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A STUDY ON GROUP ACTION ON INTUITIONISTIC FUZZY PRIMARY AND SEMIPRIMARY IDEALS

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Abstract. Group actions are a useful technique for examining the symmetry and automorphism characteristics of rings. The concept of intuitionistic fuzzy ideals in rings has been broadened with the inclusion of the concepts of intuitionistic fuzzy primary and semiprimary ideals. The purpose of this article is to extend the group action to the intuitionistic fuzzy ideals of a ring R with group action on it and to derive a relation between the intuitionistic fuzzy G-primary (G-semiprimary) ideals and the intuitionistic fuzzy primary (semiprimary) ideals of R. We established that the largest G-invariant intuitionistic fuzzy ideal contained in an intuitionistic fuzzy primary (semiprimary) ideal is an intuitionistic fuzzy G-primary (G-semiprimary) ideal of R. Conversely, if Q is an intuitionistic fuzzy G-primary (G-semiprimary) ideal of R, then there exists an intuitionistic fuzzy primary (semiprimary) ideal P of R such that $P^G = Q$. We also investigate the relationships between the intuitionistic fuzzy G-primary (G-semiprimary) ideals of R and their level cuts under this group action. Additionally, we establish a suitable characterization of intuitionistic fuzzy G-primary (G-semiprimary) ideals of R in terms of intuitionistic fuzzy points in R under this group action. In addition to these, we also investigate the preservation of the image and pre-image of an intuitionistic fuzzy G-primary (G-semiprimary) ideal of a ring under G-homomorphism.

Keywords: Intuitionistic fuzzy primary (semiprimary) ideals; Intuitionistic fuzzy G-primary (G-semiprimary) ideals; radical of an intuitionistic fuzzy G-ideals; G-homomorphism.

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1. Introduction

The notion of a G-space [8] was introduced as a consequence of an action of a group on an ordinary set under certain rules and conditions. Over the past history of Mathematics and Algebra, the theory of group action [8] has proven to be an applicable and effective mathematical framework for the study of several types of structures and making connections among them. The applications of group action can be found in different areas of science, such as physics, chemistry, biology, computer science, game theory, cryptography, etc., which have worked out very well. The abstraction provided by group actions is

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important because it allows geometrical ideas to be applied to more abstract objects. Several objects and things have been found in mathematics that have natural group actions defined for them. Specifically, groups can act on other groups or even on themselves.

Since the pioneering work of Zadeh [20] on fuzzy sets, there has been a growing interest in this field due to its wide-ranging applications in engineering and computer science (see [12]). Initially, the focus was on fuzzy set theory and fuzzy logic. However, over the past two decades, there has been increasing interest in the development of fuzzy algebra, which generalizes the well-established properties of algebraic structures. One of the prominent generalizations of fuzzy sets theory is the theory of intuitionistic fuzzy sets introduced by Atanassov [2],[3] and [4]. Biswas introduced the notion of intuitionistic fuzzy subgroup of a group in [7]. The concepts of intuitionistic fuzzy subring and ideal were introduced and studied by Hur and others in [9]. The notion like intuitionistic fuzzy (prime, primary, semiprimary, nil radical etc.) ideals were studied in [10], [13], [14], [15] and [16]. The concepts of group action on fuzzy ideal of a ring was defined and studied by Sharma, et al. in [18]. Sharma in [17] studied the group action on intuitionistic fuzzy modules. After that Yılmaz, et al. in [19] define the intuitionistic fuzzy action of a group on a set. Ali and Smarandache in [1] studied the group action through the application of fuzzy sets and Neutrosophic sets, where as Manemaran and Nagaraja applied group action on picture fuzzy soft G-modules in [11].

In this paper, we assume that R is a commutative ring with unity and consider the definition of intuitionistic fuzzy primary and semiprimary ideals of a ring R, introduced in [13] and [16], and study G-invariant intuitionistic fuzzy primary ideal of the ring R and their properties. A suitable characterization of intuitionistic fuzzy G-primary (G-semiprimary) ideals in terms of G-invariant intuitionistic fuzzy points is established. A relationship between the intuitionistic fuzzy G-primary (semiprimary) ideal with their level cuts set is studied. Finally in section 5. we elaborate the image and pre-image under G-homomorphism of intuitionistic fuzzy (primary, semiprimary) ideals. We also studied the properties of intuitionistic fuzzy G-primary and intuitionistic fuzzy G-semiprimary ideals of R under G-homomorphism.

2. Preliminaries

Throughout this paper R is a commutative ring with identity.

Definition 2.1. *([2]) "An intuitionistic fuzzy set (IFS) A in X can be represented as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to A respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$".*

Remark 2.1. *([6])*
(i) When $\mu_A(x) + \nu_A(x) = 1$, for every $x \in X$. Then A is termed as a fuzzy set.
(ii) An IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ is briefly written as $A(x) = (\mu_A(x), \nu_A(x))$, for every $x \in X$. We designate it by $IFS(X)$ the collection of all IFSs of X.
(iii) If $p, q \in [0,1]$ so that $p + q \leq 1$. Then $A \in IFS(X)$ defined by $\mu_A(x) = p$ and $\nu_A(x) = q$, for every $x \in X$, is called a constant IFS of X. Any IFS of X defined other than this is referred to as a non-constant intuitionistic fuzzy set.

"If $A, B \in IFS(X)$, then $A \subseteq B$ contingent upon $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$, for every $x \in X$ and $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$. Given any subset Y of X, the

IF-characteristic function χ_Y is an IFS of X, described as $\chi_Y(x) = (1, 0)$, for every $x \in Y$ and $\chi_Y(x) = (0, 1)$, for every $x \in X \backslash Y$. Assume that $\alpha, \beta \in [0, 1]$ such that $\alpha+\beta \leq 1$. The the crisp set $A_{(\alpha,\beta)} = \{x \in X : \mu_A(x) \geq \alpha$ and $\nu_A(x) \leq \beta\}$ is termed as the (α, β)-level cut subset of A. Further, if $f : X \to Y$ is a mapping and A, B be respectively IFS of X and Y. The image $f(A)$ is an IFS of Y is described as $\mu_{f(A)}(y) = Sup\{\mu_A(x) : f(x) = y\}$, $\nu_{f(A)}(y) = Inf\{\nu_A(x) : f(x) = y\}$, for all $y \in Y$ and the inverse image $f^{-1}(B)$ is an IFS of X is described as $\mu_{f^{-1}(B)}(x) = \mu_B(f(x))$, $\nu_{f^{-1}(B)}(x) = \nu_B(f(x))$, for every $x \in X$, i.e., $f^{-1}(B)(x) = B(f(x))$, for every $x \in X$. Also the IFS $x_{(\alpha,\beta)}$ of X defined as $x_{(\alpha,\beta)}(y) = (\alpha, \beta)$, if $y = x$, otherwise $(0, 1)$ is called the intuitionistic fuzzy point (IFP) in X with support x. By $x_{(\alpha,\beta)} \in A$ we mean $\mu_A(x) \geq \alpha$ and $\nu_A(x) \leq \beta$" [16].

Definition 2.2. *([9]) "Let $A \in IFS(R)$. Then A is termed as an intuitionistic fuzzy ideal (IFI) of ring R, if for every $r, s \in R$, the subsequent conditions hold*
(i) $\mu_A(r - s) \geq \min\{\mu_A(r), \mu_A(s)\}$;
(ii) $\mu_A(rs) \geq \max\{\mu_A(r), \mu_A(s)\}$;
(iii) $\nu_A(r - s) \leq \max\{\nu_A(r), \nu_A(s)\}$;
(iv)$\nu_A(rs) \leq \min\{\nu_A(r), \nu_A(s)\}$."

"Note that $\mu_A(0_R) \geq \mu_A(r) \geq \mu_A(1_R), \nu_A(0_R) \leq \nu_A(r) \leq \nu_A(1_R), \forall r \in R$. The collection of all IFIs of R is abbreviated by $IFI(R)$" [16].

