $\forall\ t_j, t_k \in \acute{V}$.
$\Rightarrow deg(\mu^P(t_j)) + \mu^P_{\phi(e_i)}(t_j) \neq deg(\mu^P(t_k)) + \mu^P_{\phi(e_i)}(t_k)$ and $deg(\mu^N(t_j)) + \mu^N_{\phi(e_i)}(t_j) \neq deg(\mu^N(t_k)) + \mu^N_{\phi(e_i)}(t_k)$, $j \neq k$, $\forall\ t_j, t_k \in \acute{V}$.
$\Rightarrow deg(\mu^P[t_j]) \neq deg(\mu^P[t_k])$ and $deg(\mu^N[t_j]) \neq deg(\mu^N[t_k])$, $j \neq k$, $\forall\ t_j, t_k \in \acute{V}$.
Similarly, $deg(\nu^P[t_j]) \neq deg(\nu^P[t_k])$ and $deg(\nu^N[t_j]) \neq deg(\nu^N[t_k])$, $j \neq k$, $\forall\ t_j, t_k \in \acute{V}$.

Hence $M(e_i)$ is perfectly IREG BIF SG. Therefore Γ_I is perfectly IREG BIF SG.

Definition 5.4. *A BIF SG Γ_I is said to be perfectly edge IREG BIF SG if $M(e_i)$ is perfectly edge IREG BIF graph $\forall\ e_i \in \tilde{E}$, i.e, 1. the degrees of each edges of $M(e_i)$ are different, 2. the total degrees of each edges of $M(e_i)$ are different.*

Theorem 5.4. *If Γ_I is perfectly edge IREG BIF SG, then Γ_I is necessarily neighbourly edge IREG, totally neighbourly edge IREG BIF SG.*

Proof: Let Γ_I be a perfectly edge IREG BIF SG. Then the degrees of each edge of $M(e_i)$ is different. Thus the degree of each pair of edges is different. Therefore $M(e_i)$ is neighborly edge IREG BIF graph, $\forall\ e_i \in \tilde{E}$. Hence Γ_I is neighbourly edge IREG BIF SG. Similarly, Γ_I is a totally neighbourly edge IREG BIF SG.

6. Application of bipolar intuitionistic fuzzy soft graphs in social group

We know intuitionistic fuzzy soft graphs have numerous applications in decision-making problems and are used to deal with uncertainty from the problems of our daily life. But here, we have used the concept of bipolarity. We have applied the notion of BIFSG model on social group to find out the most influenced person in the social group with respect to

Table 2. Vertex membership values of BIF soft graph

ϕ	Tuhin	Biplab	Rasu	Saddam	Sabnam
e_1	(0.2,-0.4,0.3,-0.1)	(0.7,-0.2,0.3,-0.2)	(0.9,-0.2,0.1,-0.1)	(0.5,-0.4,0.5,-0.3)	(0.1,-0.3,0.4,-0.4)
e_2	(0.3,-0.1,0.4,-0.2)	(0.4,-0.3,0.5,-0.3)	(0.6,-0.2,0.4,-0.1)	(0.4,-0.4,0.4,-0.3)	(0.5,-0.4,0.4,-0.5)

the different attributes of persons including cooperation, team spirit, controlling power, creativeness etc.

6.1. **Model construction.** In the observation of a group of people with their group behaviour, it is often shown that some people can influence the thinking of the other people through the parameters of cooperation, team spirit, controlling power, creativeness etc. This behaviour can be modelled with the help of the model of a BIF SG. Each person in the group represents a vertex. An edge between two vertices represents the relation between two persons. All persons in the group depend on the parameters cooperation, team spirit, controlling power, creativeness etc. This graph does not contain any multiple edges and loops.

We take a social group with two parameters e_1 and e_2, where e_1=cooperation and e_2=team spirit. For the parameter cooperation (e_1) we define the vertex and edge MS values in the following way: Each vertex contains four values which are defined as (i) positive MS depends on the degree of a member with other members in the group who are benefited by cooperation, (ii) sometimes by a member; any wrong information based cooperation will harm situation of others, presented by the degree of negative MS, (iii) If there is no harm to anyone for non-cooperation of a member depends on the degree of positive NMS, (iv) If there is harm to anyone for non-cooperation of a member depends on the degree of negative NMS. Every edge membership value between two vertices v_1 and v_2 contains four values which are defined as (i) positive MS value depends on how much benefited v_1 and v_2 to each other by cooperation, (ii) sometimes, any wrong information based cooperation will harm situation to each other, presented by negative MS value, (iii) if there is no harm to each other for non-cooperation (such as non-cooperation in any bad deed is indirectly good), presented by positive NMS value, (iv) if there is harm to each other for non-cooperation (such as if the information is not delivered on time then there will be loss), presented by negative NMS value. Similarly, we can define the vertex and edge membership values for the parameter team spirit (e_2). We can find the most influenced person in the social group corresponding to the distinct attributes of persons, including cooperation, team spirit, controlling power, creativeness, etc.

