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ON T AND ST-COLORING OF n-HYPERCUBE GRAPH AND KRAGUJEVAC TREE

R. MORAN[1], N. BORA[2*], §

ABSTRACT. Let $G = (V, E)$ denotes any graph, where V represents the vertex set and E represents the edge set. Then, T-coloring of a graph is an assignment of non-negative numbers to the vertices of a graph such that the difference between the colors assigned to the adjacent vertices does not belong to a predefined set of non-negative integers known as a T set, which must include zero. Ordinary vertex coloring of a graph is also a particular type of T-coloring. In this paper, we consider the T-set of the form $T = \{0, 1, 2, \ldots, k\} \cup S$, where S is any arbitrary set that does not contain any multiple of $(k+1)$, and is termed as k-initial set. We also consider the $T-$set of the form $T = \{0, s, 2s, \ldots, ks\} \cup S$, where S is a subset of the set $\{s+1, s+2, s+3, \ldots, , ks\}, ks \geq 1$ and is termed as a k-multiple of s set. We study the T-coloring on n-Hypercube graph and Tree graphs for any k-initial set and k-multiple of s set. For measuring the efficiency of the T-coloring, we also analyze two special parameters, firstly the T-span, which is being the maximum of $\mid f(u) - f(w) \mid$ over all the vertices u and w and secondly, the edge-span, denoted as $esp_T(G)$, which is being the maximum of $\mid f(u) - f(w) \mid$ over all the edges (u, w) of G. We also study the strong T-coloring (ST-coloring) of G on Kragujevac tree.

Keywords: T-coloring, T-set, n-Hypercube graph, Kragujevac tree.

AMS Subject Classification: 05C15, 05C76

1. Introduction

Graph coloring involves assigning colors to specific components of a graph, which can include vertices, edges, regions or faces, or a combination of these elements. A variety of mathematical structures, as well as the relationships and interactions between the objects, can be depicted using graph coloring. It is a method for illustrating a variety of real-world problems, such as register allocation, circuit board testing, civil engineering, and architecture. The topic of graph coloring has become a crucial component of graph theory, since the invention of the four-color problem in 1852. Long-term research has concentrated on

[1] Department of Mathematics, Dibrugarh University, Assam-786004, India.
e-mail: rubulmoran7@gmail.com; ORCID: https://orcid.org/0000-0002-8155-9492.

[2] Department of Mathematics, Dibrugarh University Institute of Engineering and Technology, Dibrugarh University, Assam-786004, India.
e-mail: niranjanbora11@gmail.com; niranjanbora@dibru.ac.in; ORCID: https://orcid.org/0000-0002-3729-5848.

[*] Corresponding author.

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the topic of coloring problems, which is now a crucial subject in the field of graph theory.

