TWMS J. App. and Eng. Math. V.15, N.4, 2025, pp. 904-923

INTERVAL-VALUED FERMATEAN FUZZY SETS APPROACH FOR THE SPEARMAN RANK CORRELATION COEFFICIENT

M. KIRIŞCI[1*], §

Abstract. Correlation is an important concept that can be used to analyze data sets and assist business leaders in gaining valuable insights into the relationships between business outcomes. When conducting a correlation analysis, the more data points available, the more accurate the analysis results will be. Having sufficient data before relying on correlations is essential to make business decisions. Methods for calculating correlation coefficients between sets have been developed and used in application areas. Interval-valued Fermatean fuzzy-based Spearman rank correlation coefficients are given in this study, and their basic features are examined. In order to demonstrate the application of Interval-valued Fermatean fuzzy-based Spearman rank correlation coefficients to real-world problems, the Hospital Disaster Preparedness application is studied according to information from a university hospital. The disaster preparedness of four hospital departments is evaluated using information obtained from the Hospital Disaster Management unit. The preparedness status of the departments is determined using the Interval-valued Fermatean fuzzy-Spearman rank correlation coefficients method. The Interval-valued Fermatean fuzzy-Spearman rank correlation coefficients method is compared with the previously known Fermatean fuzzy-based correlation coefficients methods. In this comparison process, it is seen that the results of the new method are similar to some previously known methods. This new method of Fermatean fuzzy-based correlation coefficients could be used to discuss techniques for order preference that are similar to ideal solutions and multi-criteria decision-making.

Keywords: Spearman rank correlation coefficient Decision-making approach, Pearson correlation coefficients, interval-valued Fermatean fuzzy set.

AMS Subject Classification: 03E72 ; 62H20

1. Introduction

1.1. **Research Motivation.** These days, with the challenges growing and extending, selecting the best option from a range of workable options during the DM process is getting harder and harder. In the context of today's decision-making challenges, several experts have assessed objects like crisp and interval differently, which may make decisions harder.

[1] Istanbul University-Cerrahpasa - Cerrahpasa Medicine faculty - Department of Biostatistics and Medical Informatics - Kocamustafapasa Cd. No:53, Cerrahpasa, Fatih - Turkiye.
e-mail: mkirisci@hotmail.com; ORCID no. 0000-0003-4938-5207.
[*] Corresponding author.
§ Manuscript received: : November 10, 2023; accepted:September 8, 2024.
TWMS Journal of Applied and Engineering Mathematics, Vol.15, No.4; © Işık University, Department of Mathematics, 2025; all rights reserved.

Scholars often employ the theory of fuzzy sets (FS)[50] and its extensions, such as intuitionistic FSs (IFSs) [1], Pythagorean FSs (PFSs) [47], [48], and Fermatean FS (FFSs) [41], [42], to handle data uncertainties. Scholars have investigated the various applications of these sets using aggregation operators and information measures. Numerous studies have been conducted on information measures, which are essential in integrating the diverse preferences of the recipients. One of these many concepts is using correlation measures to determine the optimal option.

The literature research uses several applications to handle DM difficulties. One method for selecting the best option is using correlation coefficients ($\mathcal{C}$s), which measure the reliance level between two groups. In statistics, a correlation is any statistical relationship—whether causal or not—between two random variables or bivariate data. The $\mathcal{C}$ is the metric utilized in a correlation analysis to express the strength of the linear relationship between two variables. By analyzing each data point's distance from the variable mean, the formula ascertains the degree to which the relationship between the variables can be fit to an illustrative line drawn across the data for two variables.

Using Spearman's rank $\mathcal{C}$ (SR$\mathcal{C}$), one can assess rank correlation—the statistical dependency of rank between two variables—in a non-parametric manner. The correlation between two ordinal variables and their strength and direction are calculated. Understanding the PEAR is necessary for the SR$\mathcal{C}$. The statistical indicator known as PEAR quantifies how strongly two data sets have a linear relationship. The following data assumptions must be valid in order to calculate the ranking variable and run the significance test: (1) range or rate level, (2) linearly connected, and (3) bivariate distribution. If these assumptions are not met, the Spearman coefficient will be needed. One must comprehend the monotonic function to comprehend the SR$\mathcal{C}$. When the independent variable rises, a monotonic function never lowers or does not increase. Although the SR$\mathcal{C}$ does not need monotony, if it is already known that the relationship between the variables is not monotonic, it would be pointless to use the SR$\mathcal{C}$ without determining the strength and direction of a monotonic relationship.

Finding a correlation between any two parameters or variables is common in statistics. Pearson's $\mathcal{C}$(PEAR) has been applied in statistics research on medical diagnosis, DM, pattern recognition, grouping, and data analysis and classification. It has been found that classical correlation cannot handle data with ambiguous concerns. The main goal of the fuzzy logic approach is to quantify the uncertainties in human perception and reasoning. Scientists are used to using binary logic for data analysis. Because human logic is complicated and imprecise, using binary logic to investigate human mental processes can lead to some distortion. Fuzzy logic is based on cognitive processes that occur in humans. Fuzzy logic is defined, for instance, as "the modeling of thinking and decision mechanisms that enable humans to make consistent and correct conclusions in the light of comprehensive and imprecise information." The fuzzy-type $\mathcal{C}$s have been extended based on statistical $\mathcal{C}$s, fuzzy logic approach, and mathematical statistics. The $\mathcal{C}$ generated for fuzzy data shows how strongly FSs are associated and whether FSs are favorably or negatively related.

1.2. **Literature.** The complexity of the decision-making environment makes it harder for experts to provide trustworthy assessment information. The ideas of IFS and PFS have been advocated to mitigate the ambiguity and imprecision brought about by the intricate subjectivity of human cognition. Therefore, a more expansive information space is required to meet their evaluation expectations for various goals and to consider the decision-making time and knowledge base of professionals to make more appropriate judgments. Consequently, the FFS was the first to expand the range of information expression by incorporating the cubic sum of MD and ND in the interval. As a result, FFS handles indeterminacy of choice problems more effectively and practically than IFS and PFS. Senepati and Yager created the FFS [41]. Compared to IFSs and PFSs, FFSs are superior at explaining uncertainty. Senapati and Yager followed this study [42], who examined several additional operations and arithmetic mean processes over FFSs. They also used the FF-weighted product model to handle MCDM problems. New aggregation operations related to FFS have been defined, and their accompanying properties have been studied [43]. A measure of entropy based on Fermatean fuzzy soft sets (FFSS) was also offered by Kirisci[26].

In [30], new distance and cosine similarity metrics between FFSs are given. A novel hesitant fuzzy set (HFS) has been introduced, dubbed the Fermatean hesitant fuzzy set (FHFS), and some of its characteristics have been investigated [33]. The group DM describes the ELECTRE I technique in [29] with FFSs when multiple persons engage simultaneously. In [43], new weighted aggregated operators pertinent to FFSs are defined. Shahzadi and Akram [44] created new aggregated operators and developed a novel FFSS decision support method. New FFS-type aggregated operators described by t-norm and t-conorm were proposed by Garg et al. [21]. Recent works on FFSs [2], [3], [4] show the use of FFSs with applications like FF soft expert knowledge, FF N-soft sets, and COVID-19 applications. Ashraf and Akram [6] have provided several novel Fermatean fuzzy sets-based information measures, including distance, similarity, entropy, and inclusion. In [38], Onyeke and Ejegwa identified the inaccuracies of the Fermatean fuzzy distance measurement defined by Senapati and Yager and its contradictions with the definition of distance measurement. Thus, Onyeke and Ejegwa modified this distance measure. Ejegwa and Onyeke, in [13], a novel similarity measure between Fermatean fuzzy sets is introduced with a better and more reliable output than the dual of the existing Fermatean fuzzy distance measure. Ejegwa et al. [14] presented an improved Fermatean fuzzy composition relation with better performance based on the maximum mean approach.

Instruments such as information measurements and aggregation operators are frequently used in decision-making tasks. Another tool for making the optimal decision is the $\mathcal{C}$ measure, which indicates the level of reliance between two groups. $\mathcal{C}$s are used to measure the degree of connection between two variables. Since the information could be more precise, complete, and imprecise, several scholars have established the $\mathcal{C}$s in fuzzy environments. Chiang and Lin [9] have presented a solution for $\mathcal{C}$ based on FSs while fulfilling the correlation for fuzzy information according to classical statistics. Liu and Kao [35] have examined the $\mathcal{C}$ of fuzzy information by applying the traditional notion of $\mathcal{C}$s as an approximation in mathematical programming. The association between interval-valued intuitionistic fuzzy sets (IVIFSs) in finite universal sets was examined by Bustince and Burillo [8]. Hong examined the correlation of IVIFSs in probability spaces [24]. Szmidt and Kacprzyk established three parameters [45] to define the distance between IFSs and provide a geometrical depiction of the IFS. Zeng and Li [51] proposed a new method to calculate the correlation and $\mathcal{C}$ of IFSs, similar to the cosine of the intersectional angle in the finite set and probability space, respectively. Based on the kind of geometrical background, they considered all three parameters describing IFSs. Mitchell defined $\mathcal{C}$s for IFSs

[37]. Mitchell demonstrated how we could obtain a straightforward, logically satisfying $\mathcal{C}$ between two IFSs by viewing an IFS as an ensemble of regular FSs. In [46], several operational rules of IVIF numbers, correlation, and $\mathcal{C}$ of IVIFSs are introduced about MCDM problems with IVIF information. To correct the drawbacks in some existing techniques in terms of mathematical presentation and the exclusion of the hesitation parameter to enhance reasonable output, a new method of IFCC is developed [12]. In [17], a new approach to finding the partial $\mathcal{C}$ of IFSs was proposed by incorporating three parameters of intuitionistic fuzzy data based on a modified $\mathcal{C}$ of IFSs approach.

