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CONVERGENCE ANALYSIS OF INTUITIONISTIC FUZZY MATRICES WITH NEARLY MONOTONE INCREASING PROPERTY

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Abstract. Intuitionistic fuzzy matrix convergence of powers has been studied in the literature. The essential role of the main diagonal elements in the convergence of the power sequence of an intuitionistic fuzzy matrix A is exported at the level of A itself by introducing a new classification. The established theorems cover intuitionistic fuzzy matrices that increase monotonically. For matrices of this type, the convergence index is always smaller than or equal to n. The results are more essential because they lay the foundation for the convergence and oscillation of the power sequence of any intuitionistic fuzzy matrix. On the one hand, the results may be seen as a generalization of the results obtained by certain authors. Furthermore, as a typical case to consider, the necessary and sufficient conditions for an increasing intuitionistic fuzzy matrix A to have the property $A^{n-1} < A^n = A^{n+I}$ were established.

Keywords: Fuzzy logic, Dominating principle, Maximum principle, Convergence.

AMS Subject Classification: 03E72, 15B15.

1. Introduction

Uncertainty forms play a very important role in our daily life. During the time, we handle real-life problems involving uncertainty such as medical fields, engineering, industry, economics, and so on. Conventional techniques may not be enough and easy, so Zadeh [1] introduced fuzzy set theory, and this turned out to be a gift for the study of some uncertainty types wherever old techniques did not work. Fuzzy theory and the generalizations regarding it contributed to some remarkable mathematical applications in so many different problems in real life that involve uncertainties of certain types. For the purpose of handling different types of uncertainties, several generalizations and modifications regarding fuzzy set theory such as vague sets, rough sets, theory of intuitionistic fuzzy sets, soft sets, and other generalizations have also been developed.

A Fuzzy Matrix (FM) is a matrix with elements having values in a closed interval [0,1]. Kim and Roush [3] introduced the concept of FM. FM plays a vital role in various areas

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of science and engineering and solves problems involving various types of uncertainties [4]. Meenakshi [7] studied the minus ordering, space ordering, and schur complement of FM and block FM. Buckley [35] Ran and Liu [30] and Gregory et al. [34] after applying the max-min operation on FM found only two results, either the FM convergence to idempotent matrices or oscillates to finite period. Hashimoto [33] studied the convergence of the power of a fuzzy transitive matrix. Xin [8] introduced the notation of controllable FMs. He addition, [9] studied the convergence of powers of controllable FMs and developed some results on nilpotent FMs. Lur et al; [32] studied the convergence powers of FM. A detailed study on the power sequence of commonly used matrices [31]. Later, many researchers have done a lot of work on FMs [37, 38, 39]. FMs deal only with membership value, while intuitionistic fuzzy matrices (IFMs) deal with both membership and non-membership value. Khan et al., [5] introduced the concept of IFMs and several interesting properties on IFMs have been obtained in [6]. Bhowmik and Pal [11] studied the convergence of the max-min powers of an IFM. Pradhan and Pal [19] studied the mean convergence powers of IFMs. The convergence of powers and the canonical form of the s-transitive intuitionistic fuzzy matrix have been studied [26]. Several auother's [12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 27] worked on IFMs and obtained various interesting results, which are very useful in handling uncertainty problems in our daily life.

The aim of this paper is to find the essential role of the principal diagonal elements in the convergence of the power sequence of an intuitionistic fuzzy matrix. The focus of this paper will be on the intuitionistic fuzzy matrix of diagonal elements. Theorem 3.5 is the main result of this paper. The established theorems cover those of monotone or nearly monotone increasing intuitionistic fuzzy matrices.
Furthermore, we tried to give a summary of the framework for fuzzy logics of the theorems in the conclusion section. We believe that the claims could have some bearing on some aspects of fuzzy logic.

1.1. **Research Gap.** The Convergence is an interesting problem in the theory of IFM. Most of the existing research focuses on specific types of IFM. We have studied Convergence of power's of IFM using max-min operations [26, 28], In this article we look at about the Convergence of the power sequence of a nearly monotone increasing intuitionistic fuzzy matrix. We also provide some illustrations, so that the theoretical content of this paper can be understood easily.

2. BASIC DEFINITIONS

Definition 2.1. [2] *An Intuitionistic Fuzzy Set (IFS) A in X (universal set) is defined as an object of the following form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$, where the functions: $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define the membership function and non-membership function of the element $x \in X$ respectively and for every $x \in X : 0 \leq \mu_A(x) + \nu_A(x) \leq 1$.*

In short, we write $\langle x, x'\rangle$ as an intuitionistic fuzzy element with $x + x' \leq 1$. For $\langle x, x'\rangle, \langle y, y'\rangle \in$ IFS, Atanassov introduced operations,
$\langle x, x'\rangle \vee \langle y, y'\rangle = \langle max\{x, y\}, min\{x', y'\}\rangle$, $\langle x, x'\rangle \wedge \langle y, y'\rangle = \langle min\{x, y\}, max\{x', y'\}\rangle$. If $\langle x, x'\rangle \leq \langle y, y'\rangle$ means $x \leq y, x' \geq y'$ and $\langle x, x'\rangle < \langle y, y'\rangle$ if $x < y$ and $x' > y'$ in this case, we say that $\langle x, x'\rangle, \langle y, y'\rangle$ are comparable. For any two comparable elements $\langle x, x'\rangle$, $\langle y, y'\rangle$ $\in$ IFS, the operation $\langle x, x'\rangle \leftharpoondown \langle y, y'\rangle$ is defined by

$$\langle x, x'\rangle \leftharpoondown \langle y, y'\rangle = \begin{cases} \langle x, x'\rangle & \text{if } \langle x, x'\rangle > \langle y, y'\rangle, \\ \langle 0, 1\rangle & \text{if } \langle x, x'\rangle \leq \langle y, y'\rangle. \end{cases}$$

