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# AN EFFICIENT SOLUTION FOR INTERVAL VALUED TRAPEZOIDAL FUZZY BOUNDED VARIABLE PROBLEM AND ITS REAL LIFE APPLICATION

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Abstract. In this research examined a completely fuzzy Interval valued Trapezoidal Bounded Variable Linear Programming Problem (FFIVBVLPP) where all the parameters of objective functions, and resource vector decision variables are represented by Interval valued Trapezoidal fuzzy numbers. The FFIVBVLPP problem is converted into a crisp bounded variable problem using the Euclidean distance in Index Vectroial centroid Ranking. This research presents a novel and efficient solution to address the Interval-Valued Trapezoidal Fuzzy Bounded Variable Problem. The study aims to provide a comprehensive analysis of the problem and propose a method that significantly enhances the efficiency and accuracy of solutions. The approach leverages advanced mathematical techniques and fuzzy logic principles to handle uncertainty within the interval-valued trapezoidal fuzzy variables. This paper contributes to the existing literature by introducing a systematic and effective methodology for dealing with bounded variable problems in a fuzzy environment. The proposed algorithm is then illustrated with some mathematical analysis and an appropriate numerical example with case study.

Keywords: Bounded interval-valued fuzzy numbers linear programming, euclidean distance , fuzzy optimal solutions , interval valued trapezoidal fuzzy numbers.

AMS Subject Classification: 90C05, 03E72

## Abbrevation

FLP- Fuzzy Linear Programming
LP- Linear Programming
FBVP -Fuzzy Bounded Variable Problem
ICV -Index Vectorial Centroid

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## 1. Introduction

The challenge posed by the Fuzzy Bounded Variable Problem is prominent in decision-making and modelling due to the inherent uncertainties existing within specified limits. This problem emerges when variables exhibit imprecise or ambiguous boundaries, rendering conventional approaches inadequate. Researchers are actively exploring inventive methodologies, such as fuzzy logic and interval mathematics, to confront and alleviate the intricacies associated with fuzzy bounded variables. These approaches provide a nuanced comprehension of uncertainty, facilitating more precise modelling and decision support across diverse domains like finance, engineering, and artificial intelligence. In the evolving landscape of technology, the pursuit of efficient solutions to the Fuzzy Bounded Variable Problem is imperative for fortifying the resilience and dependability of systems in the presence of real-world uncertainties.The challenge posed by the Fuzzy Bounded Variable Problem is prominent in decision-making and modelling due to the inherent uncertainties existing within specified limits. This problem emerges when variables exhibit imprecise or ambiguous boundaries, rendering conventional approaches inadequate. Researchers are actively exploring inventive methodologies, such as fuzzy logic and interval mathematics, to confront and alleviate the intricacies associated with fuzzy bounded variables. These approaches provide a nuanced comprehension of uncertainty, facilitating more precise modelling and decision support across diverse domains like finance, engineering, and artificial intelligence. In the evolving landscape of technology, the pursuit of efficient solutions to the Fuzzy Bounded Variable Problem is imperative for fortifying the resilience and dependability of systems in the presence of real-world uncertainties. The challenge posed by the Fuzzy Bounded Variable Problem is prominent in decision-making and modelling due to the inherent uncertainties existing within specified limits. This problem emerges when variables exhibit imprecise or ambiguous boundaries, rendering conventional approaches inadequate. Researchers are actively exploring inventive methodologies such as fuzzy logic and interval mathematics to confront and alleviate the intricacies associated with fuzzy bounded variables. These approaches provide a nuanced comprehension of uncertainty facilitating more precise modelling and decision support across diverse domains like finance, engineering, and artificial intelligence. In the evolving landscape of technology, the pursuit of efficient solutions to the Fuzzy Bounded Variable Problem is imperative for fortifying the resilience and dependability of systems in the presence of real-world uncertainties.

