TWMS J. App. and Eng. Math. V.15, N.10, 2025, pp. 2519-2529

EQUITABLE DOMINATOR COLORING OF GRAPHS

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ABSTRACT. This paper introduces a variant of domination-related coloring of graphs, called the equitable dominator coloring of graphs, which is a combination of equitable coloring and dominator coloring of graphs. The minimum number of colors used in an equitable dominator coloring of a graph is its equitable dominator chromatic number. The equitable dominator coloring and the equitable dominator chromatic number of some standard graph classes are investigated in this paper.

Keywords: Graph coloring, dominator coloring, equitable coloring, equitable dominator coloring.

AMS Subject Classification: 05C15, 05C69.

1. Introduction

For basic terminology in graph theory, we refer to [7,8], and for topics in graph coloring, refer to [9,10]. Unless mentioned otherwise, all graphs discussed in this paper are simple, undirected, finite, and connected.

Graph coloring is the assignment of colors to the graph's vertices, edges, or faces. A *vertex coloring* of a graph G is a mapping $c : V(G) \to \mathcal{C}$, where $\mathcal{C} = \{c_1, c_2, \dots, c_k\}$, is a set of colors. A *proper vertex coloring* of G is when no two adjacent vertices are assigned the same color and the minimum number of colors required in this coloring of G is the *chromatic number* of G, denoted by $\chi(G)$. The set of all vertices assigned the color c_i in a coloring c is called a *color class*, denoted by V_i. A set $v \in V(G)$ is said to *dominate* a set $S \subseteq V(G)$, if v is adjacent to every element of S.

A proper coloring of a graph G in which the cardinalities of any two color classes differ by at most 1 is said to be an *equitable coloring* of G and the minimum number of colors used in this coloring is called the *equitable chromatic number* of G, denoted by $\chi_e(G)$ (see [6]). An extensive study on the equitable coloring of graphs can be found in [17–23] and some real-life applications of equitable coloring are mentioned in [6, 31].

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§ Manuscript received: September 24, 2024; accepted: January 06, 2025.
TWMS Journal of Applied and Engineering Mathematics, Vol.15, No.10;

A vertex v in a graph G is said to *dominate* a set $S \subset V(G)$ if $S = \{v\}$ or $uv \in E(G)$, for all $u \in S$. The *dominator coloring* of a graph G is a proper coloring of G such that every vertex in $V(G)$ dominates at least one color class, possibly its own color class. The minimum number of color classes in this coloring is called the *dominator chromatic number* of G, denoted by $\chi_d(G)$ (see [5]). The dominator coloring of trees, bipartite graphs, Petersen graph and various graph classes were studied in [25–29]. Some real-life applications of dominator coloring are mentioned in [1, 2].