Definition 2.2. [5] *Let $X = \{x_1, x_2, ...x_m\}$ be a set of alternatives and $Y = \{y_1, y_2, ...y_n\}$ be the set attribute of each element of X. An IFM is defined by $A = (\langle (x_i, y_j), \mu_A(x_i, y_j), \nu_A(x_i, y_j)\rangle)$ for $i = 1, 2...m$ and $j = 1, 2, ...n$, where $\mu_A : X \times Y \to [0, 1]$ and $\nu_A : X \times Y \to [0, 1]$ satisfy the condition $0 \leq \mu_A(x_i, y_j) + \nu_A(x_i, y_j) \leq 1$. For simplicity we denote an IFM is a matrix of pairs $A = (\left\langle a_{ij}, a'_{ij} \right\rangle)$ of non negative real numbers satisfying $a_{ij} + a'_{ij} \leq 1$ for all i, j. We denote the set of all IFM of order $m \times n$ by $\mathscr{F}_{mn}$ and $\mathscr{F}_n$ denotes the set of IFM of order $n \times n$.*

Some of the definitions and results that we apply in this paper are given below.
Let $Q = [\left\langle q_{ij}, q'_{ij} \right\rangle]$ and $S = [\left\langle s_{ij}, s'_{ij} \right\rangle]$ be $n \times n$ IFMs with elements in $[0, 1] \times [0, 1]$,
$Q \vee S = (\left\langle q_{ij} \vee s_{ij}, q'_{ij} \wedge s'_{ij} \right\rangle)$, where $\langle x, x' \rangle \vee \langle y, y' \rangle = max(\langle x, x' \rangle, \langle y, y' \rangle)$,
$Q \wedge S = [(\left\langle q_{ij} \wedge s_{ij}, q'_{ij} \vee s'_{ij} \right\rangle)]$ where $\langle x, x' \rangle \wedge \langle y, y' \rangle = min(\langle x, x' \rangle, \langle y, y' \rangle)$,
$Q \overset{c}{\leftharpoondown} S = (\left\langle q_{ij}, q'_{ij} \right\rangle \overset{c}{\leftharpoondown} \left\langle s_{ij}, s'_{ij} \right\rangle)$,
$Q \times S = [(\left\langle q_{i1} \wedge s_{1j}, q'_{i1} \vee s'_{1j} \right\rangle) \vee (\left\langle q_{i2} \wedge s_{2j}, q'_{i2} \vee s'_{2j} \right\rangle) \vee \ldots \vee (\left\langle q_{in} \wedge s_{nj}, q'_{in} \vee s'_{nj} \right\rangle)]$,
$Q^{k+1} = Q^k \times Q$, $k = 1, 2, 3...$,
Denote $Q^k = [\left\langle q^k_{ij}, q'^k_{ij} \right\rangle]$, $k = 2, 3, ...$

$$(\left\langle q^k_{ij}, q'^k_{ij} \right\rangle) =$$
$$\left\langle \bigvee_{j_1=1}^{n} \bigvee_{j_2=1}^{n} \cdots \bigvee_{j_{k-1}=1}^{n} (q_{ij_1} \wedge q_{j_1 j_2} \wedge ... \wedge q_{j_{k-1}j}), \bigwedge_{j_1=1}^{n} \bigwedge_{j_2=1}^{n} \cdots \bigwedge_{j_{k-1}=1}^{n} (q'_{ij_1} \vee q'_{j_1 j_2} \vee ... \vee q'_{j_{k-1}j}) \right\rangle,$$

$Q^T = [\left\langle q_{ji}, q'_{ji} \right\rangle]$ (the transpose),
$Q \leq S$ iff $(\left\langle q_{ij}, q'_{ij} \right\rangle \leq \left\langle s_{ij}, s'_{ij} \right\rangle$ for all $i, j \in 1, 2, ..., n)$.

3. Some properties of power sequence of intuitionistic fuzzy matrices

Definition 3.1. *An intuitionistic fuzzy matrix A is said to be diagonally dominated if for each $\left\langle a_{ij}, a'_{ij} \right\rangle$, $\left\langle a_{ij}, a'_{ij} \right\rangle \leq \langle a_{ii}, a'_{ii} \rangle \bigvee \left\langle a_{jj}, a'_{jj} \right\rangle$ for all i, j.*

Definition 3.2. *Let A be a square intuitionistic fuzzy matrix A is said to satisfy the maximum principle if and only if for each t, $1 \leq t \leq n$ either $\langle a_{tt}, a'_{tt} \rangle = \max_{1 \leq k \leq n}(\langle a_{kt}, a'_{kt} \rangle)$ or $\langle a_{tt}, a'_{tt} \rangle = \max_{1 \leq k \leq n}(\langle a_{tk}, a'_{tk} \rangle)$.*

Lemma 3.3. *Let A be a square intuitionistic fuzzy matrix. Then, $\langle a_{ii}, a'_{ii} \rangle \leq \langle a_{ii}, a'_{ii} \rangle^s \leq \max_{1 \leq k \leq n} \{\langle a_{ik}, a'_{ik} \rangle\} \bigwedge \max_{1 \leq k \leq n} \{\langle a_{ik}, a'_{ik} \rangle\}$ for all $s \geq 1$*

