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NEUTROSOPHIC $AMR-$ALGEBRA

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Abstract. In this study, the truth membership function (W_T), indeterminacy membership function (W_I), falsity membership function (W_F) are incorporated with the structure of AMR-algebra. The notion of neutrosophic AMR-algebra is proposed and some significant results were furnished. Further, some compelling properties based on α-cut(level set) are also studied. Interestingly, an algorithm has been developed to validate the conditions of neutrosophic AMR-algebra using the values in the range of $[0, 1]$.

Keywords: Neutrosophic set, AMR-algebra, Level set.

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1. Introduction

A valuable tool for dealing with the incomplete, ambiguous, and inconsistent information that exists in real life is the neurosophic set (NS), which was proposed by Smarandache[14] and is a generalisation of fuzzy sets and intuitionistic fuzzy set. Truth, indeterminacy, and falsity membership functions (T, I, and F) are the characteristics of neutrosophic sets. This theory is particularly essential in numerous domains of application since indeterminacy is precisely quantified and the truth, indeterminacy, and falsity membership functions are all independent. Broumi [8] introduced a novel idea known as the "generalised neutrosophic soft set." The advantages of both generalised neutrosophic set and soft set approaches are combined in this idea. In [3], certain classes of neutrosophic crisps structures through topology, nearly open sets are examined, and some applications are provided. In addition, the crisp topological and neutrosophic crisp investigations were generalised to the concept of neutrosophic crisp set. Neutosophic open sets, neutrosophic

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continuity, numerous preservation features, and several characterizations of neutrosophic mapping and neutrosophic connectedness were introduced in [1]. The authors of [7] described the fundamental operations of trapezoidal neutrosophic fuzzy numbers and triangular neutrosophic numbers. The neutrosophic number and certain comparative studies between the existing neutrosophic sets are also presented. Jun et al.[10] laid out the concept and properties associated with interval neutrosophic length.

In the context of neutrosophic theory, the purpose of the study[2, 4], was to provide an overview of cone metric space. Further, acquired some essential findings for weakly compatible mapping by concerting fixed points in this connection. In the neutrosophic cone metric space, the concept of (Φ, Ψ)-weak contraction is described by the concept of manipulating the distance function. K-polar extended neutrosophic set was created by Borzooei et al. [9] and has been incorporated into BCK/BCI-algebras. Takallo et al. proposed MBJ neutrosophic sets and the related studies carried out on BCK/BCI algebras[15] and $\beta-$algebra[12]. The MBJ neutrosophic T-ideal is defined and examined by Khalid et al. [11] in their research using several concepts, such as union and intersection. Further, they have examined the MBJ neutrosophic T-ideal under cartesian product and homomorphic findings using the key features. In order to describe a hyper BCK algebra, Alsubie et al. [5] used the idea of an MBJ neutrosophic structure. In[6], Amir K Amin presented the idea of AMR-algebra and its generalisation while contrasting them with other algebras like BCK, BCI, BCH, and others. Muralikrishna et al.[13] were discussed the properties of neutrosophic cubic $\beta-$subalgebra. This paper emphasizes the contrasting perception of neutrosophic AMR-algebra with some captivating results.

2. Preliminares

This section goes over some key definitions that are pertinent to our study.

Definition 2.1. [13] *In an universe ϖ, a fuzzy set μ is defined as $\mu : \varpi \to [0, 1]$.*

Definition 2.2. [12] *An Intuitionistic fuzzy set in a nonempty set $\varpi$is defined by $A = \{\langle\varsigma, \zeta_A(\varsigma), \eta_A(\varsigma)\rangle / \varsigma \in \varpi\}$ where $\zeta_A : \varpi \to [0, 1]$ is a membership function of A and $\eta_A : X \to [0, 1]$ is a non membership function of A satisfying $0 \leq \zeta_A(\varsigma) + \eta_A(\varsigma) \leq 1 \; \forall \; \varsigma \in \varpi$.*

Definition 2.3. [14] *A neutrosophic set in ϖ is the structre of the form $\Omega = \{\langle\varsigma : W_T(\varsigma), W_I(\varsigma), W_F(\varsigma)/\varsigma \in \varpi\rangle\}$, Where $W_T, W_I, W_F : \varpi \to [0, 1]$ referred as truth, indeterminate, false membership functions respectively.*

Definition 2.4. [13] *A non-empty set ϖ with a binary operation $*$ and constant 0 is called as a BF algebra, if*
$\varsigma * \varsigma = 0$
$\varsigma * 0 = \varsigma$
$0 * (\varsigma * \varepsilon) = \varepsilon * \varsigma \; \forall \varsigma, \; \varepsilon \; \in \; \varpi$.

Definition 2.5. [6] *A non-empty set ϖ with a binary operation $*$ and a constant 0 is an AMR algebra,if*
$\varsigma * 0 = \varsigma$
$(\varsigma * \varepsilon) * z = \varepsilon * (z * \varsigma) \forall \varsigma, \varepsilon, z \in \varpi$.
A binary relation can be described as $\varsigma \leq \varepsilon$ if and only if $\varsigma * \varepsilon = 0$.

