TWMS J. App. and Eng. Math. V.15, N.11, 2025, pp. 2697-2708

MORE ON CONTINUOUS AND IRRESOLUTE MAPS IN PYTHAGOREAN FUZZY TOPOLOGICAL SPACES

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Abstract. The new dimension of non-standard fuzzy sets called Pythagorean fuzzy sets which can handle the inaccurate data very strongly has been established in recent days. Even though intuitionistic fuzzy sets were generously used in decision making to handle the imprecise data the novelty and the voluminous of Pythagorean fuzzy environment gives motivation to use it in decision making process. The Pythagorean fuzzy topological spaces are the novel generalization of fuzzy topological spaces. In this paper, we develop the concept of Pythagorean fuzzy δ continuity which is stronger than Pythagorean fuzzy continuous function in Pythagorean fuzzy topological spaces and specialize some of their basic properties with examples. Also, we introduce and discuss about properties and characterization of Pythagorean fuzzy δ irresolute maps. Interrelations have been studied elaborately for the defined functions using various examples.

Keywords: Pythagorean fuzzy δ open set, Pythagorean fuzzy δ Continuous and Pythagorean fuzzy δ Irresolute.

AMS Subject Classification: 03E72, 54A40, 54C05, 94D05

1. Introduction

In 1965, Zadeh [37] familiarized the concept of fuzzy set which has several applications in decision theory, artificial intelligence, operations research, expert systems, computer science, data analytics, pattern recognition, management science and robotics. In 1968, Chang and Warren [11, 32] defined fuzzy topological spaces, the basic philosophies of topology such as open set, closed set, neighbourhood, interior set, closure, continuity, compactness to fuzzy topological spaces (FTS). Applications of fuzzy sets were studied [1, 10, 21, 26]. Later numerous fuzzy topological spaces raised which have unique properties. In 1997, Dogan Coker [6, 12, 17] introduced Intuitionistic fuzzy topological spaces and

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§ Manuscript received: September 03, 2024; accepted: November 22, 2024.
TWMS Journal of Applied and Engineering Mathematics, Vol.15, No.11;

studied its continuity and compactness. Intuitionistic fuzzy sets have many applications [26, 23] and also flagged approach to study Pythagorean fuzzy sets. In both the sets membership and non-membership are incorporated in a different way. In Intuitionistic fuzzy set the membership μ and non-membership γ are incorporated in such a way that $\mu + \gamma \leq 1$ where as in Pythagorean fuzzy set it is $\mu^2 + \gamma^2 \leq 1$. In 2013, Yager [34] introduced the non-standard fuzzy sets called Pythagorean fuzzy sets in comparison with Intuitionistic fuzzy sets. He gave the basic definition of Pythagorean fuzzy set (PFS) and its application in decision making [3, 36, 35]. PFS has its applications in career placements based on academic performance [18], selection of mask during COVID-19 pandemic using Pythagorean TOPSIS technique [20], etc. Later Murat et.al [16] introduced the conception of Pythagorean fuzzy topological space ($PFTS$) by provoking from the conviction of FTS [13, 14, 19]. He defined Pythagorean fuzzy continuous function between $PFTS$.

Saha [22] defined δ-open sets in fuzzy topological spaces. In 2019, Acikgoz and Esenbel [2] defined neutrosophic soft δ-topology. Aranganayagi et al., Surendra et al. and Vadivel et al. [4, 5, 15, 24, 25, 28, 29, 30, 31] introduced δ-open sets in neutrosophic, neutrosophic soft, neutrosophic hypersoft and neutrosophic nano topological spaces and studied its maps and separation axioms.

Research Gap: No investigation on some stronger and weaker forms of Pythagorean fuzzy continuous and irresolute maps such as Pythagorean fuzzy δ continuous map, Pythagorean fuzzy δ-semi continuous map, Pythagorean fuzzy δ-pre continuous map, Pythagorean fuzzy $\delta\alpha$ continuous map and Pythagorean fuzzy $\delta\beta$ continuous maps and their respective irresolute functions on Pythagorean fuzzy topological space has been reported in the Pythagorean fuzzy literature.

This leads to encompass the notion of $PFTS$ by introducing Pythagorean fuzzy δ continuous map, pythagorean fuzzy δ-semi continuous map, pythagorean fuzzy δ-pre continuous map, pythagorean fuzzy $\delta\alpha$ continuous map and pythagorean fuzzy $\delta\beta$ continuous maps and discuss its properties. Also, we introduce the concept of Pythagorean fuzzy irresoluteness called Pythagorean fuzzy δ irresolute map, pythagorean fuzzy δ-semi irresolute map, pythagorean fuzzy δ-pre irresolute map, pythagorean fuzzy $\delta\alpha$ irresolute map and pythagorean fuzzy $\delta\beta$ irresolute maps and study some of their basic properties. This enables us to obtain conditions under which maps and inverse maps preserve respective open sets.

2. Preliminaries

We recall some basic notions of fuzzy sets, IFS's and pfs's .

Definition 2.1. [37] Let X be a nonempty set. A fuzzy set A in X is characterized by a membership function $\mu_A : X \to [0, 1]$. That is:

$$\mu_A(x) = \begin{cases} 1, & \text{if} \quad x \in X \\ 0, & \text{if} \quad x \notin X \\ (0,1) & \text{if } x \text{ is partly in } X. \end{cases}$$

Alternatively, a fuzzy set A in X is an object having the form $A = \{< x, \mu_A(x) > | x \in X\}$ or $A = \left\{ \left\langle \frac{\mu_A(x)}{x} \right\rangle | x \in X \right\}$, where the function $\mu_A(x) : X \to [0, 1]$ defines the degree of membership of the element, $x \in X$.

The closer the membership value $\mu_A(x)$ to 1, the more x belongs to A, where the grades 1 and 0 represent full membership and full nonmembership. Fuzzy set is a collection of objects with graded membership, that is, having degree of membership. Fuzzy set is an extension of the classical notion of set. In classical set theory, the membership of elements in a set is assessed in a binary terms according to a bivalent condition; an element either belongs or does not belong to the set. Classical bivalent sets are in fuzzy set theory called crisp sets. Fuzzy sets are generalized classical sets, since the indicator function of classical sets is special cases of the membership functions of fuzzy sets, if the latter only take values 0 or 1. Fuzzy sets theory permits the gradual assessment of the membership of element in a set; this is described with the aid of a membership function valued in the real unit interval $[0, 1]$.