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The aim of the optimization problem big part is referred to linear programming is to solve a problem under a variety of linear constraints. The LP issue is traditionally considered to have exact coefficients and variables, yet it is highly challenging for decision-makers to express their choices in such precision figures. Fuzzy LP (FLP) issues are the name given to the corresponding problem because, in order to manage it, a theory of fuzzy sets plays

a vital part in the procedure for determining decisions.
Russak [22] addressed challenges associated with bounded state variables, specifically focusing on the general control problem of Bolza with bounded state constraints. The paper explored necessary conditions for solutions to satisfy vector differential equations, even when the solution encompasses an infinite number of intervals meeting the state constraints. Notably, the paper relaxed the typical assumption regarding the rank of the matrix in addressing such problems. Overall, the insights provided valuable perspectives on optimal control problems involving bounded state variables. Bitran and Hax [2] introduced and discussed the application of convex knapsack problems with bounded variables in the context of disaggregation and resource allocation. The authors delved into the utilization of this mathematical approach to allocate resources effectively, optimizing the use of limited resources. The paper explored specific methodologies, models, and findings related to the application of convex knapsack problems. Kostina[4] proposed a modification of the dual simplex algorithm and presented its computational results on NETLIB and MIPLIB problems. The authors emphasized the advantages of the dual simplex method in the context of integer and combinatorial optimization. Xia and Wang [31] , [29]explained the extension of the generalized weight adaptation algorithm for 2-D feedforward neural networks. The paper detailed the adaptation algorithm for 2-D madaline and 2-D two-layer FNNs, with a focus on the error vector at specific points and the weight matrix adaptation. The aim was to address the extended weight adaptation algorithm for 2-D neural networks employing bounded variables.Stefanov [23] discussed the extension of the generalized weight adaptation algorithm for 2-D feedforward neural networks. The paper presented the adaptation algorithm for 2-D madaline and 2-D two-layer FNNs, concentrating on the error vector at specific points and the weight matrix adaptation. The focus was on addressing the extended weight adaptation algorithm for 2-D neural networks.
Charnes [4] explored the lower-bounded and partially upper-bounded distribution model, a distribution problem with limited total supply and lower bounds on origin's supply and destination's demand. The paper proposed an approach to accommodate this distribution model, demonstrating its equivalence to a certain bounded variable. Cerone [3] discussed the identification of linear models in the presence of noise affecting all observed variables, known as the errors-in-variables problem. The paper provided proof of a previous result on the description of the feasible parameter region for linear models with bounded errors in all variables. It also discussed topological features of the feasible parameter region, such as convexity and connectedness, and highlighted drawbacks in parameter estimation using statistical frameworks.Ebrahimnejad and Verdegay [9] described the application of fuzzy linear programming in bounded linear programming problems where some or all variables are subject to fuzzy bounds. They proposed a method based on a specific linear ranking function applicable to such situations, including a real-life problem application. Ebrahimnejad [10] introduced a method based on the bounded dual simplex method to determine the optimal solution of fuzzy variable linear programming problems with lower and upper bounds on some or all variables.
Zhangchun and Zhenzhou [11] discussed the need for reliability-based design optimization in structures due to uncertainties, proposing a reliability model for handling mixed uncertainties involving fuzzy variables and uncertain-but-bounded variables.Ebrahimnejad and Nasser [12] presented a dual simplex method for solving bounded linear programming problems with fuzzy parameters, addressing practical bounds on all variables but not applicable to decision variables represented by TRFNs.Ebrahimnejad et al. [13] proposed a method for solving auxiliary problems in fuzzy linear programming problems with bounded decision variables, serving as a tool in sensitivity or post-optimality analysis in

bounded fuzzy number linear programming problems.Ebrahimnejad [14] discussed various approaches to fuzzy linear programming and their development over time. The article introduced methods like a primal-dual method and a fuzzy primal simplex algorithm but did not designate a single approach as the definitive one.Ebrahimnejad [15] addressed the solution to bounded interval-valued fuzzy numbers linear programming problems using a signed distance ranking approach, providing background and discussion on fuzzy linear programming and related problem-solving techniques.
Bharati and Singh [1] explained the use of interval-valued intuitionistic fuzzy numbers to handle uncertainty in linear programming problems. The paper introduced concepts like IV-IFN and IV-IFLPP, presenting solutions and comparing them with existing methods. Ebrahimnejad [17] introduced and discussed basic concepts of fuzzy linear programming, proposing a method based on the comparison of fuzzy numbers using ranking functions to solve fuzzy linear programming problems.Radjef, S., and Bibi [21] focused on developing a method for finding all efficient extreme points in multiobjective linear programming with bounded variables. The paper proposed an efficiency test for nonbasic variables, a procedure to find a first efficient extreme point, and an algorithm to find all efficient extreme points, integrating a suboptimal criterion for desired accuracy.Tanaka and Asai [26]initially developed the conceptualization of Fuzzy Linear Programming (FLP) by considering the decision factors of difficulties as fuzzy numbers . Kumar and Kaur [19] proposed a solution for fully fuzzy linear programming problems with mixed constraints. The method involves algebraic modelling, introducing slack variables, converting inequality constraints into equality constraints, and solving the resulting linear programming problem using a ranking function [22].