Example 2.1. [6] *Considering $\varpi = \{0, \wp_1, \wp_2, \wp_3\}$ as a set with the binary operation $*$ defined by*

$*$	$\acute{\varrho_0}$	$\acute{\varrho_1}$	$\acute{\varrho_2}$	$\acute{\varrho_3}$
$\acute{\varrho_0}$	$\acute{\varrho_0}$	$\acute{\varrho_1}$	$\acute{\varrho_2}$	$\acute{\varrho_3}$
$\acute{\varrho_1}$	$\acute{\varrho_1}$	$\acute{\varrho_2}$	$\acute{\varrho_3}$	$\acute{\varrho_0}$
$\acute{\varrho_2}$	$\acute{\varrho_2}$	$\acute{\varrho_3}$	$\acute{\varrho_0}$	$\acute{\varrho_1}$
$\acute{\varrho_3}$	$\acute{\varrho_3}$	$\acute{\varrho_0}$	$\acute{\varrho_1}$	$\acute{\varrho_2}$

$(\varpi, *, 0)$ *is then an AMR-algebra.*

Definition 2.6. [6] *A non-empty subset I of an AMR-algebra ϖ is called a subalgebra of ϖ if $\acute{\varsigma} * \acute{\varepsilon} \in I$ whenever $\acute{\varsigma}, \acute{\varepsilon} \in I$.*

Definition 2.7. [6] *Let $(\varpi, *, 0) and (Y, *, 0)$ are AMR-algebra. A function $f : \varpi \to Y$ is called as a AMR homomorphism if $f(\acute{\varsigma} * \acute{\varepsilon}) = f(\acute{\varsigma}) * f(\acute{\varepsilon})$.*

Definition 2.8. [13] *A supremum property of the fuzzy set T in ϖ, $\mu(\acute{\varsigma_0}) = \sup_{\acute{\varsigma} \in T} \mu(\acute{\varsigma})$ if there exist $\acute{\varsigma}, \acute{\varsigma_0} \in T$.*

Definition 2.9. [12] *a neutrosophic fuzzy set A in any set ϖ is considered to possess the $sup - sup - inf$ property if for subset T of ϖ $\exists$ $\acute{\varsigma_0} \in T$ such that*
$W_T(\acute{\varsigma_0}) = \sup_{\acute{\varsigma} \in T} W_T(\acute{\varsigma})$, $W_I(\acute{\varsigma_0}) = \sup_{\acute{\varsigma} \in T} W_I(\acute{\varsigma})$, $W_F(\acute{\varsigma_0}) = \sup_{\acute{\varsigma} \in T} W_F(\acute{\varsigma})$ *respectively.*

3. Neutrosophic AMR-algebra

Here the aspects of a neutrosophic AMR-subalgebra and some relevant results are discussed.

Definition 3.1. *Let ϖ be a AMR-algebra. A neutrosophic set $A = \{\acute{\varsigma}, W_{AT}, W_{AI}, W_{AF} : \acute{\varsigma} \in \varpi\}$ in ϖ is called a netrosophic AMR-subalgebra of ϖ if for all $\acute{\varsigma}, \acute{\varepsilon} \in A$,*
$(i) W_{AT}(\acute{\varsigma} * \acute{\varepsilon}) \geq max\{W_{AT}(\acute{\varsigma}), W_{AT}(\acute{\varepsilon})\}$
$(ii) W_{AI}(\acute{\varsigma} * \acute{\varepsilon}) \geq min\{W_{AI}(\acute{\varsigma}), W_{AI}(\acute{\varepsilon})\}$
$(iii) W_{AF}(\acute{\varsigma} * \acute{\varepsilon}) \leq max\{W_{AF}(\acute{\varsigma}), W_{AF}(\acute{\varepsilon})\}$.

Example 3.1. *let $\varpi = \{0, \acute{\varrho_1}, \acute{\varrho_2}, \acute{\varrho_3}\}$ be a set with a binary operation $*$ defined by:*

$*$	$\acute{\varrho_0}$	$\acute{\varrho_1}$	$\acute{\varrho_2}$	$\acute{\varrho_3}$
0	0	$\acute{\varrho_1}$	$\acute{\varrho_2}$	$\acute{\varrho_3}$
$\acute{\varrho_1}$	$\acute{\varrho_1}$	$\acute{\varrho_2}$	$\acute{\varrho_3}$	$\acute{\varrho_1}$
$\acute{\varrho_2}$	$\acute{\varrho_2}$	$\acute{\varrho_3}$	$\acute{\varrho_1}$	$\acute{\varrho_2}$
$\acute{\varrho_3}$	$\acute{\varrho_3}$	$\acute{\varrho_1}$	$\acute{\varrho_2}$	$\acute{\varrho_3}$

Then $(\varpi, *, 0)$ is a AMR-algebra.

Example 3.2. *Consider a example 3.2,A is a neutrosophic AMR-subalgebra, define by the membership function*

$$W_{AT}(\acute{\varsigma}) = \begin{cases} 0.7 : & \acute{\varsigma} = \acute{\varrho_1}, \acute{\varrho_2}, \acute{\varrho_3} \\ 0.5 : & \acute{\varsigma} = \acute{\varrho_0} \end{cases}$$

$$W_{AI}(\acute{\varsigma}) = \begin{cases} 0.6 : & \acute{\varsigma} = \acute{\varrho_0}, \acute{\varrho_3} \\ 0.4 : & \acute{\varsigma} = \acute{\varrho_2}, \acute{\varrho_1} \end{cases}$$

$$W_{AF}(\acute{\varsigma}) = \begin{cases} 0.2: & \acute{\varsigma} = \acute{\varrho}_1, \acute{\varrho}_2, \acute{\varrho}_3 \\ 0.8: & \acute{\varsigma} = \acute{\varrho}_0 \end{cases}$$

Then it is observed that, A is a neutrosophic AMR-subalgebra.