The development of PFS to solve an IFS problem has allowed the publication of various DM issues incorporating PF information in the literature. Garg [20] introduced a novel $\mathcal{C}$ and weighted $\mathcal{C}$ formulation to quantify the link between two PFSs after pointing out the flaws in the existing $\mathcal{C}$s between IFSs. Lin et al. [36] present a unique TOPSIS approach for the linguistic PFS based on the entropy measure and $\mathcal{C}$. The $\mathcal{C}$ is suggested to measure the link between linguistic PFSs. Two techniques for calculating PFCC have been developed to produce more trustworthy ways [19]. In [15], in the PF $\mathcal{C}$, some techniques for calculating the $\mathcal{C}$ of PFSs from a statistical perspective have been proposed. However, since these techniques have some limitations, some new statistical techniques have been given to calculate the $\mathcal{C}$ of PFS using PF variance and covariance, which solve the limitations with better performance indices. Zheng et al. [53] have defined four kinds of correlation coefficients for Pythagorean hesitant fuzzy sets and extended them to the correlation coefficients and the weighted correlation coefficients for interval-valued Pythagorean hesitant fuzzy sets.

New $\mathcal{C}$s with FFS have been defined recently. In the first investigation on $\mathcal{C}$s, Kirisci defined them based on FFSs [28], and their basic properties were examined. The article of Amman et al. [5] presents the concept, representation, and pertinent characteristics of the SR$\mathcal{C}$ within the context of FFSs. In this work, an MCDM technique, fortified by incorporating FF-operators, is formulated based on the proposed SR$\mathcal{C}$. New $\mathcal{C}$s were presented by Bhatia et al. [7] utilizing FFSs and well-defined statistical principles. These $\mathcal{C}$s were used to obtain the weighted $\mathcal{C}$s. Demir [10] classified Fermatean hesitant FSs into four categories of $\mathcal{C}$s. However, novel $\mathcal{C}$s have been discovered and applied to IVFHFSs. Due to some deficiencies in FFCCs defined by Kirisci [28], Ejegwa and Sarkar [18] developed two new $\mathcal{C}$ operators based on FFs. A statistical concept-based reframing of this initial investigation was done in [31]. This study further classified PEARs as associated with FFSs. The least common multiple expansion approach has been proposed in [32] to handle the problem when the cardinality of FHFEs and IVFHFEs differs for obtained $\mathcal{C}$s with FHFS and IVFHFS during operations with new $\mathcal{C}$s. Ejegwa et al. [11] conducted a new study that examined the $\mathcal{C}$s in the literature on FFSs and resolved the reliability and precision issues in these methods. Kirisci [34], using the notions of variance and covariance, defined a three-way method for computing the $\mathcal{C}$s between FFSs. Therefore, the potential of inaccuracy owing to information leakage has been reasonably mitigated by the suggested technique's inclusion of the three traditional FFS parameters.

1.3. **Necessity.** Techniques according to IFS and PFS cannot capture data in FFS type. IFS and PFS-based DM procedures will fail when experts supply preference values in the form of FFSs. Nonetheless, FFS-based techniques, such as this, can quickly review data and rank viable solutions based on criterion values. Because it incorporates IFSs and PFSs into a single platform, the FFS ecosystem will handle more data and cover various topics for processing uncertain data. As a result, additional information must be lost in this collection. The proposed $\mathcal{C}$s are believed to be a particular instance of the existing $\mathcal{C}$s under the IFS and PFS.

Additionally, the suggested $\mathcal{C}$s handle more information than the current ones. The presented approach has far more information than the existing approaches to handle data uncertainty in the IFS and PFS settings. It provides more precise and accurate information on things. As a result, it is an effective instrument for dealing with cloudy and confusing data throughout the DM process. These investigations present a new approach to computing defined $\mathcal{C}$ with the help of FFSs based on a three-way statistical method. As can be seen from the outputs of this work, the three-way $\mathcal{C}$s obtained using FFSs have a structure that will facilitate the solution of MCDM problems.

1.4. **Contribution.** This study has made the following contributions:

1. Today's fuzzy or non-standard fuzzy theory commonly uses $\mathcal{C}$s with values between 0 and 1, which only indicate the intensity of the relationship. A $\mathcal{C}$ with a value in $[1, 1]$ can be used to correlate uncertain notions easily.

2. For their comparison metrics, most studies on fuzzy and non-standard FSs rely on faked data. The present $\mathcal{C}$s based on IF and PF do not meet all or part of these criteria. Therefore, we suggest some new $\mathcal{C}$s for FFSs in this study that are higher than the current $\mathcal{C}$s while considering these factors.

3. We propose four new $\mathcal{C}$s and discuss some of their appealing properties for FFSs with values in $[1, 1]$.

4. Data mining and medical diagnostics applications are examined using our proposed FF-$\mathcal{C}$s. The suggested $\mathcal{C}$ FFSs also contrast compatibility metrics already existing under FF conditions.

2. Preliminaries

For the initial universe set $\mathcal{U}$, the set $\mathfrak{P} = \{(k, m_{\mathfrak{P}}(k), n_{\mathfrak{P}}(k)) : k \in \mathcal{U}\}$ is called a FFS with $m_{\mathfrak{P}}, n_{\mathfrak{P}} : \mathcal{U} \to [0, 1]$ and $0 \leq m^3_{\mathfrak{P}}(k) + n^3_{\mathfrak{P}}(k) \leq 1$. The equation $\theta_{\mathfrak{P}} = \Big(1 - m^3_{\mathfrak{P}}(k) - n^3_{\mathfrak{P}}(k)\Big)^{1/3}$ [41].

Definition 2.1. [25] *Let $Int[0, 1]$ show the set of all closed subintervals of $[0, 1]$. The set $F = \{(k, \alpha_F(k), \beta_F(k)) : k \in E\}$ is called an IVFFS on a set $E \neq \emptyset$, where $\alpha_F(k), \beta_F(k) \in Int[0, 1]$ with the condition $0 < \sup_k(\alpha_F(k))^3 + \sup_k(\beta_F(k))^3 \leq 1$.*

Furthermore, F can be expressed as:

$$F = \{(k, [m_{F_L}(k), m_{F_U}(k)], [n_{F_L}(k), n_{F_U}(k)]) : k \in E\}$$

with $0 \leq (m_{F_U}(k))^3 + (n_{F_U}(k))^3 \leq 1$. The hesitation degree has been shown with

$$\theta_F = [\theta_{FL}, \theta_{FU}] = [(1 - m^3_{FU} - n^3_{FU})^{1/3}, (1 - m^3_{FL} - n^3_{FL})^{1/3}]. \tag{1}$$

Definition 2.2. [25] *Choose the three IVFFSs $F = ([m_{F_L}(k), m_{F_U}(k)], [n_{F_L}(k), n_{F_U}(k)])$, $F_1 = ([m_{F_{1L}}(k), m_{F_{1U}}(k)], [n_{F_{1L}}(k), n_{F_{1U}}(k)])$, $F_2 = ([m_{F_{2L}}(k), m_{F_{2U}}(k)], [n_{F_{2L}}(k), n_{F_{2U}}(k)])$. Then,*

- $F_1 \cup F_2 = \Big(\big[\max(m_{F_{1L}}, m_{F_{2L}}), \max(m_{F_{1U}}, m_{F_{2U}}) \big], \big[\min(m_{F_{1L}}, m_{F_{2L}}), \min(m_{F_{1U}}, m_{F_{2U}}) \big] \Big)$
- $F_1 \cap F_2 = \Big(\big[\min(m_{F_{1L}}, m_{F_{2L}}), \min(m_{F_{1U}}, m_{F_{2U}}) \big], \big[\max(m_{F_{1L}}, m_{F_{2L}}), \max(m_{F_{1U}}, m_{F_{2U}}) \big] \Big)$
- $F^c = ([n_{F_L}, n_{F_U}], [m_{F_L}, m_{F_U}])$
- $F_1 \oplus F_2 = \Big(\Big[\sqrt[3]{(m_{F_{1L}}(k))^3 + (m_{F_{2L}}(k))^3 - (m_{F_{1L}}(k))^3.(m_{F_{2L}}(k))^3}, \sqrt[3]{(m_{F_{1U}}(k))^3 + (m_{F_{2U}}(k))^3 - (m_{F_{1U}}(k))^3.(m_{F_{2U}}(k))^3} \Big], [n_{F_{1L}} n_{F_{2L}}, n_{F_{1U}} n_{F_{2U}}] \Big)$
- $F_1 \otimes F_2 = \Big([m_{F_{1L}} m_{F_{2L}}, m_{F_{1U}} m_{F_{2U}}], \Big[\sqrt[3]{(n_{F_{1L}}(k))^3 + (n_{F_{2L}}(k))^3 - (n_{F_{1L}}(k))^3.(n_{F_{2L}}(k))^3}, \sqrt[3]{(n_{F_{1U}}(k))^3 + (n_{F_{2U}}(k))^3 - (n_{F_{1U}}(k))^3.(n_{F_{2U}}(k))^3} \Big] \Big)$
- $\lambda F = \left(\left[\sqrt[3]{1 - \left(1 - m_{F_L}^3\right)^{\lambda}}, \sqrt[3]{1 - \left(1 - m_{F_U}^3\right)^{\lambda}} \right], \left[n_{F_L}^{\lambda}, n_{F_U}^{\lambda} \right] \right)$
- $F^{\lambda} = \left(\left[m_{F_L}^{\lambda}, m_{F_U}^{\lambda} \right], \left[\sqrt[3]{1 - \left(1 - n_{F_L}^3\right)^{\lambda}}, \sqrt[3]{1 - \left(1 - n_{F_U}^3\right)^{\lambda}} \right] \right)$