Let Z^+ denotes the set of integers. For a given graph G and a T-set of non negative integers which must include zero, a T-coloring is a function $f : V(G) \to Z^+ \bigcup\{0\}$ such that for any two distinct vertices $u \neq w$ in $V(G)$, where $\mid f(u) - f(w) \mid \notin T$ [10, 20]. The Chromatic number of G is the minimum number of colors required to color the G, whereas the T-chromatic number of G is the minimum number of colors required to T-color the G. It was W. K. Hale [8], who first proposed the general setting of T-coloring of graphs in 1980 and made a connection between the frequency assignment problem (FAP) and graphs. He adopted the minimum-order method of assignment of frequency. FAP is considered one of the well-known problems of telecommunication that uses T-coloring in their modeling. The expansion of wireless communication systems globally has created a challenge for the FAP [19], which effectively assigns frequencies. The FAP is used to design models for permanent frequency allocation, licensing, and regulating that maximize the effective use of all radio spectrum. It is intrinsically related to the graph coloring problem, in which transmitters are treated as vertices and any interference between transmitters is treated as an edge. Cozzens and Roberts [2] showed that the T-chromatic number is equal to the chromatic number and in addition to these, they also presented the problem of computing T-edge span of non-perfect graphs for k-initial sets. An extensive survey on variation and generalization of T-colorings can be found in the works of Roberts [20]. The conditions necessary and sufficient on the equality between the T-span of any graph and the T-span of a complete graph are found in Liu's [11] works. Jansen [9] showed that the T-span of a complete graph is NP-complete. Any arbitrary graph G is termed a distance graph if and only if G is isomorphic to $G(A)$ for some suitable distance set A. Gräf [4] developed the class of distance graphs to solve a constrained version of the T-coloring problem. For a finite set A of positive integers (referred to as a distance set), consider the graph $G(A)$ denote the graph with vertex set A and there is an edge between vertices u and v if and only if $\mid a - b \mid$ belongs to A. Moreover, he discussed certain difficulties of T-coloring. He used the equivalence between T-coloring of a full graph and a distance graph clique problem to prove that the complete T-coloring problem is NP-complete. Juan et al.'s [10] work on T-coloring of folded hypercubes included a discussion of the T-edge span of this graph and some of the findings. The T-span and T-edge span of the swing graph, theta mesh, and shadow graph of a cycle, as well as the crown graph, series-parallel graph, extended theta graph, and wrapped butterfly network were estimated by Sivagami in his works [26, 27]. Recently, Roselin and Raj [22] provided various results relating to these characteristics of various non-perfect graphs such as Petersen graphs, Double Wheel graphs, Helm graphs, Flower graphs, and Sun Flower graphs. They also pinpointed the graphs containing subgraphs with T-span and T-edge spans smaller than those of the corresponding graph in [25]. The T-coloring was developed by Roselin in [21] for several graph operations, including Union, Join, Cartesian product, and Tensor product. She also discussed the relationship between restricted edge span and restricted span. By generalizing the concept of T-coloring, Roseline et al. [22] presented a new sort of coloring, so-called Strong T-coloring (ST-coloring) and accordingly generalized the definitions of the parameters viz ST-span, ST-edge span, and ST-chromatic number. Inspired by the works of Roseline et al., Moran et al. [13, 14] investigated ST-coloring of several graph operations including Union, Join, Cartesian product, Tensor product, and Corona product of graphs. They calculated the ST-chromatic numbers, ST-span, and ST-edge span of these composite graphs. They also estimated ST-chromatic numbers of certain non-perfect graphs in the works [15], including the Petersen graph, Wheel graph, Helm graph, closed Helm Flower graph, and Sun Flower graph. Moreover, in the recent works [16], they reported some results on T and ST-Coloring of the Windmill graph.

The term $k-$initial set was introduced by Cozzen and Roberts [2] and that of k-multiple of s set was introduced by Raychaudhuri [19]. In the current paper, we study T-coloring on n-Hypercube graph and Tree graphs for any k-initial set and k-multiple of s set. A n-dimensional Hypercube, or n-Hypercube, can be formed by taking one vertex for each binary n-tuple, two vertices being adjacent exactly when the Hamming distance between the corresponding n-tuples is 1. Before this, Östergård [17] studied the chromatic number of the k^{th} power of the n-hypercube graph. Juan et al. [10] discussed the T-edge span of the folded hypercube network of dimension n for the k-multiple of s set. Again, in 2014, Pai et al. [18] studied incidence coloring on Hypercubes and based on the technique of Hamming codes, they presented an algorithm to obtain upper bounds of incidence coloring numbers for n in certain forms of integers. Then in 2018, Goldwasser et al. [5] also studied Polychromatic Colorings on the n-Hypercube graphs. With these studies on n-hypercube graphs, we are interested in studying T-coloring on hypercube graphs. A set of graphs known as the Kragujevac trees developed during the investigation of the atom-bond connectivity index. In 2014, Gutman et al. [7] created a generic combinatorial expression Nirmala index of Kragujevac trees. The extremal values of the energy, the Wiener index, and many vertex-degree-based topological indices were discovered by Cruz et al. [3] in 2014 across a collection of Kragujevac trees with a fixed degree center vertex. Again, in the year 2019, Mirajkar and Deshpande [12] obtained the total chromatic number of the Kragujevac Tree $Kg_n\{k_i\}, i = 1$ to n is $n + 1$ for $n > 2$ and $k_i + 2$ for $n > k_i$. A total coloring of a graph, involves assigning colors to all elements in the graph, including vertices and edges. This assignment must adhere to the condition that no two adjacent vertices or incident elements (vertices and edges) share the same color. The total chromatic number of a graph G is the minimum number of colors needed to achieve a valid total coloring of G. In 2021, Boregowda [1] calculated the Wiener and Terminal Wiener indices for the Gutman trees and Kragujevac Trees classes of trees. These research works motivated us to address the ST-chromatic number of Kragujevac tree.