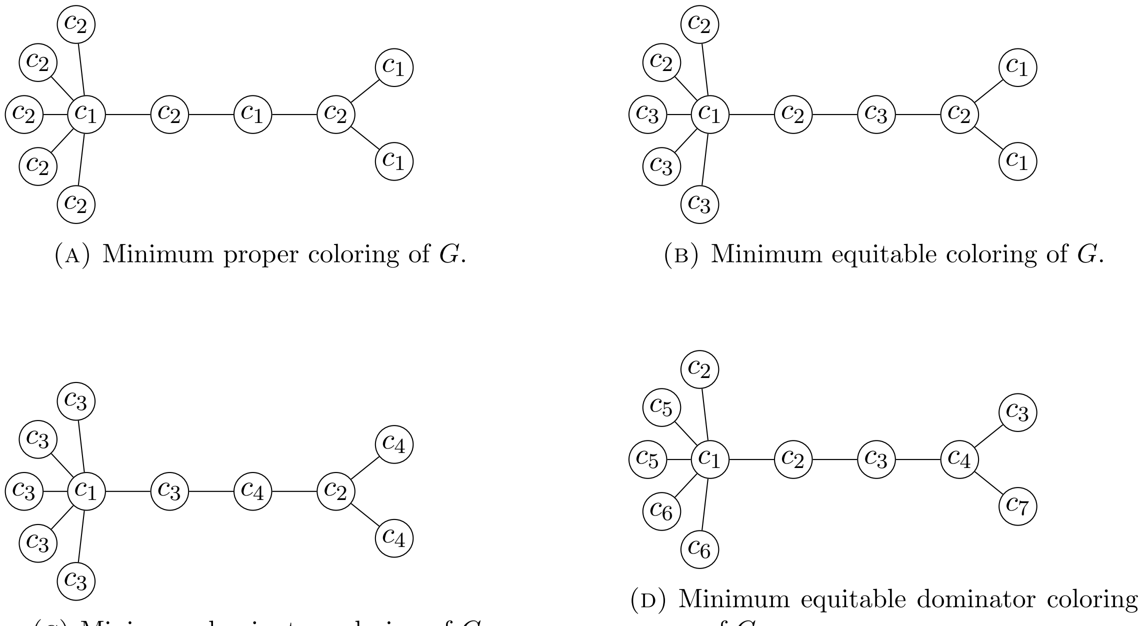
Motivated by the above mentioned types of graph coloring, a variant of domination-related coloring, called equitable dominator coloring of graphs is introduced and studied in this paper.

2. Equitable Dominator Coloring of Graphs

The notion of equitable dominator coloring of a graph is defined as follows.

Definition 2.1. An *equitable dominator coloring* of a graph G is a proper coloring of G such that every vertex in $V(G)$ dominates at least one color class, possibly its own color class and the cardinalities of the color classes differ by at most one. The minimum number of colors used in an equitable dominator coloring of G is the *equitable dominator chromatic number* of G, and we denote it by $\chi_{ed}(G)$.

An example of equitable dominator coloring of graphs is given in Figure 1, where it can be seen that $\chi(G) = 2$, $\chi_e(G) = 3$, $\chi_d(G) = 4$, and $\chi_{ed}(G) = 7$.



(A) Minimum proper coloring of G.

(B) Minimum equitable coloring of G.

(C) Minimum dominator coloring of G.

(D) Minimum equitable dominator coloring of G.

Figure 1 A graph G with $\chi(G) < \chi_e(G) < \chi_d(G) < \chi_{ed}(G)$.

As the concepts of coloring and domination in graphs are used to optimise resource allocation, or conflict-free work scheduling, in huge networks such as transportation and communication networks, biological networks, social networks and so on, and equitablity ensures such allocation or scheduling to be done in an equitable manner, the equitable dominator coloring in graphs ensures the simultaneous availability of equitable resources to all the members of a network, in an optimal manner.

Based on the definitions of proper coloring, equitable coloring, dominator coloring and equitable dominator coloring of graphs, it follows that

(i) $\chi(G) \leq \chi_e(G) \leq \chi_{ed}(G)$,

(ii) $\chi_d(G) \leq \chi_{ed}(G)$.

Figure 1 gives an example of a graph G where the inequalities are sharp. The following propositions gives the conditions when the inequalities are strict.
Since finding equitable chromatic number of graphs and dominator chromatic number of graphs are NP-complete problems (see [3, 4]), it can be observed that the problem of determining the equitable dominator chromatic number of graphs is also NP-complete.
Recall that a vertex v in a graph G of order n is said to be a *universal vertex* of G, if $deg(v) = n - 1$.

Proposition 2.1. *Every equitable coloring of a graph G with at least one universal vertex is its equitable dominator coloring, and* $\chi_e(G) = \chi_{ed}(G)$.

Proof. In any proper coloring of a graph G with at least one universal vertex v, every vertex of G dominates the color class $\{v\}$. Also, in every equitable coloring of G, as the cardinalities of the color classes differ by at most 1, the result follows. □

The converse of Proposition 2.1 does not hold, as we identify a family of graphs without a universal vertex for which $\chi_{ed}(G) = \chi_e(G)$, in the following result.