Definition 2.3. *([5]) Suppose $A, B \in IFI(R)$. Then the intuitionistic fuzzy product AB of A and B is describe as: For every $x \in R$*

$$(\mu_{AB}(x), \nu_{AB}(x)) = \begin{cases} (\sup_{x=rs}(\min\{\mu_A(r), \mu_B(s))\}, \inf_{x=rs}(\max\{\nu_A(r), \nu_B(s))\}, & \text{if } x = rs \\ (0, 1), & \text{otherwise} \end{cases}$$

where as usual supremum and infimum of an empty set are taken to be 0 and 1 respectively.

Remark 2.2. *([16]) Let R be a commutative ring. Then for any $a_{(p,q)}, b_{(t,s)} \in IFP(R)$*
(i) $a_{(p,q)} + b_{(t,s)} = (a + b)_{(p\wedge t, q\vee s)}$;
(ii) $a_{(p,q)}b_{(t,s)} = (ab)_{(p\wedge t, q\vee s)}$.

Theorem 2.1. *([6]) Let $A \in IFS(R)$. Then A is an intuitionistic fuzzy ideal if and only if $A_{(\alpha,\beta)}$ is an ideal of R, for all $\alpha \leq \mu_A(0), \beta \geq \nu_A(0)$ with $\alpha + \beta \leq 1$. In particular, if A is an IFI of R, then $A_* = \{x \in R : \mu_A(x) = \mu_A(0), \nu_A(x) = \nu_A(0)\}$ is always an ideal of R.*

Definition 2.4. *([16]) Let P be a non constant IFI of a ring R. Then P is designate to be an IF prime (primary) ideal of R, if given any two IFIs A, B of R with*

$AB \subseteq P \Rightarrow A \subseteq P$ or $B \subseteq P$ ($A \subseteq P$ or $B \subseteq \sqrt{P}$), where $\sqrt{P}$ is the radical of P defined by

$\mu_{\sqrt{P}}(x) = \sup\{\mu_P(x^n) : n \in \mathbb{N}\}$ *and* $\nu_{\sqrt{P}}(x) = \inf\{\nu_P(x^n) : n \in \mathbb{N}\}$.

Theorem 2.2. *([16], Theorem (2.8) and Theorem (3.2)) Let P be an IFI of a ring R. Then given any $a_{(p,q)}, b_{(t,s)} \in IFP(R)$ the subsequent conditions are identical:*
(i) P is an IF prime (primary) ideal of R
(ii) $a_{(p,q)}b_{(t,s)} \subseteq P \Rightarrow a_{(p,q)} \subseteq P$ or $b_{(t,s)} \subseteq P$ ($a_{(p,q)} \subseteq P$ or $b_{(t,s)} \subseteq \sqrt{P}$).

Theorem 2.3. *([16], Theorem (2.9) and Theorem (3.7)) If P is an IF prime (primary) ideal of a ring R, then the subsequent conditions hold:*
(i) $P(0_R) = (1, 0)$,

(ii) P_* *is a prime (primary) ideal of* R,
(iii) $Img(P) = \{(1,0),(t,s)\}$, *here* $t,s \in [0,1)$ *with* $t+s \leq 1$.

Definition 2.5. *([13]) Let* P *be a non constant IFI of a ring* R. *Then* P *is said to be an IF semiprimary ideal of* R, *if given any two IFIs* A, B *of* R *with* $AB \subseteq P$ *infer that* $A \subseteq \sqrt{P}$ *or* $B \subseteq \sqrt{P}$.

Remark 2.3.
(i) From the definition it is clear that if A *is an IF semiprimary ideal of* R, *then* $\sqrt{A}$ *is an IF prime ideal of* R.
(ii) If A *is an IF primary ideal of* R, *then* A *is also an IF semiprimary ideal of* R. *But converse of it is not true. Check the subsequent example*

Example 2.1. *Consider* $R = \mathbb{Z}_8$ *be the ring of integers modulo 8. Define the IFS* A *of* R *by*

$$\mu_A(r) = \begin{cases} 1, & \textit{if } r = 0 \\ 0.6, & \textit{if } r = 2,4,6 \\ 0, & \textit{if } r = 1,3,5,7. \end{cases} \quad ; \quad \nu_A(r) = \begin{cases} 0, & \textit{if } r = 0 \\ 0.3, & \textit{if } r = 2,4,6 \\ 1, & \textit{if } r = 1,3,5,7. \end{cases}$$

Now, it is easy to verify that A *is an IFI of* R *such that* $\sqrt{A} = \chi_I$, *where* $I = \{0,2,4,6\}$ *is a prime ideal of* R *and by Theorem (2.3)* $\sqrt{A}$ *is an IF prime ideal of* R. *Then by definition* A *is an IF semiprimary ideal of* R. *Moreover, by Theorem (2.3)* A *is not an IF primary ideal of* R, *for* $|img(A)| = 3$.

Definition 2.6. *([14]) Let* X *and* Y *are any sets,* $f : X \to Y$ *be any function. An IFS* A *of* X *is called* f*-invariant if* $f(x_1) = f(x_2)$ *implies that* $\mu_A(x_1) = \mu_A(x_2)$ *and* $\nu_A(x_1) = \nu_A(x_2)$. *for every* $x_1, x_2 \in X$.

Proposition 2.1. *Let* $f : R \to R'$ *be a ring homomorphism from a ring* R *onto a ring* R' *and* $A \in IFS(R)$ *which is a constant on* $kerf$. *Then*

$$\mu_{f(A)}(f(r)) = \mu_A(r) \textit{ and } \nu_{f(A)}(f(r)) = \nu_A(r), \forall r \in R.$$

Proof. Assume that A is constant on $kerf$ and $r \in R$. Then, for $r' \in R'$, $f(r) = r'$. Moreover, for any $s \in f^{-1}(r')$, we have $f(s) = r' = f(r)$ and consequently, $\mu_A(s) = \mu_A(r)$ and $\nu_A(s) = \nu_A(r)$. Now

$$\begin{aligned} \mu_{f(A)}(f(r)) &= \mu_{f(A)}(r') \\ &= \sup\{\mu_A(s) : s \in f^{-1}(r')\} \\ &= \sup\{\mu_A(r)\} \\ &= \mu_A(r). \end{aligned}$$

Likewise, it can be revealed that $\nu_{f(A)}(f(r)) = \nu_A(r)$. This complete the proof. □

Theorem 2.4. *([16]) Let* R *and* R' *be rings. Let* $f : R \to R'$ *be a homomorphism. If* P *be an IF primary ideal of* R', *then* $f^{-1}(P)$ *is an IF primary ideal of* R.

Theorem 2.5. *([16]) Let* R *and* R' *be rings. Let* $f : R \to R'$ *be an epimorphism. If* P *is an IF primary ideal which is constant on* $Kerf$ *of* R, *then* $f(P)$ *is an IF primary ideal of* R'.

3. Group action on intuitionistic fuzzy ideal of a ring

Definition 3.1. *([7]) Let G be a group and S a non-empty set. Then the map $\phi : G \times S \to S$, with $\phi(g, x)$ written as $g * x$, is an action of G on S if and only if for all $g, h \in G, x \in S$*

(1) $g * (h * x) = (gh) * x$,
(2) $e * x = x$, *where e is the identity element of the group G.*

We assume that R be a ring and G be a finite group such that G acts on a subset S of R (For example: $\forall g \in G, x \in S, x^g = gxg^{-1} \in S$, where x^g define the action of the element g on the element x of S). Here in this section, we define the group action of G on an IFS A of a ring R.