Let us consider two examples:

(i) all employees of XYZ who are over $1.8m$ in height; (ii) all employees of XYZ who are tall. The first example is a classical set with a universe (all XYZ employees) and a membership rule that divides the universe into members (those over $1.8m$) and nonmembers. The second example is a fuzzy set, because some employees are definitely in the set and some are definitely not in the set, but some are borderline.

This distinction between the ins, the outs, and the borderline is made more exact by the membership function, μ. If we return to our second example and let A represent the fuzzy set of all tall employees and x represent a member of the universe X (i.e. all employees), then $\mu_A(x)$ would be $\mu_A(x) = 1$ if x is definitely tall or $\mu_A(x) = 0$ if x is definitely not tall or $0 < \mu_A(x) < 1$ for borderline cases.

Definition 2.2. [6, 7, 8, 9] Let a nonempty set X be fixed. An IFS A in X is an object having the form: $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ or $A = \left\{\left\langle \frac{\mu_A(x),\nu_A(x)}{x} \right\rangle | x \in X\right\}$, where the functions $\mu_A(x) : X \to [0, 1]$ and $\nu_A(x) : X \to [0, 1]$ define the degree of membership and the degree of nonmembership, respectively, of the element $x \in X$ to A, which is a subset of X, and for every $x \in X : 0 \leq \mu_A(x) + \nu_A(x) \leq 1$. For each A in X: $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is the intuitionistic fuzzy set index or hesitation margin of x in X. The hesitation margin $\pi_A(x)$ is the degree of nondeterminacy of $x \in X$ to the set A and $\pi_A(x) \in [0, 1]$. The hesitation margin is the function that expresses lack of knowledge of whether $x \in X$ or $x \notin X$. Thus: $\mu_A(x) + \nu_A(x) + \pi_A(x) = 1$.

Example 2.1. Let $X = \{x, y, z\}$ be a fixed universe of discourse and $A = \left\{\left\langle \frac{0.6,0.1}{x} \right\rangle, \left\langle \frac{0.8,0.1}{y} \right\rangle, \left\langle \frac{0.5,0.3}{z} \right\rangle\right\}$, be the intuitionistic fuzzy set in X. The hesitation margins of the elements x, y, z to A are as follows: $\pi_A(x) = 0.3$, $\pi_A(y) = 0.1$ and $\pi_A(z) = 0.2$.

Definition 2.3. [33, 34, 36] Let X be a universal set. Then, a Pythagorean fuzzy set A, which is a set of ordered pairs over X, is defined by the following: $A = \{< x, \mu_A(x), \nu_A(x) | x \in X\}$ or $A = \left\{\left\langle \frac{\mu_A(x),\nu_A(x)}{x} \right\rangle | x \in X\right\}$, where the functions $\mu_A(x) : X \to [0, 1]$ and $\nu_A(x) : X \to [0, 1]$ define the degree of membership and the degree of nonmembership, respectively, of the element $x \in X$ to A, which is a subset of X, and for every $x \in X$, $0 \leq (\mu_A(x))^2 + (\nu_A(x))^2 \leq 1$. Supposing $(\mu_A(x))^2 + (\nu_A(x))^2 \leq 1$, then there is a degree of indeterminacy of $x \in X$ to A defined by $\pi_A(x) = \sqrt{1 - [(\mu_A(x))^2 + (\nu_A(x))^2]}$ and $\pi_A(x) \in [0, 1]$. In what follows, $(\mu_A(x))^2 + (\nu_A(x))^2 + (\pi_A(x))^2 = 1$. Otherwise, $\pi_A(x) = 0$ whenever $(\mu_A(x))^2 + (\nu_A(x))^2 = 1$. We denote the set of all PFS's over X by $pfs(X)$.

Definition 2.4. [36] Let A and B be pfs's of the forms $A = \{< a, \lambda_A(a), \mu_A(a) > | a \in X\}$ and $B = \{< a, \lambda_B(a), \mu_B(a) > | a \in X\}$. Then

(i) $A \subseteq B$ if and only if $\lambda_A(a) \leq \lambda_B(a)$ and $\mu_A(a) \geq \mu_B(a)$ for all $a \in X$.

(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

(iii) $\bar{A} = \{< a, \mu_A(a), \lambda_A(a) > | a \in X\}$.

(iv) $A \cap B = \{< a, \lambda_A(a) \wedge \lambda_B(a), \mu_A(a) \vee \mu_B(a) > | a \in X\}$.
(v) $A \cup B = \{< a, \lambda_A(a) \vee \lambda_B(a), \mu_A(a) \wedge \mu_B(a) > | a \in X\}$.
(vi) $\phi = \{< a, \phi, X > | a \in X\}$ and $X = \{< a, X, \phi > | a \in X\}$.
(vii) $\bar{X} = \phi$ and $\bar{\phi} = X$.

Definition 2.5. [16] An Pythagorean fuzzy topology by subsets of a non-empty set X is a family τ of pfs's satisfying the following axioms.
(i) $\phi,\ X \in \tau$.
(ii) $G_1 \cap G_2 \in \tau$ for every $G_1,\ G_2 \in \tau$ and
(iii) $\bigcup G_i \in \tau$ for any arbitrary family $\{G_i | i \in j\} \subseteq \tau$. The pair (X, τ) is called an Pythagorean fuzzy topological space ($pfts$ in short) and any pfs G in τ is called an Pythagorean fuzzy open set ($pfos$ in short) in X. The complement $\bar{A}$ of an Pythagorean fuzzy open set A in an $pfts(X, \tau)$ is called an Pythagorean fuzzy closed set ($pfcs$ in short).

Definition 2.6. [16] Let (X, τ) be an $pfts$ and $A = \{< a, \lambda_A(a), \mu_A(a) > | a \in X\}$ be an pfs in X. Then the interior and the closure of A are denoted by $pfint(A)$ and $pfcl(A)$ and are defined as follows: $pfcl(A) = \cap\{K | K$ is an $pfcs$ and $A \subseteq K\}$ and $pfint(A) = \cup\{G | G$ is an $pfos$ and $G \subseteq A\}$. Also, it can be established that $pfcl(A)$ is an $pfcs$ and $pfint(A)$ is an $pfos$, A is an $pfcs$ if and only if $pfcl(A) = A$ and A is an $pfos$ if and only if $pfint(A) = A$. We say that A is pf-dense if $pfcl(A) = X$.

Lemma 2.1. [27] For any Pythagorean fuzzy set A in (X, τ), we have $X - pfint(A) = pfcl(X - A)$ and $X - pfcl(A) = pfint(X - A)$.

