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ON THE COVERING RADIUS OF SOME CLASSES OF DNA CODE OVER FINITE RING

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Abstract. In this correspondance, investigate the covering radius of different types of repetition codes over finite ring(R) with Bachoc distance. Derive the lower bound and upper bound on the covering radius of block repetition DNA codes over R. Also determined are the covering radius of various repetition DNA codes, simplex DNA code type α and simplex DNA code type β and bounds on the covering radius for macdonald DNA codes of both types over R.

Keywords: DNA code, finite ring, covering radius, simplex codes.

AMS Subject Classification: 94B25, 94B05, 11H31, 11H71.

1. Introduction

DNA is found in all living beings as a double stranded molecule, with a form similar to a twisted ladder. This double stranded helix consists of an alternating chains of sugars, phosphates and nitrogenous bases like Adenine(A), Thiamine(T), Cytosine (C) and Guanine (G). The association between these two strands are a alternating combinations of these four nitrogenous bases.

The two ends of the strand are distinct and are conventionally denoted as $3'$ end and $5'$ end. Two strands of DNA can form (under suitable conditions) a double strand if the respective bases complement of each other, where A matches with T and C matches with G [18].

The problem of designing DNA codes (sets of words of fixed length n over the alphabets $\{A, C, G, T\}$) that satisfy certain combinatorial constraints has applications for reliably storing and retrieving information in synthetic DNA strands. These codes can be used in particular for DNA computing[1] or as molecular bar-codes.

There are many researchers doing research on code over finite rings. In particular, codes over $\mathbb{Z}_4$ received much attention [2, 4, 5, 10, 12, 16, 17, 6]. The covering radius of binary

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linear codes were studied [4, 5], Recently the covering radius of codes over $\mathbb{Z}_4$ has been investigated with various distances [13]. In [2] Sole et al. gave many upper and lower bounds on the covering radius of a code over $\mathbb{Z}_4$ with different distances. In [15, 6], the covering radius of some particular codes over $\mathbb{Z}_4$ have been investigated.

This paper, investigates the covering radius of the simplex DNA codes of both types and macdonald DNA codes of both types alone with repetition DNA codes over R. Also, it generalizes some of the known bounds in [2].

2. Preliminary

Coding theory has several applications in genetics and bioengineering. The problem of designing DNA codes (sets of that words of fixed length n over the alphabet of finite ring$(R) = \{A, C, G, T\}$ that satisfy certain combinatorial constraints) has applications for reliably storing and retrieving information in synthetic DNA strands.

A DNA code is a subset of R^n, where n is a length of DNA code. The codewords of DNA code is $(x_1, x_2, \cdots, x_n)$ with $x_i \in R$(representing the four nucleotides in DNA). Use a hat to denote the Watson-Crick complement of a nucleotide, so A matches with T and C matches with G. Let $R = \{A, C, G, T\}$ be the finite alphabet and the DNA codes are sets of codeword with fixed length n over the alphabet. That is, $C = \{AAAA, AACC, CCGG, GGTT, TTTT\}$, therefore each codeword of length is 4. It follows the map $A \to 0, C \to 1, T \to 2$ and $G \to 3$. Therefore the problem of the DNA codes is corresponding to the problem of the $\mathbb{Z}_4$-linear codes. These transpositions do not affect the GC-weight of the codeword (the number of entries that are C or G). In my work, by using the above map in $\mathbb{Z}_4$ with bachoc weight, so obtain the covering radius for repetition DNA codes.

Let $d = (d_1, d_2, \cdots, d_n) \in R^n$ and n be its length. Let b be an element of $\{A, C, G, T\}$. For all $d = (d_1, d_2, \cdots, d_n) \in R^n$, define the weight of d at b to be $w_b(d) = |\{i|x_i = b\}|$.

