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FUZZY PRIMELY FILTERS IN BL-ALGEBRAS

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Abstract. This study introduces and rigorously examines the concept of fuzzy primely filters (FPYFs) in BL-algebras, marking a significant advancement in the field. The investigation extends beyond conceptualization to elucidate the complex interrelationships between FPYFs and established fuzzy filter classes within the BL-algebraic framework. Through comprehensive analysis, we uncover intricate connections and potential hierarchical structures among these diverse filter types. Our findings not only expand the theoretical landscape of BL-algebras but also provide a robust foundation for further exploration of fuzzy filter relationships. This research contributes to a deeper understanding of the algebraic structures underpinning fuzzy logic systems, offering new insights into the fundamental properties of BL-algebras and their associated filters.

Furthermore, by leveraging the concept of the complement set, we embark on a rigorous investigation into the interplay between FPYFs and fuzzy prime ideals (FPEIs). This investigation seeks to elucidate the nature of their interaction and potential implications for the broader theory of BL-algebras.

Keywords: fuzzy primely filters, fuzzy prime ideals, BL-algebra.

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1. Introduction

Hájek's introduction of BL-algebras [1] revolutionized the algebraic investigation of many-valued logic. These structures, leveraging continuous triangular norms, a cornerstone of fuzzy logic, provide an algebraic perspective on logic. Filters within BL-algebras play a crucial role, corresponding to sets of demonstrably valid formulas in the associated logic system. Hájek [1] established filters and prime filters (PEFs), ultimately demonstrating the completeness of basic logic (BL) using PEFs. Subsequent research by Turunnen [2, 3, 4] delved into the properties of filters and PEFs in BL-algebras. Notably, Turunnen [3] introduced Boolean filters and characterized them, further establishing a connection

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between their existence and the bipartiteness of BL-algebras. Haveshki et al. [5] furthered the analysis by introducing positive implicative filters.

Within the realm of mathematics, Zadeh's (1965) introduction of fuzzy sets marked a significant departure from classical set theory and paved the way for Liu et al. [7] to introduce fuzzy filters in BL-algebras. They subsequently extended this concept by introducing fuzzy Boolean and positive implicative filters [8].

Fuzzy logic, a comprehensive theory encompassing diverse motivations from the early 20th century, has emerged from many-valued logic. It bridges distinct disciplines like quantum mechanics, probability theory, and computer science. This rich tapestry of applications motivates our investigation into novel filter structures within BL-algebras, aiming to expand and establish new conceptual frameworks for their exploration.

This paper contributes to the ongoing research by introducing the concept of fuzzy primely filters (FPYFs). We build upon the recent work by Behzadi et al. [9], who introduced primely filters (PYFs) in BL-algebras and explored their connections with other filter types. We present this novel fuzzy filter within the BL-algebraic framework, formally establish its key properties, and investigate its relationships with established classes of fuzzy filters.

The remainder of this article is meticulously structured to facilitate comprehension and engagement. Section 3 presents a comprehensive background on BL-algebras, establishing the essential terminology and foundational results that serve as the bedrock for our subsequent investigation. Section 4 delves into the crux of this paper: the introduction of FPYF within the BL-algebraic framework. We rigorously establish its key properties and embark on a comprehensive investigation into its relationships with established classes of fuzzy filters. Through this meticulous exploration, we aim to contribute meaningfully to the ongoing development of the mathematical framework for studying fuzzy logic and its diverse applications.

The following figure (1) illustrates the structural organization of the paper.

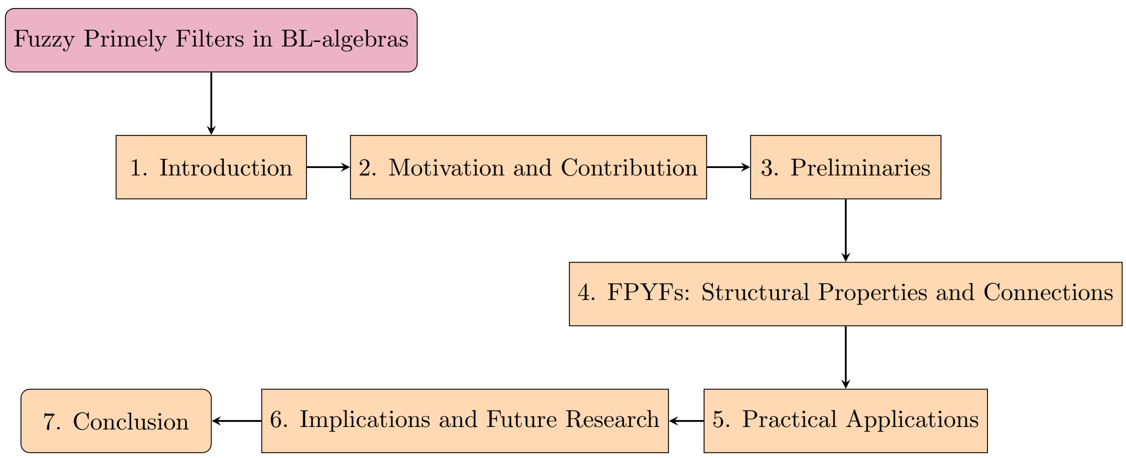

FIGURE 1. Organization of the Paper

Throughout this paper, we employ a variety of symbolic notations to represent specific concepts. Table 1 provides a comprehensive list of these symbols and their associated definitions.

Table 1. Table of Abbreviations

Abbreviation	Full Form
PYF	Primely Filter
PEF	Prime Filter
PEI	Prime Ideal
PRF	Primary Filter
FPYF	Fuzzy Primely Filter
FPEF	Fuzzy Prime Filter
FPEI	Fuzzy Prime Ideal
FPRF	Fuzzy Primary Filter

2. Motivation and Contribution

The study of fuzzy algebraic structures has been a vibrant area of research, offering novel perspectives and generalizations of classical results. Our work on FPYFs in BL-algebras represents a significant advancement in this field. Table (2) summarizes the key motivations behind our research and the primary contributions of this paper.