Definition 2.7. [27] Let (X, τ) be an $pfts$ and A be an pfs. Then A is said to be an Pythagorean fuzzy (i) regular open set ($pfros$ in short) if $A = pfint(pfcl(A))$. (ii) regular closed set ($pfrcs$ in short) if $A = pfcl(pfint(A))$. By Lemma 2.1, it follows that A is an $pfros$ iff $\bar{A}$ is an $pfrcs$.

Definition 2.8. [14] Let (X_1, Γ_P) & (X_2, Ψ_P) be a $pfts$'s. A mapping $h_P : (X_1, \Gamma_P) \to (X_2, \Psi_P)$ is said to be a Pythagorean fuzzy continuous (briefly, $pfCts$) if the inverse image of every $pfos$ in (X_2, Ψ_P) is a $pfos$.

3. Pythagorean Fuzzy δ-continuous mappings

In this section, we introduce Pythagorean fuzzy δ-continuous mappings and discuss some of their properties.

Definition 3.1. Let (X, τ) be an $pfts$ and $A = \{< a, \lambda_A(a), \mu_A(a) > | a \in X\}$ be an pfs in X. Then the δ-interior and the δ-closure of A are denoted by $pf\delta int(A)$ and $pf\delta cl(A)$ and are defined as follows. $pf\delta cl(A) = \cap\{K | K$ is an $pfrcs$ and $A \subseteq K\}$, $(pf\delta int(A) = \cup\{G | G$ is an $pfros$ and $G \subseteq A\}$.

Definition 3.2. Let (X, τ) be an $pfts$ and $A = \{< a, \lambda_A(a), \mu_A(a) > | a \in X\}$ be an pfs in X. A set A is said to be pf
(i) δ-open set (briefly, $pf\delta os$) if $A = pf\delta int(A)$,
(ii) δ-pre open set (briefly, $pf\delta\mathcal{P}os$) if $A \subseteq pfint(pf\delta cl(A))$.
(iii) δ-semi open set (briefly, $pf\delta\mathcal{S}os$) if $A \subseteq pfcl(pf\delta int(A))$.
(iv) δ-α open set or a-open set (briefly, $pf\delta\alpha os$ or $pfaos$) if $A \subseteq pfint(pfcl(pf\delta int(A)))$.
(v) δ-β open set or e^*-open set (briefly, $pf\delta\beta os$ or pfe^*os) if $A \subseteq pfcl(pfint(pf\delta cl(A)))$.
(vi) δ (resp. δ-pre, δ-semi, δ-α and δ-β) dense if $pf\delta cl(A)$ (resp. $pf\delta pcl(A)$, $pf\delta\mathcal{S}cl(A)$, $pf\delta\alpha cl(A)$ and $pf\delta\beta cl(A)) = X$.

The complement of an $pf\delta os$ (resp. $pf\delta\mathcal{P}os$, $pf\delta\mathcal{S}os$, $pf\delta\alpha os$ and $pf\delta\beta os$) is called an $pf\delta$ (resp. $pf\delta\mathcal{P}$, $pf\delta\mathcal{S}, pf\delta\alpha$ and $pf\delta\beta$) closed set (briefly, $pf\delta cs$ (resp. $pf\delta\mathcal{P}cs$, $pf\delta\mathcal{S}cs$, $pf\delta\alpha cs$ and $pf\delta\beta cs$ in X.

The family of all $pf\delta os$ (resp. $pf\delta cs$, $pf\delta\mathcal{P}os$, $pf\delta\mathcal{P}cs$, $pf\delta\mathcal{S}os$, $pf\delta\mathcal{S}cs$, $pf\delta\alpha os$, $pf\delta\alpha cs$, $pf\delta\beta os$ and $pf\delta\beta cs$) of X is denoted by $pf\delta OS(X)$, (resp. $pf\delta CS(X)$, $pf\delta\mathcal{P}OS(X), pf\delta\mathcal{P}CS(X), pf\delta\mathcal{S}OS(X), pf\delta\mathcal{S}CS(X), pf\delta\alpha OS(X), pf\delta\alpha CS(X), pf\delta\beta OS$ (X) and $pf\delta\beta CS(X)$).

Definition 3.3. Let (X,τ) be an $pfts$ and $A = \{< a, \lambda_A(a), \mu_A(a) > | a \in X\}$ be an pfs in X. Then the $pf\delta$-pre (resp. $pf\delta$-semi, $pf\delta\alpha$ and $pf\delta\beta$)-interior and the $pf\delta$-pre (resp. $pf\delta$-semi, $pf\delta\alpha$ and $pf\delta\beta$)-closure of A are denoted by $pf\delta\mathcal{P}int(A)$ (resp. $pf\delta\mathcal{S}int(A)$, $pf\delta\alpha int(A)$ and $pf\delta\beta int(A)$) and the $pf\delta\mathcal{P}cl(A)$ (resp. $pf\delta\mathcal{S}cl(A), pf\delta\alpha cl(A)$ and $pf\delta\beta$ $cl(A)$ and are defined as follows:

$pf\delta\mathcal{P}int(A)$ (resp. $pf\delta\mathcal{S}int(A), pf\delta\alpha int(A)$ and $pf\delta\beta int(A) = \cup\{G|G$ in a $pf\delta\mathcal{P}os$ (resp. $pf\delta\mathcal{S}os$, $pf\delta\alpha os$ and $pf\delta\beta os$)

and $G \subseteq A\}$ and $pf\delta\mathcal{P}cl(A)$ (resp. $pf\delta\mathcal{S}cl(A), pf\delta\alpha cl(A)$ and $pf\delta\beta cl(A) = \cap\{K|K$ is an $pf\delta\mathcal{P}cs$ (resp. $pf\delta\mathcal{S}cs$, $pf\delta\alpha cs$, $pf\delta\beta cs$) and $A \subseteq K\}$.

Example 3.1. Let $X = \{a,b\}$ and the pfs's A_1, A_2, A_3, A_4 and A_5 are defined as $\mu_{A_1}(a) = 0.5$, $\gamma_{A_1}(a) = 0.7$, $\mu_{A_1}(b) = 0.2$, $\gamma_{A_1}(b) = 0.4$; $\mu_{A_2}(a) = 0.6$, $\gamma_{A_2}(a) = 0.5$, $\mu_{A_2}(b) = 0.3$, $\gamma_{A_2}(b) = 0.9$; $\mu_{A_3}(a) = 0.4$, $\gamma_{A_3}(a) = 0.8$, $\mu_{A_3}(b) = 0.1$, $\gamma_{A_3}(b) = 0.95$; $\mu_{A_4}(a) = 0.6$, $\gamma_{A_4}(a) = 0.5$, $\mu_{A_4}(b) = 0.3$, $\gamma_{A_4}(b) = 0.4$; $\mu_{A_5}(a) = 0.5$, $\gamma_{A_5}(a) = 0.7$, $\mu_{A_5}(b) = 0.2$, $\gamma_{A_5}(b) = 0.9$.