A DNA linear code C of length n over R is an additive subgroup of R^n. An element of C is called a DNA codeword of C and a generator matrix of C is a matrix whose rows generate C. In [3], the Bachoc weight $w(x)$ of a vector x is 0 if $x_i = 0$; 1 if $x_i = 1$ and 2 if $x_i = 2,\ 3$. A linear Gray map φ from $R \to \mathbb{Z}_2^2$ is defined by $\varphi(x + 2y) = (y, x + y)$, for all $x + 2y \in R$, that is, $\varphi(0) = (0, 0), \varphi(1) = (0, 1), \varphi(2) = (1, 1), \varphi(3) = (1, 0)$. The image $\varphi(C)$, of a linear code C over R of length n by the Gray map is a binary code of length $2n$ with same cardinality [16].

Any DNA linear code C over R is equivalent to a code with Generator Matrix(GM) of the form

$$GM = \begin{bmatrix} I_{k_0} & A & B \\ 0 & 2I_{k_1} & 2D \end{bmatrix}, \text{ where } A, B \text{ and } D \text{ are matrices over } R.$$

Then the DNA code C contain all DNA codewords $[v_0, v_1]\, GM$, where v_0 is a vector of length k_1 over R and v_1 is a vector of length k_2 over $\mathbb{Z}_2$. Thus C contains a total of

$4^{k_1}2^{k_2}$ codewords. The parameters of C are given $\left[n, 4^{k_1}2^{k_2}, d\right]$, where d represents the minimum Bachoc distance of C.

A DNA linear code C over R of length n, 2-dimension k, minimum bachoc distance d is called an $[n, k, d_b]$ or simply an $[n, k, d]$ code.

Section 3, give a covering radius of repetition DNA codes, type repetition DNA codes and determines the covering radius of different types of repetition DNA codes and two block, three block repetition DNA codes with bachoc weight. Obtain the covering radius of simplex DNA codes α type and β type of R are section 4 and finally section 5 explain to macDonald DNA codes α type and β type macDonald DNA codes α type and β type determines the bounds on the covering radius of macDonald DNA codes α type and β type of R.

3. Covering Radius of Repetition DNA Codes

Let d be a bachoc distance of DNA code C over R. Thus, the covering radius of C :

$$r_d(C) = \max_{u\in R^n}\left\{\min_{c\in C}\{d(c,u)\}\right\}.$$

The following result of Mattson [7] is useful for computing covering radius of codes over rings generalized easily from codes over finite fields.

Proposition 3.1. *If C_0 and C_1 are codes over R generated by matrices GM_0 and GM_1 respectively and if C is the code generated by $GM = \left(\begin{array}{c|c} 0 & GM_1 \\ \hline GM_0 & A \end{array}\right)$ then $r_d(C) \leq r_d(C_0) + r_d(C_1)$ and the covering radius of D (concatenation of C_0 and C_1) satisfy the following $r_d(D) \geq r_d(C_0) + r_d(C_1)$, here d is a Bachoc distance in R.*

A q-ary repetition code C over a finite field $\mathbb{F}_q = \{\alpha_0 = 0, \alpha_1 = 1, \alpha_2, \alpha_3, \cdots, \alpha_{q-1}\}$ is an $[n, 1, n]$ code $C = \{\bar{\alpha} \mid \alpha \in \mathbb{F}_q\}$, where $\bar{\alpha} = (\alpha, \alpha, \cdots, \alpha)$. The covering radius of C is $\left\lceil \frac{n(q-1)}{q} \right\rceil$ [11]. Using this, it can be seen easily that the covering radius of block of size n repetition code $[n(q-1), 1, n(q-1)]$ generated by

$$GM = [\overbrace{11\cdots1}^{n}\overbrace{\alpha_2\alpha_2\cdots\alpha_2}^{n}\overbrace{\alpha_3\alpha_3\cdots\alpha_3}^{n}\cdots\overbrace{\alpha_{q-1}\alpha_{q-1}\cdots\alpha_{q-1}}^{n}]$$

is $\left\lceil \frac{n(q-1)^2}{q} \right\rceil$, since it will be equivalent to a repetition code of length $(q-1)n$.

Consider the repetition DNA code over R. There are two types of them of length n viz.

- cytosine repetition code $C_\beta : [n, 1, n]$ generated by $GM_\beta = [\overbrace{1\ 1\cdots1}^{n}]$
- thymine repetition code $C_\alpha : (n, 2, 2n)$ generated by $GM_\alpha = [\overbrace{2\ 2\cdots2}^{n}]$.