Definition 3.2. *The group action of G on an IFS A of a ring R is denoted by A^g and is defined as $A^g = \{\langle x, \mu_{A^g}(x), \nu_{A^g}(x)\rangle : x \in R\}$, where $\mu_{A^g}(x) = \mu_A(x^g)$ and $\nu_{A^g}(x) = \nu_A(x^g)$ for every $x \in R, g \in G$.*

From the definition of group action on IFS, following results are easy to derive

Lemma 3.1. *Let A and B be two IFSs of a ring R and G be a finite group which acts on A and B. Then*

(1) $(A \cap B)^g = A^g \cap B^g, \forall g \in G$;
(2) $(A \cup B)^g = A^g \cup B^g, \forall g \in G$;
(3) $(A \times B)^g = A^g \times B^g, \forall g \in G$;
(4) *If $A \subseteq B$, then $A^g \subseteq B^g, \forall g \in G$;*
(5) $(A^g)^h = A^{gh}, \forall g, h \in G$;
(6) $(A^g)^{g^{-1}} = A^e, \forall g \in G$.

Lemma 3.2. *Let G be a finite group which acts on a ring R. Then for $x, y \in R, g \in G$, we have*

(1) $(x - y)^g = x^g - y^g$;
(2) $(xy)^g = x^g y^g$;
(3) $(x, y)^g = (x^g, y^g)$;

Proposition 3.1. *Let A be an IFI of a ring R and G be a finite group which acts on A, then A^g is also an IFI of R.*

Proof. Let $x, y \in R, g \in G$ be any elements. Then

$$\begin{aligned} \mu_{A^g}(x-y) &= \mu_A((x-y)^g) = \mu_A(x^g - y^g) \\ &\geq \min\{\mu_A(x^g), \mu_A(y^g)\} \\ &= \min\{\mu_{A^g}(x), \mu_{A^g}(y)\}. \end{aligned}$$

Thus $\mu_{A^g}(x - y) \geq \min\{\mu_{A^g}(x), \mu_{A^g}(y)\}$. Likewise, it can be revealed that $\nu_{A^g}(x - y) \leq \max\{\nu_{A^g}(x), \nu_{A^g}(y)\}$. Also,

$$\begin{aligned} \mu_{A^g}(xy) &= \mu_A((xy)^g) = \mu_A(x^g y^g) \\ &\geq \max\{\mu_A(x^g), \mu_A(y^g)\} \\ &= \max\{\mu_{A^g}(x), \mu_{A^g}(y)\}. \end{aligned}$$

Thus $\mu_{A^g}(xy) \geq max\{\mu_{A^g}(x), \mu_{A^g}(y)\}$. Likewise, it can be revealed that $\nu_{A^g}(xy) \leq \min\{\nu_{A^g}(x), \nu_{A^g}(y)\}$. Hence A^g is an IFI of R. □

Remark 3.1. *The converse of Proposition (3.1) not necessarily be true, i.e., A^g may be an IFI of R even though A is not an IFI of R. See the following example.*

Example 3.1. *Let $R = (\{0, 1, 2, 3\} : +_4, \times_4)$ be a ring, $G = (\{1, 2, 3, 4\} : \times_5)$ be a finite group. Consider the IFS A of R designated by $A = \{\langle 0, 1, 0\rangle, \langle 1, 0.4, 0.4\rangle, \langle 2, 0.6, 0.2\rangle, \langle 3, 0.5, 0.3\rangle\}$. Clearly, A is not an IFI of R, for $\mu_A(3 - 2) = \mu_A(1) = 0.4 < 0.5 = \min\{\mu_A(3), \mu_A(2)\}$. Take $g = 2$ so that $g^{-1} = 3$, then $x^g = gxg^{-1} = 2 \times x \times 3 = 6x(mod4) = 2x(mod4)$, we get*

$$\mu_{A^g}(x) = \begin{cases} 1, & \text{if } x = 0, 2 \\ 0.6, & \text{if } x = 1, 3 \end{cases}; \quad \nu_{A^g}(x) = \begin{cases} 0, & \text{if } x = 0, 2 \\ 0.2, & \text{if } x = 1, 3. \end{cases}$$

Now, it is easy to check that A^g is an IFI of R.

Definition 3.3. *Let A be an IFI of a ring R and G be a finite group which acts on A. Then A is called an intuitionistic fuzzy G-ideal of R if A^g is an IFI of R for all $g \in G$.*

Proposition 3.2. *If A, B are two IFIs of a ring R and G be a finite group which acts on A and B, then $(A \cap B)^g$ is also an IFI of R.*

Proof. Follows from Lemma (3.1)(1) and Proposition (3.1). □

Proposition 3.3. *If A, B are two IFIs of a ring R and G be a finite group which acts on A and B, then $(A \times B)^g$ is also an IFI of R.*

Proof. Follows from Lemma (3.1)(3) and Proposition (3.1). □

Proposition 3.4. *If $\sqrt{A}$ is an IF radical of an IFI A of a ring R and G be a finite group which acts on A, then $\sqrt{A^g} = (\sqrt{A})^g$, for all $g \in G$.*

Proof. Let $x \in R, g \in G$ be any element. Then

$$\begin{aligned} \mu_{\sqrt{A^g}}(x) &= \sup\{\mu_{A^g}(x^n) : n \in \mathbb{N}\} \\ &= \sup\{\mu_A((x^n)^g) : n \in \mathbb{N}\} \\ &= \sup\{\mu_A((x^g)^n) : n \in \mathbb{N}\} \\ &= \mu_{\sqrt{A}}(x^g) \\ &= \mu_{(\sqrt{A})^g}(x). \end{aligned}$$

Likewise, it can be revealed that $\nu_{\sqrt{A^g}}(x) = \nu_{(\sqrt{A})^g}(x)$. Hence $\sqrt{A^g} = (\sqrt{A})^g$, for every $g \in G$. □

Proposition 3.5. *If P is an intuitionistic fuzzy primary ideal of a ring R, then P^g is also an intuitionistic fuzzy primary ideal of R, where $g \in G$ be any element.*

Proof. Let A, B be IFIs of ring R with $AB \subseteq P^g$, where $g \in G$ be any element.
Now, we claim that $A^{g^{-1}}B^{g^{-1}} \subseteq P$. It is sufficient to show that $\mu_{A^{g^{-1}}B^{g^{-1}}}(x) \leq \mu_P(x)$ and $\nu_{A^{g^{-1}}B^{g^{-1}}}(x) \geq \nu_P(x)$, $\forall x \in R$.

$$\begin{aligned} \mu_{A^{g^{-1}}B^{g^{-1}}}(x) &= \sup_{x=ab}\{min(\mu_{A^{g^{-1}}}(a), \mu_{B^{g^{-1}}}(b))\} \\ &= \sup_{x^{g^{-1}}=a^{g^{-1}}b^{g^{-1}}}\{min(\mu_A(a^{g^{-1}}), \mu_B(b^{g^{-1}}))\} \\ &= \mu_{AB}(x^{g^{-1}}) \\ &\leq \mu_{P^g}(x^{g^{-1}}) \\ &= \mu_P(x). \end{aligned}$$

Thus $\mu_{A^{g^{-1}}B^{g^{-1}}}(x) \leq \mu_P(x)$. Likewise, it can be revealed that $\nu_{A^{g^{-1}}B^{g^{-1}}}(x) \geq \nu_P(x)$. Hence $A^{g^{-1}}B^{g^{-1}} \subseteq P$ which implies that either $A^{g^{-1}} \subseteq P$ or $B^{g^{-1}} \subseteq \sqrt{P}$. If $A^{g^{-1}} \subseteq$

P, then $\mu_A(x) = \mu_A((x^g)^{g^{-1}}) = \mu_{A^{g^{-1}}}(x^g) \leq \mu_P(x^g) = \mu_{P^g}(x)$. Similarly, we have $\nu_A(x) \geq \nu_{P^g}(x)$. Thus $A \subseteq P^g$. In a same way we can achieve that, if $B^{g^{-1}} \subseteq \sqrt{P}$, then $B \subseteq (\sqrt{P})^g = \sqrt{P^g}$. Hence P^g is an IF primary ideal of R. □

In the sequel, we can prove the following

Proposition 3.6. *If P is an IF semiprimary ideal of a ring R, then P^g is also an IF semiprimary ideal of R, where $g \in G$ be any element.*

4. Intuitionistic fuzzy G-primary and G-semiprimary ideal

By using the definition of G-invariant ideal of a ring R, we define G-invariant intuitionistic fuzzy ideal and G-invariant intuitionistic fuzzy primary and semiprimary ideal of R.