The current paper is organized as follows: In Section 2, some basic definitions will be presented which will be used in the sections to follow. Section 3 contains certain results of T and $ST-$coloring of n-Hypercube Graph and Kragujevac Tree. Finally, in Section 4 a conclusion is provided on the whole work.

2. Preliminary

The following symbols and definitions will be used in the entire works.
The symbols $\chi(G)$, $\chi_T(G)$ and $\chi_{ST}(G)$ respectively, represent the Chromatic number, T-chromatic number and ST-chromatic number of the graph G.

Definition 2.1. [20] *For a T-coloring f, the f_T-span, denoted as $sp_T^f(G)$ is the maximum value of $\mid f(u) - f(w) \mid$ over all the vertices. The minimum of $sp_T^f(G)$ over all T-coloring f of G is known as T-span and is denoted as $sp_T(G)$.*

Definition 2.2. [20] *For a T-coloring f, the f_T-edge span, denoted as $esp_T^f(G)$ is the maximum value $\mid f(u) - f(w) \mid$ over all the edges (u, w). The minimum of $esp_T^f(G)$ over all T-coloring f of G is known as T edge-span and is denoted as $esp_T(G)$.*

Formal definition of ST-Coloring is given by

Definition 2.3. *(ST-Coloring)* [22] *A Strong T-coloring of G is a function $c : V(G) \to Z^+ \bigcup\{0\}$ such that for all $u \neq w$ in $V(G)$,*

(i) *$(u, w) \in E(G)$ then $| c(u) - c(w) | \notin T$ and*

(ii) *$| c(u) - c(w) | \neq | c(x) - c(y) |$ for any two distinct edges (u, w) and (x, y) in $E(G)$*

Definition 2.4. [20] *For a ST-coloring c, the c_{ST}-span, denoted as $sp^c_{ST}(G)$ is the maximum value $| c(u) - c(w) |$ over all the vertices. The minimum of $sp^c_{ST}(G)$ over all ST-coloring c of G is known as $sp_{ST}(G)$.*

Definition 2.5. [20] *The c_{ST}-edge span, denoted as $esp^c_{ST}(G)$ is the maximum value $| c(u) - c(w) |$ over all the edges (u, w). The minimum of $esp^c_{ST}(G)$ over all ST-coloring c of G is known as $esp_{ST}(G)$.*

Definition 2.6. *(The ST-chromatic number)* [22] *The ST-chromatic number of a graph is the least number of colors such that the absolute difference between the colors assigned to the vertices of any edge of the graph does not belong to a fixed set T of non negative integers including zero and that absolute difference between the vertices of any two distinct edges are distinct.*

Definition 2.7. [17] *A n-dimensional hypercube or n-hypercube, can be formed by taking one vertex for each binary n-tuple, two vertices being adjacent exactly when the Hamming distance (number of positions where two vertices differ in their binary representations) between the corresponding n-tuples is 1.*

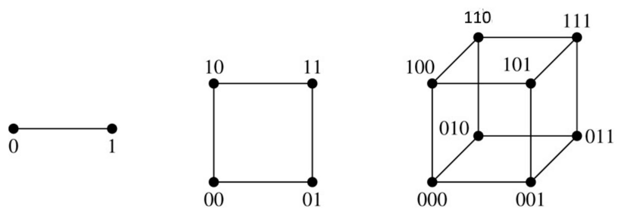

FIGURE 1. Hypercube Graphs

Definition 2.8. *(Kragujevac Tree)* [6, 7] *Let $k_i, i = 1$ to n, be non-negative integers, such that $0 \leq k_1 \leq k_2 \leq ...... \leq k_n$ and $K = \sum k_i$. Then the Kragujevac tree $Kg_n\{k_i\}$ is the tree obtained from $B_{k_1}, B_{k_2}, B_{k_3}, ..., B_{k_n}$ by connecting their roots to a new central vertex. Then the number of vertices of the Kragujevac tree $Kg_n\{k_i\}$ is $1 + n + 2K$.*

To explain Kragujevac trees [6], consider the branches B_0, B_1, B_2, B_3 and B_k, whose structure is shown in Figure 2. Then, the respective Kragujevac trees is shown in Figure 3.