Farhadinia [14] introduced sensitivity analysis in interval-valued trapezoidal fuzzy number linear programming problems. The formulation for fuzzy linear programming problems was presented, where (hL, hU)-interval-valued trapezoidal fuzzy numbers are parameters. The study suggests a method for solving these problems and concludes that sensitivity analysis provides similar results as those obtained for trapezoidal fuzzy number linear programming problems .Wei , Chen [30]introduced a novel method for fuzzy risk analysis based on similarity measures between interval-valued fuzzy numbers. The method includes a new similarity measure, a division operator, and an adjustment algorithm for dealing with fuzzy risk analysis problems by Sivakumar [24]. Figueroa-García[25] discussed optimal solutions for group matrix games involving interval-valued fuzzy numbers and proposed an uncertain-based matrix games model. Traditional game theory applications to real-life problems with incomplete or uncertain information were also considered .

Chiang [5] explored fuzzy linear programming problems using statistical data and statistical confidence intervals. The study derived fuzzy numbers and employed signed distance ranking to defuzzify linear programming in a fuzzy context. Ganesan and Veeramani [16] introduced a new fuzzy arithmetic for symmetric trapezoidal fuzzy numbers and proposed a method for solving fuzzy linear programming problems without converting them to crisp linear programming problems. The paper defines and explains the significant features of symmetric trapezoidal fuzzy numbers and proposes fuzzy analogies to important theorems of linear programming .Su [24] discussed the concept of fuzzy programming based on interval-valued fuzzy numbers and ranking. The paper presented three cases of fuzzy linear programming based on using interval-valued fuzzy numbers to fuzzify coefficients in the objective function, constraints, or both .Verdegay's [27] paper discussed a concept of fuzzy objective based on the Fuzzification Principle for solving Fuzzy Linear Programming problems. The proposed dual approach defines the dual problem of a Fuzzy Linear Programming problem based on costs, not on the value of the objective function. Wei and Chen [31]presented a method for fuzzy risk analysis based on similarity measures between interval-valued fuzzy numbers. The paper proposed a similarity measure combining geometric distance, perimeter, height, and center-of-gravity points of interval-valued fuzzy numbers. Additionally, a division operator and an interval-valued fuzzy number adjustment algorithm were introduced, leading to a new fuzzy risk analysis algorithm. Yager's[32] delved into fuzzy subsets of the unit interval, exploring a function (F) for ordering fuzzy subsets of the unit interval. The proposed ordering function is applicable to continuous, discrete, and crisp members. Ebrahimnejad[6]presented a new approach for solving linear programming problems with fuzzy cost coefficients, known as the bounded fuzzy primal simplex algorithm. The algorithm maintains primal feasibility throughout while moving towards achieving primal optimality, starting with a primal feasible basis .Prakash and Appaswamy[20], [33] proposed a solution for the fuzzy linear programming problem by introducing spherical fuzzy sets and using them as parameters. It suggests a method to convert these fuzzy numbers into crisp interval numbers, employing the Best Worst Method to solve the resulting clear-cut Linear Programming Problem. Additionally, a spherical fuzzy optimization model is introduced to address the challenges of the Spherical Fuzzy Linear Programming Problem.

**Aim :**

- Develop an efficient solution methodology for interval-valued trapezoidal fuzzy bounded variable problems.
- Enhance computational efficiency and accuracy in solving these types of fuzzy linear programming problems.
- Extend the theoretical framework of fuzzy linear programming by incorporating interval-valued trapezoidal fuzzy numbers.
- Apply the proposed solution approach to real-life decision-making scenarios involving uncertainty and imprecision.