Theorem 3.1. *Let C_β and C_α be the DNA code of types β and α in generator matrices GM_β and GM_α. Then, $2\left\lfloor \frac{n}{2} \right\rfloor \leq r(C_\alpha) \leq 2n$ and $n \leq r(C_\beta) \leq 2n$.*

Proof. Let $x = \overbrace{22\cdots2}^{\lfloor\frac{n}{2}\rfloor}\overbrace{00\cdots0}^{\lceil\frac{n}{2}\rceil}$ and the code of $C = \{00\cdots0, 22\cdots2\}$ is generated by $[22\cdots2]$ is an $[n, 1, 2n]$ code. Then, $d(x, 00\cdots0) = wt(x{-}00\cdots0) = 2\left\lceil \frac{n}{2} \right\rceil$ and $d(x, 22\cdots2) = wt(x - 22\cdots2) = 2\left\lfloor \frac{n}{2} \right\rfloor$. Therefore $d(x, C_\alpha) = \min\left\{2\left\lceil \frac{n}{2} \right\rceil, 2\left\lfloor \frac{n}{2} \right\rfloor\right\}$. Thus, by definition of covering radius

$$r(C_\alpha) \geq 2\left\lfloor \frac{n}{2} \right\rfloor \tag{1}$$

Let x be any word in R^n. Let us take x has ω_0 coordinates as 0's, ω_1 coordinates as 1's, ω_2 coordinates as 2's, ω_3 coordinates as 3's, then $\omega_0+\omega_1+\omega_2+\omega_3=n$. Since $C_\alpha=\{00\cdots 0, 22\cdots 2\}$ and Bachoc weight of $R:0$ is $0,1$ is 1, 2 and 3 are 2. Therefore, $d(x,00\cdots 0)=n-\omega_0+\omega_2+\omega_3$ and $d(x,TT\cdots T)=n-\omega_2+\omega_0+\omega_3$.

Thus $d\left(x,C_\alpha\right)=\min\left\{n-\omega_0+\omega_2+\omega_3, n-\omega_2+\omega_0+\omega_3\right\}$ and hence,

$$d\left(x,C_\alpha\right)\leq n+n=2n. \tag{2}$$

Hence, from the Equation (1) and (2), so $2\left\lfloor\frac{n}{2}\right\rfloor\leq r\left(C_\alpha\right)\leq 2n$.

Now, obtain the covering radius of C_β covering with respect to the bachoc weight. Then $d(x,00\cdots 0)=n-\omega_0+\omega_2+\omega_3, d(x,11\cdots 1)=n-\omega_1+\omega_2+\omega_3, d(x,22\cdots 2)=n-\omega_2+\omega_0+\omega_3$ and $d(x,33\cdots 3)=n-\omega_3+\omega_0+\omega_1$, for any $x\in R^n$.

This implies $d(x,C_\beta)=\min\{n-\omega_0+\omega_2+\omega_3, n-\omega_1+\omega_2+\omega_3, n-\omega_2+\omega_0+\omega_3, n-\omega_3+\omega_0+\omega_1\}\leq 2n$ and hence $r(C_\beta)\leq 2n$.

Let $x=\overbrace{00\cdots 1}^{t}\overbrace{11\cdots 1}^{t}\overbrace{22\cdots 2}^{t}\overbrace{33\cdots 3}^{n-3t}$, where $t=\left\lfloor\frac{n}{4}\right\rfloor$,
then $d(x,00\cdots 0)=2n-3t, d(x,11\cdots 1)=2n-4t, d(x,22\cdots 2)=n$ and $d(x,33\cdots 3)=5t$.

Therefore $r\left(C_\beta\right)\geq\min\{2n-3t, 2n-4t, n, 5t\}\geq n$.