Proposition 2.2. $\chi_{ed}(K_{a_1,a_2,\ldots,a_s}) = \chi(K_{a_1,a_2,\ldots,a_s})$, *when* $|a_i - a_j| \leq 1; 1 \leq i \neq j \leq s$.

Proof. Any minimum proper coloring of $K_{a_1,a_2,\ldots,a_s}$ with $|a_i - a_j| \leq 1; 1 \leq i \neq j \leq s$, is its minimum equitable coloring. Furthermore, as every vertex of the graph dominates $s - 1$ color classes, the result follows. □

The converse of Proposition 2.2 does not hold as we can see that $\chi_{ed}(C_5) = \chi(C_5) = 3$. For a graph G of order n, $\chi_{ed}(G) = n$ if and only if G is either a K_n or $\overline{K}_n$. In the following proposition, we characterise the graphs for which $\chi_{ed}(G) = 2$.

Proposition 2.3. *For a graph G,* $\chi_{ed}(G) = 2$ *if and only if* $G = K_{a,b}$ *such that* $|a-b| \leq 1$.

Proof. In the case when $G = K_{a,b}$, it follows that $\chi_{ed}(G) = 2$ from Proposition 2.2. To prove the converse, let $\chi_{ed}(G) = 2$ for some graph G. Since $\chi_{ed}(G) = 2$, there is an independent set of vertices assigned the color c_1, say V_1, and an independent set of vertices assigned color c_2, say V_2. Also, every vertex of V_1 is adjacent to every vertex of V_2 and vice-versa. In order to satisfy the condition of equitability $b = a - 1, a, a + 1$ and this concludes the result. □

Theorem 2.1. *For any integer* $j \geq 0$, *there exists at least one graph G such that* $\chi_{ed}(G) - \chi_d(G) = j$.

Proof. Consider the graph $G = K_{2,2}$. We know that the dominator chromatic number for any complete bipartite graph is 2 and by Proposition 2.3, $\chi_{ed}(G) = 2$.

Now consider the graph $K_{2,4}$. Here, $|V_1| = 2$ and hence the partite set V_2 can be partitioned into equitable parts with respect to $|V_1|$. As $\chi_{ed}(K_{2,4}) = 3$, on adding three vertices to V_2 in each iteration and making them adjacent to all the vertices of V_1, we get a complete bipartite graph $K_{2,4+3i}$, with equitable dominator chromatic number $3+i; 1 \leq i \leq n-3$. This proves the result. □

A graph realisation mentioned in Theorem 2.1 is illustrated in Figure 3, in which the dotted vertices and edges represent the added vertices and edges based on given value of j.

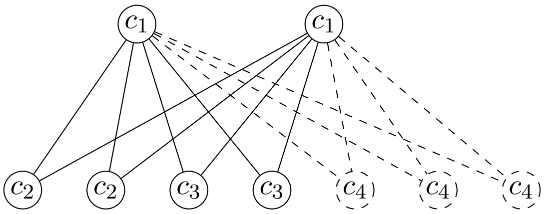


FIGURE 2 Construction of graph G such that $\chi_{ed}(G) - \chi_d(G) = 2$.

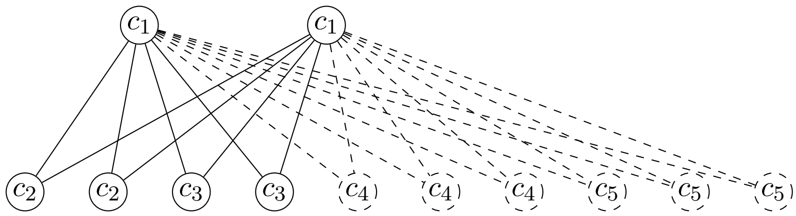


FIGURE 3 Construction of graph G such that $\chi_{ed}(G) - \chi_d(G) = 7$.