Proof. The proof of the first part follows from:
$\langle a_{ii}, a'_{ii} \rangle = \bigvee_{1 \leq l_1, ..., l_{s-1} \leq n}(\langle a_{ii}, a'_{ii} \rangle \bigwedge \langle a_{ii}, a'_{ii} \rangle \bigwedge ... \bigwedge \langle a_{ii}, a'_{ii} \rangle)$
$\leq \bigvee_{1 \leq l_1, ..., l_{s-1} \leq n}(\left\langle a_{il_1}, a'_{il_1} \right\rangle \bigwedge \left\langle a_{l_1 l_2}, a'_{l_1 l_2} \right\rangle \bigwedge \cdots \bigwedge \left\langle a_{l_{s-1}i}, a'_{l_{s-1}i} \right\rangle)$
$= \langle a_{ii}, a'_{ii} \rangle^s \leq \bigvee_{1 \leq l_1, ..., l_{s-1} \leq n}(\left\langle a_{il_1}, a'_{il_1} \right\rangle \bigwedge \left\langle a_{l_{s-1}i}, a'_{l_{s-1}i} \right\rangle)$
$\leq \max_{1 \leq k \leq n} \{\langle a_{ik}, a'_{ik} \rangle\} \bigwedge \max_{1 \leq k \leq n} \{\langle a_{ik}, a'_{ik} \rangle\}$ □

Theorem 3.4. *Let A be a square intuitionistic fuzzy matrix with A satisfying the dominating principle. Then A increases monotonically.*

Proof. Suppose that $max\{\langle a_{ii}, a'_{ii} \rangle \bigwedge \left\langle a_{ij}, a'_{ij} \right\rangle, \left\langle a_{ij}, a'_{ij} \right\rangle \bigwedge \left\langle a_{jj}, a'_{jj} \right\rangle\} = \left\langle a_{ij}, a'_{ij} \right\rangle$ for each pair i, j we have

$\left\langle a_{ij}, a'_{ij} \right\rangle^2 = \max_{1\le k\le n}(\langle a_{ik}, a'_{ik}\rangle \bigwedge \left\langle a_{kj}, a'_{kj} \right\rangle) \geq$
$\max_{1\le k\le n}\{\langle a_{ii}, a'_{ii}\rangle \bigwedge \left\langle a_{ij}, a'_{ij} \right\rangle, \left\langle a_{ij}, a'_{ij} \right\rangle \bigwedge \left\langle a_{jj}, a'_{jj} \right\rangle\} = \left\langle a_{ij}, a'_{ij} \right\rangle$
So A increasing monotonically. □

Theorem 3.5. *If A satisfies the maximum principle, then*
1. $A^2 \leq A^3 \leq ... \leq A^{n-1} \leq ...$
2. $\langle a_{ii}, a'_{ii}\rangle = \langle a_{ii}, a'_{ii}\rangle^2 = \langle a_{ii}, a'_{ii}\rangle^3 = ... = \langle a_{ii}, a'_{ii}\rangle^n$
3.$A^{n-1} = A^n$
4. $A \leq A^2$ *not holds necessarily.*

Proof. (1) In order to prove the first part of theorem we have to show $A^2 \leq A^3$.
For each given ordered pair (i, j) there is t such that,
$\left\langle a_{ij}, a'_{ij} \right\rangle^2 = \langle a_{it}, a'_{it}\rangle \bigwedge \left\langle a_{tj}, a'_{tj} \right\rangle$
if $\langle a_{tt}, a'_{tt}\rangle = \max_{1\le k\le n}\{\langle a_{kt}, a'_{kt}\rangle\}$ then
$\left\langle a_{ij}, a'_{ij} \right\rangle^2 = \langle a_{it}, a'_{it}\rangle \bigwedge \left\langle a_{tj}, a'_{tj} \right\rangle = (\langle a_{it}, a'_{it}\rangle \bigwedge \langle a_{tt}, a'_{tt}\rangle) \bigwedge \left\langle a_{tj}, a'_{tj} \right\rangle \leq \left\langle a_{ij}, a'_{ij} \right\rangle^3$
if $\langle a_{tt}, a'_{tt}\rangle = \max_{1\le k\le n}\{\langle a_{tk}, a'_{tk}\rangle\}$ then
$\left\langle a_{ij}, a'_{ij} \right\rangle^2 = \langle a_{it}, a'_{it}\rangle \bigwedge \left\langle a_{tj}, a'_{tj} \right\rangle = \langle a_{it}, a'_{it}\rangle \bigwedge(\langle a_{tt}, a'_{tt}\rangle \bigwedge \left\langle a_{tj}, a'_{tj} \right\rangle) \leq \left\langle a_{ij}, a'_{ij} \right\rangle^3$
Hence $A^2 \leq A^3$ implies $A^2 \leq A^2 \leq A^3 \leq ... \leq A^{n-1} \leq ...$
2. Let t be any positive integer then,
$\langle a_{ii}, a'_{ii}\rangle^{t+1} = \max_{1\le l_1 ... lt\le n}$
$\{\left\langle a_{il_1}, a'_{il_1} \right\rangle \bigwedge ... \bigwedge \left\langle a_{l_ti}, a'_{l_ti} \right\rangle\} \leq \max_{1\le l_1\le n}\{\left\langle a_{il_1}, a'_{il_1} \right\rangle\} \bigwedge \max_{1\le l_t\le n}\{\left\langle a_{l_ti}, a'_{l_ti} \right\rangle\}$.
Combing with the lemma 3.3 second part of theorem is proved.
3. To prove the third part we just need to show $\left\langle a_{ij}, a'_{ij} \right\rangle^{n-1} \geq \langle a_{ij}, a'_{ii}\rangle^n$
for all $1 \leq i \neq j \leq n$ for each ordered pair for $i \neq j$ there is a sequence of
$l_1 = i, l_2, ...ln, l_{n+1} = j$, such that,

$$\langle a_{ij}, a'_{ij}\rangle^n = \langle a_{l_1l_2}, a'_{l_1l_2}\rangle \bigwedge \langle a_{l_2l_3}, a'_{l_2l_3}\rangle \bigwedge ... \bigwedge \left\langle a_{l_nl_{n+1}}, a'_{l_nl_{n+1}} \right\rangle. \tag{1}$$