Definition 4.1. *Let A be an IFS of a ring R and G be a group which acts on R. Then A is said to be G-invariant IFS of R if and only if*

$$\mu_{A^g}(x) = \mu_A(x^g) \geq \mu_A(x), \nu_{A^g}(x) = \nu_A(x^g) \leq \nu_A(x), \forall x \in R \text{ and } \forall g \in G.$$

Note that if A is a G-invariant IFS (or IFI) of R, then $\mu_A(x) = \mu_A((x^g)^{g^{-1}}) \geq \mu_A(x^g) = \mu_{A^g}(x)$ implies that $\mu_{A^g}(x) = \mu_A(x)$. Similarly, we have $\nu_A(x) = \nu_A((x^g)^{g^{-1}}) \leq \nu_A(x^g) = \nu_{A^g}(x)$ implies that $\nu_{A^g}(x) = \nu_A(x)$.

Proposition 4.1. *Let A be an IFS of a ring R and G be a finite group which acts on R. Then A is G-invariant IFS of R if and only if $A^g = A$, for all $g \in G$.*

Proof. Follows immediately from the Definition (3.1). □

Theorem 4.1. *Let A be an IFS of a ring R and G be a finite group which acts on R. Let $A^G = \bigcap_{g\in G} A^g$. Then $A^G = (\mu_{A^G}, \nu_{A^G})$, where $\mu_{A^G}(x) = \min\{\mu_A(x^g) : g \in G\}$ and $\nu_{A^G}(x) = \max\{\nu_A(x^g) : g \in G\}, \forall x \in R$. Moreover, A^G is the largest G-invariant IFS of R contained in A.*

Proof. Since A be an IFS of R and so A^g is IFS of R for all $g \in G$. Also, intersection of IFSs of R is an IFS of R and so A^G is an IFS of R. Next, we show that A^G is G-invariant IFS of R. Now,

$$\begin{aligned}
\mu_{A^G}(x^g) &= \min\{\mu_{A^h}(x^g) : h \in G\} \\
&= \min\{\mu_A\{(x^g)^h\} : h \in G\} \\
&= \min\{\mu_A(x^{gh}) : h \in G\} \\
&= \min\{\mu_A(x^{g'}) : g' \in G\} \\
&= \mu_{A^G}(x).
\end{aligned}$$

Likewise, it can be revealed that $\nu_{A^G}(x^g) = \nu_{A^G}(x), \forall x \in R$. Thus A^G is G-invariant IFS of R.

Further, let B be any G-invariant IFS of R such that $B \subseteq A$. Then for any $x \in R, g \in G$, we get $\mu_B(x^g) = \mu_B(x) \leq \mu_A(x)$ and $\nu_B(x^g) = \nu_B(x) \geq \nu_A(x)$.
Now, $\mu_B(x^g) = \mu_B(x) = \mu_B\{(x^g)^{g^{-1}}\} \leq \mu_A(x^g)$
$\Rightarrow \mu_B(x) \leq \min\{\mu_A(x^g) : g \in G\} = \mu_{A^G}(x)$.
Similarly, $\nu_B(x^g) = \nu_B(x) = \nu_B\{(x^g)^{g^{-1}}\} \geq \nu_A(x^g)$
$\Rightarrow \nu_B(x) \geq \max\{\nu_A(x^g) : g \in G\} = \nu_{A^G}(x)$.
Thus $B \subseteq A^G$. Hence A^G is the largest G-invariant IFS of R contained in A. □

Proposition 4.2. *Let A be an IFI of a ring R and G be a finite group which acts on R. Then A^G is the largest G-invariant IFI of R contained in A.*

Proof. Let $x, y \in R, g \in G$ be any element, then

$$\begin{aligned}\mu_{A^G}(x-y) &= \min\{\mu_A((x-y)^g) : g \in G\}\\ &= \min\{\mu_A(x^g - y^g) : g \in G\}\\ &\geq \min\{min\{\mu_A(x^g), \mu_A(y^g)\} : g \in G\}\\ &= \min\{\min\{\mu_A(x^g) : g \in G\}, \min\{\mu_A(y^g) : g \in G\}\}\\ &= \min\{\mu_{A^G}(x), \mu_{A^G}(y)\}.\end{aligned}$$

Likewise, it can be revealed that $\nu_{A^G}(x-y) \leq max\{\nu_{A^G}(x), \nu_{A^G}(y)\}$. Also,

$$\begin{aligned}\mu_{A^G}(xy) &= \min\{\mu_A((xy)^g) : g \in G\}\\ &= \min\{\mu_A(x^g y^g) : g \in G\}\\ &\geq \min\{\max\{\mu_A(x^g), \mu_A(y^g)\} : g \in G\}\\ &= \max\{\min\{\mu_A(x^g), \mu_A(y^g)\} : g \in G\}\\ &= \max\{\min\{\mu_A(x^g) : g \in G\}, \min\{\mu_A(y^g) : g \in G\}\}\\ &= \max\{\mu_{A^G}(x), \mu_{A^G}(y)\}.\end{aligned}$$

Likewise, it can be revealed that $\nu_{A^G}(xy) \leq \min\{\nu_{A^G}(x), \nu_{A^G}(y)\}$. Hence A^G is an IFI of R. Further, A^G is the largest G-invariant IFI of R contained in A can be proved similar to Theorem (4.1). □

Proposition 4.3. *An IFI A of a ring R is G-invariant IFI of R if and only if $A^G = A$.*

Proof. From Proposition (4.2) we get $A^G \subseteq A$. Also, because A is G-invariant IFI of R and $A \subseteq A$. But A^G is the largest G-invariant IFI of R contained in A implies that $A \subseteq A^G$. Hence $A^G = A$. □

Proposition 4.4. *If A is a G-invariant IFI of a ring R and G be a finite group which acts on A, then*

$$(\sqrt{A})^G = \sqrt{A^G}.$$

Proof. Suppose $x \in R, g \in G$ be any element. Then

$$\begin{aligned}\mu_{(\sqrt{A})^G}(x) &= \min\{\mu_{\sqrt{A}}(x^g) : g \in G\}\\ &= \min\{\sup\{\mu_A((x^g)^m) : m \in \mathbb{N}\} : g \in G\}\\ &= \sup\{\min\{\mu_A((x^m)^g) : g \in G\} : m \in \mathbb{N}\}\\ &= \sup\{\mu_{A^G}(x^m) : m \in \mathbb{N}\}\\ &= \mu_{\sqrt{A^G}}(x).\end{aligned}$$

Likewise, it can be revealed that $\nu_{(\sqrt{A})^G}(x) = \nu_{\sqrt{A^G}}(x), \forall x \in R$. This implies that $(\sqrt{A})^G = \sqrt{A^G}$. □

Theorem 4.2. *If A, B are G-invariant IFIs of a ring R, then $A + B$ is also G-invariant IFI of R.*

Proof. Suppose $x \in R, g \in G$ be any elements, then

$$\begin{aligned} \mu_{(A+B)^g}(x) &= \mu_{A+B}(x^g) \\ &= \sup_{x^g=a+b}\{\mu_A(a), \mu_B(b)\} \\ &= \sup_{x^g=a+b}\{\mu_A(a^{g^{-1}}), \mu_B(b^{g^{-1}})\}[\text{As A and B are G-invariant IFIs}] \\ &= \sup_{x=a^{g^{-1}}+b^{g^{-1}}}\{\mu_A(a^{g^{-1}}), \mu_B(b^{g^{-1}})\} \\ &= \mu_{A+B}(x). \end{aligned}$$

Likewise, it can be revealed that $\nu_{(A+B)^g}(x) = \nu_{A+B}(x)$. Thus $(A+B)^g = A+B, \forall g \in G$. Hence $A+B$ is also G-invariant IFI of R. □

Theorem 4.3. *If A, B are G-invariant IFIs of a ring R, then AB is also G-invariant IFI of R.*