□

Block Repetition Code. Let $GM=[\overbrace{11\cdots 1}^{n}\overbrace{22\cdots 2}^{n}\overbrace{33\cdots 3}^{n}]$ be a generator matrix of R in each block of repetition code length is n. Then, the parameters of Block Repetition Code(BRC) is $[3n,1,4n]$. The code of $BRC=\{c_0=0\cdots 00\cdots 00\cdots 0, c_1=1\cdots 12\cdots 23\cdots 3, c_2=2\cdots 20\cdots 02\cdots 2, c_3=3\cdots 32\cdots 22\cdots 1\}$, dimension of BRC is 1 and bachoc weight is $4n$. Obtain, the following

Theorem 3.2. $2\lfloor\frac{n}{2}\rfloor+2n\leq r\left(BRC^{3n}\right)\leq 4n$.

Proof. Let $x=11\cdots 1\in R^{3n}$. Then, $d(x,BRC^{3n})=4n$. Hence by definition, $r\left(BRC^{3n}\right)\geq 2\lfloor\frac{n}{2}\rfloor+2n$.

Let $x=(u|v|w)\in R^{3n}$ with u,v and w have compositions $(r_0,r_1,r_2,r_3),(s_0,s_1,s_2,s_3)$ and (t_0,t_1,t_2,t_3) respectively such that $\sum_{i=0}^{3}r_i=n, \sum_{i=0}^{3}s_i=n$ and $\sum_{i=0}^{3}t_i=n$, then

$$d(x,c_0)=3n-r_0+r_2+r_3-s_0+s_2+s_3-t_0+t_2+t_3,$$
$$d(x,c_1)=3n-r_1+r_2+r_3-s_2+s_0+s_1-t_3+t_0+t_1,$$
$$d(x,c_2)=3n-r_2+r_0+r_1-s_0+s_2+s_3-t_2+t_0+t_1$$

and

$$d(x,c_3)=3n-r_3+r_0+r_2-s_2+s_0+s_1-t_1+t_2+t_3.$$

Thus, $d(x,BRC^{3n})=\min\{3n-r_0+r_2+r_3-s_0+s_2+s_3-t_0+t_2+t_3, 3n-r_1+r_2+r_3-s_2+s_0+s_1-t_3+t_0+t_1, 3n-r_2+r_0+r_1-s_0+s_2+s_3-t_2+t_0+t_1, 3n-r_3+r_0+r_2-s_2+s_0+s_1-t_1+t_2+t_3\}\leq 4n$ and hence, $r\left(BRC^{3n}\right)\leq 4n$. □

Define a two block repetition DNA code over R of each of length is n and the parameters of two block repetition code $\mathrm{BRC}^{2n}:[2n,1,2n]$ is generated by $G=[\overbrace{11\cdots 1}^{n}\overbrace{22\cdots 2}^{n}]$. Use the above and obtain a following

Theorem 3.3. $2\lfloor \frac{n}{2} \rfloor + n \le r\left(BRC^{2n}\right) \le 4n$.

Let $GM = [\overbrace{11\cdots 1}^{m}\overbrace{22\cdots 2}^{n}]$ be the generalized generator matrix for two different block repetition dna code of length are m and n respectively. In the parameters of two different block repetition code(BRC^{m+n}) are $[m+n, 1, min\{m, 2m+n\}]$ and Theorem 3.3 can be easily generalized for two different length using similar arguments to the following.

Theorem 3.4. $2\lfloor \frac{n}{2} \rfloor + m \le r\left(BRC^{2n}\right) \le 2n + 2m$.

4. Simplex DNA Code of type α and type β over R

In ref.[4] has been studied of quaternary simplex codes of type α and type β. Type α Simplex code S_k^α is a linear DNA code over R with parameters $\left[4^k, k\right]$ and an inductive generator matrix given by

$$GM_k^\alpha = \left[\begin{array}{c|c|c|c} 0\cdots 0 & 1\cdots 1 & 2\cdots 2 & 3\cdots 3 \\ \hline GM_{k-1}^\alpha & GM_{k-1}^\alpha & GM_{k-1}^\alpha & GM_{k-1}^\alpha \end{array}\right] \tag{3}$$

with $GM_1^\alpha = [0\ 1\ 2\ 3]$. Type simplex code S_k^β is a punctured version of S_k^α with parameters $[2^{k-1}, \left(2^k - 1\right), k]$ and an inductive generator matrix given by