Suppose a group contains five persons, viz. Tuhin, Biplab, Rasu, Saddam and Sabnam. Thus, the set of vertices of the graph constructed by these five people is $\acute{V} = \{\text{Tuhin } (t_1), \text{Biplab } (t_2), \text{Rasu } (t_3), \text{Saddam } (t_4), \text{Sabnam } (t_5)\}$ and the parameter set is $\tilde{A} = \{e_1 = cooperation, e_2 = team\ spirit\}$.

A BIF SG is denoted by $\Gamma_I = \{M(e_i) = (\phi(e_i), \psi(e_i)) : e_i \in \tilde{A}\}$ and the vertex and edge MS values are shown in Table 2 and Table 3 respectively. A BIF graph $M(e_1) = (\phi(e_1), \psi(e_1))$ with respect to e_1 is displayed in Figure 6 and same with respect to e_2 is displayed in Figure 7.

6.2. **Determination of score values.** For finding the most influenced person, the score values for this group of people are calculated by the respective five steps:

Step 1: The sum of positive MS and NMS values of each vertex (persons) with respect to e_1 and e_2 are calculated by the formula $S^P(t_i) = \sum_{j=1}^{2}(\mu^P_{\phi(e_j)}(t_i) + \nu^P_{\phi(e_j)}(t_i))$, for each $i = 1, 2, 3, 4, 5$, where $S^P(t_i)$ is the sum of positive MS and NMS values of the vertex t_i

Table 3. Edge membership values of BIF soft graph

ψ	t_1t_5	t_2t_3	t_1t_2
e_1	(0.1,-0.3,0.4,-0.4)	(0.7,-0.2,0.3,-0.2)	(0.2,-0.2,0.3,-0.2)
ψ	t_1t_3	t_2t_4	t_2t_5
e_1	(0.2,-0.2,0.3,-0.1)	(0.5,-0.2,0.5,-0.3)	(0.1,-0.2,0.4,-0.4)
ψ	t_4t_3	t_5t_3	t_5t_4
e_1	(0.5,-0.2,0.5,-0.3)	(0.1,-0.2,0.4,-0.4)	(0.1,-0.3,0.5,-0.4)
ψ	t_1t_2	t_4t_1	t_1t_5
e_2	(0.3,-0.1,0.5,-0.3)	(0.3,-0.1,0.4,-0.3)	(0.3,-0.1,0.6,-0.5)
ψ	t_3t_2	t_4t_2	t_5t_2
e_2	(0.4,-0.2,0.5,-0.3)	(0.4,-0.3,0.5,-0.3)	(0.4,-0.3,0.5,-0.5)
ψ	t_4t_3	t_4t_5	
e_2	(0.4,-0.2,0.4,-0.3)	(0.4,-0.1,0.6,-0.5)	

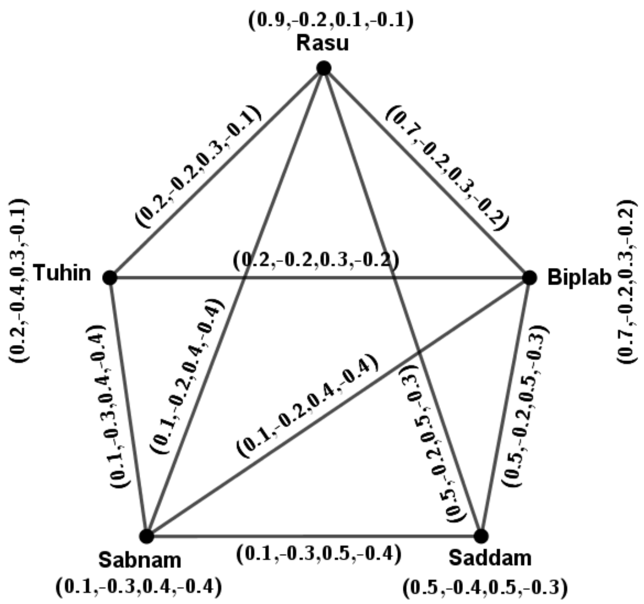


Figure 6. BIF graph corresponding to attribute e_1.