3. MAIN RESULTS

In this section, we give results on T-coloring and ST-coloring of the Tree graph and n-Hypercube graph. To get the main conclusions about any 2-chromatic graphs (a graph with chromatic number 2), such as n-Hypercube graphs and the Tree graphs, we first derive certain extended findings of T-coloring of any 2-chromatic graph where the T sets are any k-initial set or k-multiple of s sets. These findings will be used to investigate the T-coloring of any 2-chromatic graphs while taking into account the T sets as k-initial sets and k-multiples of s-sets.

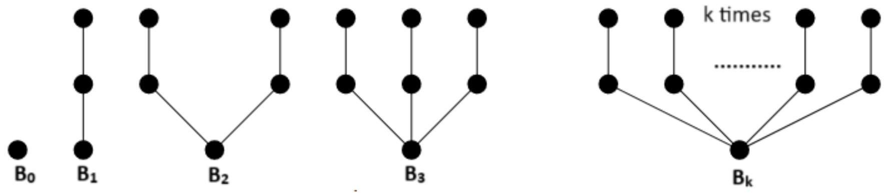

FIGURE 2. The rooted trees B_0, B_1, B_2, B_3 and B_k respectively

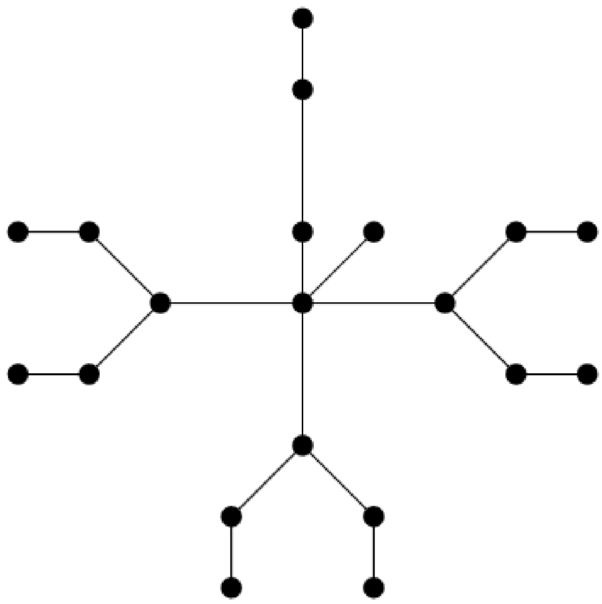

FIGURE 3. Kragujevac tree $Kg_5(0, 1, 2, 2, 4)$ for which $n = 5$ and $K = 9$

3.1. T-coloring of n-Hypercube graphs and Tree graphs.

Theorem 3.1. *For any 2-chromatic graph and for any k-initial set $T = \{0, 1, 2, \ldots, k\} \bigcup S$, where $n(k+1) \notin S$; the span of the graph G, $sp_T(G) = k+1$.*

Proof. Given that $\chi_T(G) = 2$. Let, $f : V(G) \to Z^+ \bigcup \{0\}$ be a function that assigns non-negative integers to the vertices of G, defined as 0 and $k+1$ in such a way that if 0 is assigned to any vertex then all the vertices adjacent to that vertex will be assigned the color $k+1$ and if $k+1$ is assigned to any vertex then all the vertices adjacent to that vertex will be assigned the color 0. Then, f is a T-coloring as $\mid f(v_i) - f(v_j) \mid \notin T$ for any two vertices v_i and v_j. Now, f-span, ${sp^f}_T(G)$ of any graph G is the maximum difference between the non-negative integers assigned to all the vertices, and $sp_T(G)$ is the $\min sp^f{}_T(G)$ of all such T-colorings of G. Which implies, $sp^f{}_T(G) = k+1$ and hence, $sp_T(G) \leq (k+1)$. Now, since, T is a k-initial set, which means $\mid f(v_i) - f(v_j) \mid$ can not be less than $k+1$ as $\mid f(v_i) - f(v_j) \mid \notin T$. Hence, there does not exist any T-coloring for which $sp^f{}_T(G) < k+1$. Hence, $k+1$ is the minimum of all such $sp^f{}_T(G)$. Hence, $sp_T(G) = k+1$. □