Lemma 3.1. *Let A be a netrosophic AMR-subalgebra of* ϖ*, then*
$(i) W_{AT} \geq W_{AT}(\acute{\varsigma}), W_{AI} \geq W_{AI}(\acute{\varsigma}), W_{AF} \leq W_{AF}(\acute{\varsigma}),$
$(ii) W_{AT}(0) \geq W_{AT}(\acute{\varsigma}^*) \geq W_{AT}(\acute{\varsigma}), W_{AI}(0) \geq W_{AI}(\acute{\varsigma}^*) \geq W_{AI}(\acute{\varsigma})$ &
$W_{AF}(0) \leq W_{AF}(\acute{\varsigma}^*) \leq W_{AF}(\acute{\varsigma})$ *where* $\acute{\varsigma}^* = 0 - \acute{\varsigma}$, $\forall \acute{\varsigma} \in \varpi$.

Proof. (1) For every $\acute{\varsigma} \in \varpi$,

$$\begin{aligned} W_{AT}(0) &= W_{AT}(\acute{\varsigma} - \acute{\varsigma}) \\ &\geq max\{W_{AT}(\acute{\varsigma}), W_{AT}(\acute{\varsigma})\} \\ &= W_{AT}(\acute{\varsigma}) \end{aligned}$$

$$\begin{aligned} W_{AI}(0) &= W_{AI}(\acute{\varsigma} - \acute{\varsigma}) \\ &\geq min\{W_{AI}(\acute{\varsigma}), W_{AI}(\acute{\varsigma})\} \\ &= W_{AI}(\acute{\varsigma}) \end{aligned}$$

and

$$\begin{aligned} W_{AF}(0) &= W_{AF}(\acute{\varsigma} - \acute{\varsigma}) \\ &\leq max\{W_{AF}(\acute{\varsigma}), W_{AF}(\acute{\varsigma})\} \\ &= W_{AF}(\acute{\varsigma}) \end{aligned}$$

therefore $W_{AF}(0) \leq W_{AF}(\acute{\varsigma})$
(ii) Also for $\acute{\varsigma} \in \varpi$,

$$\begin{aligned} W_{AT}(\acute{\varsigma}^*) &= W_{AT}(0 - \acute{\varsigma}) \\ &\geq max\{W_{AT}(0), W_{AT}(\acute{\varsigma})\} \\ &= W_{AT}(\acute{\varsigma}) \end{aligned}$$

$$\begin{aligned} W_{AI}(\acute{\varsigma}^*) &= W_{AI}(0 - \acute{\varsigma}) \\ &\geq min\{W_{AI}(0), W_{AI}(\acute{\varsigma})\} \\ &= W_{AI}(\acute{\varsigma}) \end{aligned}$$

and

$$\begin{aligned} W_{AF}(\acute{\varsigma}^*) &= W_{AF}(0 - \acute{\varsigma}) \\ &\leq max\{W_{AF}(0), W_{AF}(\acute{\varsigma})\} \\ &= W_{AF}(\acute{\varsigma}) \end{aligned}$$

$Thus\ W_{AF}(0) \leq W_{AF}(\acute{\varsigma}^*)$. □

Theorem 3.1. *If* $A = \{\langle \acute{\varsigma}, W_{AT}(\acute{\varsigma}), W_{AI}(\acute{\varsigma}), W_{AF}(\acute{\varsigma}) \rangle : \acute{\varsigma} \in \varpi\}$ *is an a neutrosophic AMR-subalgebra of* ϖ*. Then the sets* $\chi_{W_{AT}} = \{\acute{\varsigma} \in \varpi \ / \ W_{AT}(\acute{\varsigma}) = W_{AT}(0)\}$ $, \chi_{W_{AI}} = \{\acute{\varsigma} \in \varpi \ / \ W_{AI}(\acute{\varsigma}) = W_{AI}(0)\}$ *&* $\chi_{W_{AF}} = \{\acute{\varsigma} \in \varpi / \ W_{AF}(\acute{\varsigma}) = W_{AF}(0)\}$ *are subalgebra of* ϖ.