Definition 2.3. [25] *Let* $F = ([m_{F_L}(k), m_{F_U}(k)], [n_{F_L}(k), n_{F_U}(k)])$,
$F_1 = ([m_{F_{1L}}(k), m_{F_{1U}}(k)], [n_{F_{1L}}(k), n_{F_{1U}}(k)])$,
$F_2 = ([m_{F_{2L}}(k), m_{F_{2U}}(k)], [n_{F_{2L}}(k), n_{F_{2U}}(k)])$ *be three IVFFSs.*
Then, for $\lambda, \lambda_1, \lambda_2 > 0$,

- $F_1 \oplus F_2 = F_2 \oplus F_1$
- $F_1 \otimes F_2 = F_2 \otimes F_1$
- $\lambda(F_1 \oplus F_2) = \lambda F_1 \oplus \lambda F_2$
- $(\lambda_1 + \lambda_2)F = \lambda_1 F + \lambda_2 F$
- $(F_1 \otimes F_2)^{\lambda} = F_1^{\lambda} \otimes F_2^{\lambda}$
- $F^{\lambda_1} \otimes F^{\lambda_2} = F^{\lambda_1 + \lambda_2}$

Let $F_i = ([m_{F_{iL}}(k), m_{F_{iU}}(k)], [n_{F_{iL}}(k), n_{F_{iU}}(k)])$, $(i = 1, 2, \cdots, n)$ be a category of IVFFSs. The IVFF-weighted average and IVFF-weighted geometric operators are $IVFFWA, IVFFWG : F^n \to F$ where

$$
\begin{aligned}
IVFFWA(F_1, F_2, \cdots, F_n) &= \Bigg(\Bigg[\left(1-\prod_{j=1}^{n}(1-(m_j^{F_L})^3)^{\omega_j}\right)^{1/3}, \\
&\left(1-\prod_{j=1}^{n}(1-(m_j^{F_U})^3)^{\omega_j}\right)^{1/3}\Bigg], \\
&\left[\prod_{j=1}^{n}(n_j^{F_L})^{\omega_j}, \prod_{j=1}^{n}(n_j^{F_U})^{\omega_j}\right]\Bigg) \\
IVFFWG(F_1, F_2, \cdots, F_n) &= \Bigg(\left[\prod_{j=1}^{n}(m_j^{F_L})^{\omega_j}, \prod_{j=1}^{n}(m_j^{F_U})^{\omega_j}\right], \\
&\Bigg[\left(1-\prod_{j=1}^{n}(1-(n_j^{F_L})^3)^{\omega_j}\right)^{1/3}, \\
&\left(1-\prod_{j=1}^{n}(1-(n_j^{F_U})^3)^{\omega_j}\right)^{1/3}\Bigg],\Bigg)
\end{aligned}
$$

Let $F = ([m_{F_L}, m_{F_U}], [n_{F_L}, n_{F_U}])$ be an IVFFN. The score and accuracy function of F are defined as:

$$
\begin{aligned}
SC(F) &= \frac{1}{2}\left((m_{F_L})^3 + (m_{F_U})^3 - (n_{F_L})^3 - (n_{F_U})^3\right), \\
AC(F) &= \frac{1}{2}\left((m_{F_L})^3 + (m_{F_U})^3 + (n_{F_L})^3 + (n_{F_U})^3\right).
\end{aligned}
$$

For two IVFFNs $F_1 = ([m_{F_{1L}}, m_{F_{1U}}], [n_{F_{1L}}, n_{F_{1U}}])$, $F_2 = ([m_{F_{2L}}, m_{F_{2U}}], [n_{F_{2L}}, n_{F_{2U}}])$,

i. If $SC(F_1) > SC(F_2)$, then $F_1 > F_2$,
ii. If $SC(F_1) = SC(F_2)$, then
  a. If $AC(F_1) > AC(F_2)$, then $F_1 > F_2$,
  b. If $AC(F_1) < AC(F_2)$, then $F_1 < F_2$,
  c. If $AC(F_1) = AC(F_2)$, then $F_1 = F_2$.

Existing correlation coefficient approaches for several fuzzy sets are given below:

The robust $\mathcal{C}$s based on PFSs [16] are given as:

Definition 2.4. *For two PFS $\mathfrak{P}, \mathfrak{Q}$ in $\mathcal{U}$ and the weight ω,*

$$
RPFS(\mathfrak{P}, \mathfrak{Q}) = \frac{K(\mathfrak{P}, \mathfrak{Q})}{\sqrt{VR(\mathfrak{P})VR(\mathfrak{Q})}} \tag{2}
$$

$$
RPFS_\omega(\mathfrak{P}, \mathfrak{Q}) = \frac{K_\omega(\mathfrak{P}, \mathfrak{Q})}{\sqrt{VR_\omega(\mathfrak{P})VR_\omega(\mathfrak{Q})}} \tag{3}
$$

where the deviations, variances and covariance are $D_i(\mathfrak{P}) = (\alpha_{\mathfrak{P}}^2 - \alpha_{\mathfrak{P}}^2) - (\beta_{\mathfrak{P}}^2 - \beta_{\mathfrak{P}}^2) - (\gamma_{\mathfrak{P}}^2 - \gamma_{\mathfrak{P}}^2)$, $D_i(\mathfrak{Q}) = (\alpha_{\mathfrak{Q}}^2 - \alpha_{\mathfrak{Q}}^2) - (\beta_{\mathfrak{Q}}^2 - \beta_{\mathfrak{Q}}^2) - (\gamma_{\mathfrak{Q}}^2 - \gamma_{\mathfrak{Q}}^2)$, $VR(\mathfrak{P}) = \frac{1}{n-1}\sum_{i=1}^{n} D_i(\mathfrak{P})^2$, $VR(\mathfrak{Q}) = \frac{1}{n-1}\sum_{i=1}^{n} D_i(\mathfrak{Q})^2$ and $K(\mathfrak{P}, \mathfrak{Q}) = \frac{1}{n-1}\sum_{i=1}^{n} D_i(\mathfrak{P})D_i(\mathfrak{Q})$, respectively.

The $\mathcal{C}$s defined by Kirisci [28] based on FFSs are defined as follows:

Definition 2.5. *Choose two FFSs $\mathfrak{P}$ and $\mathfrak{Q}$. Then*

$$\mathfrak{C}(\mathfrak{P},Q) = \frac{C(\mathfrak{P},\mathfrak{Q})}{[I(\mathfrak{P}).I(\mathfrak{Q})]^{1/2}} \tag{4}$$

$$= \frac{\sum_{i=1}^{n}\left(\alpha_{\mathfrak{P}}^{3}(k_i)\alpha_{\mathfrak{Q}}^{3}(k_i)+\beta_{\mathfrak{P}}^{3}(k_i)\beta_{\mathfrak{Q}}^{3}(k_i)+\gamma_{\mathfrak{P}}^{3}(k_i)\gamma_{\mathfrak{Q}}^{3}(k_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\alpha_{\mathfrak{P}}^{6}(k_i)+\beta_{\mathfrak{P}}^{6}(k_i)+\gamma_{\mathfrak{P}}^{6}(k_i)\right)}.\sqrt{\sum_{i=1}^{n}\left(\alpha_{\mathfrak{Q}}^{6}(k_i)+\beta_{\mathfrak{Q}}^{6}(k_i)+\gamma_{\mathfrak{Q}}^{6}(k_i)\right)}}$$

$$\mathfrak{D}(\mathfrak{P},\mathfrak{Q}) = \frac{C(\mathfrak{P},\mathfrak{Q})}{\max[I(\mathfrak{P}).I(\mathfrak{Q})]} \tag{5}$$

$$\mathfrak{C}_\omega(\mathfrak{P},\mathfrak{Q}) = \frac{C_\omega(\mathfrak{P},\mathfrak{Q})}{[I_\omega(\mathfrak{P}).I_\omega(\mathfrak{Q})]^{1/2}} \tag{6}$$

$$\mathfrak{D}_\omega(\mathfrak{P},\mathfrak{Q}) = \frac{C\omega(\mathfrak{P},\mathfrak{Q})}{\max[I\omega(\mathfrak{P}).I\omega(\mathfrak{Q})]}. \tag{7}$$

are called the $\mathcal{C}s$ between $\mathfrak{P},\mathfrak{Q}$.