The novelty of this research lies in its proposition of an innovative optimization framework tailored specifically to handle interval-valued trapezoidal fuzzy bounded variables. This approach pioneers a method to effectively model and solve problems characterized by imprecise and bounded uncertainties, contributing a unique perspective to the realm of fuzzy logic-based optimization techniques.

The primary aim of this study is to introduce a comprehensive and efficient solution that addresses the complexities inherent in interval-valued trapezoidal fuzzy bounded variables. By bridging the gap in existing methodologies, the aim is to offer a robust, scalable, and adaptable optimization framework capable of handling uncertainty with precision in decision-making processes across various domains.

This research contributes a novel optimization algorithm or methodology tailored specifically for interval-valued trapezoidal fuzzy bounded variables, advancing the theoretical foundations of fuzzy logic-based optimization techniques.

The proposed solution offers practical value by providing a tool that can be applied across diverse domains, facilitating more accurate decision-making in scenarios where imprecise and bounded uncertainties are prevalent.

By addressing a niche problem domain, this study contributes to a deeper understanding of interval-valued trapezoidal fuzzy bounded variables, paving the way for further research and development in this specialized area.

The remaining of the article is structured as follows: The manuscript begins with an

Table 1. Literature Review table

| Author | Crisp parameter | Interval -valued fuzzy parameter | Linear bounded variable programming | Method | solution |
|---|---|---|---|---|---|
| [8] | no | yes | Yes | Estimating parameters within a statistical framework is conducted, employing fuzzy programming for solution. | Crisp |
| [29] | no | yes | no | Solving through fuzzy programming | Fuzzy |
| [27] | no | yes | yes | The approach involves utilizing the fuzzy simplex method, despite its failure to satisfy bounded conditions. | fuzzy |
| [23] | no | yes | no | Solving the fuzzy linear programming problem (LPP) yields a fuzzy optimal solution, and further analysis includes sensitivity analysis. | crisp |
| [20] | no | yes | no | The resolution of the fuzzy linear programming problem (LPP) is addressed through methods such as the best and worst-case scenarios for multi-criteria decision-making (MCDM), | crisp |
| [19] | no | yes | no | The application of a ranking technique has resulted in obtaining a crisp solution. | crisp |
| [15] | no | yes | yes | the problem is approached using the primal simplex algorithm. | fuzzy |
| Proposed Method | no | yes | yes | The defuzzification process includes applying an equivalent ranking function and transforming it into membership with the assistance of the Zimmerman technique, resulting in the attainment of a fuzzy solution. | Fuzzy\crsip |

Introduction that provides the background of the study, clearly articulates the problem statement, and outlines the research objectives. Following this, the Literature Review offers a summary of related work in the field, highlighting key contributions and identifying the research gap that the current study seeks to address. In the Methodology section, a detailed explanation of the methods employed in the research is presented, including the rationale behind the chosen approaches. The Results section follows, where the findings of the study are presented, often accompanied by relevant data and analysis. In the Discussion section, the results are interpreted, compared with previous research, and their broader implications are explored. Finally, the Conclusion provides a summary of the research findings, discusses their significance, and suggests directions for future work.

## 2. Literature Review Table

The literature review table encompasses a range of studies. Authors have delved into the challenges of interval-valued trapezoidal fuzzy bounded variable problems, proposing methods for solution.The literature review reveals a range of existing methods proposed for achieving optimal solutions, each employing different ranking techniques. However, these methods do not necessarily yield fuzzy optimal solutions. Although Ebirahimnejad introduced the primal duality fuzzy approach for interval-valued trapezoidal bounded variable problems, this method comes with certain limitations in table 1

## 3. Methodology

The methodology proposed in this research involves a careful combination of interval mathematics, trapezoidal fuzzy logic, and efficient computational techniques. The interval-valued trapezoidal fuzzy variables are rigorously defined, and a systematic approach is presented to handle the complexities associated with bounded variable problems. The research introduces a set of mathematical formulations and algorithms designed to optimize solutions while accommodating the uncertainty inherent in real-world applications.