Among these $l_t^{'s}$ there has to exist indices having the same value,
say $l_s = l_t$ with $s < t$. Since $l_1 \neq l_{n+1}$ we have $1 \leq t - s \leq n - 1$.
There are only two possible cases:
(a). $t - s < n - 1$, then $2 \leq n - t + s \leq n - 1$.
In equation (1) deleting $\left\langle a_{l_sl_{s+1}}, a'_{l_sl_{n+1}} \right\rangle$ we get
$\left\langle a_{ij}, a'_{ij} \right\rangle^n \leq \left\langle a_{ij}, a'_{ij} \right\rangle^{n-t+s} \leq \left\langle a_{ij}, a'_{ij} \right\rangle^{n-1}$
(b). If $t - s < n - 1$, then we consider two cases
(i) $s = 1, t = 1$ then
$\left\langle a_{ij}, a'_{ij} \right\rangle^n \leq$
$\langle a_{ii}, a'_{ii}\rangle^{n-1} \bigwedge \left\langle a_{ij}, a'_{ij} \right\rangle = \langle a_{ii}, a'_{ii}\rangle \bigwedge \left\langle a_{ij}, a'_{ij} \right\rangle \leq \left\langle a_{ij}, a'_{ij} \right\rangle^2 \leq \left\langle a_{ij}, a'_{ij} \right\rangle^{n-1}$
(ii) $s = 2, t = n + 1$ then
$\left\langle a_{ij}, a'_{ij} \right\rangle^n \leq$
$\left\langle a_{ij}, a'_{ij} \right\rangle \bigwedge \left\langle a_{ij}, a'_{ij} \right\rangle^{n-1} = \left\langle a_{ij}, a'_{ij} \right\rangle \bigwedge \left\langle a_{jj}, a'_{jj} \right\rangle \leq \left\langle a_{ij}, a'_{ij} \right\rangle^2 \leq \left\langle a_{ij}, a'_{ij} \right\rangle^{n-1}$

4. Let $A = \begin{pmatrix} \langle 0.5, 0.5\rangle & \langle 1, 0\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0.5, 0.5\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0, 1\rangle & \langle 0.5, 0.5\rangle \end{pmatrix}$,

Let $A^2 = \begin{pmatrix} \langle 0.5, 0.5\rangle & \langle 0.5, 0.5\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0.5, 0.5\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0, 1\rangle & \langle 0.5, 0.5\rangle \end{pmatrix} = A^3$.

This proves fourth part. □

Corollary 3.6. *Let A be a square fuzzy matrix with A satisfying the strong row or column maximum principle, that is*
$\langle a_{ii}, a'_{ii}\rangle = \max_{1\leq k\leq n} \langle a_{ik}, a'_{ik}\rangle$ *for all* $1 \leq i \leq n$
or

$\langle a_{ii}, a'_{ii}\rangle = \max_{1\leq k\leq n} \langle a_{ki}, a'_{ki}\rangle$ *for all* $1 \leq i \leq n$. *Then,*
1. A increases monotonically.
2. The principal elements are stable, that is
$\langle a_{ii}, a'_{ii}\rangle = \langle a_{ii}, a'_{ii}\rangle^k$ *for all* $1 \leq i \leq n, k = 1, 2...$
3. There is a $s \leq n-1$ *such that* $A^s = A^{s+1}$.

Proof. The first part follows from Theorem 3.4 and the second and third part follows from Theorem 3.5. □

4. The conditions for $A^{n-1} < A$ monotone increasing intuitionistic fuzzy matrices

In section 3, we have proved that if A is monotone increasing matrix, generally, A converges to $A^s, s \leq n$. Also we have established the condition that A converges to A^n with $s \leq n-1$. In this section, we will discuss under the conditions that a monotonic increasing matrix A converges to A^n exactly.
We begin with a square fuzzy matrix, denoted as
$A^{(k)} = A + A^2 + ... + A^k = (\left\langle a_{ij}, a'_{ij}\right\rangle^k)$ for all $k \geq 1$.

Lemma 4.1. $A^{(n)} > A^{(n-1)}$ *iff there is at least one* i_0 *such that*
$\left\langle a_{i_0i_0}, a'_{i_0i_0}\right\rangle^n > \max_{1\leq k\leq n-1} \left\langle a_{i_0i_0}, a'_{i_0i_0}\right\rangle^n$.

Proof. From Lemma 3.3 we see that
$\left\langle a_{ij}, a'_{ij}\right\rangle^n \leq \left\langle a_{ij}, a'_{ij}\right\rangle^{(n-1)}$ for all $1 \leq i \neq j \leq n$.
So the lemma holds if
$\left\langle a_{i_0i_0}, a'_{i_0i_0}\right\rangle^n > \max_{1\leq k\leq n-1} \left\langle a_{i_0i_0}, a'_{i_0i_0}\right\rangle^k$. □

Lemma 4.2. *Let P be an* n^{th} *order permutation intuitionistic fuzzy matrix and* $B = P'AP = (\left\langle b_{ij}, b'_{ij}\right\rangle)$. *Then*
$(P'AP)^k = P'A^kP$ *for all* $k \geq 1$.