Proof. For any $\acute{\varsigma}, \acute{\varepsilon} \in \chi_{W_{AT}}$.
$W_{AT}(\acute{\varsigma}) = W_{AT}(0)$, $W_{AT}(\acute{\varepsilon}) = W_{AT}(0)$ Now

$$\begin{aligned} W_{AT}(\acute{\varsigma} * \acute{\varepsilon}) &\geq max\{W_{AT}(\acute{\varsigma}), W_{AT}(\acute{\varepsilon})\} \\ &= max\{W_{AT}(0), W_{AT}(0)\} \\ &= W_{AT}(0) \end{aligned}$$

$\acute{\varsigma} * \acute{\varepsilon} \in \chi_{W_{AT}}$
For any $\acute{\varsigma}, \acute{\varepsilon} \in \chi_{W_{AI}}$.
$W_{AI}(\acute{\varsigma}) = W_{AI}(0)$, $W_{AI}(\acute{\varepsilon}) = W_{AI}(0)$ Now

$$\begin{aligned} W_{AI}(\acute{\varsigma} * \acute{\varepsilon}) &\geq min\{W_{AI}(\acute{\varsigma}), W_{AI}(\acute{\varepsilon})\} \\ &= min\{W_{AI}(0), W_{AI}(0)\} \\ &= W_{AI}(0) \end{aligned}$$

$\acute{\varsigma} * \acute{\varepsilon} \in \chi_{W_{AI}}$
therefore $\chi_{W_{AT}}$ and $\chi_{W_{AI}}$is a subalgebra of ϖ.

Consider, $\acute{\varsigma}, \acute{\varepsilon} \in \chi_{W_{AF}}$ then $W_{AF}(\acute{\varsigma}) = W_{AF}(0)$, $W_{AF}(\acute{\varepsilon}) = W_{AF}(0)$ Now

$$\begin{aligned} W_{AF}(\acute{\varsigma} * \acute{\varepsilon}) &\geq min\{W_{AF}(\acute{\varsigma}), W_{AF}(\acute{\varepsilon})\} \\ &= min\{W_{AF}(0), W_{AF}(0)\} \\ &= W_{AF}(0) \end{aligned}$$

$\acute{\varsigma} * \acute{\varepsilon} \in \chi_{W_{AF}}$
therefore $\chi_{W_{AF}}$ is a subalgebra of ϖ. □

Definition 3.2. *Let $A = \{\langle\acute{\varsigma}, W_{AT}(\acute{\varsigma}), W_{AI}(\acute{\varsigma}), W_{AF}(\acute{\varsigma})\rangle : \acute{\varsigma} \in \varpi\}$ be a neutrosophic set in ϖ and f be a function from a set ϖ Y, then the image $f(A)$ of A under f, is defined as $f(A) = \{\langle\acute{\varsigma}, f_{inf}(W_{AT}), f_{sup}(W_{AI}), f_{inf}(W_{AF})\rangle : \acute{\varsigma} \in Y\}$, where*

$$f_{inf}(W_{AT,I})(\acute{\varepsilon}) = \begin{cases} inf_{\acute{\varsigma} \in f^{-1}(\acute{\varepsilon})} W_{AT}(\acute{\varsigma}), & if\, f^{-1}(\acute{\varepsilon}) \neq \emptyset \\ 1, & otherwise \end{cases}$$

$$f_{sup}(W_{AI})(\acute{\varepsilon}) = \begin{cases} sup_{\acute{\varsigma} \in f^{-1}(\acute{\varepsilon})} W_{AI}(\acute{\varsigma}), & if\, f^{-1}(\acute{\varepsilon}) \neq \emptyset \\ 0, & otherwise \end{cases}$$

$$f_{inf}(W_{AF})(\acute{\varepsilon}) = \begin{cases} inf_{\acute{\varsigma} \in f^{-1}(\acute{\varepsilon})} W_{AF}(\acute{\varsigma}), & if\, f^{-1}(\acute{\varepsilon}) \neq \emptyset \\ 1, & otherwise \end{cases}$$

Definition 3.3. *For two neutrosophic AMR sets A and B in ϖ and Y, a function $f : \varpi \to Y$, an inverse image of B under f is defined by $f^{-1}(B) = \{\acute{\varsigma}, f^{-1}(W_{BT}(\acute{\varsigma})), f^{-1}(W_{BI}(\acute{\varsigma})), f^{-1}(W_{BF}(\acute{\varsigma})) : \acute{\varsigma} \in \varpi\}$ such that $f^{-1}(W_{BT}(\acute{\varsigma})) = W_{BT}(f(\acute{\varsigma}))$,$f^{-1}(W_{BI}(\acute{\varsigma})) = W_{BI}(f(\acute{\varsigma}))$ and $f^{-1}(W_{BF}(\acute{\varsigma})) = (W_{BF}(f(\acute{\varsigma}))$.*

Theorem 3.2. *Let ϖ and Y be two AMR-algebras and $f : \varpi \rightarrow Y$ be a homomorphism. If A is a neutrosophic AMR-subalgebra of ϖ, define $f(A) = \{\langle \acute{\varsigma}, (W_{AT}(\acute{\varsigma})) = W_{AT}(f(\acute{\varsigma})), (W_{AI}(\acute{\varsigma})) = W_{AI}(f(\acute{\varsigma})), (W_{AF}(\acute{\varsigma})) = W_{AF}(f(\acute{\varsigma}))\rangle : \acute{\varsigma} \in \ varpi\}$. Then f(A) is a neutrosophic AMR-subalgebra of Y.*