Theorem 3.2. *For any 2-chromatic graph and for any k-initial set $T = \{0, 1, 2, \ldots, k\} \bigcup S$, where $n(k+1) \notin S$, the edge span of the graph G, $esp_T(G) = k+1$.*

Proof. Given that $\chi_T(G) = 2$. Let, $f : V(G) \to Z^+ \bigcup \{0\}$ be a function that assigns non-negative integers to the vertices of G, defined as: 0 and $k+1$ in such a way that if 0 is assigned to any of the vertex then all the vertices adjacent to that vertex will be assigned the color $k+1$ and if $k+1$ is assigned to any of the vertex then all the vertices adjacent to that vertex will be assigned the color 0. Then, f is a T-coloring as $\mid f(v_i) - f(v_j) \mid \notin T$ for any edge (v_i, v_j). Now, $f-$ *edge span*, $esp^f{}_T(G)$ of any graph G is the maximum difference between the non-negative integers assigned to the vertices of an edge over all the edges of G and $esp_T(G)$ is the $\min esp^f{}_T(G)$ of all such T-colorings of G. Which implies, $esp^f{}_T(G) = k+1$ and hence, $sp_T(G) \leq (k+1)$. Now, since, T is a k-initial set, which means, for any edge (v_i, v_j), $\mid f(v_i) - f(v_j) \mid$ can not be less than $k+1$ as $\mid f(v_i) - f(v_j) \mid \notin T$. Hence, there doesn't exist any other T-coloring for which $esp^f{}_T(G) < k+1$. Hence, $k+1$ is the minimum of all such $esp^f{}_T(G)$, where the minimum is taken over all the edges (v_i, v_j) of G. Hence, $esp_T(G) = k+1$. □

Now from Theorem 3.1 and Theorem 3.2, the following result can be derived.

Corollary 3.1. *For any 2-chromatic graph and for any k-initial set $T = \{0, 1, 2, ..., k\} \bigcup S$, where $n(k+1) \notin S$, the result $sp_T(G) = esp_T(G) = k+1$ holds.*

Corollary 3.2. *For any 2-chromatic graph G and for any k-multiple of s set $T = \{0, s, 2s, 3s, \ldots, ks\} \bigcup S$, where $S \subseteq \{s+1, s+2, s+3, ..., ks-1\}, s, k > 1$, Then, the span of the graph G, $sp_T(G) = 1$.*

Proof. Since, the chromatic number of the graph G, $\chi_T(G) = 2$. Let, $f : V(G) \to Z^+ \bigcup \{0\}$ be a function that assigns non-negative integers to the vertices of G, defined as 0 and 1 in such a way that if 0 is assigned to any vertex then all the vertices adjacent to that vertex will be assigned the color 1 and if 1 is assigned to any vertex then all the vertices adjacent to that vertex will be assigned the color 0. Then, f is a T-coloring as $\mid f(v_i) - f(v_j) \mid \notin T$ for any two arbitrary vertices v_i and v_j. Now, by the definition of $f-$ *span* and *span* of any graph, $sp^f{}_T(G) = 1$ and hence, $sp_T(G) \leq (1)$. Now, $\mid f(v_i) - f(v_j) \mid$ can not be less than 1 for any k-multiple of s- set. Hence, there does not exist any T-coloring for which $sp^f{}_T(G) < 1$. Hence, 1 is the minimum of all such $sp^f{}_T(G)$. Hence, $sp_T(G) = 1$. □

Theorem 3.3. *For any 2-chromatic graph G and for any k-multiple of s set $T = \{0, s, 2s, 3s, \ldots, ks\} \bigcup S$, where $S \subseteq \{s+1, s+2, s+3, ..., ks-1\}, s, k > 1$, Then, the edge span of the graph G, $esp_T(G) = 1$.*