## 4. Premilinaries

**Definition 4.1.** *If $X$ is a collection of objects denoted generically by $x$, then the fuzzy set $\tilde{A}(x)$ in $X$ is defined be a set of ordered pairs. Where $\mu_{\tilde{A}(x)}$ is called the membership function for the fuzzy set. The membership function maps each element of $x$ to a value between (0,1).*

**Definition 4.2.** *Let $\tilde{\tilde{A}} = \left[\tilde{\breve{A}}, \tilde{\widehat{A}},\right] = \left\langle \left(\breve{a}_1, \breve{a}_2, \breve{a}_3; \breve{h}\right), \left(\widehat{a}_1, \widehat{a}_2, \widehat{a}_3; \widehat{h}\right)\right\rangle$ be the Interval-Valued Triangular Fuzzy Number the lower and upper membership function is defined as ,*

$$\mu_{\tilde{\breve{A}}} = \begin{cases} \breve{h}\frac{x-\breve{a}_1}{\breve{a}_2-\breve{a}_{11}}, \breve{a}_1 \leq x \leq \breve{a}_2 \\ \breve{h}\frac{\breve{a}_3-x}{\breve{a}_3-\breve{a}_2}, \breve{a}_2 \leq x \leq \breve{a}_3 \\ 0, otherwise \end{cases}$$

$$\mu_{\tilde{\widehat{A}}} = \begin{cases} \widehat{h}\ \frac{x-\widehat{a}_1}{\widehat{a}_2-\widehat{a}_1}, \widehat{a}_1 \leq x \leq \widehat{a}_2 \\ \widehat{h}\ \frac{\widehat{a}_3-x}{\widehat{a}_3-\widehat{a}_2}, \widehat{a}_2 \leq x \leq \widehat{a} \\ 0, otherwise \end{cases}$$

*where* $\breve{a}_1 \leq \breve{a}_2 \leq \breve{a}_3$, $\widehat{a}_1 \leq \widehat{a}_2 \leq \widehat{a}_3$, $0 \prec \breve{h} \leq \widehat{h} \leq 1$, $\mu_{\tilde{\breve{A}}} \leq \mu_{\tilde{\widehat{A}}}$

**Definition 4.3.** *Let $\tilde{\tilde{A}} = \left[\tilde{\breve{A}}, \tilde{\widehat{A}},\right] = \left\langle \left(\breve{a}_1, \breve{a}_2, \breve{a}_3, \breve{a}_4; \breve{h}\right), \left(\widehat{a}_1, \widehat{a}_2, \widehat{a}_3, \widehat{a}_4; \widehat{h}\right)\right\rangle$ be the Interval-Valued Trapezoidal Fuzzy Number the lower and upper membership function is defined as ,*

$$\mu_{\tilde{\breve{A}}} = \begin{cases} \breve{h}\frac{x-\breve{a}_1}{\breve{a}_2-\breve{a}_1}, \breve{a}_1 \leq x \leq \breve{a}_2 \\ \breve{h}, \breve{a}_2 \leq x \leq \breve{a}_3 \\ \breve{h}\frac{\breve{a}_4-x}{\breve{a}_4-\breve{a}_3}, \breve{a}_3 \leq x \leq \breve{a}_4 \\ 0, otherwise \end{cases}$$

$$\mu_{\tilde{\widehat{A}}} = \begin{cases} \widehat{h}\ \frac{x-\widehat{a}_1}{\widehat{a}_2-\widehat{a}_1}, \widehat{a}_1 \leq x \leq \widehat{a}_2 \\ \widehat{h}, \widehat{a}_2 \leq x \leq \widehat{a}_3 \\ \widehat{h}\ \frac{\widehat{a}_4-x}{\widehat{a}_4-\widehat{a}_3}, \widehat{a}_3 \leq x \leq \widehat{a}_4 \\ 0, otherwise \end{cases}$$

*where* $\underset{\sim}{a}_1 \leq \underset{\sim}{a}_2 \leq \underset{\sim}{a}_3 \leq \underset{\sim}{a}_4$, $\widehat{a}_1 \leq \widehat{a}_2 \leq \widehat{a}_3 \leq \widehat{a}_4$, $0 \prec \underset{\smile}{h} \leq \widehat{h} \leq 1$, $\mu_{\underset{\approx}{A}} \leq \mu_{\underset{\approx}{\widehat{A}}}$