Proof. Let $x \in R, g \in G$ be any elements, then

$$\begin{aligned} \mu_{(AB)^g}(x) &= \sup_{x^g=ab}\min\{\mu_A(a), \mu_B(b)\} \\ &= \sup_{x^g=ab}\min\{\mu_A(a^{g^{-1}}), \mu_B(b^{g^{-1}})\}[\text{As A and B are G-invariant IFIs}] \\ &= \sup_{x=a^{g^{-1}}b^{g^{-1}}}\min\{\mu_A(a^{g^{-1}}), \mu_B(b^{g^{-1}})\} \\ &= \mu_{AB}(x). \end{aligned}$$

Likewise, it can be revealed that $\nu_{(AB)^g}(x) = \nu_{AB}(x)$. Thus $(AB)^g = AB, \forall g \in G$. Hence AB is also G-invariant IFI of R. □

Definition 4.2. *Let P be a non-constant IFI of a ring R and G be a finite group which acts on P. Then P is termed as an IF G-primary ideal of R if P is G-invariant IF primary ideal of R.*

Definition 4.3. *Let P be a non-constant IFI of a ring R and G be a finite group which acts on P. Then P is said to be an IF G-semiprimary ideal of R if P is G-invariant IF semiprimary ideal of R.*

Proposition 4.5. *Let P be an IF G-primary ideal of R. Then $P_{(s,t)}$ is a G-primary ideal of R, where $s \in [\mu_P(1), \mu_P(0)]$ and $t \in [\nu_P(0), \nu_P(1)]$ such that $s + t \leq 1$.*

Proof. It is easy to show that $P_{(s,t)}$ is an ideal of R. We show that $P_{(s,t)}$ is G-invariant. Let $x \in P_{(s,t)}, g \in G$ be any element. Since P is G-invariant intuitionistic fuzzy primary ideal of R, so $\mu_P(x^g) = \mu_P(x) \geq s$ and $\nu_P(x^g) = \nu_P(x) \leq t, \forall g \in G$ implies that $x^g \in P_{(s,t)}, \forall g \in G$. Hence $P_{(s,t)}$ is G-invariant.

Next we show that $P_{(s,t)}$ is primary ideal of R. Let I and J be two G-invariant ideals of R such that $IJ \subseteq P_{(s,t)}$. Define two IFSs $A = \chi_I$ and $B = \chi_J$. It is easy to check that A and B are G-invariant IFIs of R (as I and J are G-invariant ideals). We claim that $AB \subseteq P$. Let $x \in R$ be any element. If $AB(x) = (0, 1)$, there is nothing to prove. If $AB(x) \neq (0, 1)$. Then $AB(x) = (s, t)$, but $\mu_{AB}(x) = \sup_{x=yz}(\mu_A(y) \wedge \mu_B(z)) = \sup_{x=yz}(\chi_I(y) \wedge \chi_J(z)) \neq 0$ and $\nu_{AB}(x) = \inf_{x=yz}(\nu_A(y) \vee \nu_B(z)) = \inf_{x=yz}(\chi_I(y) \vee \chi_J(z)) \neq 1$. This implies that there exist $y \in I, z \in J$ such that $x = yz$. Thus $x = yz \in IJ \subseteq P_{(s,t)}$. So $\mu_P(x) \geq s, \nu_P(x) \leq t$. Hence $AB \subseteq P$. Since P is an intuitionistic fuzzy G-primary ideal of R, either $A \subseteq P$

or $B \subseteq \sqrt{P}$. Suppose that, $A \subseteq P$, then $I \subseteq P_{(s,t)}$. Because, if $I \supset P_{(s,t)}$, then there is an element $a \in R$ such that $a \in I$, but $a \notin P_{(s,t)}$. This implies that $\mu_A(a) = \mu_{\chi_I}(a) = 1$ and $\nu_A(a) = \nu_{\chi_I}(a) = 0$, but $\mu_P(a) < s$ and $\nu_P(a) > t$. Thus $\mu_A(a) = 1 > \mu_P(a)$ and $\nu_A(a) = 0 < \nu_P(a)$. Hence $A \supset P$, a contradiction. Similarly, if $B \subseteq \sqrt{P}$, then $J \subseteq \sqrt{P_{(s,t)}}$. Hence $P_{(s,t)}$ is G-primary ideal of R. □

similarly, we can prove the following

Proposition 4.6. *Let P be an IF G-semiprimary ideal of R. Then $P_{(s,t)}$ is a G-semiprimary ideal of R, where $s \in [\mu_P(1), \mu_P(0)]$ and $t \in [\nu_P(0), \nu_P(1)]$ such that $s + t \leq 1$.*

Proposition 4.7. *If P is an intuitionistic fuzzy G-semiprimary ideal of R, then $P = (\sqrt{P})^G$.*

Proof. Let P is an intuitionistic fuzzy G-semiprimary ideal of R. Therefore, P is G-invariant intuitionistic fuzzy semiprimary ideal of R and so, $P^G = P$.

$$\begin{aligned}
\mu_{(\sqrt{P})^G}(x) &= \min\{\mu_{\sqrt{P}}(x^g) : g \in G\} \\
&= \min\{\sup\{\mu_P((x^g)^m) : m \in \mathbb{N}\} : g \in G\} \\
&= \sup\{min\{\mu_P((x^m)^g) : g \in G\} : m \in \mathbb{N}\} \\
&= \sup\{\mu_P(x^m) : m \in \mathbb{N}\} \\
&\leq \mu_P(x).
\end{aligned}$$

Likewise, it can be revealed that $\nu_{(\sqrt{P})^G}(x) \geq \nu_P(x)$. Thus $(\sqrt{P})^G \subseteq P$.

For the other inclusion we have $\mu_P(x) = \mu_{P^G}(x) = min\{\mu_P(x^g) : g \in G\}$.
As A is an IFI of R, we have $\mu_A(x) \leq \mu_A(x^m), \forall m \in \mathbb{N}, x \in R$ and so

$$\begin{aligned}
\min\{\mu_P(x^g) : g \in G\} &\leq \min\{\mu_P((x^g)^m) : g \in G\} \\
&\leq \min\{\sup\{\mu_P((x^g)^m) : m \in \mathbb{N}\} : g \in G\} \\
&= \mu_{(\sqrt{P})^G}(x).
\end{aligned}$$

Thus $\mu_P(x) \leq \mu_{(\sqrt{P})^G}(x)$. Likewise, it can be revealed that $\nu_P(x) \geq \nu_{(\sqrt{P})^G}(x)$. Thus we have $P \subseteq (\sqrt{P})^G$. Hence the result proved. □

From the above discussion on the results on intuitionistic fuzzy primary (semiprimary) ideals that are G-invariant also. We can also define intuitionistic fuzzy G-primary (G-semiprimary) ideals in the following ways too

Definition 4.4. *A non-constant G-invariant IFI P of a ring R is said to be G-primary IFI if for any two G-invariant IFIs A and B of R such that $AB \subseteq P$ implies either $A \subseteq P$ or $B \subseteq \sqrt{P}$.*

Definition 4.5. *A non-constant G-invariant IFI P of a ring R is said to be G-semiprimary IFI if for any two G-invariant IFIs A and B of R such that $AB \subseteq P$ implies either $A \subseteq \sqrt{P}$ or $B \subseteq \sqrt{P}$.*

Proposition 4.8. *Let P be an intuitionistic fuzzy G-invariant ideal of R. Then the following are equivalent*

(1) *P is an intuitionistic fuzzy G-primary ideal of R;*
(2) *For any $x_{(p,q)}, y_{(s,t)} \in IFP(R)$, where $x, y \in R$ are G-invariant points such that $x_{(p,q)}y_{(s,t)} \subseteq P$ implies that either $x_{(p,q)} \subseteq P$ or $y_{(s,t)} \subseteq \sqrt{P}$.*