$$GM_2^\beta = \left[\begin{array}{cccc|c|c} 1 & 1 & 1 & 1 & 0 & 2 \\ \hline 0 & 1 & 2 & 3 & 1 & 1 \end{array}\right] \tag{4}$$

$$GM_k^\beta = \left[\begin{array}{c|c|c} 11\cdots 1 & 00\cdots 0 & 22\cdots 2 \\ \hline GM_{k-1}^\alpha & GM_{k-1}^\beta & GM_{k-1}^\beta \end{array}\right] \tag{5}$$

and for $k > 2$, where GM_{k-1}^α is the generator matrix of S_{k-1}^α. For details the reader is refered to [4]. Type α DNA code with minimum bachoc weight is 4.

Theorem 4.1. $r(S_k^\alpha) \le \frac{2^{2(k+1)}-1}{3}$.

Proof. Let $x = 11\cdots 1 \in R^n$. By equation(3), the result of Mattson for finite rings and using Theorem 3.2, then

$$\begin{aligned} r\left(S_k^\alpha\right) &\le r\left(S_{k-1}^\alpha\right) + r(<\overbrace{11\cdots 1}^{2^{2(k-1)}}\overbrace{22\cdots 2}^{2^{2(k-1)}}\overbrace{33\cdots 3}^{2^{2(k-1)}}>) \\ &= r\left(S_{k-1}^\alpha\right) + 4.2^{2(k-1)} \\ &= 4.2^{2(k-1)} + 4.2^{2(k-2)} + 4.2^{2(k-3)} + \ldots\ldots + 4.2^{2.1} + r\left(S_1^\alpha\right) \\ r\left(S_k^\alpha\right) &\le \frac{2^{2(k+1)}-1}{3}, (\text{since } r\left(S_1^\alpha\right) = 5) \end{aligned}$$

$\square$

Theorem 4.2. $r(S_k^\beta) \le \frac{2^{2k+1}+3\cdot 4^{k-1}-9\cdot 2^{k-2}-20}{3}$

Proof. By equation(5), Proposition 3.1 and Theorem 3.4, thus

$$r\left(S_k^\beta\right) \le r\left(S_{k-1}^\beta\right) + r(<\overbrace{11\cdots 1}^{4^{(k-1)}}\quad\overbrace{22\cdots 2}^{2^{(2k-3)}-2^{(k-2)}}\quad>)$$

$$r\left(S_k^\beta\right) = r\left(S_{k-1}^\beta\right) + 2^{(2k-2)} + 2^{(2k-3)} - 2^{(k-2)}$$

$$r\left(S_k^\beta\right) \leq 2\left(2^{(2k-2)} + 2^{(2k-4)} + \ldots + 2^4\right) + 2\left(2^{(2k-3)} + 2^{(2k-5)} + \ldots + 2^3\right) - 2\left(2^{(k-2)} + 2^{(k-3)} + \ldots + 2\right) + r\left(S_2^\beta\right)$$

$$r\left(S_k^\beta\right) \leq \frac{2^{2k+1}+3\cdot 4^{k-1}-9\cdot 2^{k-2}-20}{3}, \left(\text{since } r\left(S_2^\beta\right) = 5\right).$$

□

5. MacDonald DNA codes of type α and type β Over R

Let $\mathbb{F}_q$ be a finite field with q element and the q-ary MacDonald code C in $\mathbb{F}_q$ is a unique parameters $\left[\frac{q^k-q^t}{q-1}, k, q^{k-1} - q^{t-1}\right]$ and it is denoted by $M_{k,t}(q)$ code. In which every non-zero codeword has weight either q^{k-1} or $q^{k-1} - q^{t-1}$[11]. In [14], the author studied the covering radius of MacDonald codes over a finite field and also given many exact values for smaller dimension. In [9], authors have defined the macdonald codes over a ring using the generator matrices of simplex codes. For $2 \leq t \leq k-1$, let $GM_{k,t}^\alpha$ be the matrix obtained from GM_k^α by deleting columns corresponding to the columns of GM_t^α. That is,