Table 2. Motivation and Contribution of Research on FPYFs

Motivation	**Contribution**
• Fuzzification of algebraic structures provides new insights and generalizations • Recent work by Behzadi et al. [9] introduced PYFs in BL-algebras • Need to extend PYFs to fuzzy domain for enhanced understanding and applications	• Introduction of FPYFs as a novel concept • Theoretical exploration of FPYFs' properties and characteristics • Investigation of FPYFs' interrelations with other fuzzy filter types • Examination of potential applications of FPYFs in algebraic structures
Impact: Enriches the existing body of knowledge in algebraic structures and provides a deeper understanding of the role and significance of fuzzy filters in BL-algebras and related fields.	

As illustrated in Table (2), our research is driven by recent advancements in the fuzzification of algebraic structures, notably influenced by Motamed et al. [9], who introduced PYFs in BL-algebras. Their work on the relationships between PYFs and other filters forms the basis of our study.

We extend these concepts into the fuzzy domain by introducing FPYFs, a novel contribution that offers a more refined approach to uncertainty in algebraic structures. Our theoretical investigation aims to reveal the unique properties and relationships of FPYFs with other fuzzy filters.

Additionally, we explore the potential applications of FPYFs in various algebraic contexts, highlighting their practical relevance. This research enriches the existing knowledge in algebraic structures, particularly within BL-algebras, and provides a deeper understanding of fuzzy filters, opening new avenues for theoretical and practical advancements.

Introducing FPYFs in BL-algebras advances fuzzy logic, bridging theory, and real-world applications. This research enhances modeling of complex systems and decision-making under uncertainty, potentially impacting fields from AI to quantum mechanics. FPYFs offer new tools for handling imprecision while contributing to theoretical completeness in fuzzy logic. This work expands our understanding of algebraic structures and offers practical applications in reasoning with uncertainty.

The following sections will discuss the theoretical foundations of FPYFs, present our main results, and consider their implications for fuzzy algebraic structures.

3. Preliminaries

Section 3 embarks on a foundational review, meticulously dissecting the essential terminology and results germane to BL-algebras. This groundwork serves as the springboard for our subsequent investigation.

Definition 3.1 ([1]). *A BL-algebra is an algebra* $\mathsf{A} = (\mathsf{A}, \wedge, \vee, \odot, \rightarrow, 0, 1)$ *such that:*

(1) $\mathsf{A} = (\mathsf{A}, \wedge, \vee, 0, 1)$ *is a bounded lattice ,*
(2) $\mathsf{A} = (\mathsf{A}, \odot, 1)$ *is a commutative monoid,*
(3) *The following assertions hold for every* $\iota, \varsigma, w \in \mathsf{A}$:
  a. $w \leq \iota \rightarrow \varsigma \iff \iota \odot w \leq \varsigma$,
  b. $\iota \wedge \varsigma = \iota \odot (\iota \rightarrow \varsigma)$,
  c. $(\iota \rightarrow \varsigma) \vee (\varsigma \rightarrow \iota) = 1$.

Proposition 3.1 ([1]). *For any BL-algebra* A,*the subsequent properties are valid for each* $\iota, \varsigma, w \in \mathsf{A}$ *and will be employed in subsequent sections:*

BL1. $0^- = 1$ *and* $1^- = 0$;
BL2. $1 \rightarrow \iota = \iota$ *and* $0 \rightarrow \iota = \iota \rightarrow 1 = \iota \rightarrow \iota = 1$;
BL3. $\iota \odot \varsigma \leq \iota \wedge \varsigma$;
BL4. $\iota \leq \varsigma \iff \iota \rightarrow \varsigma = 1$;
BL5. $(\iota \rightarrow \varsigma) \rightarrow w = (\varsigma \rightarrow \iota) \rightarrow w = (\iota \odot \varsigma) \rightarrow w$;
BL6. $\iota \leq \varsigma \rightarrow (\iota \odot \varsigma)$;
BL7. $\iota \odot (\iota \rightarrow \varsigma) \leq \varsigma$;
BL8. $\iota \rightarrow \varsigma \leq (\varsigma \rightarrow w) \rightarrow (\iota \rightarrow w)$;
BL9. $\iota \rightarrow \varsigma \leq (w \rightarrow \iota) \rightarrow (w \rightarrow \varsigma)$;
BL10. $\iota \leq \iota^{--}$, $\iota^- = \iota^{---}$, $\iota \odot \iota^- = 0$ *and* $\iota \odot 0 = 0$;
BL11. $(\iota \odot \varsigma)^{--} = \iota^{--} \odot \varsigma^{--}$,$(\iota \vee \varsigma)^- = \iota^- \wedge \varsigma^-$ *and* $(\iota \wedge \varsigma)^- = \iota^- \vee \varsigma^-$;
BL12. $\iota \rightarrow \varsigma \leq \iota \odot w \rightarrow \varsigma \odot w$ *and* $\iota \odot (\varsigma \vee w) = \iota \odot \varsigma \wedge \iota \odot w$.

Definition 3.2 ([1]). *Let* A *be a BL-algebra. A subset* $\mathscr{F}$ *of* A *is called a filter of* A *if it satisfies*

$\mathscr{F}1$. $1 \in \mathscr{F}$;
$\mathscr{F}2$. $\iota \in \mathscr{F}$ and $\iota \to \varsigma \in \mathscr{F}$ imply $\varsigma \in \mathscr{F}$.

Definition 3.3 ([7]). *Let Ξ be a fuzzy set in A. Ξ is called a fuzzy filter if for each $\mathtt{t} \in [0,1]$, $\Xi_{\mathtt{t}}$ is either empty or a filter of A. The set of all fuzzy sets is represented by $F(\mathsf{A})$. The set of all fuzzy filters in A is denoted by $FF(\mathsf{A})$.*

Theorem 3.1 ([7]). *If $\Xi \in F(\mathsf{A})$, then Ξ is called a fuzzy filter of A if, for each $\iota, \varsigma \in \mathsf{A}$*
F$\mathscr{F}1$. $\Xi(1) \geq \Xi(\iota)$
F$\mathscr{F}2$. $\Xi(\varsigma) \geq \Xi(\iota) \wedge \Xi(\iota \to \varsigma)$.