Proof. Let $P'P = I$ we get the lemma directly. □

Theorem 4.3. *Let A be an arbitrary square intuitionistic fuzzy matrix. Then* $A^{(n)} > A^{(n-1)}$ *iff there exists a permutation matrix P such that* $B = P'AP$, *Let*

$$B_\lambda = \begin{pmatrix} \langle 0,1\rangle & \langle 1,0\rangle & \langle 0,1\rangle & ...\langle 0,1\rangle \\ \langle 0,1\rangle & C_{22} & \langle 1,0\rangle & ...\langle 0,1\rangle \\ \langle 0,1\rangle & C_{32} & C_{33} & ...\langle 0,1\rangle \\ ... & ... & ... & ... \\ \langle 0,1\rangle & C_{n-12} & C_{n-13} & ...\langle 1,0\rangle \\ \langle 1,0\rangle & C_{n2} & C_{n3} & ...c_{nn} \end{pmatrix}, \tag{2}$$

where, $\lambda = \langle b_{12}, b'_{12}\rangle \wedge \langle b_{23}, b'_{23}\rangle \wedge ... \wedge \left\langle b_{n-1n}, b'_{n-1n}\right\rangle \wedge \langle b_{n1}, b'_{n1}\rangle$ *and* $B_\lambda = (\left\langle c_{ij}, c'_{ij}\right\rangle)$,

$$\left\langle c_{ij}, c'_{ij}\right\rangle = \begin{cases} \langle 1,0\rangle & if \left\langle b_{ij}, b'_{ij}\right\rangle \geq \lambda, \\ \langle 0,1\rangle & if \left\langle b_{ij}, b'_{ij}\right\rangle < \lambda. \end{cases} for\ all\ 1 \leq i,j \leq n\ .$$

Proof. If $A^n > A^{n-1}$ According to lemma 4.2 there is an i_0 such that

$$\left\langle a_{i_0i_0}, a'_{i_0i_0}\right\rangle^n > \max_{1\leq k\leq n-1} \left\langle a_{i_0i_0}, a'_{i_0i_0}\right\rangle^k. \tag{3}$$

Then there are $1 \leq l_2, l_3..., l_n \leq n$ such that

$$\left\langle a_{i_0i_0}, a'_{i_0i_0}\right\rangle^n = \left\langle a_{l_1l_2}, a'_{l_1l_2}\right\rangle \bigwedge \left\langle a_{l_2l_3}, a'_{l_2l_3}\right\rangle \bigwedge ... \bigwedge \left\langle a_{l_nl_l}, a'_{l_nl_l}\right\rangle \tag{4}$$

where $l_1 = i_0$
Now we claim in equation (4) that $l_s \neq l_t$ for all $s \neq t$. If it not the case, say there are s and t such that $1 \leq s < t \leq n$ and $l_s = l_t$ then equation (4) deleting
$\left\langle a_{l_sl_{s+1}}, a'_{l_sl_{s+1}}\right\rangle \bigwedge ... \bigwedge \left\langle a_{l_{t-1}l_t}, a'_{l_{t-1}l_t}\right\rangle \bigwedge$ leads to
$\left\langle a_{i_0i_0}, a'_{i_0i_0}\right\rangle^n \leq \left\langle a_{i_0i_0}, a'_{i_0i_0}\right\rangle^{n-t+s}$, $1 \leq n-t+s \leq n-1$
but $n-t+s \leq n-1$ and it contradicts inequality (3) Thus permutation matrix P can be well defined as
$P: l_t \rightarrow$ for all $t = 1,2,3...n$
Now we set
$B = P'AP$, that is $\left\langle b_{ij}, b'_{ij}\right\rangle = \left\langle a_{l_il_j}, a'_{l_il_j}\right\rangle$
and by lemma 6
$\left\langle b_{ij}, b'_{ij}\right\rangle^k = \left\langle a_{l_il_j}, a'_{l_il_j}\right\rangle^k$ foe all $1 \leq i,j \leq n, k \geq 1$ also from equation (4)
$\langle b_{11}, b'_{11}\rangle^n = \left\langle a_{i_0i_0}, a'_{i_0i_0}\right\rangle^n$
$= \left\langle a_{l_1l_2}, a'_{l_1l_2}\right\rangle \bigwedge \left\langle a_{l_2l_3}, a'_{l_2l_3}\right\rangle \bigwedge ... \bigwedge \left\langle a_{l_nl_1}, a'_{l_nl_1}\right\rangle$
$= \langle b_{12}, b'_{12}\rangle \bigwedge \langle b_{23}, b'_{23}\rangle \bigwedge ... \bigwedge \langle b_{n1}, b'_{n1}\rangle$
Setting $\lambda = \{\langle b_{11}, b'_{11}\rangle\}^n$, we try to determine the elements in B_λ as much as we can from equation (3).
Let $\langle c_{11}, c'_{11}\rangle = \langle 0,1\rangle$. If not then, $\langle c_{11}, c'_{11}\rangle = \langle 1,0\rangle$. implies $\langle b_{11}, b'_{11}\rangle = \langle a_{i_0i_0}, a'_{i_0i_0}\rangle \geq \langle a_{i_0i_0}, a'_{i_0i_0}\rangle^n$ which contradicts to the equation (3).
If there exists $1 \leq k \leq n-1$, $\langle c_{k1}, c'_{k1}\rangle = \langle 1,0\rangle$. Then
$\langle a_{i_0i_0}, a'_{i_0i_0}\rangle^k = \langle b_{11}, b'_{11}\rangle^k \geq \langle b_{12}, b'_{12}\rangle \bigwedge \langle b_{23}, b'_{23}\rangle \bigwedge ... \bigwedge \langle b_{k-11}, b'_{k-11}\rangle \bigwedge \langle b_{k1}, b'_{k1}\rangle \geq \lambda = \langle b_{11}, b'_{11}\rangle^n = \langle a_{i_0i_0}, a'_{i_0i_0}\rangle^n$ Which is contradiction to the inequality (3). So the first column in B_λ has to be of the form as in equation (2).
Examine the $\langle b_{st}, b'_{st}\rangle$ for $1 \leq s \leq n-2$ and $t \geq s+2$. If there exists a $\langle C_{st}, a'_{st}\rangle = \langle 1,0\rangle$ then $\langle b_{st}, b'_{st}\rangle \geq \lambda$. It leads to
$\lambda = \langle b_{11}, b'_{11}\rangle^n = \langle b_{11}, b'_{11}\rangle^n \bigwedge \langle b_{st}, b'_{st}\rangle$
$= \langle b_{12}, b'_{12}\rangle \bigwedge ... \bigwedge \left\langle b_{s-1s}, b'_{s-1s}\right\rangle \bigwedge \left\langle b_{ss+1}, b'_{ss+1}\right\rangle \bigwedge ... \bigwedge \left\langle b_{t-1t}, b'_{t-1t}\right\rangle \bigwedge \left\langle b_{tt+1}, b'_{tt+1}\right\rangle \bigwedge ...$