Proof. let $\acute{\varsigma}, \acute{\varepsilon} \in \varpi$.
Now

$$\begin{aligned} W_{AT}(\acute{\varsigma} * \acute{\varepsilon}) &= W_{AT}(f(\acute{\varsigma} * \acute{\varepsilon})) \\ &= W_{AT}(f(\acute{\varsigma}) * f(\acute{\varepsilon})) \\ &\geq max\{W_{AT}(f(\acute{\varsigma})), W_{AT}(f(\acute{\varepsilon}))\} \\ &= max\{(W_{AT,I_f})(\acute{\varsigma}), (W_{AT,I_f})(\acute{\varepsilon})\} \end{aligned}$$

and

$$\begin{aligned} W_{AI}(\acute{\varsigma} * \acute{\varepsilon}) &= W_{AI}(f(\acute{\varsigma} * \acute{\varepsilon})) \\ &= W_{AI}(f(\acute{\varsigma}) * f(\acute{\varepsilon})) \\ &\geq min\{W_{AI}(f(\acute{\varsigma})), W_{AI}(f(\acute{\varepsilon}))\} \\ &= min\{(W_{AIf})(\acute{\varsigma}), (W_{AIf})(\acute{\varepsilon})\} \end{aligned}$$

Similarly,

$$\begin{aligned} W_{AF}(\acute{\varsigma} * \acute{\varepsilon}) &= W_{AF}(f(\acute{\varsigma} * \acute{\varepsilon})) \\ &= W_{AF}(f(\acute{\varsigma}) * f(\acute{\varepsilon})) \\ &\leq max\{W_{AF}(f(\acute{\varsigma})), W_{AF}(f(\acute{\varepsilon}))\} \\ &= max\{(W_{AFf})(\acute{\varsigma}), (W_{AFf})(\acute{\varepsilon})\} \end{aligned}$$

Hence $f(A)$ is a neutrosophic AMR-subalgebra of Y. □

Theorem 3.3. *Let $f : \varpi \rightarrow Y$ be a homomorphism of AMR-algebra of ϖ into a AMR-algebra Y. If $A = \{\langle \acute{\varsigma}, W_{AT}(\acute{\varsigma}), W_{AI}(\acute{\varsigma}), W_{AF}(\acute{\varsigma})\rangle : \acute{\varsigma} \in \varpi\}$ is a neutrosophic AMR-subalgebra of ϖ, then the image $f(A) = \{\langle \acute{\varsigma}, f_{inf}W_{AT}, f_{sup}W_{AI}, f_{inf}W_{AF}\rangle : \acute{\varsigma} \in \varpi\}$ of A under f is a neutrosophic AMR-subalgebra of Y.*

Proof. Let $A = \{\langle \acute{\varsigma}, W_{AT}(\acute{\varsigma}), W_{AI}(\acute{\varsigma}), W_{AF}(\acute{\varsigma})\rangle : \acute{\varsigma} \in \varpi\}$ is a neutrosophic AMR-subalgebra of ϖ and let $\acute{\varepsilon}_1, \acute{\varepsilon}_2 \in Y$.
therefore $\{\acute{\varsigma}_1 * \acute{\varsigma}_2 : \acute{\varsigma}_1 \in f^{-1}(\acute{\varepsilon}_1), \acute{\varsigma}_2 \in f^{-1}(\acute{\varepsilon}_2)\} \subseteq \{\acute{\varsigma} \in \varpi : \acute{\varsigma} \in f^{-1}(\acute{\varepsilon}_1 * \acute{\varepsilon}_2)\}$.
Now,

$$\begin{aligned} f_{inf}\{W_{AT}(\acute{\varepsilon}_1 * \acute{\varepsilon}_2)\} &= inf\{W_{AT}(\acute{\varsigma})/\acute{\varsigma} \in f^{-1}(\acute{\varepsilon}_1 * \acute{\varepsilon}_2)\} \\ &\geq inf\{W_{AT}(\acute{\varsigma}_1 * \acute{\varsigma}_2)/\acute{\varsigma}_1 \in f^{-1}(\acute{\varepsilon}_1), \acute{\varsigma}_2 \in f^{-1}(\acute{\varepsilon}_2)\} \\ &\geq inf\{max\{W_{AT}(\acute{\varsigma}_1), W_{AT}(\acute{\varsigma}_2)\}, \acute{\varsigma}_1 \in f^{-1}(\acute{\varepsilon}_1), \acute{\varsigma}_2 \in f^{-1}(\acute{\varepsilon}_2)\} \\ &= max\{inf\{W_{AT}(\acute{\varsigma}_1)/\acute{\varsigma}_1 \in f^{-1}(\acute{\varepsilon}_1)\}, inf\{W_{AT}(\acute{\varsigma}_2)/\acute{\varsigma}_2 \in f^{-1}(\acute{\varepsilon}_2)\}\} \\ &= max\{f_{inf}(W_{AT}(\acute{\varepsilon}_1)), f_{inf}(W_{AT}(\acute{\varepsilon}_2))\} \end{aligned}$$