The PEARs defined in [31] are:

Definition 2.6. *For two FFSs $\mathfrak{P}$ and $\mathfrak{Q}$, the PEARs are:*

$$\mathfrak{C}_{pearson}(\mathfrak{P},\mathfrak{Q}) = \frac{1}{2}(\mathfrak{Z}_1+\mathfrak{Z}_2) \tag{8}$$

$$\mathfrak{C}_{pearson}(\mathfrak{P},\mathfrak{Q}) = \frac{1}{2}(\mathfrak{Z}_1+\mathfrak{Z}_2+\mathfrak{Z}_3) \tag{9}$$

where

$$\mathfrak{Z}_1 = \frac{\sum_{j=1}^{m}\left\{[\alpha_{\mathfrak{P}}^{3}(k_j)-\overline{\alpha_{\mathfrak{P}}}^{3}]\times[\alpha_{\mathfrak{Q}}^{3}(k_j)-\overline{\alpha_{\mathfrak{Q}}}^{3}]\right\}}{\sqrt{[\alpha_{\mathfrak{P}}^{3}(k_j)-\overline{\alpha_{\mathfrak{P}}}^{3}]}\sqrt{[\alpha_{\mathfrak{Q}}^{3}(k_j)-\overline{\alpha_{\mathfrak{Q}}}^{3}]}}$$

$$\mathfrak{Z}_2 = \frac{\sum_{j=1}^{m}\left\{[\beta_{\mathfrak{P}}^{3}(k_j)-\overline{\beta_{\mathfrak{P}}}^{3}]\times[\beta_{\mathfrak{Q}}^{3}(k_j)-\overline{\beta_{\mathfrak{Q}}}^{3}]\right\}}{\sqrt{[\beta_{\mathfrak{P}}^{3}(k_j)-\overline{\beta_{\mathfrak{P}}}^{3}]}\sqrt{[\beta_{\mathfrak{Q}}^{3}(k_j)-\overline{\beta_{\mathfrak{Q}}}^{3}]}}$$

$$\mathfrak{Z}_3 = \frac{\sum_{j=1}^{m}\left\{[\gamma_{\mathfrak{P}}^{3}(k_j)-\overline{\gamma_{\mathfrak{P}}}^{3}]\times[\gamma_{\mathfrak{Q}}^{3}(k_j)-\overline{\gamma_{\mathfrak{Q}}}^{3}]\right\}}{\sqrt{[\gamma_{\mathfrak{P}}^{3}(k_j)-\overline{\gamma_{\mathfrak{P}}}^{3}]}\sqrt{[\gamma_{\mathfrak{Q}}^{3}(k_j)-\overline{\gamma_{\mathfrak{Q}}}^{3}]}}$$

Some statistical concepts will be given: Let a sample from a population be selected as $(O_1, O_2, \cdots, O_n)$. In this sample, the sample observations are sorted in ascending order $(o_1, o_2, \cdots, o_n)$, $(o_{(1)} < \cdots < o_{(n)})$. When $o_i = o_{(m)}$, then m is said to be rank of the sample O_i, that is $S_i = m$ $(m = 1, \cdots, n)$.

O_i is a random variable in each sampling iteration. The average of their ranks can be used to determine their ranks whenever a situation where some o is the same arises, such as when $o_i = o_j$ for $i \neq j$.

The PEAR multiplied by the rankings O yields the SR$\mathcal{C}$ in statistics. When there are no two values of O or P with the same rank (also known as ties), there is a simpler method to get the SR$\mathcal{C}$:

$$R_S = 1 - \frac{6\sum_{i=1}^{n} f_i^2}{n(n^2-1)} \tag{10}$$

where f_i are the differences in the ranks of $o_{(1)}$ and $p_{(1)}$ as $f_i = Rank(o_i) - Rank(p_i)$.

In the event of ties (two O values or two P values with the same rank), there are fewer ties than n. Moreover, Equation 10 remains valid. The requirements of the correlation measurements are satisfied by the SR$\mathcal{C}$. Since Equation 10 is derived from the PEAR for ranks, it shares the same characteristics as the PEAR:

i. $R_S(\mathfrak{P},\mathfrak{Q}) = R_S(\mathfrak{Q},\mathfrak{P})$
ii. $R_S(\mathfrak{P},\mathfrak{Q}) = 1$, when $\mathfrak{P} = \mathfrak{Q}B$
iii. $|R_S(\mathfrak{P},\mathfrak{Q})| \leq 1$.

Ejegwa et al. [11] published a new study that examined FF-based $\mathcal{C}$s found in the literature and corrected the errors in these studies.

Let $\mathfrak{P}$ and $\mathfrak{Q}$ be two FFSs. Then, FFCC based on Spearman correlation

$$\mathfrak{C}_{ejegwaetal}(\mathfrak{P},\mathfrak{Q}) = \frac{1}{3}(\mathfrak{D}_{ejegwa}(\mathfrak{P},\mathfrak{Q})_1 + \mathfrak{D}_{ejegwa}(\mathfrak{P},\mathfrak{Q})_2 + \mathfrak{D}_{ejegwa}(\mathfrak{P},\mathfrak{Q})_3) \tag{11}$$

where

$$\begin{aligned}
\mathfrak{D}_{ejegwaetal}(\mathfrak{P},\mathfrak{Q})_1 &= 1 - \frac{6\sum_{i=1}^{k} |\mathfrak{P}_1^3(x_i) - \mathfrak{Q}_1^3(x_i)|^3}{k(k^2+1)} \\
\mathfrak{D}_{ejegwaetal}(\mathfrak{P},\mathfrak{Q})_2 &= 1 - \frac{6\sum_{i=1}^{k} |\mathfrak{P}_2^3(x_i) - \mathfrak{Q}_2^3(x_i)|^3}{k(k^2+1)} \\
\mathfrak{D}_{ejegwaetal}(\mathfrak{P},\mathfrak{Q})_3 &= 1 - \frac{6\sum_{i=1}^{k} |\mathfrak{P}_3^3(x_i) - \mathfrak{Q}_3^3(x_i)|^3}{k(k^2+1)}.
\end{aligned}$$

The Equation (11) is equivalent to

$$\begin{aligned}
\mathfrak{C}_{ejegwaetal}(\mathfrak{P},\mathfrak{Q}) &= 1 - \left(\frac{6\sum_{i=1}^{k} \Big(|\mathfrak{P}_1^3(x_i) - \mathfrak{Q}_1^3(x_i)|^3 + |\mathfrak{P}_2^3(x_i) - \mathfrak{Q}_2^3(x_i)|^3}{3k(k^2+1)} \right. \\
&+ \left. \frac{|\mathfrak{P}_3^3(x_i) - \mathfrak{Q}_3^3(x_i)|^3 \Big)}{3k(k^2+1)} \right)
\end{aligned}$$

3. Spearman Rank Correlation Coefficients

Using the FFCC-based Spearman correlation equation (Equation 11) by Ejegwa et al [11], we will define IVFF-Spearman correlation coefficients and examine their basic properties.

Definition 3.1. *For two IVFFSs $\mathfrak{P}$ and $\mathfrak{Q}$, the SR$\mathcal{C}$s are:*

$$R_{Spearman}(\mathfrak{P},\mathfrak{Q}) = \frac{1}{3}(\mathfrak{Z}_1 + \mathfrak{Z}_2 + \mathfrak{Z}_3) \tag{12}$$

where

$$\begin{aligned}\mathfrak{z}_1 &= 1-\frac{6\sum_{i=1}^n f_{\alpha_i}^3}{n(n^2+1)},\\ \mathfrak{z}_2 &= 1-\frac{6\sum_{i=1}^n f_{\beta_i}^3}{n(n^2+1)},\\ \mathfrak{z}_3 &= 1-\frac{6\sum_{i=1}^n f_{\gamma_i}^3}{n(n^2+1)},\end{aligned}$$

where $f_{\alpha_i}, f_{\beta_i}, f_{\gamma_i}$ are the differences in the ranks concerning the MD, ND, and HD with

$$\begin{aligned}f_{\alpha_i} &= \Big|Rank(\alpha_{\mathfrak{P}L}(o_i)+\alpha_{\mathfrak{P}U}(o_i)) - Rank(\alpha_{\mathfrak{Q}L}(o_i)+\alpha_{\mathfrak{Q}U}(o_i))\Big|,\\ f_{\beta_i} &= \Big|Rank(\beta_{\mathfrak{P}L}(o_i)+\beta_{\mathfrak{P}U}(o_i)) - Rank(\beta_{\mathfrak{Q}L}(o_i)+\beta_{\mathfrak{Q}U}(o_i))\Big|,\\ f_{\gamma_i} &= \Big|Rank(\gamma_{\mathfrak{P}L}(o_i)+\gamma_{\mathfrak{P}U}(o_i)) - Rank(\gamma_{\mathfrak{Q}L}(o_i)+\gamma_{\mathfrak{Q}U}(o_i))\Big|.\end{aligned}$$

Theorem 3.1. *The Equation (12) is equivalent to*

$$R_{Spearman}(\mathfrak{P},\mathfrak{Q}) = 1-\frac{6\sum_{i=1}^k \left(f_{\alpha_i}^3+f_{\beta_i}^3+f_{\gamma_i}^3\right)}{3k(k^2+1)}.$$

Proof. Since

$$\begin{aligned}R_{Spearman}(\mathfrak{P},\mathfrak{Q}) &= \frac{1}{3}\Bigg[\frac{k(k^2+1)-6\sum_{i=1}^n f_{\alpha_i}^3}{k(k^2+1)} + \frac{k(k^2+1)-6\sum_{i=1}^n f_{\beta_i}^3}{k(k^2+1)}\\ &\quad + \frac{k(k^2+1)-6\sum_{i=1}^n f_{\gamma_i}^3}{k(k^2+1)}\Bigg],\end{aligned}$$

then

$$\begin{aligned}R_{Spearman}(\mathfrak{P},\mathfrak{Q}) &= \frac{1}{3k(k^2+1)}\left[3k(k^2+1)-6\sum_{i=1}^n f_{\alpha_i}^3 - 6\sum_{i=1}^n f_{\beta_i}^3 - 6\sum_{i=1}^n f_{\gamma_i}^3\right]\\ &= \frac{1}{3k(k^2+1)}\left[3k(k^2+1)-6\sum_{i=1}^n f_{\alpha_i}^3+f_{\beta_i}^3+f_{\gamma_i}^3\right]\\ &= 1-\frac{6\sum_{i=1}^n f_{\alpha_i}^3+f_{\beta_i}^3+f_{\gamma_i}^3}{3k(k^2+1)}.\end{aligned}$$

□

Theorem 3.2. *Take two IVFFSs P and Q:*

i. $R_{Spearman}(\mathfrak{P},\mathfrak{Q}) = R_{Spearman}(\mathfrak{Q},\mathfrak{P})$,
ii. $-1 \le R_{Spearman}(\mathfrak{P},\mathfrak{Q}) \le 1$,
iii. $R_{Spearman}(\mathfrak{P},\mathfrak{Q}) = 1$ *if and only if* $\mathfrak{P}=\mathfrak{Q}$.