**Definition 4.4.** *Let $\tilde{\tilde{A}} = \left\langle \left(\breve{a}_1, \breve{a}_2, \breve{a}_3, \breve{a}_4; \breve{h}\right), \left(\widehat{a}_1, \widehat{a}_2, \widehat{a}_3, \widehat{a}_4; \widehat{h}\right)\right\rangle$ and $\tilde{\tilde{B}} = \left\langle \left(\breve{b}_1, \breve{b}_2, \breve{b}_3, \breve{b}_4; \breve{h}\right), \left(\widehat{b}_1, \widehat{b}_2, \widehat{b}_3, \widehat{b}_4; \widehat{h}\right)\right\rangle$ be the two Interval-Valued Trapezoidal Fuzzy*

*Number , belong to* $F_{IVTN}\left(\underset{\sim}{h},\widehat{h}\right)$ *and K be the non negative real number . then the addition and scalar multiplication is defined as*

$$\underset{\sim}{A}=\left\langle \begin{array}{c}\left(\breve{a}_1+\breve{b}_1,\breve{a}_2+\breve{b}_2,\breve{a}_3+\breve{b}_3,\breve{a}_4+\breve{b}_4;\breve{h}\right),\\ \left(\widehat{a}_1+\widehat{b},\widehat{a}_2+\widehat{b}_2,\widehat{a}_3+\widehat{b}_3,\widehat{a}_4+\widehat{b};\widehat{h}\right)\end{array}\right\rangle$$

$$k\tilde{\tilde{A}}=\begin{cases}\left\langle\left(k\breve{a}_1,k\breve{a}_2,k\breve{a}_3,k\breve{a}_4;\breve{h}\right),\left(k\,\widehat{a}_1,k\,\widehat{a}_2,k\,\widehat{a}_3,k\,\widehat{a}_4;\widehat{h}\right)\right\rangle,k\succ 0\\ \left\langle\left(k\breve{a}_1,k\breve{a}_2,k\breve{a}_3,k\breve{a}_4;\breve{h}\right),\left(k\,\widehat{a}_1,k\,\widehat{a}_2,k\,\widehat{a}_3,k\,\widehat{a}_4;\widehat{h}\right)\right\rangle,k\prec 0\\ \left\langle\left(0,0,0,0;\breve{h}\right),\left(0,0,0,0;\widehat{h}\right)\right\rangle,k=0\end{cases}$$

**Definition 4.5.** *Let F(R) is a set of fuzzy numbers defined on the set of real numbers and the ranking of a fuzzy number is actually a function* ***R*** *from F(R) to R, which maps each fuzzy number into the real line.If* $\tilde{A}$ *and* $\tilde{B}$*, are any two fuzzy numbers then the relation between those two fuzzy numbers are, given by*

(1) *If* $R(\tilde{A})\leq R(\tilde{B})$ *Then* $\tilde{A}\leq\tilde{B}$
(2) *If* $R(\tilde{A})\geq R(\tilde{B})$ *Then* $\tilde{A}\geq\tilde{B}$
(3) *If* $R(\tilde{A})= R(\tilde{B})$ *Then* $\tilde{A}=\tilde{B}$

## 5. Fuzzy Bounded Variable Problem

Fuzzy Linear programming problems with simple upper and lower bounds on the variables have been well explored. Here, we look into issues that include a system of relative upper and/or lower limits. We offer computational methods that take use of the unique triangular nature of these restrictions. The fuzzy bounded-variables approach, However,due to uncertainty ,ambiguous and vagueness ,the fuzzy bounded-variables problem, a variable represents that is either fixed at its lower bound or upper bound. Let
The fuzzy bounded variable problem is defined as
Max $\tilde{\tilde{Z}}\approx\tilde{\tilde{c}}x$
Subject to constraints

$$\begin{array}{c}\tilde{\tilde{A}}x_j\leq,=,\geq\tilde{\tilde{b}},j=1,2,..n\\ \tilde{\tilde{l}}\leq x_j\leq\tilde{\tilde{u}}\\ x_j\geq 0,j=1,..,n\end{array}\tag{1}$$

Where $\tilde{\tilde{c}}^T,\tilde{\tilde{l}},\tilde{\tilde{u}}\in\left(F_{IVTrN}\left(\breve{h},\widehat{h}\right)\right)^n$, $\tilde{\tilde{A}}\in\left(F_{IVTrN}\left(\breve{h},\widehat{h}\right)\right)^{m\times n}$ and $\tilde{\tilde{b}}\in\left(F_{IVTrN}\left(\breve{h},\widehat{h}\right)\right)^n$ are given and $x\in\mathrm{R}^n$is to be determined . $\left(F_{IVTrN}\left(\breve{h},\widehat{h}\right)\right)^n$be an fuzzy interval valued trapezoidal number.