and

$$\begin{aligned}
f_{sup}\{W_{AI}(\acute{\varepsilon}_1 * \acute{\varepsilon}_2)\} &= sup\{W_{AI}(\acute{\varsigma})/\acute{\varsigma} \in f^{-1}(\acute{\varepsilon}_1 * \acute{\varepsilon}_2)\} \\
&\geq sup\{W_{AI}(\acute{\varsigma}_1 * \acute{\varsigma}_2)/\acute{\varsigma}_1 \in f^{-1}(\acute{\varepsilon}_1), \acute{\varsigma}_2 \in f^{-1}(\acute{\varepsilon}_2)\} \\
&\geq sup\{min\{W_{AI}(\acute{\varsigma}_1), W_{AI}(\acute{\varsigma}_2)\}, \acute{\varsigma}_1 \in f^{-1}(\acute{\varepsilon}_1), \acute{\varsigma}_2 \in f^{-1}(\acute{\varepsilon}_2)\} \\
&= min\{sup\{W_{AI}(\acute{\varsigma}_1)/\acute{\varsigma}_1 \in f^{-1}(\acute{\varepsilon}_1)\}, sup\{W_{AI}(\acute{\varsigma}_2)/\acute{\varsigma}_2 \in f^{-1}(\acute{\varepsilon}_2)\}\} \\
&= min\{f_{sup}(W_{AI}(\acute{\varepsilon}_1)), f_{sup}(W_{AI}(\acute{\varepsilon}_2))\}
\end{aligned}$$

$$\begin{aligned}
f_{inf}\{W_{AF}(\acute{\varepsilon}_1 * \acute{\varepsilon}_2)\} &= inf\{W_{AF}(\acute{\varsigma})/\acute{\varsigma} \in f^{-1}(\acute{\varepsilon}_1 * \acute{\varepsilon}_2)\} \\
&\leq inf\{W_{AF}(\acute{\varsigma}_1 * \acute{\varsigma}_2)/\acute{\varsigma}_1 \in f^{-1}(\acute{\varepsilon}_1), \acute{\varsigma}_2 \in f^{-1}(\acute{\varepsilon}_2)\} \\
&\leq inf\{max\{W_{AF}(\acute{\varsigma}_1), W_{AF}(\acute{\varsigma}_2)\}, \acute{\varsigma}_1 \in f^{-1}(\acute{\varepsilon}_1), \acute{\varsigma}_2 \in f^{-1}(\acute{\varepsilon}_2)\} \\
&= max\{inf\{W_{AF}(\acute{\varsigma}_1)/\acute{\varsigma}_1 \in f^{-1}(\acute{\varepsilon}_1)\}, inf\{W_{AF}(\acute{\varsigma}_2)/\acute{\varsigma}_2 \in f^{-1}(\acute{\varepsilon}_2)\}\} \\
&= max\{f_{inf}(W_{AF}(\acute{\varepsilon}_1)), f_{inf}(W_{AF}(\acute{\varepsilon}_2))\}
\end{aligned}$$

□

Theorem 3.4. *Let ϖ and Y be two neutrosophic sets. Let $f : \varpi \to Y$ be a homomorphism. If $B = \{\langle \acute{\varsigma}, W_{BT}(\acute{\varsigma}), W_{BI}(\acute{\varsigma}), W_{BF}(\acute{\varsigma})\rangle : \acute{\varsigma} \in \varpi\}$ is a neutrosophic AMR-subalgebra of Y, then $f^{-1}(B)$ is a neutrosophic AMR-subalgebra of ϖ.*

Proof. Let B be a neutrosophic AMR-subalgebra of Y.
Let $\acute{\varsigma}, \acute{\varepsilon} \in Y$.
Then

$$\begin{aligned}
f^{-1}(W_{AT})(\acute{\varsigma} * \acute{\varepsilon}) &= W_{AT}(f(\acute{\varsigma} * \acute{\varepsilon})) \\
&= W_{AT}(f(\acute{\varsigma}) * f(\acute{\varepsilon})) \\
&= max\{W_{AT}(f(\acute{\varsigma})), W_{AT}(f(\acute{\varepsilon}))\} \\
&\geq max\{f^{-1}(W_{AT}), f^{-1}(W_{AT}(\acute{\varepsilon}))\}
\end{aligned}$$

and,

$$\begin{aligned}
f^{-1}(W_{AI})(\acute{\varsigma} * \acute{\varepsilon}) &= W_{AI}(f(\acute{\varsigma} * \acute{\varepsilon})) \\
&= W_{AI}(f(\acute{\varsigma}) * f(\acute{\varepsilon})) \\
&= min\{W_{AI}(f(\acute{\varsigma})), W_{AI}(f(\acute{\varepsilon}))\} \\
&\geq min\{f^{-1}(W_{AI}), f^{-1}(W_{AI}(\acute{\varepsilon}))\}
\end{aligned}$$

$$\begin{aligned}
f^{-1}(W_{AF})(\acute{\varsigma} * \acute{\varepsilon}) &= W_{AF}(f(\acute{\varsigma} * \acute{\varepsilon})) \\
&= W_{AF}(f(\acute{\varsigma}) * f(\acute{\varepsilon})) \\
&\leq max\{W_{AF}(f(\acute{\varsigma})), W_{AF}(f(\acute{\varepsilon}))\} \\
&= max\{f^{-1}(W_{AF})(\acute{\varsigma}), f^{-1}(W_{AF})(\acute{\varepsilon})\}
\end{aligned}$$

therefore
$f^{-1}(B)$ is a neutrosophic AMR-subalgebra of ϖ. □

Theorem 3.5. *The R_1 and R_2 are two neutrosophic AMR-subalgebras of ϖ, then $R_1 \cap R_2$ is a neutrosophic AMR-subalgebra of ϖ.*