3. Equitable Dominator Chromatic Number for Certain Graph Classes

Theorem 3.1. *For $n \geq 4$,*

$$\chi_{ed}(P_n) = \begin{cases} 3\lfloor \frac{n}{5} \rfloor + 1, & n \equiv 0, 1 \pmod 5; \\ 3\lceil \frac{n}{5} \rceil - 1, & n \equiv 2, 3 \pmod 5; \\ 3\lceil \frac{n}{5} \rceil, & n \equiv 4 \pmod 5). \end{cases}$$

Proof. For $P_n := v_1 - v_2 - \ldots - v_n$, consider the following coloring patterns.

Case 1:- When $n \equiv 1 \pmod 5$, let $c : V(P_n) \to \{c_1, c_2, \ldots\}$ be a coloring such that

$$c(v_j) = \begin{cases} c_{3\lfloor \frac{j}{5} \rfloor + k_1}, & j = 5k + k_1, k_1 = 1, 2, 3; \\ c(v_{j-2}), & j \equiv 0, 4 \pmod 5). \end{cases}$$

With respect to this coloring c of P_n, the vertices $v_j; j \equiv 2 \pmod 5$, dominate the color class $\{v_{j-1}\}$ and the vertices $v_j; j \equiv 0 \pmod 5$, dominate the color class $\{v_{j+1}\}$. Also, the vertices $v_j; j \equiv 1 \pmod 5$, dominate their own color classes and the vertices $v_j; j \equiv 3, 4 \pmod 5$, dominate the color classes $\{v_{j-1}, v_{j+1}\}$. As the cardinality of the color classes of the colors used in c is at most 2, it is an equitable dominator coloring of P_n with $3(\frac{n-1}{5}) + 1$ colors.

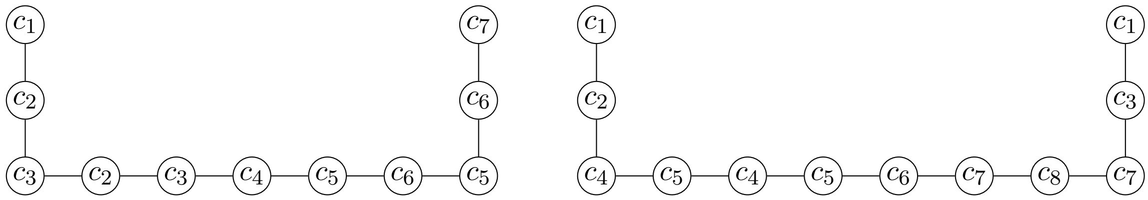
Assume that there exists a coloring c^* of $P_n; n = 5k + 1$, using $3(\frac{n-1}{5})$ colors. Therefore, with respect to c^*, there are k color classes of cardinality 1 and $2(\frac{n-1}{5})$ color classes of cardinality 2, because a pendant vertex of P_n can either dominate its own color class or the color class of its adjacent vertex. Hence, there exists no color class having cardinality greater than 2 in any equitable dominator coloring of P_n. Two vertices in the same color class are at least at a distance 2, because if the vertices are at distance greater than 2, then at least two vertices adjacent to them need to have unique colors such that they dominate their own color classes; yielding a contradiction.