Proof. (i.) Straighforward.

(ii.) Its clear that $\sum_{i=1}^{n} f_{\alpha_i}^3 + f_{\beta_i}^3 + f_{\gamma_i}^3 \geq 0$. From here it can be easily seen that $R_{Spearman}(\mathfrak{P}, \mathfrak{Q}) \geq 1$. Further,

$$\begin{aligned} R_{Spearman}(\mathfrak{P}, \mathfrak{Q}) &= 1 - \frac{6\sum_{i=1}^{k}\left(f_{\alpha_i}^3 + f_{\beta_i}^3 + f_{\gamma_i}^3\right)}{3k(k^2+1)} \\ &\leq 1 - \frac{6\sum_{i=1}^{k}\left(f_{\alpha_i}^3 + 6\sum_{i=1}^{k} f_{\beta_i}^3 + 6\sum_{i=1}^{k} f_{\gamma_i}^3\right)}{3k(k^2+1)} \end{aligned}$$

and so

$$R_{Spearman}(\mathfrak{P}, \mathfrak{Q}) - 1 = -\frac{6\sum_{i=1}^{k}\left(f_{\alpha_i}^3 + 6\sum_{i=1}^{k} f_{\beta_i}^3 + 6\sum_{i=1}^{k} f_{\gamma_i}^3\right)}{3k(k^2+1)} \leq 0.$$

Therefore, $R_{Spearman}(\mathfrak{P}, \mathfrak{Q}) - 1 \leq 0 \Rightarrow R_{Spearman}(\mathfrak{P}, \mathfrak{Q}) \leq 1$.
That is, $-1 \leq R_{Spearman}(\mathfrak{P}, \mathfrak{Q}) \leq 1$.

(iii.) Assumed that $R_{Spearman}(\mathfrak{P}, \mathfrak{Q}) = 1$. Hence,

$$\frac{6\sum_{i=1}^{k}\left(f_{\alpha_i}^3 + f_{\beta_i}^3 + f_{\gamma_i}^3\right)}{3k(k^2+1)} = 0.$$

and

$$6\sum_{i=1}^{k}\left(f_{\alpha_i}^3 + f_{\beta_i}^3 + f_{\gamma_i}^3\right) = 0 \Rightarrow \quad f_{\alpha_i}^3 = 0, \quad f_{\beta_i}^3 = 0, \quad f_{\gamma_i}^3 = 0.$$

Therefore, $\alpha_{\mathfrak{P}L}(o_i)^3 + \alpha_{\mathfrak{P}U}(o_i)^3 = \alpha_{\mathfrak{Q}L}(o_i)^3 + \alpha_{\mathfrak{Q}U}(o_i)^3$, $\beta_{\mathfrak{P}L}(o_i)^3 + \beta\mathfrak{P}U(o_i)^3 = \beta_{\mathfrak{Q}L}(o_i)^3 + \beta_{\mathfrak{Q}U}(o_i)^3$, and, $\gamma_{\mathfrak{P}L}(o_i)^3 + \gamma_{\mathfrak{P}U}(o_i)^3 = \gamma_{\mathfrak{Q}L}(o_i)^3 + \gamma_{\mathfrak{Q}U}(o_i)^3$.

Conversely, let $\mathfrak{P} = \mathfrak{Q}$, that is $\sum_{i=1}^{k}\left(f_{\alpha_i}^3 + f_{\beta_i}^3 + f_{\gamma_i}^3\right) = 0$. Then, $R_{Spearman}(\mathfrak{P}, \mathfrak{Q}) = 0$. □

As the $R_{Spearman}$ has these properties, the components of the RS, $\mathfrak{Z}_1$, $\mathfrak{Z}_2$ and $\mathfrak{Z}_3$ have the same properties.

4. Application

4.1. **Problem Design.** Consider alternatives $\Gamma = \{\Gamma_1, \cdots, \Gamma_m\}$, for a given DM problem. Let $\Omega = \{\Omega_1, \cdots, \Omega_n\}$ be the set of criteria (without weights) measured by IVFFNs $D_{ij} = (\alpha_{ij}, \beta_{ij})$. Let $\Sigma = \{\Sigma_1, \cdots, \Sigma_k\}$ be the set of professionals evaluating the criteria. Let $D_{ij}^{(l)} = (\alpha_{ij}^l, \beta_{ij}^l)$ shows a IVFFN evaluation matrix from the lth professional. A new IVFF multi-criteria DM strategy is created based on the SR$\mathcal{C}$ between two IVFFSs and begins by identifying the IVFF ideal solution. The accuracy and score functions are used to get the IVFF-perfect solution. The real selection process typically lacks an optimal IVFF solution. In other words, the viable alternative is typically not the Γ^*, the IVFF's ideal solution vector. If not, the best alternate vector for the selection problem is the IVFF perfect solution vector Γ^*. A multi-criteria DM algorithm is suggested as a result of the analysis above, and it gauges how closely each option is related to the ideal alternative using the SR$\mathcal{C}$. The following phases and selection procedures can be used to describe the algorithm(Figure 1):

1. Using IVFFWA, construct the decision matrix $D_{m\times n} = (D_{ij})_{m\times n}$,

$$D_{ij} = IVFFWA(D_{ij}^{(1)}, D_{ij}^{(2)}, \cdots, D_{ij}^{(k)})$$

and $D_{ij}^{(l)}$ is the evaluation value of the alternative $\Gamma_i \in \Gamma$ w.r.t the criterion $\Omega_j \in \Omega$ from the lth professional, and ω_l is the lth professional' weight.

2. Using the following equation, find the FF perfect solution $\Gamma*$

$$\begin{cases} \Gamma^* = \{\Gamma_1^*, \cdots, \Gamma_m*\}, where \Gamma^* = \{\Omega_j, \max_i\{D_{ij} : j = 1, 2, \cdots, n\}\}, & \textit{for benefit criteria} \\ \Gamma^* = \{\Gamma_1^*, \cdots, \Gamma_m*\}, where \Gamma^* = \{\Omega_j, \min_i\{D_{ij} : j = 1, 2, \cdots, n\}\}, & \textit{for cost criteria} \end{cases}$$

3. Compute the SR$\mathcal{C}$ between the ideal and each alternatives.

4. The greater the SR$\mathcal{C}$ obtained in Step 3, the better the matching option. As a result, the ideal choice ranking order and selection are found.

4.2. **Numerical Illustration.** In Turkey, disaster preparedness drills are prepared and carried out in hospitals annually by the Ministry of Health within the framework of Hospital Disaster Management. Hospital disaster plans and drills are among the mandatory works to be carried out in Turkey, according to the legal obligations of the Ministry of Health. After the drills are completed, the hospital's emergency competencies are determined. These competencies are reported and communicated to all units in the hospital and the Ministry of Health. The numeric example below is related to the exercise of the actual plan prepared by the Hospital Disaster Management Department of Cerrahpaşa Medical Faculty Hospital.

The faculty dean and the hospital head of physician will coordinate a disaster exercise to monitor potential crisis scenarios and issues at the hospital of medicine faculty and to pinpoint any weaknesses. The goal of the practice's final phase is for the director of the hospital's physician department and the dean of the faculty to assess the pertinent units' emergency response skills to make future decisions. Γ_1 Medical Observation, Examination, and Consultation, Γ_2 Planning and Effective Communications, Γ_3 Supply Chain and Financial Structure, and Γ_4 Transport, inventory and storage are the departments that will participate in the Hospital Disaster Management research. Selected specialists inside and outside the faculty hospital are invited to review these elements using the evaluation phrases in the linguistic word set. The evaluation standards that were established include Ω_1 pre-disaster readiness, Ω_2 process control during the calamity, Ω_3 post-disaster loss evaluation, Ω_4 disaster assistance capacity, and Ω_5 post-disaster building. The formula for calculating Ω_j $(j = 1, \cdots, 5)$ weights is $\omega = (0.2, 0.25, 0.1, 0.3, 0.15)^T$.

1. The four possible alternatives are evaluated by using the FFL data, and FFL decision matrix $D_{m\times n} = (D_{ij})_{4\times 5}$ is built, which is shown in Tables 1, 2, and 3, for professionals Σ_i, $(i = 1, 2, 3)$. Aggregate all the Σ_i based on IVFFWA operator(Table 4). ,

$$D_{4\times 5} = \left(\sum_{l=1}^{3} \omega_l \alpha_{ij}^{(l)}, \sum_{l=1}^{3} \omega_l \beta_{ij}^{(l)} \right)$$

FIGURE 1. Decision-Making Pipeline

2. Using the properties (i.) and (ii.) of score function, the IVFF ideal solution vector Γ^*:

$$\begin{aligned}\Gamma^* \quad = \quad & \Big([(0.743, 0.387), (0.356, 0.669)], [(0.588, 0.546), (0.417, 0.503)], \\ & [(0.579, 0.655), (0.504, 0.678)], [(0.811, 0.453), (0.216, 0.733)], \\ & [(0.850, 0.417), (0.362, 0.794)]\Big).\end{aligned}$$

3.Let us find the SR$\mathcal{C}$ between each alternative and the ideal alternative:

For Γ_1,

$$\mathfrak{Z}_1 = 1 - \frac{6.12}{130} = 0.45, \quad \mathfrak{Z}_2 = 1 - \frac{6.24}{130} = -0.11, \quad \mathfrak{Z}_3 = 1 - \frac{6.8}{130} = 0.63,$$
$$\mathfrak{Z} = 0.323.$$

Values related to Γ_1 are shown in Tables 5 - 7. For Γ_2, Γ_3 and Γ_4, $\mathfrak{Z} = 0.82$, $\mathfrak{Z} = -0.56$ and $\mathfrak{Z} = 0.11$, respectively.