$\bigwedge \langle b_{n1}, b'_{n1}\rangle \bigwedge \langle b_{st}, b'_{st}\rangle$
$\langle b_{12}, b'_{12}\rangle \bigwedge ... \bigwedge \langle b_{s-1s}, b'_{s-1s}\rangle \bigwedge \langle b_{st}, b'_{st}\rangle \bigwedge \langle b_{tt+1}, b'_{tt+1}\rangle \bigwedge ... \bigwedge \langle b_{n1}, b'_{n1}\rangle$
$\langle b_{11}, b'_{11}\rangle^{n-t+s+1}$, and $n - t + s + 1 \leq n - 1$ is the contradiction to the equation (3).
From the definition of λ it is easy to see $\langle c_{n1}, c'_{n1}\rangle = \langle 1, 0\rangle$ and $\langle c_{ii+1}, c'_{ii+1}\rangle = \langle 1, 0\rangle$ for $i = 1, 2, 3, ..., (n-1)$. Thus, the necessary part of the theorem is proved.
Sufficient Condition: Next if B_λ is of the form given in equation (2),. we claim $B^{(n)} > B^{(n-1)}$ by claiming first that $\langle b_{11}, b'_{11}\rangle^n > \langle b_{11}, b'_{11}\rangle^{n-1}$. It is easy see that
$\langle b_{11}, b'_{11}\rangle^n \geq \langle b_{12}, b'_{12}\rangle \bigwedge \langle b_{23}, b'_{23}\rangle \bigwedge \langle b_{n1}, b'_{n1}\rangle = \lambda$
Also it is not hard to figure out that $\langle b_{11}, b'_{11}\rangle^n = \lambda$ if there is an integer k such that $\langle b_{11}, b'_{11}\rangle^k \geq \lambda$ then there are integers $1 \leq m_1, m_2, ...m_k \leq n$ such that
$\langle b_{11}, b'_{11}\rangle^k = \langle b_{m_1m_2}, b'_{m_1m_2}\rangle \bigwedge \langle b_{m_2m_3}, b'_{m_2m_3}\rangle \bigwedge \langle b_{m_km_1}, b'_{m_km_1}\rangle \geq \lambda$
where $m_1 = 1$ then $\left\langle b_{m_tm_{t+1}}, b'_{m_tm_{t+1}}\right\rangle \lambda$. But the form of B_λ implies that
$m_{t+1} \leq m_t + 1$ for all $1 \leq t \leq k-1$ and $m_k = n$.
They lead to
$n = m_k \leq m_{k-1} + 1 \leq m_{k-2} + 2... \leq m_1 + k - 1 = k$,
that is $k \geq n$ consequently,
$\langle b_{11}, b'_{11}\rangle^n > \max_{1\leq k\leq n-1} \langle b_{11}, b'_{11}\rangle^k, B^{(n)} > B^{(n-1)}$.
Note that $A = PBP'$. Let P' permute $1 \to i_1$; $\langle a_{i_1i_1}, a'_{i_1i_1}\rangle^k = \langle b_{11}, b'_{11}\rangle^k$ Also
$\langle a_{i_1i_1}, a'_{i_1i_1}\rangle^n > \max_{1\leq k\leq n-1} \langle a_{i_1i_1}, a'_{i_1i_1}\rangle^k, A^n > A^(n-1)$;
Hence Proved. $\square$