Let $\tau = \{0_P, 1_P, A_1, A_2, A_3, A_4, A_5\}$ be a $pfts$ on X. Then the pfs

(i) A_2 is $pfos$ (resp. $pf\delta\mathcal{P}os$) but not $pf\delta os$.
(ii) A_1^c is $pf\delta\mathcal{S}os$ but not $pf\delta os$.
(iii) A_2 is $pf\delta\beta os$ but not $pf\delta\mathcal{P}os$.
(iv) A_1^c is $pf\delta\beta os$ but not $pf\delta\mathcal{S}os$.
(v) A_5 is $pf\delta\mathcal{S}os$ but not $pf\delta\alpha os$.
(vi) A_5 is $pf\delta\mathcal{P}os$ but not $pf\delta\alpha os$.

Definition 3.4. Let (X_1,Γ_P) & (X_2,Ψ_P) be a $pfts$'s. A mapping $h_P : (X_1,\Gamma_P) \to (X_2,\Psi_P)$ is said to be a Pythagorean fuzzy δ (resp. $\delta\alpha$, $\delta\mathcal{S}$, $\delta\mathcal{P}$ & $\delta\beta$ or e^*)-continuous (briefly, $pf\delta Cts$ (resp. $pf\delta\alpha Cts$, $pf\delta\mathcal{S}Cts$, $pf\delta\mathcal{P}Cts$ & $pf\delta\beta Cts$ or pfe^*Cts)) if the inverse image of every $pfos$ in (X_2,Ψ_P) is a $pf\delta os$ (resp. $pf\delta\alpha os$, $pf\delta\mathcal{S}os$, $pf\delta\mathcal{P}os$ & $pf\delta\beta os$ or pfe^*os) in (X_1,Γ_P).

Theorem 3.1. Let (X_1,Γ_P) & (X_2,Ψ_P) be a $pfts$'s. Let $h_P : (X_1,\Gamma_P) \to (X_2,\Psi_P)$ be a mapping. Then the following statements are hold for $pfts$, but not conversely.

(i) Every $pf\delta Cts$ is a $pfCts$.
(ii) Every $pf\delta Cts$ is a $pf\delta\mathcal{S}Cts$.
(iii) Every $pf\delta Cts$ is a $pf\delta\mathcal{P}Cts$.
(iv) Every $pf\delta\mathcal{S}Cts$ is a $pf\delta\beta Cts$.
(v) Every $pf\delta\mathcal{P}Cts$ is a $pf\delta\beta Cts$.
(vi) Every $pf\delta\alpha Cts$ is a $pf\delta\mathcal{S}Cts$.
(vii) Every $pf\delta\alpha Cts$ is a $pf\delta\mathcal{P}Cts$.

Proof. (i) Let h_P be a $pf\delta Cts$ and K is a $pfos$ in X_2. Then $h_P^{-1}(K)$ is $pf\delta os$ in X_1. Since for each $pf\delta os$ is $pfos$, $h_P^{-1}(K)$ is $pfos$ in X_1. Therefore, h_P is $pfCts$.

(ii) Let h_P be a $pf\delta Cts$ and K is a $pfos$ in X_2. Then $h_P^{-1}(K)$ is $pf\delta os$ in X_1. Since for each $pf\delta os$ is $pf\delta\mathcal{S}os$, $h_P^{-1}(K)$ is $pf\delta\mathcal{S}os$ in X_1. Therefore, h_P is $pf\delta\mathcal{S}Cts$.

(iii) Let h_P be a $pfCts$ and K is a $pfos$ in X_2. Then $h_P^{-1}(K)$ is $pf\delta os$ in X_1. Since for each $pf\delta os$ is $pf\delta\mathcal{P}os$, $h_P^{-1}(K)$ is $pf\delta\mathcal{P}os$ in X_1. Therefore, h_P is $pf\delta\mathcal{P}Cts$.

(iv) Let h_P be a $pf\delta\mathcal{S}Cts$ and K is a $pfos$ in X_2. Then $h_P^{-1}(K)$ is $pf\delta\mathcal{S}os$ in X_1. Since for each $pf\delta\mathcal{S}os$ is $pf\delta\beta os$, $h_P^{-1}(K)$ is $pf\delta\beta os$ in X_1. Therefore, h_P is $pf\delta\beta Cts$.

(v) Let h_P be a $pf\delta\mathcal{P}Cts$ and K is a $pfos$ in X_2. Then $h_P^{-1}(K)$ is $pf\delta\mathcal{P}os$ in X_1. Since for each $pf\delta\mathcal{P}os$ is $pf\delta\beta os$, $h_P^{-1}(K)$ is $pf\delta\beta os$ in X_1. Therefore, h_P is $pf\delta\beta Cts$.

(vi) Let h_P be a $pf\delta\alpha Cts$ and K is a $pfos$ in X_2. Then $h_P^{-1}(K)$ is $pf\delta\alpha os$ in X_1. Since for each $pf\delta\alpha os$ is $pf\delta\mathcal{S}os$, $h_P^{-1}(K)$ is $pf\delta\mathcal{S}os$ in X_1. Therefore, h_P is $pf\delta\mathcal{S}Cts$.

(vii) Let h_P be a $pf\delta\alpha Cts$ and K is a $pfos$ in X_2. Then $h_P^{-1}(K)$ is $pf\delta\alpha os$ in X_1. Since for each $pf\delta\alpha os$ is $pf\delta\mathcal{P}os$, $h_P^{-1}(K)$ is $pf\delta\mathcal{P}os$ in X_1. Therefore, h_P is $pf\delta\mathcal{P}Cts$. □

Remark 3.1. The following Figure shows the relations among the different types of Pythagorean fuzzy δ continuous mappings that were studied in this section.