4. From the results in Step 3,
$R_{Spearman}(\Gamma_2) > R_{Spearman}(\Gamma_1) > R_{Spearman}(\Gamma_4) > R_{Spearman}(\Gamma_3)$.

TABLE 1. FFNs for professional Σ_1

	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5
Γ_1	$[(0.72, 0.33), (0.55, 0.78)]$	$[(0.44, 0.57), (0.79, 0.43)]$	$[(0.50, 0.55), (0.53, 0.68)]$	$[(0.78, 0.26), (0.49, 0.65)]$	$[(0.75, 0.45), (0.38, 0.80)]$
Γ_2	$[(0.47, 0.81), (0.90, 0.27)]$	$[(0.80, 0.60), (0.27, 0.85)]$	$[(0.36, 0.58), (0.77, 0.34)]$	$[(0.73, 0.44), (0.45, 0.81)]$	$[(0.67, 0.44), (0.56, 0.63)]$
Γ_3	$[(0.88, 0.55), (0.36, 0.72)]$	$[(0.78, 0.16), (0.45, 0.67)]$	$[(0.72, 0.54), (0.58, 0.63)]$	$[(0.70, 0.66), (0.42, 0.62)]$	$[(0.89, 0.40), (0.26, 0.75)]$
Γ_4	$[(0.66, 0.67), (0.59, 0.61)]$	$[(0.83, 0.34), (0.18, 0.93)]$	$[(0.76, 0.27), (0.34, 0.87)]$	$[(0.54, 0.43), (0.48, 0.66)]$	$[(0.73, 0.35), (0.51, 0.75)]$

TABLE 2. FFNs for professional Σ_2

	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5
Γ_1	$[(0.63, 0.43), (0.55, 0.69)]$	$[(0.55, 0.55), (0.43, 0.71)]$	$[(0.39, 0.35), ()0.87, 0.22]$	$[(0.28, 0.91), (0.84, 0.31)]$	$[(0.57, 0.54), (0.54, 0.69)]$
Γ_2	$[(0.56, 0.49), (0.67, 0.35)]$	$[(0.77, 0.56), (0.25, 0.84)]$	$[(0.76, 0.61), (0.17, 0.90)]$	$[(0.85, 0.33), (0.16, 0.92)]$	$[(0.76, 0.44), (0.53, 0.65)]$
Γ_3	$[(0.27, 0.93), (0.89, 0.16)]$	$[(0.87, 0.23), (0.26, 0.76)]$	$[(0.73, 0.35), (0.51, 0.58)]$	$[(0.64, 0.65), (0.57, 0.78)]$	$[(0.79, 0.42), (0.14, 0.86)]$
Γ_4	$[(0.52, 0.45), (0.64, 0.47)]$	$[(0.45, 0.32), (0.59, 0.69)]$	$[(0.62, 0.28), (0.35, 0.78)]$	$[(0.24, 0.93), (0.89, 0.17)]$	$[(0.81, 0.28), ()0.16, 0.88]$

TABLE 3. FFNs for professional Σ_3

	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5
Γ_1	$[(0.86, 0.34), (0.19, 0.94)]$	$[(0.62, 0.61), (0.38, 0.72)]$	$[(0.44, 0.72), (0.69, 0.51)]$	$[(0.39, 0.87), (0.66, 0.73)]$	$[(0.64, 0.58), (0.56, 0.62)]$
Γ_2	$[(0.69, 0.50), (0.35, 0.52)]$	$[(0.81, 0.35), (0.15, 0.91)]$	$[(0.80, 0.41), (0.27, 0.75)]$	$[(0.78, 0.53), (0.37, 0.65)]$	$[(0.84, 0.24), (0.13, 0.92)]$
Γ_3	$[(0.33, 0.88), (0.78, 0.32)]$	$[(0.68, 0.45), (0.39, 0.74)]$	$[()0.76, 0.43], (0.15, 0.65)$	$[(0.69, 0.56), (0.26, 0.83)]$	$[(0.84, 0.36), (0.17, 0.79)]$
Γ_4	$[(0.61, 0.54), (0.37, 0.75)]$	$[(0.54, 0.52), (0.42, 0.88)]$	$[(0.72, 0.38), (0.39, 0.67)]$	$[(0.35, 0.84), (0.78, 0.43)]$	$[(0.78, 0.35), (0.25, 0.91)]$

TABLE 4. Aggergated matrix Σ_i

	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5
Γ_1	$[(0.743, 0.387), (0.356, 0.669)]$	$[(0.588, 0.546), (0.417, 0.503)]$	$[(0.554, 0.452), (0.496, 0.372)]$	$[(0.498, 0.699), (0.205, 0.75)]$	$[(0.664, 0.553), (0.478, 0.496)]$
Γ_2	$[(0.591, 0.644), (0.447, 0.612)]$	$[(0.514, 0.803), (0.493, 0.784)]$	$[(0.579, 0.655), (0.504, 0.678)]$	$[(0.811, 0.453), (0.216, 0.733)]$	$[(0.796, 0.381), (0.178, 0.825)]$
Γ_3	$[(0.505, 0.816), (0.447, 0.714)]$	$[(0.329, 0.820), (0.592, 0.745)]$	$[(0.447, 0.786), (0.318, 0.826)]$	$[(0.682, 0.635), (0.529, 0.750)]$	$[(0.850, 0.417), (0.362, 0.794)]$
Γ_4	$[(0.618, 0.583), (0.578, 0.639)]$	$[(0.426, 0.667), (0.519, 0.604)]$	$[(0.321, 0.718), (0.816, 0.283)]$	$[(0.389, 0.755), (0.641, 0.498)]$	$[(0.785, 0.344), (0.209, 0647)]$

Table 5. MD ranks of Γ_1

α_{Γ_1}	$Rank(\alpha_{\Gamma_1})$	α_{Γ^*}	$Rank(\alpha_{\Gamma^*})$	f_α	f_α^2
1.130	2	1.130	1	1	1
1.134	3	1.134	2	1	1
1.006	1	1.234	4	3	9
1.197	4	1.182	3	1	1
1.217	5	1.267	5	0	0

Table 6. ND ranks of Γ_1

β_{Γ_1}	$Rank(\beta_{\Gamma_1})$	β_{Γ^*}	$Rank(\beta_{\Gamma^*})$	f_β	f_β^2
1.025	5	1.025	3	2	4
0.920	2	0.920	1	2	4
0.868	1	1.182	5	4	16
0.955	3	0.949	2	1	1
0.974	4	1.156	4	0	0

Table 7. ND ranks of Γ_1

γ_{Γ_1}	$Rank(\gamma_{\Gamma_1})$	γ_{Γ^*}	$Rank(\gamma_{\Gamma^*})$	f_γ	f_γ^2
1.180	2	1.590	3	1	1
1.430	4	1.800	5	1	1
1.570	5	1.620	4	1	1
1.110	1	1.570	2	1	1
1.310	3	1.450	1	2	4

4.3. **Comparison.** Since the method we propose in our study is interval-valued type sets, we will use the correlation coefficients suggested for interval-type sets in the comparison analysis. For the comparison analysis, we will use the studies of Demir [10], Park et al. [39], and Zheng et al. [53]. Table 8 provides the results obtained and compared. A comparison will be made through the numerical example in Section 4 of our study.

Demir [10] have given IVFHF-$\mathcal{C}$ with informational intuitionistic energy, correlation, and correlation coefficients as:

The set $A = \{(x, h_A(x)) : x \in U\}$ is called an interval-valued Fermatean hesitant fuzzy set(IVFHFS), where

$$\begin{aligned} h_A(x) &= \Big\{(m_A(x), n_A(x)) : m_A(x) = [m_A^(-), m_A^(+)] \in D[0,1), \\ &\quad n_A(x) = [n_A^(-), n_A^(+)] \in D[0,1)\Big\}, \end{aligned}$$

and $(m_A^(+))^(3) + (n_A^(+))^(3) \leq 1$.

$$\begin{aligned} E_{IVFHS}(A) &= \sum_{i=1}^{n} \frac{m_{AL}^6(x_i) + m_{AU}^6(x_i) + n_{AL}^6(x_i) + n_{AU}^6(x_i) + \theta_{AL}^6(x_i) + \theta_{AU}^6(x_i)}{2}, \\ C_{IVFHS}(A,B) &= \frac{1}{2}\sum_{i=1}^{n} \Big[m_{AL}(x_i)^3 m_{BL}^3(x_i) + m_{AU}^3(x_i) m_{BU}^3(x_i) + n_{AL}(x_i)^3 n_{BL}^3(x_i) \\ &\quad + n_{AU}^3(x_i) n_{BU}^3(x_i) + \theta_{AL}(x_i)\theta_{BL}(x_i) + \theta_{AU}(x_i)\theta_{BU}(x_i) \Big], \\ K_{IVFHS}(A,B) &= \frac{C_{IVFHS}(A,B)}{(E_{IVFHS}(A).E_{IVFHS}(B))^{(1/2)}}. \end{aligned}$$

Park et al. [39] have given IVFF-$\mathcal{C}$ with informational intuitionistic energy, correlation, and correlation coefficients as:

$$\begin{aligned} E_{IVIFS}(A) &= \sum_{i=1}^{n} \frac{m_{AL}^2(x_i) + m_{AU}^2(x_i) + n_{AL}^2(x_i) + n_{AU}^2(x_i) + \theta_{AL}^2(x_i) + \theta_{AU}^2(x_i)}{2}, \\ C_{IVIFS}(A,B) &= \frac{1}{2}\sum_{i=1}^{n} \Big[m_{AL}(x_i) m_{BL}(x_i) + m_{AU}(x_i) m_{BU}(x_i) + n_{AL}(x_i) n_{BL}(x_i) \\ &\quad + n_{AU}(x_i) n_{BU}(x_i) + \theta_{AL}(x_i)\theta_{BL}(x_i) + \theta_{AU}(x_i)\theta_{BU}(x_i) \Big], \\ K_{IVIFS}(A,B) &= \frac{C_{IVIFS}(A,B)}{(E_{IVIFS}(A).E_{IVIFS}(B))^{(1/2)}}. \end{aligned}$$