Proof. (1) $\Rightarrow$ (2) Assume that P is an intuitionistic fuzzy G-primary ideal of R.
Let $x_{(p,q)}, y_{(s,t)} \in IFP(R)$, where $x, y \in R$ are G-invariant points such that $x_{(p,q)}y_{(s,t)} \subseteq P$. Then $x_{(p,q)}y_{(s,t)} = (xy)_{(p\wedge s, q\vee t)}$, where $\mu_P(xy) \geq p \wedge s$ and $\nu_P(xy) \leq q \vee t$.
Let us define IFSs A, B of R by $A = \chi_{<x>}$ and $B = \chi_{<y>}$. Clearly, A and B are G-invariant IFIs of R. Now $\mu_{AB}(z) = Sup_{z=uv}(\mu_A(u) \wedge \mu_B(v)) = p \wedge s$ and $\nu_{AB}(z) = \inf_{z=uv}(\nu_A(u) \vee \nu_B(v)) = q \vee t$, where $u \in < x >$ and $v \in < y >$. Thus $\mu_{AB}(z) = p \wedge s \leq \mu_P(z)$ and $\nu_{AB}(z) = q \vee t \geq \nu_P(z)$, when $z = uv$, where $u \in < x >$ and $v \in < y >$. Otherwise $AB(z) = (0, 1)$, i.e., $AB \subseteq P$. As P is intuitionistic fuzzy G-primary ideal so either $A \subseteq P$ or $B \subseteq \sqrt{P}$. Then $x_{(p,q)} \subseteq A \subseteq P$ or $y_{(s,t)} \subseteq \sqrt{B} \subseteq \sqrt{P}$.

(2) $\Rightarrow$ (1) Let A and B be two G-invariant IFIs of R such that $AB \subseteq P$. Suppose $A \supset P$. Then there exists G-invariant element $x \in R$ such that $\mu_A(x) > \mu_P(x)$ and $\nu_A(x) < \nu_P(x)$. Let $\mu_A(x) = p, \nu_A(x) = q$. Let $y \in R$ be G-invariant element of R such that $\mu_B(y) = s, \nu_B(y) = t$. If $z = xy$, then $x_{(p,q)}y_{(s,t)} = (xy)_{(p\wedge s,q\vee t)}$. Hence $\mu_P(z) = \mu_P(xy) \geq \mu_{AB}(xy) \geq [\mu_A(x) \wedge \mu_B(y)] = p \wedge s = \mu_{(xy)_{(p\wedge s,q\vee t)}}(z)$. Similarly, we have $\nu_P(z) \leq \nu_{(xy)_{(p\wedge s,q\vee t)}}(z)$. Hence $x_{(p,q)}y_{(s,t)} \subseteq P$, then by (2), we get either $x_{(p,q)} \subseteq P$ or $y_{(s,t)} \subseteq \sqrt{P}$, i.e., either $\mu_P(x) \geq p$, $\nu_P(x) \leq q$ or $\mu_{\sqrt{P}}(x) \geq s$, $\nu_{\sqrt{P}}(x) \leq t$. Since $\mu_P(x) < p, \nu_P(x) > q$ and $\mu_B(y) = s \leq \mu_{\sqrt{P}}(y)$, $\nu_B(y) = t \geq \nu_{\sqrt{P}}(y)$. So, $B \subseteq \sqrt{P}$. Hence P is an intuitionistic fuzzy G-primary ideal of R. □

Similarly, we can prove the following

Proposition 4.9. *Let P be an intuitionistic fuzzy G-invariant ideal of R. Then the following are equivalent*

(1) *P is an intuitionistic fuzzy G-semiprimary ideal of R;*
(2) *Given any $x_{(p,q)}, y_{(s,t)} \in IFP(R)$, where $x, y \in R$ are G-invariant points such that $x_{(p,q)}y_{(s,t)} \subseteq P$ infer that either $x_{(p,q)} \subseteq \sqrt{P}$ or $y_{(s,t)} \subseteq \sqrt{P}$.*

Proposition 4.10. *If P is an IF primary ideal of a ring R, then P^G is G-primary IFI ideal of R. Conversely, if Q is an intuitionistic fuzzy G-primary ideal of R, then there exists an IF primary ideal P of R such that $P^G = Q$.*

Proof. Let P be an intuitionistic fuzzy primary ideal of the ring R and let A and B be two G-invariant IFIs of R such that $AB \subseteq P^G$. Then $AB \subseteq P$ (since $P^G \subseteq P$ always). So, either $A \subseteq P$ or $B \subseteq \sqrt{P}$. Since A and B are G-invariant IFIs of R, but P^G is the largest G-invariant IFI of R contained in P. So, either $A \subseteq P^G$ or $B \subseteq (\sqrt{P})^G = \sqrt{P^G}$. Hence P^G is G-primary IFI ideal of R.

For the converse part, suppose that Q is an IF G-primary ideal of R. Therefore, $Q^G = Q$. Let $\mathcal{S} = \{P | P \text{ is an IFI of } R \text{ with } P^G \subseteq Q\}$. By Zorn's lemma, there exist intuitionistic fuzzy maximal ideal P such that $P^G \subseteq Q$. Let A and B be two IFIs of R such that $AB \subseteq P$. Then $(AB)^G \subseteq P^G \subseteq Q$. Since A^G and B^G are largest IFIs of R contained in A and B respectively. We claim that $A^G B^G \subseteq AB$ is a G-invariant.

$$\begin{aligned}\mu_{A^G B^G}(x^g) &= \sup_{x^g=uv} \min\{\mu_{A^G}(u), \mu_{B^G}(v)\} \\ &= \sup_{x=u^{g^{-1}}v^{g^{-1}}} \min\{\mu_{A^G}(u^{g^{-1}}), \mu_{B^G}(v^{g^{-1}})\} \\ &= \mu_{A^G B^G}(x).\end{aligned}$$

Similarly, we can show that $\nu_{A^G B^G}(x^g) = \nu_{A^G B^G}(x)$. Hence $A^G B^G \subseteq (AB)^G \subseteq Q$. Since Q is an IF G primary ideal of R, then we have either $A^G \subseteq Q$ or $B^G \subseteq \sqrt{Q}$. By maximality of P either $A \subseteq P$ or $B \subseteq P$. This implies that P is an IF prime and hence an IF primary ideal of R. As $Q^G = Q$, we have $Q \in \mathcal{S}$. But maximality of P gives that $Q \subseteq P$. Since P and Q^G are G invariant and P^G is largest in P, we get $Q \subseteq P^G$. Hence $P^G = Q$. □

Similarly, we can prove the following

Proposition 4.11. *If P is an intuitionistic fuzzy semiprimary ideal of a ring R, then P^G is G-semiprimary IFI ideal of R. Conversely, if Q is an intuitionistic fuzzy G-semiprimary ideal of R, then there exists an intuitionistic fuzzy semiprimary ideal P of R such that $P^G = Q$.*

5. G-Homomorphism of intuitionistic fuzzy G–ideals

In this part of paper, we explore the image and pre image of intuitionistic fuzzy G-ideals under the ring homomorphism.

Definition 5.1. *A homomorphism $\phi : R \to R'$ from a ring R to a ring R' with unity is called G-homomorphism, if for all $g \in G, x \in R, \phi(g * x) = g * \phi(x)$, where group G acts on both the rings.*

Lemma 5.1. *Let R and R' be rings and G be a finite group which acts on R and R'. Let $f : R \to R'$ is a function defined by $f(x^g) = (f(x))^g, \forall x \in R, g \in G$. Then f is a ring homomorphism. We designate the function f as G-homomorphism.*

Proof. Let $x, y \in R, g \in G$ be any elements, then we have

$$f(x^g + y^g) = f((x+y)^g) = (f(x) + f(y))^g = (f(x))^g + (f(y))^g = f(x^g) + f(y^g) \text{ and}$$
$$f(x^g y^g) = f((xy)^g) = (f(xy))^g = (f(x)f(y))^g = (f(x))^g (f(y))^g = f(x^g)f(y^g).$$

Therefore f is a G-homomorphism. □

Lemma 5.2. *Let R and R' be rings and G be a finite group which acts on R and R'. Let $f : R \to R'$ be a G-homomorphism and A, B are IFSs of R and R' respectively. Then*
(1) *$f^{-1}(B^g) = (f^{-1}(B))^g, \forall g \in G$;*
(2) *$f(A^g) = (f(A))^g, \forall g \in G$.*

Proof. **(1)** Let $x \in R$ and $g \in G$ be any element. Then

$$f^{-1}(B^g)(x) = B^g(f(x)) = B((f(x))^g) = B(f(x^g))) = f^{-1}(B)(x^g) = (f^{-1}(B))^g(x)$$

Hence $f^{-1}(B^g) = (f^{-1}(B))^g, \forall g \in G$.