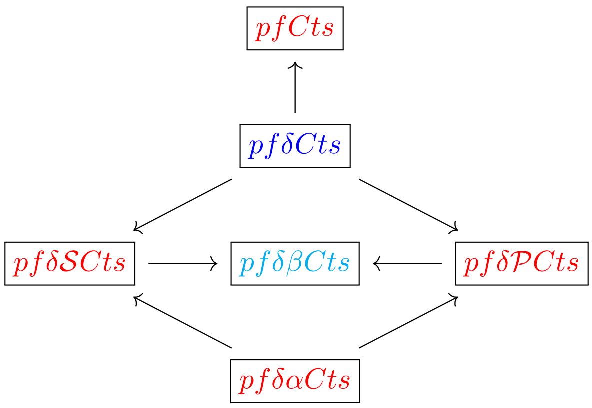

Figure : $pf\delta cts$ mappings in $pfts$

Example 3.2. Let $X = X_1 = X_2 = X_3 = X_4 = X_5 = \{x_1, x_2\}$ and the pfs's A_1, A_2 and A_3 are defined as

$$A_1 = \{< x_1, 0.020, 0.040 >, < x_2, 0.050, 0.050 >\}$$
$$A_2 = \{< x_1, 0.010, 0.040 >, < x_2, 0.050, 0.050 >\}$$
$$A_3 = \{< x_1, 0.020, 0.030 >, < x_2, 0.050, 0.050 >\}$$

Here we have $\tau_1 = \{0_{X_1}, 1_{X_1}, A_1, A_2\}$, $\tau_2 = \{0_{X_2}, 1_{X_2}, A_2\}$, $\tau_3 = \{0_{X_3}, 1_{X_3}, A_1^c\}$, $\tau_4 = \{0_{X_4}, 1_{X_4}, A_2^c\}$ and $\tau_5 = \{0_{X_5}, 1_{X_5}, A_3\}$ be a $pfts$'s on X. Let $h1_P : (X_1, \tau_1) \to (X_2, \tau_2)$, $h2_P : (X_1, \tau_1) \to (X_3, \tau_3)$, $h3_P : (X_1, \tau_1) \to (X_4, \tau_4)$, $h4_P : (X_1, \tau_1) \to (X_5, \tau_5)$ be an identity mapping. Then

(i) $h1_P$ is $pfCts$ (resp. $pf\delta\beta Cts$ and $pf\delta\mathcal{P}Cts$) but not $pf\delta Cts$ (resp. $pf\delta\mathcal{S}Cts$ and $pf\delta\alpha Cts$), because the set A_2 is a $pfos$ in X_2 but $h1_P^{-1}(A_2) = A_2$ is not $pf\delta os$ (resp. $pf\delta\mathcal{S}os$ and $pf\delta\alpha os$) in X_1.

(ii) $h2_P$ is $pf\delta\mathcal{S}Cts$ but not $pf\delta Cts$, because the set A_1^c is a $pfos$ X_3 but $h2_P^{-1}(A_1^c) = A_1^c$ is not $pf\delta os$ in X_1.

(iii) $h3_P$ is $pf\delta\mathcal{P}Cts$ but not $pf\delta Cts$, because the set A_2^c is a $pfos$ X_4 but $h3_P^{-1}(A_2^c) = A_2^c$ is not $pf\delta\mathcal{P}os$ in X_1.

(iv) $h4_P$ is $pf\delta\beta Cts$ (resp. $pf\delta\mathcal{S}Cts$) but not $pf\delta\mathcal{P}Cts$ (resp. $pf\delta\alpha Cts$), because the set A_3 is a $pfos$ X_5 but $h4^{-1}(A_3) = A_3$ is not $pf\delta\mathcal{P}os$ (resp. $pf\delta\alpha os$) in X_1.

Theorem 3.2. Let (X_1, Γ_P) & (X_2, Ψ_P) be a $pfts$'s. A mapping $h_P : (X_1, \Gamma_P) \to (X_2, \Psi_P)$ satisfies the following conditions are equivalent.

(i) h_P is $pf\delta\beta Cts$;

(ii) The inverse $h_P^{-1}(K)$ of all $pf\delta os$ K in X_2 is $pf\delta\beta os$ in X_1.

Proof. The proof is directly, since $h_P^{-1}(\overline{K}) = \overline{h_P^{-1}(K)}$ for all $pf\delta os$ K of X_2. □

Theorem 3.3. Let (X_1, Γ_P) & (X_2, Ψ_P) be a $pfts$'s. A mapping $h_P : (X_1, \Gamma_P) \to (X_2, \Psi_P)$ satisfies the following conditions are hold.

(i) $h_P(pf\delta\beta cl(L)) \subseteq pf\delta cl(h_P(L))$, for all $pfcs$ L in X_1.

(ii) $pf\delta\beta cl(h_P^{-1}(K)) \subseteq h_P^{-1}(pf\delta cl(K))$, for all $pfcs$ K in X_2.

Proof. (i) Since $pf\delta cl(h_P(L))$ is a $pf\delta cs$ in X_2 and h_P is $pf\delta\beta Cts$, then $h_P^{-1}(pf\delta cl(h_P(L)))$ is $pf\delta\beta c$ in X_1. Now, since $L \subseteq h_P^{-1}(pf\delta cl(h_P(L)))$, $pf\delta\beta cl(L) \subseteq h_P^{-1}(pf\delta\ cl(h_P(L)))$. Therefore, $h_P(pf\delta\beta cl(L)) \subseteq pf\delta cl(h_P(L))$.

(ii) By replacing L with K in (i), we obtain $h_P(pf\delta\beta cl(h_P^{-1}(K))) \subseteq pf\delta cl(h_P(h_P^{-1}(K))) \subseteq pf\delta cl(K)$. Hence, $pf\delta\beta cl(h_P^{-1}(K)) \subseteq h_P^{-1}(pf\delta cl(K))$. □

Remark 3.2. Let (X_1, Γ_P) & (X_2, Ψ_P) be a $pfts$'s. Let $h_P : (X_1, \Gamma_P) \to (X_2, \Psi_P)$ be a mapping. If h_P is $pf\delta\beta Cts$, then

(i) $h_P(pf\delta\beta cl(L))$ is not necessarily equal to $pf\delta cl(h_P(L))$ where $L \in X_1$.

(ii) $pf\delta\beta cl(h_P^{-1}(K))$ is not necessarily equal to $h_P^{-1}(pf\delta cl(K))$ where $K \in X_2$.

Example 3.3. Let $X = Y = \{x_1, x_2\}$ and the pfs's A is defined as $A = B = \{< x_1, 0.8, 0.3 >, < x_2, 0.9, 0.3 >\}$ Here we have $\tau_P = \{0_P, 1_P, A\}$ is $pfts$ on X. Let $h_P : (X, \tau_P) \to (Y, \tau_P)$ be an identity mapping. Then h_P is $pf\delta\beta Cts$.