Theorem 4.4. *Suppose that A is a monotone increasing intuitionistic fuzzy matrix. Then the necessary and sufficient condition for $A^n > A^{n-1}$ is that there exists a permutation matrix P such that $B = P'AP = \left\langle b_{ij}, b'_{ij}\right\rangle$ satisfies*
$\langle b_{k1}, b'_{k1}\rangle < \lambda$, $k = 1, 2, 3, ..., n-1$
$\langle b_{kl}, b'_{kl}\rangle < \lambda$, $k = 1, 2, 3, ..., n+2$ $l = k+2, k+3, ..., n$
where $\lambda = \langle b_{12}, b'_{12}\rangle \bigwedge \langle b_{23}, b'_{23}\rangle \bigwedge ... \bigwedge \langle b_{n-1n}, b'_{n-1n}\rangle \bigwedge \langle b_{n1}, b'_{n1}\rangle$.
Further more B also satisfies
$\langle b_{kk+1}, b'_{kk+1}\rangle \leq \langle b_{kk}, b'_{kk}\rangle \bigvee \langle b_{k+1k+1}, b'_{k+1k+1}\rangle$ *for all* $1 \leq k \leq n-1$, $\langle b_{n1}, b'_{n1}\rangle < \langle b_{nn}, b'_{nn}\rangle$.

Proof. Since A is monotone increasing with $A^{(k)} = A^k$, then from theorem 5 there is a permutation matrix P such that $B = PA'P$, $B\lambda$ is of the form in equation (2) and λ is defined to be
$\lambda = \lambda = \langle b_{12}, b'_{12}\rangle \bigwedge \langle b_{23}, b'_{23}\rangle \bigwedge ... \bigwedge \langle b_{n-1n}, b'_{n-1n}\rangle \bigwedge \langle b_{n1}, b'_{n1}\rangle$.
But the first part is a consequence of the form of $B\lambda$.
We need to show the second part only. That A is monotone increasing implies that so is B. Hence,
$\langle b_{kk+1}, b'_{kk+1}\rangle^2 = \max_{1\leq i\leq n} \{\langle b_{ki}, b'_{ki}\rangle \bigwedge \langle b_{ik+1}, b'_{ik+1}\rangle\} \geq \langle b_{kk+1}, b'_{kk+1}\rangle \geq \lambda$ for all $1 \leq k \leq n-1$.
From the first part we have
$\max_{1\leq i\leq k-1} \{\langle b_{ki}, b'_{ki}\rangle \bigwedge \langle b_{ik+1}, b'_{ik+1}\rangle\} \leq \max_{1\leq i\leq k-1} \{\langle b_{ik+1}, b'_{ik+1}\rangle\} < \lambda$ and
$\max_{k+2\leq i\leq n} \{\langle b_{ki}, b'_{ki}\rangle \bigwedge \langle b_{ik+1}, b'_{ik+1}\rangle\} \leq \max_{k+2\leq i\leq n} \{\langle b_{ki}, b'_{ki}\rangle\} < \lambda$.
These two equations force
$\langle b_{kk+1}, b'_{kk+1}\rangle^2 = \max_{i=k,k+1} \{\langle b_{ki}, b'_{ki}\rangle \bigwedge \langle b_{ik+1}, b'_{ik+1}\rangle\}$
$= (\langle b_{kk}, b'_{kk}\rangle \bigwedge \langle b_{kk+1}, b'_{kk+1}\rangle) \bigvee (\langle b_{kk+1}, b'_{kk+1}\rangle \bigwedge \langle b_{k+1k+1}, b'_{k+1k+1}\rangle)$
$= \langle b_{kk+1}, b'_{kk+1}\rangle \bigwedge (\langle b_{kk}, b'_{kk}\rangle \bigwedge \langle b_{k+1k+1}, b'_{k+1k+1}\rangle)$.

Combing with $\left\langle b_{kk+1}, b'_{kk+1}\right\rangle^2 \geq \left\langle b_{kk+1}, b'_{kk+1}\right\rangle$ we get
$\left\langle b_{kk+1}, b'_{kk+1}\right\rangle^2 = \left\langle b_{kk+1}, b'_{kk+1}\right\rangle$, $\left\langle b_{kk+1}, b'_{kk+1}\right\rangle \leq \langle b_{kk}, b'_{kk}\rangle \bigvee \left\langle b_{k+1k+1}, b'_{k+1k+1}\right\rangle$.

□

Corollary 4.5. *For each symmetric monotone increasing fuzzy matrix A we have* $A^{n-1} = A^n$.

Corollary 4.6. *For each symmetric fuzzy matrix A we have* $A^{2n-2} = A^{2n}$.

Theorem 4.7. *Let* M_n *be the set of all Boolean matrices of order n,* $S = \left\{A \in M_n | A \leq A^2, \langle a_{11}, a'_{11}\rangle^{n-1}\right\} < \langle a_{11}, a'_{11}\rangle^n$ *and* $\|S\|$ *be the coordinate number S. Then*
$\|S\| \geq 2^{(n-1)\frac{(n-2)}{2}}$.