Proof. Since, the chromatic number of the graph G, $\chi_T(G) = 2$. Let, $f : V(G) \to Z^+ \bigcup \{0\}$ be a function that assigns non-negative integers to the vertices of G, defined as 0 and 1 in such a way that if 0 is assigned to any vertex then all the vertices adjacent to that vertex will be assigned the color 1 and if 1 is assigned to any vertex then all the vertices adjacent to that vertex will be assigned the color 0. Then, f is a T-coloring as $\mid f(v_i) - f(v_j) \mid \notin T$ for any arbitrary edge (v_i, v_j). Now, by the definition of $f-$ *span* and *span* of any graph, $esp^f{}_T(G) = 1$ and hence, $esp_T(G) \leq (1)$. Now, $\mid f(v_i) - f(v_j) \mid$ can not be less than 1 for any k-multiple of s- set. Hence, there doesn't exist any T-coloring for which $esp^f{}_T(G) < 1$. Hence, 1 is the minimum of all such $esp^f{}_T(G)$. Hence, $esp_T(G) = 1$. □

Now from Theorem 3.2 and Theorem 3.3, the following result can be derived.

Theorem 3.4. *For any 2-chromatic graph G and for any k-multiple of s set $T = \{0, s, 2s, 3s, ...ks\} \bigcup S$, where $S \subseteq \{s+1, s+2, s+3, ...ks-1\}, s, k > 1$, Then, span of the graph G, $sp_T(G) = esp_T(G) = 1$.*

Theorem 3.5. *For a n-Hypercube graph Q_n, and for any k-initial set $T = \{0, 1, 2, 3, ... k\} \bigcup S$, where $n(k+1) \notin S$, Then, $sp_T(Q_n) = esp_T(Q_n) = k+1$.*

Proof. Since, $\chi(Q_n) = 2$. Hence, from Theorem 3.1, for any k-initial set $T = \{0, 1, 2, \dots, k\} \bigcup S$, where $n(k+1) \notin S$. Then, $sp_T(Q_n) = esp_T(Q_n) = k+1$. □

Theorem 3.6. *For a n-hypercube graph Q_n and for any k-multiple of s set $T = \{0, s, 2s, 3s, ...ks\} \bigcup S$, where $S \subseteq \{s+1, s+2, s+3, ...ks-1\}, s, k > 1$, Then, span of the graph Q_n, $sp_T(Q_n) = esp_T(Q_n) = 1$.*

Proof. Since, $\chi(Q_n) = 2$. Hence, from Theorem 3.4, for any k-multiple of s set $T = \{0, s, 2s, 3s, \dots, ks\} \bigcup S$, where $S \subseteq \{s+1, s+2, s+3, ...ks-1\}, s, k > 1$, Then, span and edge span of the graph Q_n, $sp_T(Q_n) = esp_T(Q_n) = 1$. □

Similarly, the following two theorems are obvious from Theorem 3.1 and Theorem 3.4.

Theorem 3.7. *For any tree graph G, and for any k-initial set $T = \{0, 1, 2, 3, ...k\} \bigcup S$, where $n(k+1) \notin S$, Then, $sp_T(G) = esp_T(G) = k+1$.*

Theorem 3.8. *For any tree graph G, and for any k-multiple of s set $T = \{0, s, 2s, 3s, ...ks\} \bigcup S$, where $S \subseteq \{s+1, s+2, s+3, ...ks-1\}, s, k > 1$, Then, span of the graph G, $sp_T(G) = esp_T(G) = 1$.*

3.2. ***ST*-chromatic number of Kragujevac Tree.** Recall the Definition 2.8 and consider the Kragujevac tree $Kg_n\{k_i\}$ obtained from branches $B_{k_1}, B_{k_2}, B_{k_3}, ..., B_{k_n}$. Let us name the vertices of the Kragujevac tree as: v_0 be the central root vertex, v_is are the connected vertices of the central root vertex v_0 $v_{i,j}$ are the adjacent vertices of V_i and $v_{i,j'}$ are the pendant vertices connected to $v_{i,j}$, where $i = 1$ to n, and $j, j' = 1$ to r_i. Let, $f : V(Kg_n) \to Z^+ \bigcup\{0\}$ be a T-coloring of Kg_n, which assigns non negative integers to the vertices of Kg_n, as: $f(v_0) = f(v_{i,j'}) = 0$, $f(v_i) = i$, and $f(v_{i,j}) = k^j + i^j$. Then for any two edges (a, b) and (c, d), $\mid f(a) - f(b) \mid \neq \mid f(c) - f(d) \mid$. Which shows that f is ST-coloring of Kg_n.
Now we estimate the ST-chromatic number of Kragujevac tree. For this, we use the following theorem.