(i) $h_P(pf\delta\beta cl(A)) = A$. But $pf\delta cl(h_P(A)) = 1$.
Thus $h_P(pf\delta\beta cl(A)) \neq pf\delta cl(h_P(A))$.

(ii) $pf\delta\beta cl(h_P^{-1}(A)) = A$. But $h_P^{-1}(pf\delta cl(A)) = 1$.
Thus $pf\delta\beta cl(h_P^{-1}(A)) \neq h_P^{-1}(pf\delta cl(A))$.

Theorem 3.4. Let (X_1, Γ_P) & (X_2, Ψ_P) be a $pfts$'s. Let $h_P : (X_1, \Gamma_P) \to (X_2, \Psi_P)$ be a mapping. If h_P is $pf\delta\beta Cts$, then $h_P^{-1}(pf\delta int(L)) \subseteq pf\delta\beta int(h_P^{-1}(L))$, for all pfs L in X_2.

Proof. If h_P is $pf\delta\beta Cts$ and $L \subseteq X_2$. $pf\delta int(L)$ is $pf\delta o$ in X_2 and hence, $h_P^{-1}(pf\delta\ int(L))$ is $pf\delta\beta o$ in X_1. Therefore $pf\delta\beta int(h_P^{-1}(pf\delta int(L))) = h_P^{-1}(pf\delta int(L))$. Also, $pf\delta int(L) \subseteq L$, implies that $h_P^{-1}(pf\delta int(L)) \subseteq h_P^{-1}(L)$. Therefore $pf\delta\beta int(h_P^{-1}(pf\delta\ int(L))) \subseteq pf\delta\beta int(h_P^{-1}(L))$. That is $h_P^{-1}(pf\delta int(L)) \subseteq pf\delta\beta int\ (h_P^{-1}(L))$.

Conversely, let $h_P^{-1}(pf\delta int(L)) \subseteq pf\delta\beta int(h_P^{-1}(L))$ for all subset L of X_2. If L is $pf\delta o$ in X_2, then $pf\delta int(L) = L$. By assumption, $h_P^{-1}(pf\delta int(L)) \subseteq pf\delta\beta int(h_P^{-1}(L))$. Thus $h_P^{-1}(L) \subseteq pf\delta\beta int(h_P^{-1}(L))$. But $pf\delta\beta int\ (h_P^{-1}(L)) \subseteq h_P^{-1}(L)$. Therefore $pf\delta\beta int(h_P^{-1}(L)) = h_P^{-1}(L)$. That is, $h_P^{-1}(L)$ is $pf\delta\beta o$ in X_1, for all $pf\delta os$ L in X_2. Therefore h_P is $pf\delta\beta Cts$ on X_1. □

Remark 3.3. Let (X_1, Γ_P) & (X_2, Ψ_P) be a $pfts$'s. Let $h_P : (X_1, \Gamma_P) \to (X_2, \Psi_P)$ be a mapping. If h_P is $pf\delta\beta Cts$, then $pf\delta\beta int(h_P^{-1}(K))$ is not necessarily equal to $h_P^{-1}(pf\delta int(K))$ where $K \in X_2$.

Example 3.4. In Example 3.3, h_P is a $pf\delta\beta Cts$.

Then $pf\delta\beta int(h_P^{-1}(A)) = A$. But $h_P^{-1}(pf\delta int(A)) = 0$. Thus $pf\delta\beta int(h_P^{-1}(K)) \neq h_P^{-1}(pf\delta int(K))$.

Remark 3.4. Theorems 3.2, 3.3, 3.4 and Remarks 3.2, 3.3 are true for $pf\delta\mathcal{P}os$, $pf\delta\mathcal{S}os$ and $pf\delta\alpha os$.

4. Pythagorean fuzzy δ-irresolute maps

In this section, we introduce the concept of Pythagorean fuzzy irresoluteness called Pythagorean fuzzy δ irresolute map, pythagorean fuzzy δ-semi irresolute map, pythagorean fuzzy δ-pre irresolute map, pythagorean fuzzy $\delta\alpha$ irresolute map and pythagorean fuzzy $\delta\beta$ irresolute maps and study some of their basic properties. This enables us to obtain conditions under which maps and inverse maps preserve respective open sets.

Definition 4.1. A map $h_P : (X_1, \Gamma_P) \to (X_2, \Psi_P)$ is known as a pythagorean fuzzy (resp. δ, $\delta\mathcal{P}$, $\delta\mathcal{S}$, $\delta\alpha$ and $\delta\beta$)-irresolute (in short, $pfIrr$ (resp. $pf\delta Irr$, $pf\delta\mathcal{P}Irr$, $pf\delta\mathcal{S}Irr$, $pf\ \delta\alpha Irr$ and $pf\delta\beta Irr$)) map if $h_P^{-1}(K)$ is a $pf\mathcal{S}os$ (resp. $pf\delta os$, $pf\delta\mathcal{P}os$, $pf\delta\mathcal{S}os$, $pf\delta\alpha os$ and $pf\delta\beta os$) in (X_1, Γ_P) for each $pf\mathcal{S}os$ (resp. $pf\delta os$,$pf\delta\mathcal{P}os$, $pf\delta\mathcal{S}os$, $pf\delta\alpha os$ and $pf\delta\beta os$) K of (X_2, Ψ_P).

Theorem 4.1. Let (X_1, Γ_P) & (X_2, Ψ_P) be a $pfts$'s. Let $h_P : (X_1, \Gamma_P) \to (X_2, \Psi_P)$ be a mapping. Then the following statements are hold for $pfts$, but not conversely.

(i) Every $pfIrr$ map is a $pf\mathcal{S}Cts$.
(ii) Every $pf\delta\mathcal{S}Irr$ map is a $pf\delta\mathcal{S}Cts$.
(iii) Every $pf\delta\mathcal{P}Irr$ map is a $pf\delta\mathcal{P}Cts$.
(iv) Every $pf\delta\alpha Irr$ map is a $pf\delta\alpha Cts$.
(v) Every $pf\delta\beta Irr$ map is a $pf\delta\beta Cts$.

But the converse is not true.

Proof. (i) Consider a $pfIrr$ map h_P and a $pfos$ K in X_2. As each $pfos$ is a $pf\mathcal{S}os$, K is a $pf\mathcal{S}os$ in X_2. By presumption, $h_P^{-1}(K)$ is a $pf\mathcal{S}os$ in X_1. Thus f is a $pf\mathcal{S}Cts$ map.