Case 2:- Consider a coloring c' of $P_n; n \not\equiv 1 \pmod 5$, such that $c'(v_1) = c'(v_n) = c_1$, $c'(v_2) = c_2, c'(v_{n-1}) = c_3$, and for $3 \le j \le n-2$,

$$c'(v_j) = \begin{cases} c_{3\lfloor \frac{j}{5} \rfloor + k_1 + 1}, & j = 5k + k_1, k_1 = 2, 3, 4; \\ c'(v_{j-2}), & j \equiv 0, 1 \pmod 5. \end{cases}$$

With respect to the coloring c' of P_n, the vertices v_1, v_2 dominate the color class $\{v_2\}$ and the vertices v_{n-1}, v_n dominate the color class $\{v_{n-1}\}$. Also, the vertices $v_j; j \equiv 3 \pmod 5$, dominate the color class $\{v_{j-1}\}$, the vertices $v_j; j \equiv 2 \pmod 5$, dominate their own color classes, the vertices $v_j; j \equiv 0, 4 \pmod 5$, dominate the color class $\{v_{j-1}, v_{j+1}\}$ and the vertices $v_j; j \equiv 1 \pmod 5$, dominate the color class $\{v_{j+1}\}$. As the cardinality of the color classes of the colors used in c' is at most 2, it is an equitable dominator coloring of P_n, in this case. Also, owing to the arguments mentioned in Case 1, it can be established that c' is a minimum equitable dominator coloring of P_n. Figure 4 illustrates the equitable dominator coloring protocol of P_n given in Case 1 and Case 2.

□



(A) Minimum equitable dominator coloring of P_{11}.

(B) Minimum equitable dominator coloring of P_{12}.

FIGURE 4 Minimum equitable dominator coloring of paths.

Theorem 3.2. *For $n \ge 3$,*

$$\chi_{ed}(C_n) = \begin{cases} 3\lfloor \frac{n}{5} \rfloor + 3, & n \equiv 4 \pmod 5; \\ 3\lfloor \frac{n}{5} \rfloor + r, & n \equiv r \pmod 5, 0 \le r \le 3. \end{cases}$$

Proof. Consider a cycle $C_n := v_1 - v_2 - \ldots - v_n - v_1$ whose vertices are assigned colors as follows. Note that $v_{n+j} = v_j$.

Case 1:- Let $n \equiv 0, 1, 2 \pmod 5$, and c be a coloring of C_n such that

$$c(v_j) = \begin{cases} c_{3\lfloor \frac{j}{5} \rfloor + r}, & j = 5k_1 + r, r = 0, 1, 2; \\ c(v_{j-2}), & j \equiv 3, 4 \pmod 5. \end{cases}$$

Based on the coloring protocol given, the vertices $v_j; j \equiv 1 \pmod 5$, dominate the color class of the color assigned to the vertex v_{j-1}. The vertices $v_j; j \equiv 0 \pmod 5$, dominate their own color classes. The vertices $v_j; j \equiv 2, 3 \pmod 5$, dominate the color class of the color assigned to the vertex v_{j-1} and the vertices $v_j; j \equiv 4 \pmod 5$, dominate the color class $\{v_{j+1}\}$. The cardinality of every color class in this coloring is at most 2; thus satisfying the condition of equitability. Hence, the coloring is an equitable dominator coloring and the result follows in this case.

Case 2:- Let $n \equiv 3, 4 \pmod 5$. The vertices $v_i; 1 \le i \le n-1$, are assigned colors as mentioned in Case 1 and follows the property of equitable dominator coloring as justified in Case 1. However, in this case, the vertex v_n needs to be assigned an unique color since the vertex v_1 cannot dominate the color class assigned to the vertex v_2 or its own color class and hence the result follows.

Since the vertices $v_i : 1 \leq i \leq n-1$ form a path P_{n-1} in C_n, the minimality of the coloring follows from the Proof of Theorem 3.1. The optimality condition follows in C_n as $v_1 \sim v_n$, v_n needs to be assigned a unique color so that the vertex v_1 satisfies the condition of dominator coloring, this concludes the result. For illustration, see Figure 5. □

(A) Minimum equitable dominator coloring of C_7.

(B) Minimum equitable dominator coloring of C_9.

FIGURE 5 Minimum equitable dominator coloring of cycles.

A *bi-star* $S_{a,b}$ is a graph obtained by joining the central vertices of two-star graphs $K_{1,a}$ and $K_{1,b}$ by an edge.