Definition 3.4 ([7]). *Let A be a BL-algebra and Ξ a non constant fuzzy filter of A is a FPEF of A iff for every $\iota, \varsigma \in \mathsf{A}$, $\Xi(\iota \wedge \varsigma) = \Xi(\iota) \wedge \Xi(\varsigma)$.*

Proposition 3.2 ([7]). *Ξ is FPEF iff $\Xi_{\mathtt{t}}$ is either empty or a PEF of A, for each $\mathtt{t} \in [0,1]$.*

Definition 3.5 ([10]). *Let A be a BL-algebra and Ξ a non constant fuzzy filter of A is a fuzzy integral filter of A iff for every $\iota, \varsigma \in \mathsf{A}$, $\Xi((\iota \odot \varsigma)^-) = \Xi(\iota^-) \wedge \Xi(\varsigma^-)$.*

Theorem 3.2 ([11]). *Let $\mathscr{F}$ be a filter of A. Then the subsequent conditions stand:*

- *If $\mathscr{F}$ is a proper filter of A, then $\mathscr{F}$ is a PRF iff $(\iota^n)^- \in \mathscr{F}$ or $((\iota^-)^m)^- \in \mathscr{F}$, for some $n, m \in N$, for each $\iota \in \mathsf{A}$.*
- *If $\mathscr{F}$ is a proper filter of A, then $\mathscr{F}$ is a PRF iff $(\iota \odot \varsigma)^- \in \mathscr{F} \implies (\iota^m)^- \in \mathscr{F}$ or $(\varsigma^n)^- \in \mathscr{F}$, for some $n, m \in \mathtt{N}$.*
- *Every PEF of A is a PRF of A.*

Definition 3.6 ([10]). *Let A be a BL-algebra and Ξ a non constant fuzzy filter of A is a FPRF of A iff for every $\iota, \varsigma \in \mathsf{A}$, $\Xi((\iota \odot \varsigma)^-) = \Xi(\iota^n)^-) \vee \Xi(\varsigma^n)^-), for\ some\ n \in \mathbb{N}$.*

Proposition 3.3 ([10]). *Ξ is a FPRF iff $\Xi_{\mathtt{t}}$ is either empty or a PRF of A, for each $\mathtt{t} \in [0,1]$.*

Definition 3.7 ([12]). *Let A be a BL-algebra. A subset $\mathscr{I}$ of A is called an ideal of A if it satisfies*

$\mathscr{I}1$. $0 \in \mathscr{I}$;
$\mathscr{I}2$. $\varsigma \in \mathscr{I}$ and $\iota \odot \varsigma^- \in \mathscr{I}$ imply $\iota \in \mathscr{I}$.

Definition 3.8 ([13]). *Let ξ be a fuzzy set in A. ξ is called a fuzzy ideal if for each $\mathtt{t} \in [0,1]$, $\xi_{\mathtt{t}}$ is either empty or an ideal of A. The set of all fuzzy sets is represented by $F(\mathsf{A})$. The set of all fuzzy ideals in A is denoted by $FI(\mathsf{A})$.*

Theorem 3.3 ([13]). *If $\xi \in F(\mathsf{A})$, then $\xi \in FI(\mathsf{A})$ if, for each $\iota, \varsigma \in \mathsf{A}$*
F$\mathscr{I}1$. $\xi(0) \geq \xi(\iota)$
F$\mathscr{I}2$. $\xi(\varsigma) \geq \xi(\iota) \wedge \xi(\iota \odot \varsigma^-)$.

Definition 3.9 ([13]). *Let A be a BL-algebra and ξ a non constant fuzzy ideal of A is a FPEI of A iff for every $\iota, \varsigma \in \mathsf{A}$, $\xi(\iota \vee \varsigma) = \xi(\iota) \wedge \xi(\varsigma)$.*

Remark 3.1 ([13]). *It is easy to show that if $\xi \in FI(\mathsf{A})$, then $\xi(\iota) = \xi(\iota^{--})$for all $\iota \in \mathsf{A}$.*

Definition 3.10 ([13]). *Let A be a BL-algebra an Ξ be any fuzzy subset of A. The fuzzy subsets $\mathscr{N}(\Xi)$ and $\mathbb{D}(\Xi)$ from $\mathsf{A} \to [0,1]$ are characterized by: for each $\iota \in \mathsf{A}$, $\mathscr{N}(\Xi)(\iota) = \Xi(\iota^-)$ and $\mathbb{D}(\Xi)(\iota) = \Xi(\iota^{--})$ are called the set of complement and the set of double complement of the fuzzy subset Ξ.*

Lemma 3.1 ([13]). *Let $\xi \in FI(\mathsf{A})$ and $\omega \in FF(\mathsf{A})$. Then the subsequent properties hold :*

(i) *for every* $\iota, \varsigma \in \mathsf{A}$, *if* $\iota \leq \varsigma$ *then* $\xi(\varsigma) \leq \xi(\iota)$ *and* $\omega(\iota) \leq \omega(\varsigma)$.
(ii) $\mathscr{N}(\xi) \in FF(\mathsf{A})$ *and* $\mathscr{N}(\omega) \in FI(\mathsf{A})$.

Definition 3.11 ([6]). *Assume that* A *and* $\mathscr{B}$ *are two BL-algebras ,* ξ *and* ω *fuzzy subsets of* A *and* $\mathscr{B}$ *, respectivelv , and* $\varphi : \mathsf{A} \to \mathscr{B}$ *a homomorphism. The image of* ξ *under* φ *represented by* $\varphi(\xi)$ *is a fuzzy set of* $\mathscr{B}$ *described by : For every* $\varsigma \in \mathscr{B}$,

$$\varphi(\xi)(\varsigma) = \begin{cases} \sup_{\iota \in \varphi^{-1}(\varsigma)} \xi(\iota) & if \quad \varphi^{-1}(\varsigma) \neq \phi \\ 0 & if \quad \varphi^{-1}(\varsigma) = \phi. \end{cases}$$

The preimage of ω *under* φ *represented by* $\varphi^{-1}(\omega)$ *is a fuzzy set of* A *described by: for every* $\iota \in \mathsf{A}, \varphi^{-1}(\omega)(\iota) = \omega(\varphi(\iota))$.

Theorem 3.4 ([10]). *Let* $\varphi : \mathsf{A} \to \mathscr{B}$ *be a BL-homomorphism and let* $\Xi \in FF(\mathscr{B})$*. Then* $\varphi^{-1}(\Xi) \in FF(\mathsf{A})$.

Lemma 3.2 ([10]). *Let* $\varphi : \mathsf{A} \to \mathscr{B}$ *be a BL-algebra isomorphism and* $\Xi \in FF(\mathsf{A})$*. Then* $\varphi(\Xi) \in FF(\mathscr{B})$.