(ii) Consider a $pf\delta\mathcal{S}Irr$ map h_P and a $pf\delta os$ K in X_2. As each $pf\delta os$ is a $pfos$ and $pf\delta\mathcal{S}os$, K is a $pf\delta os$ and $pf\delta\mathcal{S}os$ in X_2. By presumption, $h_P^{-1}(K)$ is a $pf\delta\mathcal{S}os$ in X_1. Thus f is a $pf\delta\mathcal{S}Cts$ map.

(iii) Consider a $pf\delta\mathcal{P}Irr$ map h_P and a $pf\delta os$ K in X_2. As each $pf\delta os$ is a $pfos$ and $pf\delta\mathcal{P}os$, K is a $pf\delta os$ and $pf\delta\mathcal{P}os$ in X_2. By presumption, $h_P^{-1}(K)$ is a $pf\delta\mathcal{P}os$ in X_1. Thus f is a $pf\delta\mathcal{P}Cts$ map.

(iv) Consider a $pf\delta\alpha Irr$ map h_P and a $pf\delta os$ K in X_2. As each $pf\delta os$ is a $pfos$ and $pf\delta\alpha os$, K is a $pf\delta os$ and $pf\delta\alpha os$ in X_2. By presumption, $h_P^{-1}(K)$ is a $pf\delta\alpha os$ in X_1. Thus f is a $pf\delta\alpha Cts$ map.

(v) Consider a $pf\delta\beta Irr$ map h_P and a $pf\delta os$ K in X_2. As each $pf\delta os$ is a $pfos$ and $pf\delta\beta os$, K is a $pf\delta os$ and $pf\delta\beta os$ in X_2. By presumption, $h_P^{-1}(K)$ is a $pf\delta\beta os$ in X_1. Thus f is a $pf\delta\beta Cts$ map. □

Example 4.1. Let $X = Y = \{x_1, x_2\}$ and the pfs's A_1, A_2, A_3, A_4, A_5, A_6 and A_7 are defined as

$$A_1 = \{< x_1, 0.020, 0.080 >, < x_2, 0.040, 0.060 >\}$$
$$A_2 = \{< x_1, 0.010, 0.090 >, < x_2, 0.030, 0.070 >\}$$
$$A_3 = \{< x_1, 0.090, 0.010 >, < x_2, 0.070, 0.030 >\}$$
$$A_4 = \{< x_1, 0.020, 0.080 >, < x_2, 0.030, 0.070 >\}$$
$$A_5 = \{< x_1, 0.020, 0.080 >, < x_2, 0.030, 0.060 >\}$$
$$A_6 = \{< x_1, 0.040, 0.020 >, < x_2, 0.040, 0.040 >\}$$
$$A_7 = \{< x_1, 0.080, 0.020 >, < x_2, 0.060, 0.040 >\}.$$

Here we have $\tau_1 = \{0_X, 1_X, A_1, A_2, A_3, A_4\}$, $\tau_2 = \{0_X, 1_X, A_2\}$ and $\tau_3 = \{0_X, 1_X, A_3\}$ be a $pfts$'s on X.

(i) Let $h_P : (X, \tau_1) \to (Y, \tau_2)$ be an identity mapping. Then h_P is $pf\mathcal{S}cts$ (resp. $pf\delta\mathcal{S}cts$) but not $pf\mathcal{S}Irr$ (resp. $pf\delta\mathcal{S}Irr$), because the set A_4^c (resp. A_4) is a $pf\mathcal{S}os$ (resp. $pf\delta\mathcal{S}os$) in Y but $h_P^{-1}(A_4^c) = A_4^c$ (resp. $h_P^{-1}(A_4) = A_4$) is not $pf\mathcal{S}os$ (resp. $pf\delta\mathcal{S}os$) in X.

(ii) Let $h_P : (X, \tau_1) \to (Y, \tau_3)$ be an identity mapping. Then h_P is $pf\delta\mathcal{P}cts$ but not $pf\delta\mathcal{P}Irr$, because the set A_1^c is a $pf\delta\mathcal{P}os$ in Y but $h_P^{-1}(A_1^c) = A_1^c$ is not $pf\delta\mathcal{P}os$ in X.

Definition 4.2. A $pfts$ (X_1, Γ_P) is known as a Pythagorean fuzzy $\delta\mathcal{S}U_{\frac{1}{2}}$ (resp. $\delta\mathcal{P}U_{\frac{1}{2}}$, $\delta\alpha U_{\frac{1}{2}}$ and $\delta\beta U_{\frac{1}{2}}$) (in short, $pf\delta\mathcal{S}U_{\frac{1}{2}}$ (resp. $pf\delta\mathcal{P}U_{\frac{1}{2}}$, $pf\delta\alpha U_{\frac{1}{2}}$ and $pf\delta\beta U_{\frac{1}{2}}$))-space, if each $pf\delta\mathcal{S}os$ (resp. $pf\delta\mathcal{P}os$, $pf\delta\alpha os$ and $pf\delta\beta os$) in X is $pfos$ in X.

Theorem 4.2. Let $h_P : (X_1, \Gamma_P) \to (X_2, \Psi_P)$ and $g_P : (X_2, \Psi_P) \to (X_3, \Phi_P)$ be $pf\delta Irr$ (resp. $pf\delta\mathcal{S}Irr$, $pf\delta\mathcal{P}Irr$, $pf\delta\alpha Irr$ and $pf\delta\beta Irr$) maps, then $g_P \circ h_P : (X_1, \Gamma_P) \to (X_3, \Phi_P)$ is a $pf\delta Irr$ (resp. $pf\delta\mathcal{S}Irr$, $pf\delta\mathcal{P}Irr$, $pf\delta\alpha Irr$ and $pf\delta\beta Irr$) map.

Proof. Consider a $pf\delta os$ K in X_3. So $g_P^{-1}(K)$ is a $pf\delta os$ in X_2. As h_P is a $pf\delta Irr$ map, $f_P^{-1}(g_P^{-1}(K))$ is a $pf\delta os$ in X_1. Thus $g_P \circ h_P$ is a $pf\delta Irr$ map. The other cases are similar. □

Theorem 4.3. Consider a $pf\delta Irr$ (resp. $pf\delta\mathcal{S}Irr$, $pf\delta\mathcal{P}Irr$, $pf\delta\alpha Irr$ and $pf\delta\beta Irr$) map $h_P : (X_1, \Gamma_P) \to (X_2, \Psi_P)$ and a $pf\delta Cts$ (resp. $pf\delta\mathcal{S}Cts$, $pf\delta\mathcal{P}Cts$, $pf\delta\alpha Cts$ and $pf\delta\beta Cts$) map $g_P : (X_2, \Psi_P) \to (X_3, \Phi_P)$. Then $g_P \circ h_P : (X_1, \Gamma_P) \to (X_3, \Phi_P)$ is a $pf\delta Cts$ (resp. $pf\delta\mathcal{S}Cts$, $pf\delta\mathcal{P}Cts$, $pf\delta\alpha Cts$ and $pf\delta\beta Cts$) map.