Theorem 3.3. *For* $2 \leq a \leq b$, $\chi_{ed}(S_{a,b}) = 2 + \lceil \frac{a+b}{2} \rceil$.

Proof. Let u, v to be the support vertices of $S_{a,b}$ and $u_i; 1 \leq i \leq a$, and $v_j; 1 \leq j \leq b$, to be the pendant vertices adjacent to u and v, respectively. Define a coloring $c : V(S_{a,b}) \to \{c_1, c_2, \ldots, c_{2+\lceil \frac{a+b}{2} \rceil}\}$ as follows. For a vertex $w \in V(S_{a,b})$,

$$c(w) = \begin{cases} c_1, & w = u; \\ c_2, & w = v; \\ c_{i+2}, & w \in \{u_i, v_i; 1 \leq i \leq a\}; \\ c_{a+2+\lceil \frac{i}{2} \rceil}, & w = v_{a+i}; 1 \leq i \leq b-a. \end{cases}$$

We observe that, with respect to c, the vertices in $\{u\} \cup \{u_i : 1 \leq i \leq a\}$, dominate the color class $\{u\}$ and the vertices in $\{v\} \cup \{v_i : 1 \leq i \leq b\}$ dominate the color class $\{v\}$. Here, the cardinality of all the color classes is at most 2 and hence, $\chi_{ed}(S_{a,b}) \leq 2 + \lceil \frac{a+b}{2} \rceil$.

Assume there exists an equitable dominator coloring of $S_{a,b}$, say c', with fewer colors. In this case, either three pendant vertices are assigned the same color or one of the support vertex is assigned the same color as that of a pendant vertex, leading to a contradiction as a pendant vertex can dominate only its own color class or the color class of its adjacent vertex. □

For $t \geq 3$, a wheel graph $W_{1,t}$ is obtained by making a vertex, say v, adjacent to all the vertices of a cycle C_t. As a consequence of Proposition 2.1, the following result is immediate.

Corollary 3.1. *For* $t \geq 3$, $\chi_{ed}(W_{1,t}) = 1 + \lceil \frac{t}{2} \rceil$.

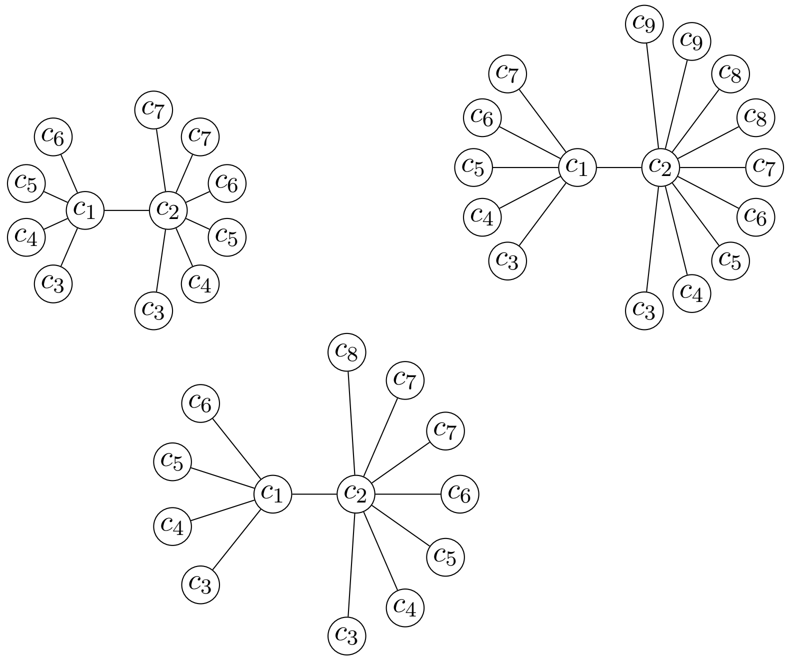


FIGURE 6 Equitable dominator coloring of some bi-stars.