Theorem 3.5 ([9]). *(Extension property for PYFs). Let* $\mathscr{F}$ *and* $\mathscr{G}$ *be two proper filters of BL-algebra* A *such that* $\mathscr{F} \subseteq \mathscr{G}$ *and let* $\mathscr{F}$ *be a PYF. Then* $\mathscr{G}$ *is a PYF.*

Proposition 3.4 ([9]). *In any BL-algebra,*
(1) *every PEF is a PYF.*
(2) *every integral filter is a PYF.*

4. FPYFs: Structural Properties and Connections:

This section offers an elucidation of FPYFs within the domain of BL-algebras, accompanied by an exploration of various properties evinced by these filters.

Definition 4.1. *Let* Ξ *be a non-constant fuzzy filter of* A *. Then* Ξ *is said to be a FPYF of* A *if for every* $\mathtt{t} \in [0,1]$*, the t-cut set* $\Xi_{\mathtt{t}}$ *is either empty or a PYF if it is proper.*

Theorem 4.1. *If* Ξ *is a non-constant fuzzy filter of* A *, then* Ξ *is said to be a FPYF of* A *iff for each* $\iota, \varsigma \in \mathsf{A}$
$\Xi((\iota \wedge \varsigma)^-) \leq \Xi(\iota^-) \vee \Xi(\varsigma^-)$.

Proof. Assume that Ξ is a FPYF. Let $\iota, \varsigma \in \mathsf{A}$, and $\mathtt{t} = \Xi((\iota\wedge\varsigma)^-)$, then $(\iota\wedge\varsigma)^- \in \Xi_{\mathtt{t}}$, and since $\Xi_{\mathtt{t}}$ is a PYF, we conclude that $\iota^- \in \Xi_{\mathtt{t}}$ or $\varsigma^- \in \Xi_{\mathtt{t}}$. Thus, $\Xi(\iota^-) \geq \mathtt{t}$ or $\Xi(\varsigma^-) \geq \mathtt{t}$ from which it follows that $\Xi(\iota^-)\vee\Xi(\varsigma^-) \geq \Xi((\iota\wedge\varsigma)^-)$. To establish the converse, suppose that Ξ is a non-constant fuzzy filter and for every $\iota, \varsigma \in \mathsf{A}$, $\Xi(\iota^-) \vee \Xi(\varsigma^-) \geq \Xi((\iota \wedge \varsigma)^-)$. Let $\mathtt{t} \in [0,1]$ such that $\Xi_{\mathtt{t}}$ is non empty, and $\Xi_{\mathtt{t}} \neq \mathsf{A}$. Let $\iota, \varsigma \in \mathsf{A}$ such that $(\iota\wedge\varsigma)^- \in \Xi_{\mathtt{t}}$, then $\Xi((\iota\wedge\varsigma)^-) \geq \mathtt{t}$. We apply the hypothesis and obtain $\Xi(\iota^-)\vee\Xi(\varsigma^-) \geq \Xi((\iota\wedge\varsigma)^-) \geq \mathtt{t}$. So $\Xi(\iota^-) \geq \mathtt{t}$ or $\Xi(\varsigma^-) \geq \mathtt{t}$, which means $\iota^- \in \Xi_{\mathtt{t}}$ or $\varsigma^- \in \Xi_{\mathtt{t}}$,therefore $\Xi_{\mathtt{t}}$ is a PYF. Thus by Definition 4.1 we hold Ξ is a FPYF. □

Remark 4.1. *Let* A *be a BL-algebra and* Ξ *a non constant fuzzy filter of* A*.* Ξ *is a FPYF of* A *iff for every* $\iota, \varsigma \in \mathsf{A}$, $\Xi((\iota \wedge \varsigma)^-) = \Xi(\iota^-) \vee \Xi(\varsigma^-)$.

Example 4.1 ([9]). *Let* $A = \{0, \iota, \varsigma, w, 1\}$*. Define where* $\wedge, \vee, \odot$ *and* $\to$ *. as the subsequent tables :*

Table 3. Meet Operation

$\wedge$	*0*	w	ι	ς	*1*
0	*0*	*0*	*0*	*0*	*0*
w	*0*	w	w	w	ς
ι	*0*	w	ι	w	ι
ς	*0*	w	w	ς	ς
1	*0*	w	ι	ς	*1*

Table 4. Join Operation

$\vee$	*0*	w	ι	ς	*1*
0	*0*	w	ι	ς	*1*
w	w	w	ι	ς	*1*
ι	ι	ι	ι	*1*	*1*
ς	*0*	ι	ι	*1*	*1*
1	*1*	*1*	*1*	*1*	*1*

Table 5. Product Operation

$\odot$	*0*	w	ι	ς	*1*
0	*0*	*0*	*0*	*0*	*0*
w	*0*	w	w	w	w
ι	*0*	w	ι	w	ι
ς	*0*	w	w	ς	ς
1	*0*	w	ι	ς	*1*

Table 6. Implication Operation

$\rightarrow$	*0*	w	ι	ς	*1*
0	*1*	*1*	*1*	*1*	*1*
w	*0*	*1*	*1*	*1*	*1*
ι	*0*	ς	*1*	ς	*1*
ς	*0*	ι	ι	*1*	*1*
1	*0*	w	ι	ς	*1*

A direct examination reveals that $\mathsf{A} = (\mathsf{A}, \wedge, \vee, \odot, \rightarrow, 0, 1)$ *is a BL-algebra. Let* $\Xi \in F(\mathsf{A})$ *be described as* $\Xi(1) = 0.8$, $\Xi(\iota) = \Xi(\varsigma) = \Xi(w) = \Xi(0) = 0.5$.
. It is simple to check $\Xi \in FF(\mathsf{A})$ *. It is a FPYF.*

Theorem 4.2. *Let* Ξ *be a non-constant fuzzy filter of* A*.* Ξ *is a FPYF iff* $\Xi_{\Xi(1)}$ *is a PYF.*