(2) Let $y \in R'$ and $g \in G$ be any element. Then $f(A^g)(y) = (\mu_{f(A^g)}(y), \nu_{f(A^g)}(y))$. Now

$$\begin{aligned} \mu_{f(A^g)}(y) &= \sup\{\mu_{A^g}(x) : f(x) = y\} = \sup\{\mu_A(x^g) : f(x) = y\} \\ &= \sup\{\mu_A(x^g) : f(x^g) = y^g\} \\ &= \mu_{f(A)}(f(x^g)) = \mu_{f(A)}((f(x))^g) = \mu_{(f(A))^g}(f(x)) = \mu_{(f(A))^g}(y). \end{aligned}$$

Similarly, we can show that $\nu_{f(A^g)}(y) = \nu_{(f(A))^g}(y)$. Hence $f(A^g) = (f(A))^g, \forall g \in G$. □

Theorem 5.1. *Let R and R' be rings and G be a finite group which acts on R and R'. Let $f : R \to R'$ be a G-homomorphism. If B be an intuitionistic fuzzy G-ideal of R', then $f^{-1}(B)$ is an intuitionistic fuzzy G-ideal of R.*

Proof. Let B be an intuitionistic fuzzy G-ideal of $R^{'}$. To show that $f^{-1}(B)$ is an intuitionistic fuzzy G-ideal of R. For this we show that $(f^{-1}(B))^g$ is an IFI of R for all $g \in G$.
In view of Lemma (5.2)(1), we show that $f^{-1}(B^g)$ is an IFI of R for all $g \in G$.
For $x, y \in R$ and $g \in G$, we have $f^{-1}(B^g)(x+y) = (\mu_{f^{-1}(B^g)}(x+y), \nu_{f^{-1}(B^g)}(x+y))$, where

$$\begin{aligned} \mu_{f^{-1}(B^g)}(x+y) &= \mu_{B^g}\{f(x+y)\} = \mu_{B^g}\{f(x)+f(y)\} \\ &\geq \min\{\mu_{B^g}(f(x)), \mu_{B^g}(f(y))\} \\ &= \min\{\mu_{f^{-1}(B^g)}(x), \mu_{f^{-1}(B^g)}(y)\} \end{aligned}$$

Thus $\mu_{f^{-1}(B^g)}(x+y) \geq \min\{\mu_{f^{-1}(B^g)}(x), \mu_{f^{-1}(B^g)}(y)\}$.
Likewise, it can be revealed that $\nu_{f^{-1}(B^g)}(x+y) \leq \max\{\nu_{f^{-1}(B^g)}(x), \nu_{f^{-1}(B^g)}(y)\}$.
Also, $f^{-1}(B^g)(xy) = (\mu_{f^{-1}(B^g)}(xy), \nu_{f^{-1}(B^g)}(xy))$, where

$$\begin{aligned} \mu_{f^{-1}(B^g)}(xy) &= \mu_{B^g}\{f(xy)\} = \mu_{B^g}\{f(x)f(y)\} \\ &\geq \max\{\mu_{B^g}(f(x)), \mu_{B^g}(f(y))\} \\ &= \max\{\mu_{f^{-1}(B^g)}(x), \mu_{f^{-1}(B^g)}(y)\}. \end{aligned}$$

Thus $\mu_{f^{-1}(B^g)}(xy) \geq \max\{\mu_{f^{-1}(B^g)}(x), \mu_{f^{-1}(B^g)}(y)\}$.
Likewise, it can be revealed that $\nu_{f^{-1}(B^g)}(xy) \leq \min\{\nu_{f^{-1}(B^g)}(x), \nu_{f^{-1}(B^g)}(y)\}$.
Therefore, $f^{-1}(B^g)$ and so $(f^{-1}(B))^g$ is an IFI of R.
Hence $f^{-1}(B)$ is an intuitionistic fuzzy G-ideal of R. □

Theorem 5.2. *Let R and $R^{'}$ be rings and G be a finite group which acts on R and $R^{'}$. Let $f : R \to R^{'}$ be a G-epimorphism. If A is an intuitionistic fuzzy G-ideal of R which is constant on $Kerf$ of R, then $f(A)$ is an intuitionistic fuzzy G-ideal of $R^{'}$.*

Proof. Let A be an intuitionistic fuzzy G-ideal of R. In order to establish that $f(A)$ is an intuitionistic fuzzy G-ideal of $R^{'}$, we show that $(f(A))^g$ is an IFI of $R^{'}$ for all $g \in G$.
In view of Lemma (5.2)(2), we claim that $f(A^g)$ is an IFI of $R^{'}$ for all $g \in G$.

Let $x^{'}, y^{'} \in R^{'}, g \in G$. As f is epimorphism, therefore there exists $x, y \in R$ such that $f(x) = x^{'}$ and $f(y) = y^{'}$. Now, $f(A^g)(x^{'}+y^{'}) = (\mu_{f(A^g)}(x^{'}+y^{'}), \nu_{f(A^g)}(x^{'}+y^{'}))$.

$$\begin{aligned} \mu_{f(A^g)}(x^{'}+y^{'}) &= \mu_{f(A^g)}(f(x)+f(y)) = \mu_{f(A^g)}(f(x+y)) \\ &= \mu_{A^g}(x+y)[\text{As } A \text{ is constant on } Kerf, \text{ so } \mu_{f(A^g)}(f(x+y)) = \mu_{A^g}(x+y)] \\ &\geq \min\{\mu_{A^g}(x), \mu_{A^g}(y)\} \\ &= \min\{\mu_{f(A^g)}(f(x)), \mu_{f(A^g)}(f(y))\} \\ &= \min\{\mu_{f(A^g)}(x^{'}), \mu_{f(A^g)}(y^{'})\}. \end{aligned}$$

Thus, $\mu_{f(A^g)}(x^{'}+y^{'}) \geq \min\{\mu_{f(A^g)}(x^{'}), \mu_{f(A^g)}(y^{'})\}$.
Similarly, we can show that $\nu_{f(A^g)}(x^{'}+y^{'}) \leq \max\{\nu_{f(A^g)}(x^{'}), \nu_{f(A^g)}(y^{'})\}$.
Also, $f(A^g)(x^{'}y^{'}) = (\mu_{f(A^g)}(x^{'}y^{'}), \nu_{f(A^g)}(x^{'}y^{'}))$, where

$$\begin{aligned} \mu_{f(A^g)}(x^{'}y^{'}) &= \mu_{f(A^g)}(f(x)f(y)) = \mu_{f(A^g)}(f(xy)) \\ &= \mu_{A^g}(xy) \\ &\geq \max\{\mu_{A^g}(x), \mu_{A^g}(y)\} \\ &= \max\{\mu_{f(A^g)}(f(x)), \mu_{f(A^g)}(f(y))\} \\ &= \max\{\mu_{f(A^g)}(x^{'}), \mu_{f(A^g)}(y^{'})\}. \end{aligned}$$

Thus, $\mu_{f(A^g)}(x^{'}y^{'}) \geq \max\{\mu_{f(A^g)}(x^{'}), \mu_{f(A^g)}(y^{'})\}$.
Similarly, we can show that $\nu_{f(A^g)}(x^{'}y^{'}) \leq \min\{\nu_{f(A^g)}(x^{'}), \nu_{f(A^g)}(y^{'})\}$.
Therefore, $f(A^g)$ and so $(f(A))^g$ is an IFI of $R^{'}$.
Hence $f(A)$ is an intuitionistic fuzzy G-ideal of $R^{'}$. □