Proof. To establish the conclusion we examine matrices of the form of

$$A = \begin{pmatrix} \langle 0,1\rangle & \langle 1,0\rangle & \langle 0,1\rangle & \langle 0,1\rangle & ... & \langle 0,1\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 1,0\rangle & \langle 1,0\rangle & \langle 0,1\rangle & ... & \langle 0,1\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & * & \langle 1,0\rangle & \langle 1,0\rangle & ... & \langle 0,1\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & * & * & \langle 1,0\rangle & ... & \langle 0,1\rangle & \langle 0,1\rangle \\ ... & ... & ... & ... & ... & ... & ... \\ \langle 0,1\rangle & * & * & * & ... & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & * & * & * & ... & * & \langle 0,1\rangle \end{pmatrix},$$

where * represents a number $\langle 0,1\rangle$ and $\langle 1,0\rangle$. Since $\left\langle a_{jj}, a'_{jj}\right\rangle = \langle 1,0\rangle$ for all $1 \leq i \neq j \leq n$ then the dominating principle holds and A is monotone increasing. Also using Theorem 4 $\langle a_{11}, a'_{11}\rangle^n = \langle 1,0\rangle$, $\langle a_{11}, a'_{11}\rangle^{n-1} = \langle 0,1\rangle$ So all of the A'^s of these form belong to S and it is easy to see that $\|S\| \geq 2^{(n-1)\frac{(n-2)}{2}}$ Especially, when $n = 3$ there are two and only two elements as

$$A_1 = \begin{pmatrix} \langle 0,1\rangle & \langle 1,0\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 0,1\rangle & \langle 1,0\rangle \end{pmatrix},$$

$$A_2 = \begin{pmatrix} \langle 0,1\rangle & \langle 1,0\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \end{pmatrix}.$$

□

5. Conclusion and Discussion

An interpretation of power sequence of intuitionistic fuzzy matrix in fuzzy logics: In the multivalent logics underlying intuitionistic fuzzy set theories, we always denote the truth value of a proposition P by $v(P)$, where $v(P) \in [0,1]$. Also the valuation of the negation is $v(\neg P) = 1 - v(P)$. Therefore $v(\neg\neg P) = v(P)$. The connective implication $(\rightarrow)$ is defined as $v(P \rightarrow Q) = v(\neg P \vee Q)$. In the logic associated with $(P(x), \cup, \cap, -)$, the disjunction and the conjunction underlying $\cup$ and $\cap$ are defined as $v(\neg P \vee Q) = max(v(P), v(Q))$, $V(P \wedge Q) = min(v(P), v(Q))$ respectively. This multivalent logic is usually called the K-standard sequence (K-SEQ) logic. With this logic, we consider the following problem. Let $P_1.....P_n$ be propositions, $v(P_i \rightarrow P_j) = \langle a_{ij}, a_{ij}\rangle$ and $A = (\langle a_{ij}, a_{ij}\rangle)$. Then A can be regarded to be an intuitionistic fuzzy matrix with the operations defined in Section 3. Also, $\langle a_{ij}, a_{ij}\rangle^k$ can be written as

$\langle a_{ij}, a_{ij}\rangle^k = \max_{1\leq l_1...l_{k-1}\leq n} \left\{min(\langle a_{il_1}, a_{il_1}\rangle, \langle a_{l_1l_2}, a_{l_1l_2}\rangle, ..., \langle a_{l_{k-1}j}, a_{l_{k-1}j}\rangle)\right\}$
$\langle a_{ij}, a_{ij}\rangle^k = \max_{1\leq l_1...l_{k-1}\leq n} \left\{v((P_i \rightarrow P_{l_1}) \wedge (P_{l_1} \rightarrow P_{l_2}) \wedge ... \wedge (P_{l_{k-1}} \rightarrow P_j))\right\}$
$\langle a_{ij}, a_{ij}\rangle^k = v(\wedge_{1\leq l_1...l_{k-1}\leq n} \left\{((P_i \rightarrow P_{l_1}) \wedge (P_{l_1} \rightarrow P_{l_2}) \wedge ... \wedge (P_{l_{k-1}} \rightarrow P_j)))\right\}$.

So A^k can be clearly interpreted in the K-SEQ logic. Now we claim that A satisfies the dominating principle. For each pair (i, j), for all $1 \leq i, j \leq n$, we have
$\langle a_{ij}, a_{ij}\rangle = v(P_i \to P_j) = v(\neg P_i \to P_j) = max\{1 - v(P_i), v(P_j)\}$
$\leq max\{1 - v(P_i), v(P_j), 1 - v(P_j), v(P_j)\}$
$= max\{max(1 - v(P_i), v(P_j)), max(1 - v(P_j), v(P_j))\} = max\{v(P_i \to P_i), v(P_j \to P_j)\}$
$= max\{\langle a_{ii}, a_{ii}\rangle, \langle a_{jj}, a_{jj}\rangle\}$
Hence, A satisfies the dominating principle, and A is monotone increasing. So, the power sequence of A converges. Let s be the convergence index of A that is $A^{s-1} < A^s = A^{s+1}$. Define
$\langle b_{ij}, b_{ij}\rangle = \sup_{k\geq 1}\{v(\vee_{1\leq l_1\leq,\ldots,l_{k-1}n}((P_i \to P_{l_1}) \wedge (P_{l_1} \to P_{l_2}) \wedge \ldots \wedge (P_{l_{k-1}} \to P_j))\}$
Then for each pair (i,j), there exist $l_1, l_2, ..., l_{s-1}$
$v((P_i \to P_{l_1}) \wedge (P_{l_1} \to P_{l_2}\wedge) \wedge \ldots \wedge (P_{l_{s-1}} \to P_j)) = \langle b_{ij}, b_{ij}\rangle$
Furthermore, for each $t < s$ there exists at least one pair (i_0, j_0), such that
$v((P_{i_0} \to P_{l_1}) \wedge (P_{l_1} \to P_{l_2}\wedge) \wedge \ldots \wedge (P_{l_{t-1}} \to P_{j_0})) < \langle b_{i_0 j_0}, b_{i_0 j_0}\rangle$ whatever $l_1, l_2, ..., l_{t-1}$ are.
There exists some relation between the convergence index s and the finite step proof in fuzzy logic associated with $(P(x), \cup, \cap, -)$.

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