Proof. . Obviously, First, note that if Ξ a non constant FPYF, It is evident that $\Xi_{\Xi(1)} = \{\iota \in \mathsf{A}/\Xi(\iota) = \Xi(1)\}$ is proper; and as a direct consequence of the definition 4.1 , we have $\Xi_{\Xi(1)}$ is a PYF. Since Ξ is a non-constant fuzzy filter, then $\Xi(0) \leq \Xi(1)$, i.e., $0 \notin \Xi_{\Xi(1)}$. By Definition 4.1 we hold $\Xi_{\Xi(1)}$ is a PYF. Conversely, suppose $\Xi_{\Xi(1)}$ is a primely filter . Then $\iota^- \rightarrow \varsigma^- \in \Xi_{\Xi(1)}$ or $\varsigma^- \rightarrow \iota^- \in \Xi_{\Xi(1)}$ This means that $(\iota^- \vee \varsigma^-) \rightarrow \varsigma^- = \iota^- \rightarrow \varsigma^- \in \Xi_{\Xi(1)}$ or $(\iota^- \vee \varsigma^-) \rightarrow \iota^- = \varsigma^- \rightarrow \iota^- \in \Xi_{\Xi(1)}$. So $\Xi((\iota^- \vee \varsigma^-) \rightarrow \varsigma^-) = \Xi(1)$ or $\Xi((\iota^- \vee \varsigma^-) \rightarrow \iota^-) = \Xi(1)$. Since Ξ is a fuzzy filter, we obtain $\Xi(\varsigma^-) \geq \Xi((\iota^- \vee \varsigma^-) \rightarrow$

$\varsigma^-) \wedge \Xi(\iota^- \vee \varsigma^-) = \Xi(\iota^- \vee \varsigma^-)$ or $\Xi(\iota^-) \geq \Xi((\iota^- \vee \varsigma^-) \to \iota^-) \wedge \Xi(\iota^- \vee \varsigma^-) = \Xi(\iota^- \vee \varsigma^-)$. Thus, $\Xi((\iota \wedge \varsigma)^-) = \Xi(\iota^- \vee \varsigma^-) \leq \Xi(\iota^-) \vee \Xi(\varsigma^-)$.So by Theorem 4.1 Ξ is a FPYF. □

Corollary 4.1. *Let* A *be a BL-algebra.* $\mathscr{F}$ *is a PYF of* A *iff* $\chi_{\mathscr{F}}$ *is a FPYF in* A.

Proof. Let $\mathscr{F}$ be a PYF. Then we show that $\chi_{\mathscr{F}}((\iota \wedge \varsigma)^-) = \chi_{\mathscr{F}}(\iota^-) \vee \chi_{\mathscr{F}}(\varsigma^-)$. If $(\iota \wedge \varsigma)^- \in \mathscr{F}$, then $\chi_{\mathscr{F}}((\iota \wedge \varsigma)^-) = 1$ and since $\mathscr{F}$ is a PYF, then $\iota^- \in \mathscr{F}$ or $\varsigma^- \in \mathscr{F}$ and so $\chi_{\mathscr{F}}(\iota^-) = 1$ or $\chi_{\mathscr{F}}(\varsigma^-) = 1$. Hence, $\chi_{\mathscr{F}}((\iota \wedge \varsigma)^-) = \chi_{\mathscr{F}}(\iota^-) \vee \chi_{\mathscr{F}}(\varsigma^-) = 1$. Now, let $(\iota \wedge \varsigma)^- \notin \mathscr{F}$. Then $\iota^- \notin \mathscr{F}$ and $\varsigma^- \notin \mathscr{F}$. Since $\iota \wedge \varsigma = \iota \odot (\iota \to \varsigma) = \varsigma \odot (\varsigma \to \iota)$, then $\iota \wedge \varsigma \leq \iota$ and $\iota \wedge \varsigma \leq \varsigma$, so $\iota^- \leq (\iota \wedge \varsigma)^-$ and $\varsigma^- \leq (\iota \wedge \varsigma)^-$. If $\iota^- \in \mathscr{F}$ or $\varsigma^- \in \mathscr{F}$, then $(\iota \wedge \varsigma)^- \in \mathscr{F}$ and it is impossible. Hence, $\iota^- \notin \mathscr{F}$ and $\varsigma^- \notin \mathscr{F}$ and so $\chi_{\mathscr{F}}(\iota^-) = 0$ and $\chi_{\mathscr{F}}(\varsigma^-) = 0$.Therefore, $\chi_{\mathscr{F}}((\iota \wedge \varsigma)^-) = \chi_{\mathscr{F}}(\iota^-) \vee \chi_{\mathscr{F}}(\varsigma^-) = 0$.
Conversely,let $\chi_{\mathscr{F}}$ be a FPYF we need to estabish that $\mathscr{F}$ is a PYF of A . Let $(\iota \wedge \varsigma)^- \in \mathscr{F}$, by utilizing the notion of characteristic function, we hold $\chi_{\mathscr{F}}((\iota \wedge \varsigma)^-) = 1$. Now by utilizing the fact that $\chi_{\mathscr{F}}$ is a FPYF of A , we obtain $\chi_{\mathscr{F}}((\iota \wedge \varsigma)^-) = \chi_{\mathscr{F}}(\iota^-) \vee \chi_{\mathscr{F}}(\varsigma^-) = 1$, and so $\chi_{\mathscr{F}}(\iota^-) = 1$ or $\chi_{\mathscr{F}}(\varsigma^-) = 1$. Hence , $\iota^- \in \mathscr{F}$ or $\varsigma^- \in \mathscr{F}$. Therefore,$\mathscr{F}$ is a PYF of A . □

Theorem 4.3. *Let* Ξ *be a proper filter of a BL-algebra* A *. Then* Ξ *is a FPYF iff* $\Xi(\iota^- \to \varsigma^-) = \Xi(1)$ *or* $\Xi(\varsigma^- \to \iota^-) = \Xi(1)$ *for any* $\iota, \varsigma \in \mathsf{A}$.

Proof. By Theorem 4.2, Ξ is a FPYF iff $\Xi_{\Xi(1)}$ is a PYF iff $\iota^- \to \varsigma^- \in \Xi_{\Xi(1)}$ or $\varsigma^- \to \iota^- \in \Xi_{\Xi(1)}$ iff $\Xi(\iota^- \to \varsigma^-) = \Xi(1)$ or $\Xi(\varsigma^- \to \iota^-) = \Xi(1)$. □

Our subsequent objective is to establish the extension property for FPYFs.