Theorem 5.3. *Let R and $R^{'}$ be rings and G be a finite group which acts on R and $R^{'}$. Let $f : R \to R^{'}$ be a G-homomorphism. If P be an intuitionistic fuzzy G-primary ideal of $R^{'}$, then $f^{-1}(P)$ is an intuitionistic fuzzy G-primary ideal of R.*

Proof. Since P be an intuitionistic fuzzy G-primary ideal of $R^{'}$ so by Theorem (2.4) $f^{-1}(P)$ is also an intuitionistic fuzzy primary ideal of R. So it remain to show that $f^{-1}(P)$ is G-invariant. For this consider $x \in R, g \in G$ be any elements. Then we have $\mu_{f^{-1}(P)}(x^g) = \mu_P(f(x^g)) = \mu_P((f(x))^g) = \mu_P((f(x))) = \mu_{f^{-1}(P)}(x)$. Likewise, it can be revealed that $\nu_{f^{-1}(P)}(x^g) = \nu_{f^{-1}(P)}(x)$. Thus $f^{-1}(P)$ is G-invariant. Hence $f^{-1}(P)$ is an intuitionistic fuzzy G-primary ideal of R. □

Theorem 5.4. *Let R and $R^{'}$ be rings and G be a finite group which acts on R and $R^{'}$. Let $f : R \to R^{'}$ be a G-epimorphism. If P is an intuitionistic fuzzy G-primary ideal which is constant on $Kerf$ of R, then $f(P)$ is an intuitionistic fuzzy G-primary ideal of $R^{'}$.*

Proof. Since P be an intuitionistic fuzzy G-primary ideal of R which is constant on $Kerf$ of R so by Theorem (2.5) $f(P)$ is also an intuitionistic fuzzy primary ideal of $R^{'}$. So it remain to show that $f(P)$ is G-invariant. For this consider $y \in R^{'}, g \in G$ be any element. As f is an epimorphism so, there exists $x \in R$ such that $f(x) = y$. Then we have $\mu_{f(P)}(y^g) = \mu_{(f(P))^g}(y) = \mu_{f(P^g)}(y) = \mu_{P^g}(f^{-1}(y)) = \mu_{P^g}(x) = \mu_P(x^g) = \mu_P(x) = \mu_P(f^{-1}(y)) = \mu_{f(P)}(y)$. Similarly, we can show that $\nu_{f(P)}(y^g) = \nu_{f(P)}(y)$. Thus $f(P)$ is G-invariant. Hence $f(P)$ is an intuitionistic fuzzy G-primary ideal of $R^{'}$. □

In the sequel, we can show that

Theorem 5.5. *Let R and $R^{'}$ be rings and G be a finite group which acts on R and $R^{'}$. Let $f : R \to R^{'}$ be a G-homomorphism. If P be an intuitionistic fuzzy G-semiprimary ideal of $R^{'}$, then $f^{-1}(P)$ is an intuitionistic fuzzy G-semiprimary ideal of R.*

Theorem 5.6. *Let R and $R^{'}$ be rings and G be a finite group which acts on R and $R^{'}$. Let $f : R \to R^{'}$ be a G-epimorphism. If P is an intuitionistic fuzzy G-semiprimary ideal which is constant on $Kerf$ of R, then $f(P)$ is an intuitionistic fuzzy G-semiprimary ideal of $R^{'}$.*

6. Conclusions

This paper investigates the impact of a group action on the intuitionistic fuzzy ideals of a ring. We develop a connection between the intuitionistic fuzzy G-primary (G-semiprimary) ideals and the intuitionistic fuzzy primary (semiprimary) ideals of R. We have explored the relationship between intuitionistic fuzzy G-semiprimary ideals and the radicals of intuitionistic fuzzy ideals. We analysed a suitable characterization of intuitionistic fuzzy G-primary (G-semiprimary) ideals of R in terms of intuitionistic fuzzy points of R under this group action. Finally, the behaviour of an intuitionistic fuzzy G-primary (G-semiprimary) ideal under G-homomorphism was investigated. In the near future, we are applying these concepts in the field of physics, chemistry and other related fields to find the uncertainty in symmetries.

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References

[1] Ali, M., Smarandache, F., (2015), A New Type of Group Action Through the Applications of Fuzzy Sets and Neutrosophic Sets, Math. Combin. Book Ser., 4, 33-40.

[2] Atanassov, K.T., (1983), Intuitionistic fuzzy sets, In: Sgurev v(ed) vii ITKR′s session, Central Science and Technology Library of the Bulgarian Academy of Sciences, Sofia.

[3] Atanassov, K.T., (1986), Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1), 87-96.

[4] Atanassov, K.T., (1999), Intuitionistic Fuzzy Sets Theory and Applications, Studies on Fuzziness and Soft Computing, 35, Physica-Verlag, Heidelberg.

[5] Bakhadach, I., Melliani, S., Oukessou M., and Chadli, S.L., (2016), Intuitionistic fuzzy ideal and intuitionistic fuzzy prime ideal in a ring, Notes on Intuitionistic Fuzzy Sets, 22(2), 59-63.

[6] Basnet, D.K., (2011), Topics in intuitionistic fuzzy algebra, Lambert Academic Publishing, ISBN: 978-3-8443-9147-3

[7] Biswas, R., (1989), Intuitionistic fuzzy subgroups, Mathematical Forum X, 39-44.

[8] Dummit, D. S., Foote, R. M., (2004), Abstract Algebra, 3rd Ed., John Viley and Sons Inc.

[9] Hur K., Jang, S.Y., Kang, H.W., (2005), Intuitionistic fuzzy ideal of a ring, J. Korea Soc. Math. Educ. Ser. B : Pure Appl. Math., 12(3), 193-209.

[10] Jun, Y.B., Zturk, M. A. O., and Park, C. H., (2007), Intuitionistic nil radicals of intuitionistic fuzzy ideals and Euclidean intuitionistic fuzzy ideals in rings, Information Sciences, 177(21), 4662-4677.

[11] Manemaran, S.V., and Nagaraja, R., (2019), A Group Action On Picture Fuzzy Soft G-Modules, International Journal of Applied Mathematics & Statistical Sciences, 8(5), 31-48.

[12] Negoita, C. V., and Ralescu, D. A., (1975), Applications of Fuzzy Sets and Systems Analysis, Birkhauser, Basel. ISBN : 978-3-7643-0789-9.

[13] Palanivelrajan, M., and Nandakumar S., (2012), Some operations of intuitionistic fuzzy primary and semi-primary ideal, Asian Journal of Algebra 5(2), 44-49.

[14] Sharma, P. K., and Kaur, G., (2017), Residual quotient and annihilator of intuitionistic fuzzy sets of ring and module, International Journal of Computer Science & Information Technology (IJCSIT), 9(4), 1-15.

[15] Sharma, P. K., and Kaur, G., (2017), Intuitionistic fuzzy prime spectrum of a ring, CiiT International Journal of Fuzzy Systems, 9(8), 167-175.

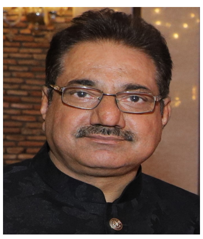
[16] Sharma, P. K., (2022), On intuitionistic fuzzy primary ideal of a ring, Pan-American Journal of Mathematics, 1, 1-11.

[17] Sharma, P. K., (2016), Group action on intuitionistic fuzzy modules, International Journal of Pure and Applied Mathematical Technologies, 1(1), 24-37.

[18] Sharma, R. P., Sharma, S., (1998), Group Action On Fuzzy Ideals, Communications in Algebra, 26(12), 4207-4220.

[19] Yılmaz, S.T., Tarsuslu, A., and Citil, M. C., (2018), Intuitionistic fuzzy action of a group on a set, Notes on Intuitionistic Fuzzy Sets, 24(2), 18-24.

[20] Zadeh, L. A., (1965), Fuzzy sets, Information and Control, 8(3), 338-353.



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