Table 4. Sum of positive MS and NMS values of each vertex

$S^P(t_1)$	$S^P(t_2)$	$S^P(t_3)$	$S^P(t_4)$	$S^P(t_5)$
1.2	1.9	2.0	1.8	1.4

and $\mu^P_{\phi(e_j)}(t_i)$ and $\nu^P_{\phi(e_j)}(t_i)$ are the positive MS and positive NMS values of the vertex t_i with respect to parameter e_j. For $i = 1$,

$$S^P(t_1) = \sum_{j=1}^{2}(\mu^P_{\phi(e_j)}(t_1) + \nu^P_{\phi(e_j)}(t_1)) = (0.2 + 0.3) + (0.3 + 0.4) = 1.2.$$

Similarly, $S^P(t_2) = 1.9, S^P(t_3) = 2, S^P(t_4) = 1.8, S^P(t_5) = 1.4$. These values are given by Table 4.

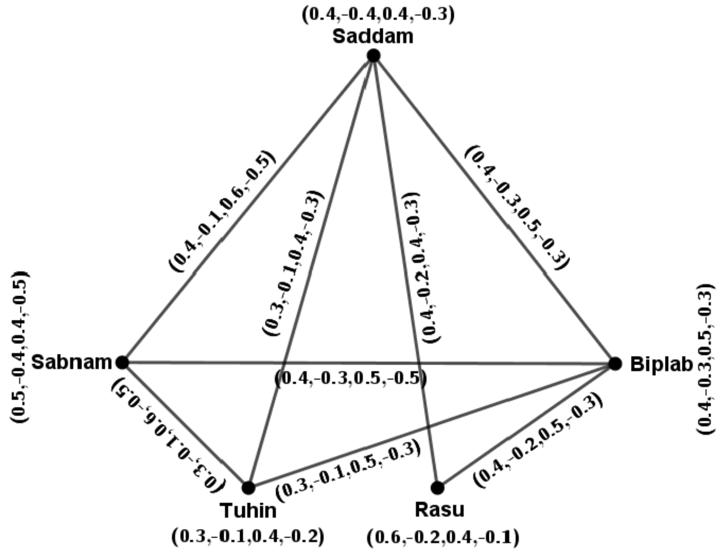


FIGURE 7. BIF graph corresponding to attribute e_2.

TABLE 5. Sum of negative MS and NMS values of each vertex

$S^N(t_1)$	$S^N(t_2)$	$S^N(t_3)$	$S^N(t_4)$	$S^N(t_5)$
-0.8	-1.0	-0.6	-1.4	-1.6

Step 2: We evaluate the sum of negative MS and negative NMS values of each vertices with respect to e_1 and e_2, using the formula $S^N(t_i) = \sum_{j=1}^{2}(\mu^N_{\phi(e_j)}(t_i) + \nu^N_{\phi(e_j)}(t_i))$, for each $i = 1, 2, 3, 4, 5$ and the calculated values are given by the Table 5, where $S^N(t_i)$ is the sum of negative MS and NMS values of the vertex t_i and $\mu^N_{\phi(e_j)}(t_i)$ and $\nu^N_{\phi(e_j)}(t_i)$ are the negative MS and negative NMS values of the vertex t_i with respect to parameter e_j.

Step 3: Here, we evaluate the sum of positive score values of each vertex t_i, influenced by another vertices, denoted by $I^P(t_i)$ with respect to the parameters e_1 (Figure 6) and e_2 (Figure 7) using the formula $I^P(t_i) = \sum_{j=1}^{2}[\sum_{l=1,l\neq i}^{5}(\mu^P_{\psi(e_j)}(t_lt_i) + \nu^P_{\psi(e_j)}(t_lt_i))]$ for each $i = 1, 2, 3, 4, 5$, where, $\mu^P_{\psi(e_j)}(t_lt_i)$ and $\nu^P_{\psi(e_j)}(t_lt_i)$ are the positive edge MS and NMS values on the vertex t_i, influenced by the vertex t_l with respect to parameter e_j. Any two vertices t and u are not related by the parameters in a group, then there is no edge between them. So, its edge membership values are 0, i.e, $\mu^P_{\psi(e_j)}(tu) = 0$, $\nu^P_{\psi(e_j)}(tu) = 0$, $\mu^N_{\psi(e_j)}(tu) = 0$, $\nu^N_{\psi(e_j)}(tu) = 0$. For $i = 1$,

$$\begin{aligned}
I^P(t_1) &= \sum_{j=1}^{2}[\sum_{l=2}^{5}(\mu^P_{\psi(e_j)}(t_lt_1) + \nu^P_{\psi(e_j)}(t_lt_1))] \\
&= [\sum_{l=2}^{5}(\mu^P_{\psi(e_1)}(t_lt_1) + \nu^P_{\psi(e_1)}(t_lt_1))] + [\sum_{l=2}^{5}(\mu^P_{\psi(e_2)}(t_lt_1) + \nu^P_{\psi(e_2)}(t_lt_1))] \\
&= [0.2 + 0.3 + 0.2 + 0.3 + 0.1 + 0.4] + [0.3 + 0.5 + 0.3 + 0.4 + 0.3 + 0.6] = 3.9
\end{aligned}$$