Theorem 4.4. *(Extention theorem of FPYFs) Suppose that* $\Xi, \varrho \in FF(\mathsf{A})$ *such that* $\Xi \leq \varrho$ *and* $\Xi(1) = \varrho(1)$. *If* Ξ *is a FPYF, then* ϱ *is also FPYF.*

Proof. Since Ξ is a FPYF, then $\Xi(\iota^- \to \varsigma^-) = \Xi(1)$ or $\Xi(\varsigma^- \to \iota^-) = \Xi(1)$ for all $\iota, \varsigma \in \mathsf{A}$. If $\Xi(\iota^- \to \varsigma^-) = \Xi(1)$, by $\Xi \leq \varrho$ and $\Xi(1) = \varrho(1)$ we hold $\varrho(\iota^- \to \varsigma^-) = \varrho(1)$. Similarly, if $\Xi(\varsigma^- \to \iota^-) = \Xi(1)$, then $\varrho(\varsigma^- \to \iota^-) = \varrho(1))$. Using Theorem 4.3, we have ϱ is a FPYF. □

Proposition 4.1. *In any BL-algebra, every FPEF is a FPYF.*

Proof. let Ξ be a FPEF on A. Then $\Xi_{\mathtt{t}}$ is a PEF on A, for any $\mathtt{t} \in [0, 1]$. So, by Proposition 3.4(i) , $\Xi_{\mathtt{t}}$ is a PYF on A . Hence, by Definition 4.1 Ξ is a FPYF on A. □

The subsequent example illustrates that the converse is not generally correct.
In example 4.1 Ξ is a FPYF on A. But not a FPEF because $\Xi(\iota \vee \varsigma) = \Xi(1) = 0.8 \neq 0.5 = \Xi(\iota) \vee \Xi(\varsigma)$.

Proposition 4.2. *In any BL-algebra, every fuzzy integral filter is a FPYF.*

Proof. let Ξ be a fuzzy integral filter on A. Then for each $\iota, \varsigma \in \mathsf{A}$,we hold

$$\begin{aligned} & \iota \odot \varsigma \leq \iota \wedge \varsigma \\ \Longrightarrow\ & (\iota \wedge \varsigma)^- \leq (\iota \odot \varsigma)^- \\ \Longrightarrow\ & \Xi((\iota \wedge \varsigma)^-) \leq \Xi((\iota \odot \varsigma)^-) \\ & \qquad = \Xi(\iota^-) \vee \Xi(\varsigma^-). \end{aligned}$$

Therefore, Ξ is a FPYF on A. □

However, the reverse is not accurate.

Example 4.2 ([14])**.** *Let $A = \{0, \iota, \varsigma, 1\}$, where $0 < \iota < \varsigma < 1$. Assume that*

TABLE 7. Product Operation

$\odot$	0	ι	ς	1
0	0	0	0	0
ι	0	0	ι	ι
ς	0	ι	ς	ς
1	0	ι	ς	1

TABLE 8. Implication Operation

$\to$	0	ι	ς	1
0	1	1	1	1
ι	ι	1	1	1
ς	0	ι	1	1
1	0	ι	ς	1

It is simple to investigate that $\mathsf{A} = (\mathsf{A}, \wedge, \vee, \odot, \to, 0, 1)$ is a BL-algebra. Let $\Xi \in F(\mathsf{A})$ be described as $\Xi(1) = 0.7, \Xi(\iota) = \Xi(\varsigma) = \Xi(0) = 0.5$.
It is straightforward to verify $\Xi \in FF(\mathsf{A})$. Ξ is a FPYF. But it is not a fuzzy integral filter because,$\Xi((\iota \odot \iota)^-) = \Xi(0^-) = \Xi(1) = 0.7 \neq 0.5 = \Xi(\iota^-) \vee \Xi(\iota^-)$.

Theorem 4.5. *Let $\Xi \in FI(\mathsf{A})$. Then Ξ is a FPEI iff $\mathscr{N}(\Xi)$ is a FPYF on A.*

Proof. Let Ξ be a FPEI on A. Then by Lemma 3.1(ii), $\mathscr{N}(\Xi) \in FF(\mathsf{A})$. Now, for each $\iota, \varsigma \in \mathsf{A}$,

$$\begin{aligned}
\mathscr{N}(\Xi)((\iota \wedge \varsigma)^-) &= \Xi((\iota \wedge \varsigma)^{--}) \\
&= \Xi(\iota \wedge \varsigma) \\
&= \Xi(\iota) \vee \Xi(\varsigma) \\
&= \Xi(\iota^{--}) \vee \Xi(\varsigma^{--}) \\
&= \mathscr{N}(\Xi)(\iota^-) \vee \mathscr{N}(\Xi)(\varsigma^-).
\end{aligned}$$

Therefore, $\mathscr{N}(\Xi)$ is a FPYF on A.
Conversely, let $\mathscr{N}(\Xi)$ be a FPYF on A. Then for each $\iota, \varsigma \in \mathsf{A}$,

$$\begin{aligned}
\Xi(\iota \wedge \varsigma) &= \Xi((\iota \wedge \varsigma)^{--}) \\
&= \mathscr{N}(\Xi)((\iota \wedge \varsigma)^-) \\
&= \mathscr{N}(\Xi)(\iota^-) \vee \mathscr{N}(\Xi)(\varsigma^-) \\
&= \Xi(\iota^{--}) \vee \Xi(\varsigma^{--}) \\
&= \Xi(\iota) \vee \Xi(\varsigma).
\end{aligned}$$

Therefore,Ξ is a FPEI on A. □

Theorem 4.6. *Let $\Xi \in FF(\mathsf{A})$. Then Ξ is a FPYF iff $\mathscr{N}(\Xi)$ is a FPEI on A.*

Proof. Let Ξ be a FPYF on A.Then by Lemma 3.1(ii), $\mathscr{N}(\Xi) \in FI(\mathsf{A})$. Now, for each $\iota, \varsigma \in \mathsf{A}$,

$$\begin{aligned}\mathscr{N}(\Xi)(\iota \wedge \varsigma) &= \Xi((\iota \wedge \varsigma)^{-}) \\ &= \Xi(\iota^{-}) \vee \Xi(\varsigma^{-}) \\ &= \mathscr{N}(\Xi)(\iota) \vee \mathscr{N}(\Xi)(\varsigma).\end{aligned}$$

Therefore, $\mathscr{N}(\Xi)$ is a FPEI on A.
Conversely, let $\mathscr{N}(\Xi)$ be a FPEI on A. Then for each $\iota, \varsigma \in \mathsf{A}$,

$$\begin{aligned}\Xi((\iota \wedge \varsigma)^{-}) &= \mathscr{N}(\Xi)(\iota \wedge \varsigma) \\ &= \mathscr{N}(\Xi)(\iota) \vee \mathscr{N}(\Xi)(\varsigma) \\ &= \Xi(\iota^{-}) \vee \Xi(\varsigma^{-}).\end{aligned}$$

Therefore, Ξ is a FPYF on A. □

Proposition 4.3. *Let* $\Xi \in FF(\mathsf{A})$*.Then* Ξ *is a FPYF on* A *iff* $\mathbb{D}(\Xi)$ *is a FPEF on* A.