Theorem 3.9. [15] *In a ST-coloring, three consecutive vertices of a path have distinct colors.*

Theorem 3.10. *ST-chromatic number of a Kragujevac tree is $\chi_{ST}(Kg_n\{k_i\}) = 1+n+K$, where $K = \sum k_i, i = 1, \dots, n$.*

Proof. By invoking Theorem 3.9, in a ST-coloring, three consecutive vertices of any path must be of distinct colors. This means, v_0, v_i and $v_{i,j}$, $i = 1$ to $n, j = 1$ to t must be assigned distinct colors as they are in the sequence of three consecutive vertices of a path. Which shows that, $\chi_{ST}(Kg_n\{k_i\}) \geq 1 + n + K$. Now, the central vertex v_0 and $v_{i,j'}$ may be colored with the same color, provided that for any two edges (a, b) and (c, d),

$| f(a) - f(b) | \neq | f(c) - f(d) |$. Hence, the required minimum number of colors required to color all the vertices of $Kg_n\{k_i\}, i = 1$ to n. Hence,

$$\chi_{ST}(Kg_n\{k_i\}) = 1 + n + K$$

□

4. Conclusions

In the paper, we discussed T-coloring of 2-chromatic graphs like n-Hypercube graphs and Tree graphs. We first obtained the span and edge span related to T-coloring of any 2-chromatic graphs and as an application of these results we obtained the T-span and T-edge span of n-hypercube graphs and tree graphs. These results may be also useful for studying T-coloring of any 2-coloring graphs. We also have obtained the ST-chromatic number of a Kragujevac tree.

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References

[1] Boregowda, H. S., (2021), Wiener Type Indices Of Certain Classes of Trees, South East Asian JournaL of Mathematics and Mathematical Sciences, 17(2), pp. 241-250.
[2] Cozzens, M. B. and Roberts, F. S., (1982), T-colorings of Graphs and the Channel Assignment Problem, Congr. Numer., 35, pp.191-208.
[3] Cruz, R., Gutman, I. and Rada, J., (2014), Topological Indices of Kragujevac Trees, Proyecciones Journal of Mathematics, 33(4), pp. 471-482.
[4] Gräf, A., (1999), Distance Graphs and the $T-$Coloring, Discrete Mathematics, 196(1-3), pp. 153-166.
[5] Goldwasser, J., Lidicky, B., Martin, R. R., Offner, D., Talbot, J. and Youngk, M., (2018), Polychromatic Colorings on the Hypercube, Journal of Combinatorics, 9(4), pp. 631–657.
[6] Gutman, I., (2014), Kragujevac Trees and Their Energy, Scientific Publications of the State University of Novi Pazar Series A: Applied Mathematics, Informatics and mechanics, 6(2), pp. 71-79.
[7] Gutman, I., Kulli, V. R. and Redžepovič, I., (2019), Nirmala Index of Kragujevac Trees, International Journal of Mathematics Trends and Technology, 67(6), pp. 44-49.
[8] Hale, W. K., (1980), Frequency Assignment: Theory and Applications, In: Proceedings of IEEE, 68, pp. 1497-1514.
[9] Jansen, K., (1996), A Rainbow about T-Colorings for Complete Graphs, Discrete Mathematics, 154, pp. 129-139.
[10] Juan, J., Sun, T. and Wu, P. X., (2009), $T-$Coloring on Folded Hypercubes, Taiwanese Journal of Mathematics, 13(4), pp. 1331-1341.
[11] Liu, D. D. F., (1992), T-Colorings of Graphs, Discrete Mathematics, 101, pp. 203-212.
[12] Mirajkar, K. G. and Deshpande, A. V., (2019), Total Coloring of Jahangir graph, Kragujevac tree and Dendrimers, Research Guru: Online Journal of Multidisciplinary Subjects, 13, pp.1-9.
[13] Moran, R., Bora, N., Baruah, A. K. and Bharali, A., (2020), $ST-$Coloring of Join and Disjoint Union of Graphs, Advances in mathematics: scientific journal, 9(11), pp. 9393-9399.
[14] Moran, R., Bora, N., Baruah, A.K. and Bharali, A., (2020), ST-Coloring of Some Products of Graphs, Journal of Mathematical and computational science, 11(1), pp. 337-347.
[15] Moran, R., Pegu, A., Gogoi, I. and Bharali, A., (2021), A Note on ST-Coloring of Some Non Perfect Graphs. In: Book Publisher International, 11, pp. 112-119.
[16] Moran, R. and Bora, N., (2023), On T and ST-Coloring of Windmill graph, Mathematics and Statistics, 11(2), pp. 335-339.
[17] Östergård, P. R. J., (2004), On a Hypercube Coloring Problem, Journal of Combinatorial Theory, A(108), pp. 199-204
[18] Pai, K. J., Chang, J. M., Yang, J. S. and Wu, R. U., (2014), Incidence Coloring on Hypercubes, Theoretical Computer Science, 557, pp. 59–65