Proof. Consider a $pfos$ K in X_3. So $g_P^{-1}(K)$ is a $pf\delta os$ in X_2. As h_P is a $pf\delta Irr$ map, $f_P^{-1}(g_P^{-1}(U))$ is a $pf\delta os$ in X_1. Thus $g_P \circ h_P$ is a $pf\delta Cts$ map. The other cases are similar. □

Theorem 4.4. Consider a map $h_P : (X_1, \Gamma_P) \to (X_2, \Psi_P)$ from a $pfts$ X_1 into a $pfts$ X_2. The following are equivalent if X_1 and X_2 are $pf\delta U_{\frac{1}{2}}$ (resp. $pf\delta\mathcal{S}U_{\frac{1}{2}}$, $pf\delta\mathcal{P}U_{\frac{1}{2}}$, $pf\delta\alpha U_{\frac{1}{2}}$ and $pf\delta\beta U_{\frac{1}{2}}$)-spaces.

(i) h_P is a $pf\delta Irr$ (resp. $pf\delta\mathcal{S}Irr$, $pf\delta\mathcal{P}Irr$, $pf\delta\alpha Irr$ and $pf\delta\beta Irr$) map.

(ii) $h_P^{-1}(K)$ is a $pf\delta os$ (resp. $pf\delta\mathcal{P}os$, $pf\delta\mathcal{S}os$, $pf\delta\alpha os$ and $pf\delta\beta os$) in X_1 for every $pf\delta os$ (resp. $pf\delta\mathcal{P}os$, $pf\delta\mathcal{S}os$, $pf\delta\alpha os$ and $pf\delta\beta os$) K in X_2.

(iii) $pfcl(h_P^{-1}(K)) \subseteq h_P^{-1}(pfcl(K))$ for every pfs K of X_2.

Proof. (i) $\rightarrow$ (ii): Consider a $pf\delta\beta cs$ K in X_2. It follows K^c is a $pf\delta\beta os$ in X_2. As h_P is $pf\delta\beta Irr$, $h_P^{-1}((K)^c)$ is a $pf\delta\beta os$ in X_1. We know that $h_P^{-1}((K)^c) = \left(h_P^{-1}(K)\right)^c$. Hence $h_P^{-1}(K)$ is a $pf\delta\beta cs$ in X_1.

(ii) $\rightarrow$ (iii): Consider a pfs K in X_2 and $K \subseteq pf\delta\beta cl(K)$. Then $h_P^{-1}(K) \subseteq h_P^{-1}(pf\delta\beta\ cl(K))$. Since $pf\delta\beta cl(K))$ is a $pf\delta\beta cs$ in X_2, $pf\delta\beta cl(K)$ is a $pf\delta\beta cs$ in X_2. Therefore $(pf\delta\beta cl(K))^c$ is a $pf\delta\beta os$ in X_2. By presumption, $h_P^{-1}((pf\delta\beta cl(K))^c)$ is a $pf\delta\beta os$ in X_1. We know that $h_P^{-1}((pf\delta\beta cl(K))^c) = \left(h_P^{-1}(pf\delta\beta cl(K))\right)^c$. So $h_P^{-1}(pf\delta\beta cl(K))$ is a $pf\delta\beta cs$ in X_1. Also, as X_1 is $pf\delta\beta U_{\frac{1}{2}}$-space, $h_P^{-1}(pf\delta\beta cl(K))$ is a $pf\delta\beta cs$ in X_1.

(iii) $\rightarrow$ (i): Consider a $pf\delta\beta cs$ K in X_2. As X_2 is $pf\delta\beta U_{\frac{1}{2}}$-space, K is $pfcs$ in X_2 and $pfcl(K) = (K)$. Thus $h_P^{-1}(K) = h_P^{-1}(pf\delta\beta cl(K)) \supseteq pf\delta\beta cl(h_P^{-1}(K)) = pfcl(h_P^{-1}(K))$. But clearly $(h_P^{-1}(K)) \subseteq pfcl(h_P^{-1}(K))$. Therefore $pfcl((h_P^{-1}(K))) = h_P^{-1}(K)$. It follows $h_P^{-1}(K)$ is a $pfcs$ and so it is a $pf\delta\beta cs$ in X_1. Hence h_p is $pf\delta\beta irr$ map. The proof is similar for other cases. □

5. Conclusions

In this paper, the notions of Pythagorean fuzzy δ-continuous maps ($pf\delta Cts$), Pythagorean fuzzy continuous maps ($pfCts$), Pythagorean fuzzy δ-semi-continuous maps ($pf\delta\mathcal{S}Cts$), Pythagorean fuzzy δ-pre-continuous maps ($pf\delta\mathcal{P}Cts$), Pythagorean fuzzy $\delta\alpha$-continuous maps ($pf\delta\alpha Cts$), and Pythagorean fuzzy $\delta\beta$-continuous maps ($pf\delta\beta Cts$) are introduced and investigated in detail. For each of these mappings, the corresponding irresolute maps are defined with respect to the sets $pf\delta o$, $pf\delta\mathcal{S}o$, $pf\delta\mathcal{P}o$, $pf\delta\alpha o$, and $pf\delta\beta o$. Their fundamental properties are analyzed and illustrated through suitable examples to provide a deeper understanding of their topological behavior.

Furthermore, a comparative study is carried out between Pythagorean fuzzy continuous maps and other generalized forms of Pythagorean fuzzy continuous mappings to highlight their interrelationships and distinctions. The concept is then extended to define and characterize Pythagorean fuzzy open and closed maps, emphasizing their structural and functional significance.

The proposed Pythagorean fuzzy continuous and irresolute functions also establish a foundation for further extensions to Fermatean fuzzy sets and Fermatean neutrosophic sets, thereby enriching the theoretical framework and expanding their potential applications in advanced research. Additionally, these mappings are examined within specific subclasses of Pythagorean fuzzy topological spaces, such as "somewhat," "regular," and "normal" spaces, to explore their specialized roles and implications in these particular contexts.

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