Proof. Let Ξ be a FPYF on A. Then for each $\iota, \varsigma \in \mathsf{A}$,

$$\begin{aligned}\mathbb{D}(\Xi)(\iota \vee \varsigma) &= \Xi((\iota \vee \varsigma)^{--}) \\ &= \Xi((\iota^{-} \wedge \varsigma^{-})^{-}) \\ &= \Xi(\iota^{--}) \vee \Xi(\varsigma^{--}) \\ &= \mathbb{D}(\Xi)(\iota) \vee \mathbb{D}(\Xi)(\varsigma).\end{aligned}$$

Therefore, $\mathbb{D}(\Xi)$ is a FPEF on A.
Conversely, let $\mathbb{D}(\Xi)$ be a FPEF on A.Then for each $\iota, \varsigma \in \mathsf{A}$,

$$\begin{aligned}\Xi((\iota \wedge \varsigma)^{-}) &= \Xi((\iota \wedge \varsigma)^{---}) \\ &= \mathbb{D}(\Xi)((\iota \wedge \varsigma)^{-}) \\ &= \mathbb{D}(\Xi)(\iota^{-} \vee \varsigma^{-}) \\ &= \mathbb{D}(\Xi)(\iota^{-}) \vee \mathbb{D}(\Xi)(\varsigma^{-}) \\ &= \Xi(\iota^{---}) \vee \Xi(\varsigma^{---}) \\ &= \Xi(\iota^{-}) \vee \Xi(\varsigma^{-}).\end{aligned}$$

Therefore, Ξ is a FPYF on A. □

Proposition 4.4. *Let* $\Xi \in FF(\mathsf{A})$*. Then* Ξ *is a FPYF on* A *iff* $\mathbb{D}(\Xi)$ *is a FPYF on* A.

Proof. Let Ξ be a FPYF on A. Then for each $\iota, \varsigma \in \mathsf{A}$,

$$\begin{aligned}\mathbb{D}(\Xi)((\iota \wedge \varsigma)^{-}) &= \Xi((\iota \wedge \varsigma)^{---}) \\ &= \Xi((\iota \wedge \varsigma)^{-}) \\ &= \Xi(\iota^{-}) \vee \Xi(\varsigma^{-}) \\ &= \Xi(\iota^{---}) \vee \Xi(\varsigma^{---}) \\ &= \mathbb{D}(\Xi)(\iota^{-}) \vee \mathbb{D}(\Xi)(\varsigma^{-}).\end{aligned}$$

Therefore, $\mathbb{D}(\Xi)$ is a FPYF on A.
Conversely, let $\mathbb{D}(\Xi)$ be a FPYF on A.Then for each $\iota, \varsigma \in \mathsf{A}$,

$$\begin{aligned}\Xi((\iota \wedge \varsigma)^-) &= \Xi((\iota \wedge \varsigma)^{---}) \\ &= \mathbb{D}(\Xi)((\iota \wedge \varsigma)^-) \\ &= \mathbb{D}(\Xi)(\iota^-) \vee \mathbb{D}(\Xi)(\varsigma^-) \\ &= \Xi(\iota^{---}) \vee \Xi(\varsigma^{---}) \\ &= \Xi(\iota^-) \vee \Xi(\varsigma^-).\end{aligned}$$

Therefore, Ξ is a FPYF on A. □

Theorem 4.7. *Let $\varphi : \mathsf{A} \to \mathscr{B}$ be a BL-homomorphism and let Ξ be a FPYF on $\mathscr{B}$. Then $\varphi^{-1}(\Xi)$ is a FPYF on A.*

Proof. First, we hold $\varphi^{-1}(\Xi)$ is a fuzzy filter on A since $\Xi \in FF(\mathscr{B})$ by Theorem 3.4 .Now assume that Ξ is a FPYF. Then for every $\iota, \varsigma \in \mathsf{A}$, we hold by Definition 3.11

$$\begin{aligned}\varphi^{-1}(\Xi)((\iota \wedge \varsigma)^-) &= \Xi(\varphi((\iota \wedge \varsigma)^-) \\ &= \Xi((\varphi(\iota) \wedge \varphi(\varsigma))^-) \\ &= \Xi(\varphi(\iota)^-) \vee \Xi(\varphi(\varsigma)^-) \\ &= \varphi^{-1}(\Xi)(\iota^-) \vee \varphi^{-1}(\Xi)(\varsigma^-).\end{aligned}$$

Therefore, $\varphi^{-1}(\Xi)$ is a FPYF . □

Lemma 4.1. *Let $\varphi : \mathsf{A} \to \mathscr{B}$ be a BL-algebra isomorphism and Ξ be a FPYF on A. Then $\varphi(\Xi)$is a FPYF on $\mathscr{B}$.*

Proof. First, we hold $\varphi(\Xi) \in FF(\mathscr{B})$ since $\Xi \in FF(\mathsf{A})$ by Lemma 3.2. In addition, setting $x, y \in \mathscr{B}$. Since φ is a BL-isomorphism , we conclude $\varphi(\iota) = x$ and $\varphi(\varsigma) = y$, for some $\iota, \varsigma \in \mathsf{A}$ and so by Definition 3.11 $\varphi(\Xi)((x \wedge y)^-) = \sup_{t \in \varphi^{-1}((x \wedge y)^-)} \Xi(t)$.
Now, if $t \in \varphi^{-1}((x \wedge y)^-)$, then $\varphi(t) = (x \wedge y)^- = (\varphi(\iota) \wedge \varphi(\varsigma))^- = \varphi((\iota \wedge \varsigma)^-)$ .Since φ is a BL-isomorphism, we get $t = (\iota \wedge \varsigma)^-$ and by utilizing the fact of Ξ is a FPYF, we hold $\varphi(\Xi)((x \wedge y)^-) = \sup_{t=(\iota \wedge \varsigma)^-} \Xi(t) = \Xi((\iota \wedge \varsigma)^-) = \Xi(\iota^-) \vee \Xi(\varsigma^-)$.
By similarly ,$\varphi(\Xi)(x^-) = \Xi(\iota^-)$ and $\varphi(\Xi)(y^-) = \Xi(\varsigma^-)$.
Hence,$\varphi(\Xi)((x \wedge y)^-) = \varphi(\Xi)(x^-) \vee \varphi(\Xi)(y^-)$.
Therefore, $\varphi(\Xi)$ is a FPYF. □

It is important to remember that a BL-algebra A is an MV-algebra if for each $\iota \in \mathsf{A}, \iota^{--} = \iota$ [1].