[19] Raychaudhuri, A., (1994), Further Results on T-Coloring and Frequency Assignment Problems, SIAM Journal on Discrete Mathematics, 7(4), pp. 605-613.
[20] Roberts, F. S., (1991), T-Colorings of Graphs: Recent Results and Open Problems, Discrete Mathematics, 93, pp. 229-245.
[21] Roselin, S. J., (2022), T-Coloring of Product Graphs, Discrete Mathematics, Algorithms and Applications, 14(02), 2150103.
[22] Roselin, S. J., Raj, L. B. M. and Germina, K. A., (2019), Strong T-Coloring of Graphs, International Journal of Innovative Technology and Exploring Engineering, 8(12), pp. 4677-4681.
[23] Roselin, S. J. and Raj, L. B. M., (2019), T-Coloring of Certain Non-perfect Graphs, Journal of Applied Science and Computations, 6(2), pp. 1456-1468.
[24] Roselin, S. J. and Raj, L. B. M., (2019), $T-$Coloring of Wheel graphs, International Journal of Information and Computing Science, 6(3), pp. 11-18.
[25] Roselin, S. J. and Raj, L. B. M., (2019), T-Span, $T-$Edge Span Critical Graphs, International Journal of Innovative Technology and Exploring Engineering, 8(19), pp. 3898-3901.
[26] Sivagami, P., (2018), T-Coloring of Certain Graphs, International Journal of Pure and Applied Mathematics, 120(8), pp. 119-127.
[27] Sivagami, P. and Rajasingh, I., (2016), T-Coloring of Certain Networks, Mathematics in Computer Science, 10(2), pp. 239-248.
[28] Tesman, B. A., (1993), List T-Colorings of Graphs, Discrete Applied Mathematics, 45, pp. 277-289.

Rubul Moran is currently working as an Assistant Professor at the Department of Mathematics, Jorhat Institute of Science and Technology, Jorhat, Assam, India. He is also pursuing PhD in Mathematics at the Department of Mathematics, Dibrugarh University, Assam, India. He completed Master in Science as well as Master of Philosophy in Mathematics from Dibrugarh University, Assam, India. His research area is focused on the intriguing domain of graph theory, with a particular emphasis on Graph Coloring. He published several insightful articles in various research journals.

Niranjan Bora is presently working as an Assistant Professor at the Department of Mathematics, Dibrugarh University Institute of Engineering and Technology, Dibrugarh University, Dibrugarh, Assam, India since 2012. He did his Master in Science in Mathematics from Gauhati University, Assam, India and Ph.D in Mathematics from Dibrugarh University, Assam, India. He has published 18 research papers in the International journals of good repute and attended academic conferences in India as well as in abroad. His area of research includes numerical linear algebra, Multiparameter eigenvalue problem and Graph Coloring.