Proof. R_1 and R_2 are two neutrosophic AMR-subalgebras of ϖ. Now,

$$\begin{aligned}(\wedge_i W_{AiT})(\acute{\varsigma} * \acute{\varepsilon}) &= max\{W_{AiT}(\acute{\varsigma} * \acute{\varepsilon}), W_{AiT}(\acute{\varsigma} * \acute{\varepsilon})\} \\ &\geq \{max\{W_{AiT}(\acute{\varsigma}), W_{AiT}(\acute{\varepsilon})\}\} \\ &= max\{inf\ W_{AiT}(\acute{\varsigma}), inf\ W_{AiT}(\acute{\varepsilon})\} \\ &= max\{\wedge_i W_{AiT}(\acute{\varsigma}), \wedge_i W_{AiT}(\acute{\varepsilon})\}\end{aligned}$$

and,

$$\begin{aligned}(\wedge_i W_{AiI})(\acute{\varsigma} * \acute{\varepsilon}) &= min\{W_{AiI}(\acute{\varsigma} * \acute{\varepsilon}), W_{AiI}(\acute{\varsigma} * \acute{\varepsilon})\} \\ &\geq \{min\{W_{AiI}(\acute{\varsigma}), W_{AiI}(\acute{\varepsilon})\}\} \\ &= min\{sup\ W_{AiI}(\acute{\varsigma}), sup\ W_{AiI}(\acute{\varepsilon})\} \\ &= min\{\wedge_i W_{AiI}(\acute{\varsigma}), \wedge_i W_{AiI}(\acute{\varepsilon})\}\end{aligned}$$

$$\begin{aligned}(\wedge_i W_{AiF})(\acute{\varsigma} * \acute{\varepsilon}) &= inf\ W_{AiF}(\acute{\varsigma} * \acute{\varepsilon}) \\ &\leq inf\ \{max\{W_{AiF}(\acute{\varsigma}), W_{AiF}(\acute{\varepsilon})\}\} \\ &= max\{inf\ W_{AiF}(\acute{\varsigma}), inf\ W_{AiF}(\acute{\varepsilon})\} \\ &= max\{\wedge_i W_{AiF}(\acute{\varsigma}), \wedge_i W_{AiF}(\acute{\varepsilon})\}\end{aligned}$$

□

4. α-cut on Neutrosophic AMR-subalgebras

This section, classifies the AMR-subalgebras by their family level on neutrosophic AMR-subalgebras of a AMR-algebra.

Definition 4.1. *Let A be a neutrosophic AMR-subalgebra of ϖ, $\alpha \in [0,1]$. Then $A_\alpha = \{\acute{\varsigma} \in \varpi : W_{AT} \geq \alpha, W_{AI} \geq \alpha, W_{AF} \leq \alpha\}$ is called an $\alpha-$cut neutrosophic AMR-subalgebras of A.*

Theorem 4.1. *If $A = \{\acute{\varsigma}, W_{AT}(\acute{\varsigma}), W_{AI}(\acute{\varsigma}), W_{AF}(\acute{\varsigma}) : \acute{\varsigma} \in \varpi\}$ is a neutrosophic AMR-subalgebra in ϖ, then A_α is a subalgebra of ϖ, for every $\alpha \in [0,1]$*

Proof. **Proof:**
For $\acute{\varsigma}, \acute{\varepsilon} \in A_\alpha$ and $W_{AT}(\acute{\varsigma}) \geq \alpha$

$$\begin{aligned}W_{AT}(\acute{\varsigma} * \acute{\varepsilon}) &\geq max\{W_{AT}(\acute{\varsigma}), W_{AT}(\acute{\varepsilon})\} \\ &\geq max\{\alpha, \alpha\} \\ &\geq \alpha\end{aligned}$$

$\Rightarrow \acute{\varsigma} * \acute{\varepsilon} \in A_\alpha$
For $\acute{\varsigma}, \acute{\varepsilon} \in A_\alpha$ and $W_{AI}(\acute{\varsigma}) \geq \alpha$

$$\begin{aligned}W_{AI}(\acute{\varsigma} * \acute{\varepsilon}) &\geq min\{W_{AI}(\acute{\varsigma}), W_{AI}(\acute{\varepsilon})\} \\ &\geq min\{\alpha, \alpha\} \\ &\geq \alpha\end{aligned}$$

$\Rightarrow \acute{\varsigma} * \acute{\varepsilon} \in A_\alpha$
For $\acute{\varsigma}, \acute{\varepsilon} \in A_\alpha$ and $W_{AF}(\acute{\varsigma}) \leq \alpha$

$$\begin{aligned} W_{AF}(\acute{\varsigma} * \acute{\varepsilon}) &\leq max\{W_{AF}(\acute{\varsigma}), W_{AF}(\acute{\varepsilon})\} \\ &\leq max\{\alpha, \alpha\} \\ &\leq \alpha \end{aligned}$$

$\Rightarrow \acute{\varsigma} * \acute{\varepsilon} \in A_\alpha$
Hence A_α is subalgebra of ϖ. □

Theorem 4.2. *Let $A = \{\acute{\varsigma}, W_{AT}(\acute{\varsigma}), W_{AI}(\acute{\varsigma}), W_{AF}(\acute{\varsigma}) : \acute{\varsigma} \in \varpi\}$ is a neutrosophic set in ϖ such that A_α is a subalgebra of ϖ for every $\alpha \in [0,1]$ & $\alpha \in [0,1]$. Then A is a neutrosophic AMR-subalgebra of ϖ.*