Proposition 4.5. *Let Ξ be a FPYF of an MV-algebra A. The subsequent statement stands: Ξ is a FPEF.*

Proof. Let Ξ be a FPYF of an MV-algebra A.Then for $\iota, \varsigma \in \mathsf{A}$.

$$\begin{aligned}\Xi((\iota \vee \varsigma)^-) &= \Xi((\iota^{--} \vee \varsigma^{--})^-), \ \textit{As } \mathsf{A} \textit{ is an } MV-algebra. \\ &= \Xi((\iota^- \wedge \varsigma^-)^-), \ \textit{by hypothesis} \\ &= \Xi(\iota^{--}) \vee \Xi(\varsigma^{--}) \\ &= \Xi(\iota) \vee \Xi(\varsigma).\end{aligned}$$

Therefore, Ξ is a FPEF. □

It is important to remember that a BL-algebra A is a semi-G-algebra if for each $\iota \in \mathsf{A}$, $(\iota^2)^- = \iota^-$ [16].

Proposition 4.6. *Let Ξ be a FPRF of a semi-G-algebra A. Then Ξ is a FPYF.*

Proof. Let Ξ be a FPRF of a semi-G-algebra A. Then for each $\iota, \varsigma \in \mathsf{A}$,we hold

$$\begin{aligned} & \iota \odot \varsigma \leq \iota \wedge \varsigma \\ \Longrightarrow\ & (\iota \wedge \varsigma)^- \leq (\iota \odot \varsigma)^- \\ \Longrightarrow\ & \Xi((\iota \wedge \varsigma)^-) \leq \Xi((\iota \odot \varsigma)^-) \\ & \qquad = \Xi((\iota^n)^-) \vee \Xi((\varsigma^n)^-), based\ on\ the\ hypothesis\ , for\ some\ n\ \in \mathbb{N} \\ & \qquad = \Xi(\iota^-) \vee \Xi(\varsigma^-). \end{aligned}$$

Therefore, Ξ is a FPYF. ☐

It should be noted that a BL-algebra A is, in fact, an SBL-algebra if for each $\iota, \varsigma \in \mathsf{A}$, $(\iota \odot \varsigma)^- = (\iota^- \vee \varsigma^-)$ [15].

Proposition 4.7. *Let Ξ be a FPYF of a SBL-algebra A. Then Ξ is a FPRF.*

Proof. Let Ξ be a FPYF of a SBL-algebra A . Then since A is a SBL-algebra, $(\iota \odot \varsigma)^- = (\iota^- \vee \varsigma^-) = (\iota \wedge \varsigma)^-$. So, based on the hypothesis, $\Xi((\iota \odot \varsigma)^-) = \Xi((\iota \wedge \varsigma)^-) = \Xi(\iota^-) \vee \Xi(\varsigma^-)$. Thus, Ξ is a FPRF. ☐

Proposition 4.8. *Let Ξ be a FPYF of a BL-algebra A and $\iota \vee \varsigma = 1, \forall \iota, \varsigma \neq 1$. Then Ξ is a FPRF.*

Proof. Let Ξ be a FPYF of a BL-algebra A and since $\iota \vee \varsigma = 1$, then $\iota \odot \varsigma = \iota \wedge \varsigma$. Hence, $(\iota \odot \varsigma)^- = (\iota \wedge \varsigma)^- = (\iota^- \vee \varsigma^-)$. So, based on the hypothesis, $\Xi((\iota \odot \varsigma)^-) = \Xi((\iota \wedge \varsigma)^-) = \Xi(\iota^-) \vee \Xi(\varsigma^-)$. Thus, Ξ is a FPRF. ☐

5. Implications and Future Directions

The introduction of FPYF opens up several avenues for future research:

(i) Topological properties: Investigate the topological properties of the space of fuzzy primely filters.
(ii) Representation theorems: Develop representation theorems for BL-algebras using fuzzy primely filters.
(iii) Applications in fuzzy logic programming: Explore potential applications in fuzzy logic programming and fuzzy constraint satisfaction problems.
(iv) Generalizations: Extend the concept to more general classes of algebras, such as MTL-algebras or residuated lattices.
(v) Relationships with other fuzzy algebraic structures: Further explore connections with fuzzy ideals, fuzzy congruences, and other fuzzy algebraic concepts.

6. Practical Applications

While primarily theoretical, fuzzy primely filters in BL-algebras have potential applications across various fields:

- **Decision Support Systems:** Enhance handling of uncertainty in multi-criteria decision-making processes.
- **Artificial Intelligence:** Improve reasoning capabilities in expert systems and natural language processing.

- **Control Systems:** Increase stability and robustness in fuzzy control systems, particularly for complex, non-linear systems.
- **Pattern Recognition:** Develop more flexible algorithms for image segmentation and object recognition, especially with unclear boundaries or noisy data.
- **Risk Analysis:** Provide more nuanced modeling of vague risk factors and their interdependencies in complex systems.

These applications leverage the ability of fuzzy primely filters to handle uncertainty and vagueness in logical and algebraic structures. However, further research is needed to bridge the gap between theory and practical implementation.

7. Conclusion

The focus of this paper's findings centered on the exploration of the FPYF concept within the domain of BL-algebras. We have presented a comprehensive characterization, elucidating several fundamental properties of this innovative category of fuzzy filters in BL-algebras. Additionally, we have devised and substantiated theorems that delineate the association between this concept and various types of fuzzy filters in a BL-algebra.

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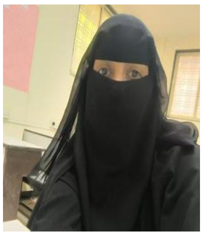

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