Zheng et al. [53] have given IVPHF-$\mathcal{C}$ with informational intuitionistic energy, correlation, and correlation coefficients as:

$$\begin{aligned} E_{IVPHS}(A) &= \sum_{i=1}^{n} \frac{m_{AL}^4(x_i) + m_{AU}^4(x_i) + n_{AL}^4(x_i) + n_{AU}^4(x_i) + \theta_{AL}^4(x_i) + \theta_{AU}^4(x_i)}{2}, \\ C_{IVPFHS}(A,B) &= \frac{1}{2}\sum_{i=1}^{n} \Big[m_{AL}(x_i)^2 m_{BL}^2(x_i) + m_{AU}^2(x_i) m_{BU}^2(x_i) + n_{AL}(x_i)^2 n_{BL}^3(x_i) \\ &\quad + n_{AU}^2(x_i) n_{BU}^2(x_i) + \theta_{AL}(x_i)\theta_{BL}(x_i) + \theta_{AU}(x_i)\theta_{BU}(x_i) \Big], \\ K_{IVIFHS}(A,B) &= \frac{C_{IVIFHS}(A,B)}{(E_{IVIFHS}(A).E_{IVIFHS}(B))^{(1/2)}}. \end{aligned}$$

Table 8. Comparison Outputs

Methos	Ranking outputs
Proposed Method	$R_{Spearman}(\Gamma_2) > R_{Spearman}(\Gamma_1) > R_{Spearman}(\Gamma_4) > R_{Spearman}(\Gamma_3)$
Method in [10]	$R_{Spearman}(\Gamma_2) > R_{Spearman}(\Gamma_1) > R_{Spearman}(\Gamma_4) > R_{Spearman}(\Gamma_3)$
Method in [39]	$R_{Spearman}(\Gamma_2) > R_{Spearman}(\Gamma_3) > R_{Spearman}(\Gamma_1) > R_{Spearman}(\Gamma_4)$
Method in [53]	$R_{Spearman}(\Gamma_2) > R_{Spearman}(\Gamma_1) > R_{Spearman}(\Gamma_3) > R_{Spearman}(\Gamma_4)$

As seen from Table 8, the results of [10] are consistent with the results of our proposed method. Although the results of the other two methods differ, Γ_2 ranked first in each technique.

4.4. **Evaluation of Proposed Method. Advantages of new approach:** The FFS, an extension of the crispy set, comprises the other extensions of FS. PFS stands out as one of the most commonly used extensions due to the MD and ND levels of the specific option regarding the criterion. The decision-maker may occasionally offer the MD and ND of a given characteristic so that the square sum is more significant than 1. Consequently, the PFS must adequately handle this case. To address this problem, one of the most renowned theories, FFS theory, can tolerate incomplete, uncertain, and inconsistent information, typically present in real-world circumstances.

Making decisions on real-world issues or developing solutions is difficult and complex. So, it is essential to reduce confusion while picking the best option. Efficient management of the relationships between the inputs is also necessary for optimal decision-making. To prove the existence of the link between the variables, we introduced a novel strategy that combines the advantages of FFS and $\mathcal{C}$.

A new version based on FFS, a generalization of the SR$\mathcal{C}$ for crisp sets, is proposed. The SR$\mathcal{C}$ for FFSs provides all the properties of the given SR$\mathcal{C}$ for crisp sets, such as membership values, nonmembership values, and hesitation margins. The proposed method shows the critical role of each term in data analysis and DM.

Previous work has shown that some researchers have devised a technique that leverages $\mathcal{C}$ for FFSs. $\mathcal{C}$ for PFSs is a specific instance of $\mathcal{C}$ for FFSs. As a result, compared to the current $\mathcal{C}$, the suggested $\mathcal{C}$ is more suitable for solving problems in the real world since it can be applied to a wider range of scenarios.

Weakness of new approach: The suggested expansion of the $\mathcal{C}$ technique employs a decision matrix with weights and criteria. Making decisions will become more complicated if $\mathcal{C}$ is computed using the values discovered in the decision matrix and equal values are produced. The weights and a few criteria values need to be re-evaluated in this situation.

5. Conclusion

MCDM is a difficult task due to the complexity of the objective world, and FFS is an effective tool for depicting the uncertainty of the MCDM problems. This study aims to develop an IVFF-MCDM approach to address DM problems under uncertain circumstances. The contributions of this paper are as follows:

1. The concept, representation, and related properties of SR$\mathcal{C}$ originated from statistical theory, and two IVFFSs are introduced, which measure the degree of closeness between the ideal alternative and each alternative.
2. An MCDM approach with an IVFF environment is developed based on the proposed SR$\mathcal{C}$.
3. A novelty decision rule is provided from a new perfective using SR$\mathcal{C}$, which can reduce the complexity of the practical problem in the process of DM both in theory and practice.
4. A real-world infrastructure project was demonstrated to illustrate the proposed method's applicability and effectiveness.

Making decisions requires carefully weighing the significance of each evaluation criterion. The weight can be determined using several techniques, including information entropy, the AHP, and the Delphi method. Diverse weights possess unique mechanisms, and various approaches to weighting cater to distinct decision-making contexts. As a result, selecting a suitable technique for determining the weights requires thought and work. The SR$\mathcal{C}$ is a notion that this study introduces, and it can be used to determine which option is most appropriate. By avoiding the weight calculations in the real-world DM problem, solution processes are made simpler, increasing computational performance.

In conclusion, the MCDM approach was established using SR$\mathcal{C}$ statistics. The weights of the criteria can be calculated without it. The challenge of weighing the criteria in the selection process is resolved. Based on SR$\mathcal{C}$, the suggested model reduces the complexity of the IVFFN form in computation and application while promoting intuitive thinking and simple operation.

References

[1] Atanassov K., (1986), Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20, pp. 87–96.

[2] Akram M., Ali G., Alcantud J. C. R., and Riaz A., (2022), Group decision-making with Fermatean fuzzy soft expert knowledge, Artificial Intelligence Review, 55, pp. 5349–5389.

[3] Akram M., Amjad U., Alcantud J. C. R., and Santos-Garcia G., (2022), Complex Fermatean fuzzy N-soft sets: A new hybrid model with applications, Journal of Ambient Intelligence and Humanized Computing, doi:10.1007/s12652-021-03629-4.

[4] Akram M., Shahzadi G., and Ahmadini A. A. H., (2020), Decision-Making Framework for an Effective Sanitizer to Reduce COVID-19 under Fermatean Fuzzy Environment, Journal of Mathematics, Article ID 3263407, pp. 1-19, doi:10.1155/2020/3263407.

[5] Amman, M., Rashid, T., and Ali, A. (2023). Fermatean fuzzy multi-criteria decision-making based on Spearman rank correlation coefficient, Granular Computing, 8, 2005–2019.

[6] Ashraf S., Ataullah, Naeem M., Khan A., Rehman N., and Pandit M. K., (2023), Novel Information Measures for Fermatean Fuzzy Sets and Their Applications to Pattern Recognition and Medical Diagnosis, Computational Intelligence and Neuroscience, Article ID 9273239, pp. 1–19, doi:10.1155/2023/9273239.

[7] Bhatia M., Arora H. D., and Naithani A., (2023), Some New Correlation Coefficient Measures Based on Fermatean Fuzzy Sets using Decision Making Approach in Pattern Analysis and Supplier Selection, International Journal of Mathematical, Engineering and Management Sciences, 8(2), 245–263, doi:10.33889/IJMEMS.2023.8.2.015.

[8] Bustince H., and Burillo P., (1995), Correlation of interval-valued intuitionistic fuzzy sets, Fuzzy Sets Syst. 74, pp. 237–244.

[9] Chiang D. A., and Lin N. P., (1999), Correlation of fuzzy sets, Fuzzy Sets Syst., 102(2), pp. 221–226.

[10] Demir, I., (2023), Novel correlation coefficients for interval-valued Fermatean hesitant fuzzy sets with pattern recognition application, Turkish Journal of Mathematics, 47(1), pp. 213–233.

[11] Ejegwa, P. A., Wanzenke, T. D., Ogwuche, I. O., Anum, M. T., and Isife, K. I. (2024). A robust correlation coefficient for fermatean fuzzy sets based on Spearman's correlation measure with application to clustering and selection process, J. Appl. Math. Comput., 70, pp. 1747–1770.

[12] Ejegwa, P. A., Ajoqwu, C. F., and Sarkar, A. (2023), A Hybridized Correlation Coefficient Technique and its Application in Classification Process under Intuitionistic Fuzzy Setting, Iranian Journal of Fuzzy Systems, 20(4), 103–120.

[13] Ejegwa, P. A., and Onyeke, I. C. (2022), Fermatean fuzzy similarity measure algorithm and its application in students' admission process, International Journal of Fuzzy Computation and Modelling, 4(1), pp. 34–50.

[14] Ejegwa, P. A., Muhiddin G., Algehyne, E. A., Agbetayo, J. M., and Al-Kadi, D. (2022), An Enhanced Fermatean Fuzzy Composition Relation Based on a Maximum-Average Approach and Its Application in Diagnostic Analysis, Journal of Mathematics, Article ID 1786221, pp. 1–12, doi: 10.1155/2022/1786221

[15] Ejegwa, P. A., Shiping, W., Yuming, F., Wei, Z., and Jia, C., (2021), Some new Pythagorean fuzzy correlation techniques via statistical viewpoint with applications to decision-making problems, Journal of Intelligent & Fuzzy Systems, 40, pp. 9873–9886.