4. Equitable Dominator Coloring of Complete Bipartite Graphs

Note that any proper coloring of $K_{a,b}$ with two colors is its dominator coloring; whereas, the property of equitability is not satisfied unless $|a-b| \leq 1$. Therefore, we use the concept of equitable partition of an integer, introduced in [30], as follows, to find the equitable dominator coloring of $K_{a.b}$. An equitable partition of an integer n is such that the integer n is expressed as the sum of one or more positive integers such that the integers differ by at most 1 (see [30]).

Finding the equitable partition of an integer b with respect to the equitable partition of an integer a is complex. Hence, a Python program is developed in order to find the equitable dominator chromatic number of a complete bipartite graph $K_{a,b}$.

```python
def integerpart(integer):
    partites = set()
    partites.add((integer, ))
    for x in range(1, integer):
        for y in integerpart(integer - x):
            partites.add(tuple(sorted((x, ) + y)))
    return partites


def equitable(partites):
    listed=list(partites)
    length=len(partites)
    L1=[]
    for i in range (length):
        element=listed[i]
        element1=list(element)
        result=all(elements-element1[0]==1 or
        elements-element1[0]==-1 or elements-element1[0]==0
```

```
        for elements in element1)
        if (result):
            L1.append(element)
        else:
            None
    return (L1)

a=int(input("Enter first integer:"))
A=integerpart(a)

B=equitable(A)
B1= sorted(B, key=lambda x: len(x))
print(B1)


c=int(input("Enter the second integer:"))
C=integerpart(c)


D=equitable(C)
D1= sorted(D, key=lambda x: len(x))
print(D1)


def bipartite(B1,D1):
    listed1=list(B1)
    listed2=list(D1)
    length1=len(B1)
    length2=len(D1)
    if length1>length2:
        length3=length2
    else:
        length3=length1

    Pairs=[]
    for i in range (length1):
        for j in range (length2):
            element1=listed1[i]
            element2=listed2[j]
            element11=list(element1)
            element22=list(element2)
            result=all(elements-element11[0]==1 or
            elements-element11[0]==-1 or elements-element11[0]==0
            for elements in element22)
            if (result):
                Pairs.append((element1,element2))
            else:
                None
    return (Pairs)

L=bipartite(B1,D1)

num=L[0]
```

```

x=[j for i in num for j in i]
x1=len(x)
print(x1)

print('The equitable dominator chromatic
integer for K_{a,b} where a is %s and j is %s is %s.'%
      (a,c,x1))
```

In the algorithm above, the function `integerpart()` is defined to find all possible integer partitions of an integer n, which is saved as a set and returned. Next, the `equitable()` returns only those partitions from the output of `integerpart()` that are equitable by comparing each i-th element of the set with all the other j-th elements of the set and is appended and returned in a list $L1$, whose elements are tuples and the elements of each tuple are the equitable parts of the given integer n. When the user inputs the value of a, all the partitions of a are returned, further the equitable partitions of a are generated. Similarly, the user inputs the value of b, from which only the equitable partitions of b are obtained. The equitable partitions of a, b is then sorted and saved in lists $B1, D1$. A function `bipartite()` is then defined in which a list `Pairs` generated by comparing each element of each tuple from the list generated for a with each element of each tuple from the list generated for b. The `Pairs` list which is sorted, and the tuple with the minimum number elements in the list `L` is saved in the variable `num` and finally, $x1$ gives us the most optimal way of partitioning a and b in an equitable manner.