Proof. Let $A = \{\acute{\varsigma}, W_{AT}(\acute{\varsigma}), W_{AI}(\acute{\varsigma}), W_{AF}(\acute{\varsigma}) : \acute{\varsigma} \in \varpi\}$ is a neutrosophic set in ϖ.
Since A_α is a subalgebra of ϖ for $\alpha \in [0,1]$
$\acute{\varsigma} * \acute{\varepsilon} \in A_\alpha$
Now, take $\alpha = max\{W_{AT}(\acute{\varsigma}), W_{AT}(\acute{\varepsilon})\}$, $\alpha = min\{W_{AI}(\acute{\varsigma}), W_{AI}(\acute{\varepsilon})\}$ and $\alpha = max\{W_{AF}(\acute{\varsigma}), W_{AF}(\acute{\varepsilon})\}$
$\Rightarrow \acute{\varsigma} * \acute{\varepsilon} \in A_\alpha \Rightarrow W_{AT}(\acute{\varsigma} * \acute{\varepsilon}) \geq \alpha$, $W_{AI}(\acute{\varsigma} * \acute{\varepsilon}) \geq \alpha$ and $W_{AF}(\acute{\varsigma} * \acute{\varepsilon}) \leq \alpha$
therefore $W_{AT}(\acute{\varsigma} * \acute{\varepsilon}) \geq max\{W_{AT}(\acute{\varsigma}), W_{AT}(\acute{\varepsilon})\}$,$W_{AI}(\acute{\varsigma} * \acute{\varepsilon}) \geq min\{W_{AI}(\acute{\varsigma}), W_{AI}(\acute{\varepsilon})\}$
Also,
$W_{AF}(\acute{\varsigma} * \acute{\varepsilon}) \leq max\{W_{AF}(\acute{\varsigma}), W_{AF}(\acute{\varepsilon})\}$ Hence A is a neutrosophic AMR-subalgebra of ϖ. □

5. Algorithm for Neutrosophic AMR-algebras

In this section, we present an algorithm to check the conditions of neutrosophic AMR-algebras using the values in between 0 and 1.

```
import random
import numpy as np

n1 = round(random.uniform(0.1, 0.9), 1)
n2 = round(random.uniform(0.1, n1), 1)
my_list = ['p1', 'p2', 'p3']
my_list1=['0','p3']
```

```
def tr(x):

    if x in my_list:
        return 0.7
    else:
        return 0.5
```

```
n1 = round(random.uniform(0.1, 0.9), 1)
n2 = round(random.uniform(0.1, n1), 1)


def In(x):

    if x in my_list1:
        return 0.6
    else:
```

```
print("Sample 5x5 Matrix")
ar = [['*', '0', 'p1', 'p2', 'p3'],
      ['0', '0', 'p1', 'p2', 'p3'],
      ['p1', 'p1', 'p2', 'p3', 'p1'],
      ['p2', 'p2', 'p3', 'p1', 'p2'],
      ['p3', 'p3', 'p1', 'p2', 'p3']]
i,j=1,1
print(np.matrix(ar))
# print the 2D character array
for i in range(rows):
    for j in range(cols):
        #print("check true portion", ar[i][j])
        y=tr(ar[i][j])
        if y >= max(tr(ar[i][0]), tr(ar[0][j])): #check true portion
            res.append("tr_true")
        else:
            res.append("tr_false")
        #print("check intermediate portion",ar[i][j])
        y=In(ar[i][j])
        if y >= min(In(ar[i][0]), In(ar[0][j])): #check intermediate portion
            res1.append("tr_true")
        else:
            res1.append("tr_false")
       # print("check false portion",ar[i][j])
        y=fl(ar[i][j])
```

```
Sample 5x5 Matrix
[['*' '0' 'p1' 'p2' 'p3']
 ['0' '0' 'p1' 'p2' 'p3']
 ['p1' 'p1' 'p2' 'p3' 'p1']
 ['p2' 'p2' 'p3' 'p1' 'p2']
 ['p3' 'p3' 'p1' 'p2' 'p3']]
Test TRUE PORTION
Input Max: 0.7 & Min:0.5
[['tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true'
  'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true'
  'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true'
  'tr_true' 'tr_true' 'tr_true' 'tr_true']]
Input Max: 0.6 & Min:0.4
Test INTERMEDIATE PORTION
[['tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true'
  'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true'
  'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true'
  'tr_true' 'tr_true' 'tr_true' 'tr_true']]
Input Max: 0.8 & Min:0.2
Test FALSE PORTION
[['tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true'
  'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true'
  'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true' 'tr_true'
  'tr_true' 'tr_true' 'tr_true' 'tr_true']]
```

6. Conclusions

This study illustrates several attractive and captivating properties of neutrosophic AMR- algebra with appropriate examples. In sequel, the findings on image, inverse image and intersection of neutrosophic AMR- algebra have been disclosed. Furthermore, the level set on neutrosophic AMR algebra and their associated outcomes were dealt. Especially, an algorithm is described to validate the criteria of the neutrosophic AMR algebra for the specific values between 0 and 1. This can be expanded to other algebraic structures in the future.

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