Similarly, $I^P(t_2) = 6.5, I^P(t_3) = 4.7, I^P(t_4) = 6, I^P(t_5) = 4.9$. These score values are given by Table 6.

Table 6. Sum of positive score values of each vertex influenced by another vertices

$I^P(t_1)$	$I^P(t_2)$	$I^P(t_3)$	$I^P(t_4)$	$I^P(t_5)$
3.9	6.5	4.7	6.0	4.9

Table 7. Sum of negative score values of each vertex influenced by another vertices

$I^N(t_1)$	$I^N(t_2)$	$I^N(t_3)$	$I^N(t_4)$	$I^N(t_5)$
-2.8	-4.2	-2.8	-3.8	-4.6

Table 8. Final score values of each vertex

$F(t_1)$	$F(t_2)$	$F(t_3)$	$F(t_4)$	$F(t_5)$
1.5	3.2	3.3	2.6	0.1

Step 4: Sum of negative score values of each vertex t_i, influenced by another vertices, denoted by $I^N(t_i)$ with respect to e_1 (Figure 6) and e_2 (Figure 7) are calculated by the formula $I^N(t_i) = \sum_{j=1}^{2}[\sum_{l=1,l\neq i}^{5}(\mu^N_{\psi(e_j)}(t_lt_i) + \nu^N_{\psi(e_j)}(t_lt_i))]$ for each $i = 1, 2, 3, 4, 5$. In Table 7, the score values are given, where, $\mu^N_{\psi(e_j)}(t_lt_i)$ and $\nu^N_{\psi(e_j)}(t_lt_i)$ are the negative edge MS and NMS values on the vertex t_i, influenced by the vertex t_l with respect to parameter e_j.

Step 5: The final score value of the vertex t_i, is calculated by the formula $F(t_i) = (S^P(t_i) + S^N(t_i)) + (I^P(t_i) + I^N(t_i))$, for each $i = 1, 2, 3, 4, 5$. In Table 8, the score values are given.

6.3. **Decision making.** By Table 8, it is meant that the highest scoring value obtained by Rasu, which is 3.3. The second and third-highest scoring values respectively obtained by Biplab and Saddam. The score values of Biplab, Saddam, Tuhin and Sabnam are 3.2, 2.6, 1.5 and 0.1, respectively. Hence the most influenced person in the group is Rasu, second-most is Biplab, third-most is Saddam, fourth is Tuhin and fifth is Sabnam.

7. Conclusion

In this article, particular types of BIF soft graphs, a combined representation of bipolar intuitionistic fuzzy soft sets and graph structure have been introduced and elaborately discussed in a new way. We have defined a BIF soft graph and shown the new concepts about their properties and the proof of some theorems based on their aspects. The concepts of regular, totally regular, perfectly regular and edge-regular, totally edge-regular, perfectly edge-regular bipolar intuitionistic fuzzy graphs have been proposed in soft environment and discussed with suitable examples. Also, some related properties have been investigated. The concept of perfectly irregular and perfectly edge-irregular bipolar intuitionistic fuzzy soft graphs has been introduced and explored a few features. The concept of BIF soft graph is applied to a real-life application as a decision-making tool to find out the most influenced person from a social group. This analysis process gives more efficient fuzziness results than other fuzzy soft graphs. As bipolar sense includes a positive and a negative sense of any parameter; so introducing such new fuzzy soft graphs will be very interesting and helpful to solve many real-life problems with uncertainties and vagueness.

In the future, we want to incorporate our research using the soft set theory on bipolar fuzzy hyper-graphs, bipolar Pythagorean fuzzy graphs and complex fuzzy graphs.

Acknowledgments. The first author is thankful to the University Grant Commission (UGC) Govt. of India for financial support under UGC-Ref. No.: 1155/(CSIR-UGC NET DEC. 2018) dated 24/07/2019.

Compliance with ethical standards.

Disclosure statement. No potential conflict of interest was reported by the authors.

Ethical approval. This article does not contain any studies with human participants or animals performed by any of the authors.

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