$$GM_{k,t}^\alpha = \left[GM_k^\alpha \backslash \frac{0}{GM_t^\alpha}\right] \tag{6}$$

and let $GM_{k,t}^\beta$ be the matrix obtained from GM_k^β by deleting columns corresponding to the columns of GM_t^β. That is,

$$GM_{k,t}^\beta = \left[GM_k^\beta \backslash \frac{0}{GM_t^\beta}\right] \tag{7}$$

where $[A\backslash B]$ denotes the matrix obtained from the matrix A by deleting the columns of the matrix B and 0 is a $(k-t) \times 2^{2t}\left((k-t) \times 2^{t-1}\left(2^t-1\right)\right)$. The code generated by the matrix $GM_{k,t}^\alpha$ is called code of type α and the code generated by the matrix $GM_{k,t}^\beta$ is called Macdonald code of type β. The type α code is denoted by $M_{k,t}^\alpha$ and the type β code is denoted by $M_{k,t}^\beta$. The $M_{k,t}^\alpha$ code is $\left[4^k - 4^t, k\right]$ code over R and $M_{k,t}^\beta$ is a $\left[\left(2^{k-1} - 2^{t-1}\right)\left(2^k + 2^t - 1\right), k\right]$ code over R. In fact, these codes are punctured code of S_k^α and S_k^β respectively.

Next Theorem gives a basic bound on the covering radius of above macdonald codes.

Theorem 5.1. $r\left(M_{k,t}^\alpha\right) \leq \frac{4^{k+1}-4^{r+1}}{3} + r\left(M_{r,t}^\alpha\right)$ *for* $t < r \leq k$.

Proof. In equation(6), Proposition 3.1 and Theorem 3.2, thus

$$\begin{aligned} r\left(M_{k,t}^\alpha\right) &\leq r(<\overbrace{11\cdots1}^{2^{2(k-1)}}\overbrace{22\cdots2}^{2^{2(k-1)}}\overbrace{33\cdots3}^{2^{2(k-1)}}>) + r\left(M_{r,t}^\alpha\right) \\ &= 4.4^{k-1} + r\left(M_{k-1,t}^\alpha\right), \text{ for } \ k \geq r > t. \\ &\leq 4.4^{k-1} + 4.4^{k-2} + \cdots + 4.4^r + r\left(M_{r,t}^\alpha\right) \text{ for } k \geq r > t \end{aligned}$$

Thus, $r\left(M_{k,t}^\alpha\right) \leq 2^{2k} - 2^{2r} + r\left(M_{r,t}^\alpha\right)$, for $k \geq r > t$.

□

Theorem 5.2. $r\left(M_{k,t}^\beta\right) \leq \frac{4^{k+1}-4^{r+1}+3(4^{k-1}-4^{r-1})+9(2^{r-1}-2^{k-1})}{6} + r\left(M_{r,t}^\beta\right)$, *for* $t < r \leq k$.

Proof. Using Proposition 3.1, Theorem 3.4 and in equation(7), obtain

$$
\begin{aligned}
r\left(M_{k,t}^{\beta}\right) &\leq r(<\overbrace{11\cdots 1}^{2^{2(k-1)}} \quad \overbrace{22\cdots 2}^{2^{2(k-1)-1}-2^{(k-1)-1}} \quad >) + r\left(M_{k-1,t}^{\beta}\right) \\
&\leq 2.2^{2(k-1)}+2.2^{2(k-1)-1} - 2.2^{(k-1)-1} + r\left(M_{k-1,t}^{\beta}\right) \\
r\left(M_{k,t}^{\beta}\right) &\leq \frac{4^{k+1}-4^{r+1}+3(4^{k-1}-4^{r-1})+9(2^{r-1}-2^{k-1})}{6} + r\left(M_{r,t}^{\beta}\right), \text{ for } \ t<r\leq k.
\end{aligned}
$$

□

6. Conclusion

This work is for finite ring with four element. This could be extended for other even numbers higher than 4. The estimation of lower bound and upper bound for each case can be through light on the nature of sets and rings of higher order. These DNA codes can be applied to complex sitution encounteded in genetic engineering.

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