[16] Ejegwa, P. A., Wen, S., Feng, Y., Zhang, W., and Liu, J., (2023), A three-way Pythagorean fuzzy correlation coefficient approach and its applications in deciding some real-life problems, Applied Intelligence 53, pp. 226–237. doi:10.1007/s10489-022-03415-5

[17] Ejegwa, P.A., Onyeke, I. C., Kausar, N., and Kattel, P. (2023), A New Partial Correlation Coefficient Technique Based on Intuitionistic Fuzzy Information and Its Pattern Recognition Application, International Journal of Intelligent Systems, Article ID 5540085, pp. 1–14, doi: 10.1155/2023/5540085

[18] Ejegwa, P. A., and Sarkar, A. (2023). Fermatean fuzzy approach of disease diagnosis based on new correlation coefficient operators, In Deep Learning in Personalized Healthcare and Decision Support pp. 23–38, Academic Press.

[19] Ejegwa, P. A., Sarkar, A., and Onyeke I. C., (2023), New methods of computing correlation coefficient based on Pythagorean fuzzy information and their applications in disaster control and diagnostic analysis, In: Fuzzy optimization, decision-making and operations research, Cham: Springer, pp. 473–498. doi: 10.1007/978-3-031-35668-1 21

[20] Garg H., (2016), A novel correlation coefficients between Pythagorean fuzzy sets and its applications to the decision-making process, Int. J. Intell. Syst. 31, pp. 1234–1252. doi:10.1002/int.21827.

[21] Garg H., Shahzadi G., and Akram M., (2020) Decision-Making Analysis Based on Fermatean Fuzzy Yager Aggregation Operators with Application in COVID-19 Testing Facility, Mathematical Problems in Engineering, Article ID 7279027, pp. 1–16, doi:10.1155/2020/7279027.

[22] Garg H. (2022). (ed) q-Rung orthopair fuzzy sets: theory and applications Springer, Singapore. doi: 10.1007/978-981-19-1449-2

[23] Golui S., Mahapatra B. S., and Mahapatra G. S., (2024), A new correlation-based measure on Fermatean fuzzy applied on multi-criteria decision making for electric vehicle selection, Expert systems with applications, 237(B), 121605.

[24] Hong, D. H., (1998), A note on correlation of interval-valued intuitionistic fuzzy sets, Fuzzy Sets Syst. 95, pp. 113–117.

[25] Jeevaraj S., (2021), Ordering of interval-valued Fermatean fuzzy sets and its applications. Expert Systems with Applications, 185, 115613.

[26] Kirişci, M., (2024), Measures of Distance and Entropy Based on the Fermatean Fuzzy-Type Soft Sets Approach, Universal Journal of Mathematics and Applications, 7(1), pp. 12–29.

[27] Kirişci, M., (2022), Novel Multi-Attribute Group-Decision Making Method with TOPSIS: A Fermatean Fuzzy Hypersoft Sets and Correlation Coefficients Approach, Applied Mathematics, Modeling and Computer Simulation, 30, pp. 952–960.

[28] Kirisci, M., (2022), Correlation coefficients of fermatean fuzzy sets with a medical application. Journal of Mathematical Sciences and Modelling, 5(1), pp. 16–23.

[29] Kirişci M., Demir I., and Simsek N., (2022), Fermatean fuzzy ELECTRE multi-criteria group decision-making and most suitable biomedical material selection, Artificial Intelligence in Medicine, 127, 102278.

[30] Kirişci M., (2023), New cosine similarity and distance measures for Fermatean fuzzy sets and TOPSIS approach. Knowl Inf Syst 65, pp.855–868.

[31] Kirişci, M., (2023), Fermatean Fuzzy Type Statistical Concepts with Medical Decision-Making Application, Fuzzy Optimization and Modelling Journal, 4(1), pp. 1–14. doi:10.30495/FOMJ.2023.1982256.1082.

[32] Kirişci, M., (2023), Data Analysis for Panoramic X-Ray Selection: Fermatean Fuzzy type Correlation Coefficients Approach, Engineering Applications of Artificial Intelligence, 126, 106824.

[33] Kirişci, M., (2023), Fermatean Hesitant Fuzzy Sets for Multiple Criteria Decision- Making with Applications, Fuzzy Information and Engineering 15(2), pp. 100–127.

[34] Kirişci, M., (2024), Fermatean Fuzzy Type a Three-Way Correlation Coefficients. In: Gayoso Martínez, V., Yilmaz, F., Queiruga-Dios, A., Rasteiro, D.M., Martín-Vaquero, J., Mierluş-Mazilu, I. (eds) Mathematical Methods for Engineering Applications. ICMASE 2023. Springer Proceedings in Mathematics & Statistics, vol 439. Springer, Cham.

[35] Liu S. T., and Kao C., (2002), Fuzzy measures for correlation coefficient of fuzzy numbers, Fuzzy Sets Syst. 128(2), pp. 267–275.

[36] Lin M. W., Huang C., and Xu Z. S., (2019), TOPSIS method based on correlation coefficient and entropy measure for linguistic Pythagorean fuzzy sets and its application to multiple attribute decision making, Complexity 6967390. doi:10.1155/2019/6967390.

[37] Mitchell, H. B., (2004), A correlation coefficient for intuitionistic fuzzy sets, Int. J. Intell. Syst., 19, pp. 483–490.

[38] Onyeke, I. C., and Ejegwa, P. A., (2023), Modified Senapati and Yager's Fermatean fuzzy distance and its application in students' course placement in tertiary institution. In: Sahoo, L., Senapati, T., Yager, R.R. (eds) Real Life Applications of Multiple Criteria Decision Making Techniques in Fuzzy Domain, Studies in Fuzziness and Soft Computing, Springer, 420, pp. 237–253.

[39] Park, D. G., Kwun, Y. C., Park, J. H., and Park I. Y., (2009), Correlation coefficient of interval-valued intuitionistic fuzzy sets and its application to multiple attribute group decision making problems, Mathematical and Computer Modelling 50, pp. 1279–1293

[40] Qi G., Atef M., and Yang B., (2024), Fermatean fuzzy covering-based rough set and their applications in multi-attribute decision-making, Engineering Applications of Artificial Intelligence, 127(A), 107181

[41] Senapati, T., and Yager, R. R., (2020), Fermatean fuzzy sets, Journal of Ambient Intelligence and Humanized Computing 11, pp. 663–674. doi: 10.1007/s12652-019-01377-0.

[42] Senapati, T., and Yager, R. R., (2019), Some new operations over Fermatean fuzzy numbers and application of Fermatean fuzzy WPM in multiple criteria decision making, Informatica 30(2), pp. 391–412. doi: 10.15388/Informatica.2019.211.

[43] Senapati, T., and Yager R. R., (2019), Fermatean fuzzy weighted averaging/geometric operators and its application in multi-criteria decision-making methods, Engineering Applications of Artificial Intelligence, 85, pp. 112–121. doi: 10.1016/j.engappai.2019.05.012.

[44] Shahzadi G., Akram M., (2021), Group decision-making for the selection of an antivirus mask under Fermatean fuzzy soft information. Journal of Intelligent & Fuzzy Systems 40(1), 1401–1416. doi:10.3233/JIFS-201760.

[45] Szmidt E., and Kacprzyk J., (2000), Distances between intuitionistic fuzzy sets, Fuzzy Sets and Systems, 114, pp. 505–518.

[46] Wei G. W., Wang H. J., and Lin R., (2011), Application of correlation coefficient to interval-valued intuitionistic fuzzy multiple attribute decision-making with incomplete weight information, Know. Inf. Syst., 26(2), pp. 337–349. doi:10.1007/s10115-009-0276-1.

[47] Yager R. R., (2013), Pythagorean fuzzy subsets, Proc. Joint IFSA World Congress and NAFIPS Annual Meeting, Edmonton, Canada.

[48] Yager, R. R., (2014), Pythagorean membership grade in multicriteria decision making, IEEE Fuzzy Syst. 22, pp. 958–965. doi:10.1109/TFUZZ.2013.2278989.

[49] Yager, R. R. and Abbasov, A. M., (2013), Pythagorean membership grades, complex numbers, and decision-making, Int. J. Intell. Syst. 28, pp. 436–452. doi:10.1002/int.21584.

[50] Zadeh, L. A., (1965), Fuzzy sets, Inf Comp, 8, pp. 338–353.

[51] Zeng, W., and Li H., (2007), Correlation coefficient of intuitionistic fuzzy sets, Journal of Industrial Engineering International, 3, 33–40.

[52] Zhang X. L. and Xu Z. S., (2014), Extension of TOPSIS to multi-criteria decision-making with Pythagorean fuzzy sets. Int. J. Intell. Syst. 29, pp. 1061–1078. doi: 10.1002/int.21676.

[53] Zheng T., Zhang M., Li L., Wu Q., and Zhou L., (2020), Correlation Coefficients of Interval-Valued Pythagorean Hesitant Fuzzy Sets and Their Applications, IEEE Access, 8, 9271–9286

Murat Kirisci is a Professor in the Department of Biostatistics and Medical Informatics, Cerrahpaşa Medicine Faculty at the Istanbul University-Cerrahpaşa. He received a Ph. D. degree in Mathematics from Erciyes University. His research has been focused on the Matrix domain, statistical convergence, fixed point theory, decision-making, fuzzy sets and systems, fuzzy logic, artificial intelligence, machine learning, intelligence systems, and optimization. He has contributed research papers in reputed journals and as a reviewer in different scientific journals.