5. Equitable Dominator Chromatic Number of Graph Complements

We begin by discussing the equitable dominator chromatic number of the complement of paths and cycles. Note that $\overline{P}_2 = 2K_1$, $\overline{P}_3 = K_1 \cup K_2$, $\overline{P}_4 = P_4$ and hence, $\chi_{ed}(\overline{P}_2) = 2$, $\chi_{ed}(\overline{P}_3) = 3$, $\chi_{ed}(\overline{P}_4) = 2$. Also, we observe that $\overline{C}_3 = 3K_1$ and $\overline{C}_4$ is $2K_2$, for which, $\chi_{ed}(\overline{C}_3) = 3$ and $\chi_{ed}(\overline{C}_4) = 4$. Therefore, we consider the following result for $n \geq 5$.

Theorem 5.1. *For $n \geq 5$, $\chi_{ed}(\overline{P}_n) = \chi_{ed}(\overline{C}_n) = \lceil \frac{n}{2} \rceil$.*

Proof. For $V(\overline{P}_n) = \{v_i : 1 \leq i \leq n\}$, consider a coloring $c : V(\overline{P}_n) \to \mathcal{C}$ such that $c(v_i) = c(v_{i+1}) = c_{\lceil \frac{i}{2} \rceil}$, where $i \equiv 1 \pmod 2$. This coloring is an equitable dominator coloring, since $\overline{P}_n$ is a $\lceil \frac{n}{2} \rceil$-partite graph with cardinality of each part at most 2. Hence, the equitable dominator chromatic number of $\overline{P}_n$ is $\lceil \frac{n}{2} \rceil$. The above argument holds for the graphs $\overline{C}_n$, as $\overline{C}_n = \overline{P}_n - v_1 v_n$. □

Note that $\overline{W}_{1,t} = \overline{C}_t \cup K_1$ and hence the following result is immediate.

Corollary 5.1. *For $t \geq 5$, $\chi_{ed}(W_{1,t}) = 1 + \chi_{ed}(\overline{C}_t)$.*

Theorem 5.2. *For $a, b \geq 2$, $\chi_{ed}(\overline{S}_{a,b}) = a + b$.*

Proof. Let u, v to be the support vertices of $S_{a,b}$ and $u_i; 1 \leq i \leq a$, and $v_j; 1 \leq j \leq b$, to be the pendant vertices adjacent to u and v, respectively. The pendant vertices in $S_{a,b}$ forms a clique K_{a+b} in $\overline{S}_{a,b}$ and hence $\chi_{ed}(\overline{S}_{a,b}) \geq a + b$. In $\overline{S}_{a,b}$, u (resp. v) being adjacent to all the $v_j; 1 \leq j \leq b$ (resp. $u_i; 1 \leq i \leq a$), u (resp. v) can be assigned any of the colors assigned to any of the $u_i; 1 \leq i \leq a$ (resp. $v_j; 1 \leq j \leq b$), in any equitable coloring of $\overline{S}_{a,b}$.

This coloring satisfies the condition of dominator coloring since the vertices $\{u\} \cup \{u_i : 1 \leq i \leq a\}$ dominate the color class of the colors assigned to the vertices $\{v_j : 1 \leq j \leq b\}$, the vertices $\{v\} \cup \{v_i : 1 \leq i \leq b\}$, dominate the color class of the colors assigned to the vertices $\{u_i : 1 \leq i \leq a\}$. Hence, $\chi_{ed}(\overline{S}_{a,b}) = a + b$. □

6. Conclusions

This paper introduces the notion of equitable dominator coloring and determines the corresponding parameter of equitable dominator chromatic number for some graph classes and their complements. The equitable dominator chromatic number for a complete bipartite graph is found using a Python program and characterizations of graphs having some specific equitable dominator chromatic number have also been done.

Some further directions of studies on equitable dominator coloring are mentioned below.

- To study the parameter for various graph operations like join of graphs, strong product, normal product, etc.
- To study the parameter for generalized Petersen graphs
- To extend the study of $\chi_{ed}(G)$ to power of graphs.

Acknowledgement. The first author would like to acknowledge her gratitude to her fellow researcher Dr. Sabitha Jose for their valuable suggestions and guidance throughout the work.

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