The constraints $\tilde{\tilde{l}}\leq x_j\leq\tilde{\tilde{u}},j=1,..,n$ are called the bounded constraints. Any vector $x\in\mathrm{R}^n$ which satisfies $x\geq 0$ is said to be a feasible solution and the feasible space is denoted by $\tilde{\tilde{S}}=\left\{x\in\mathrm{R}^n:\tilde{\tilde{A}}x_j\leq,=,\geq\tilde{\tilde{b}},\tilde{\tilde{l}}\leq x_j\leq\tilde{\tilde{u}}\right\}$

## 6. Theorem

**Theorem 6.1.** *Let* $\tilde{\tilde{A}}=\left\langle\left(\breve{a}_1,\breve{a}_2,\breve{a}_3,\breve{a}_4;\breve{h}\right),\left(\widehat{a}_1,\widehat{a}_2,\widehat{a}_3,\widehat{a}_4;\widehat{h}\right)\right\rangle$*be a non- negative interval valued trapezoidal fuzzy numbers , then*

$$R^{IVC}{}_{\underset{\sim}{A}_1}(x,y)\geq R^{IVC}{}_{\underset{\sim}{A}_2}(x,y)\tag{2}$$

*Proof.* Let $R^{IVC}{}_{\tilde{A}_1}(x,y)$ ranking under Index Vectorial Centroid using euclidean distance. Since $\tilde{\tilde{A}} = \left[\tilde{\breve{A}}, \tilde{\widehat{A}},\right] = \left\langle \left(\breve{a}_1, \breve{a}_2, \breve{a}_3; \breve{h}\right), \left(\widehat{a}_1, \widehat{a}_2, \widehat{a}_3; \widehat{h}\right)\right\rangle$be a non- negative interval valued trapezoidal fuzzy numbers . Then we have ,

$$\left(\widehat{a}_1 + \widehat{a}_2 + \widehat{a}_3 + \widehat{a}_4\right)^2 \geq \left(\widehat{a}_1 + \widehat{a}_2\right)^2 + \left(\widehat{a}_3 + \widehat{a}_4\right)^2 \tag{3}$$

$$\left(\breve{a}_1 + \breve{a}_2 + \breve{a}_3 + \breve{a}_4\right) \geq \left(\breve{a}_1 + \breve{a}_2\right)^2 + \left(\breve{a}_3 + \breve{a}_4\right)^2 \tag{4}$$

Where ,

$$\left(\widehat{a}_1 + \widehat{a}_2 + \widehat{a}_3 + \widehat{a}_4\right)^2 + 2\left(\widehat{a}_2 + \widehat{a}_3\right)^2 \geq \left(\widehat{a}_1 + \widehat{a}_2\right)^2 + \left(\widehat{a}_3 + \widehat{a}_4\right)^2 + 2\left(\widehat{a}_2\right)^2 + 2\left(\widehat{a}_3\right)^2 \tag{5}$$

$$\left(\breve{a}_1 + \breve{a}_2 + \breve{a}_3 + \breve{a}_4\right) + 2\left(\breve{a}_2 + \breve{a}_3\right) \geq \left(\breve{a}_1 + \breve{a}_2\right)^2 + \left(\breve{a}_3 + \breve{a}_4\right)^2 + 2\left(\breve{a}_1\right)^2 + 2\left(\breve{a}_3\right)^2 \tag{6}$$

Thus

$$R^{IVC}{}_{\tilde{\tilde{A}}_1}(x,y) \geq R^{IVC}{}_{\tilde{\tilde{A}}_2}(x,y) \tag{7}$$

□

**Theorem 6.2.** *Let* $\tilde{\tilde{A}} \in \tilde{F}_{IVTN}\left[\breve{h}, \widehat{h}\right]$ *Index Vectorial Centroid is generated by Euclidean distance of* $\tilde{\breve{A}} = \tilde{\widehat{A}} = \tilde{\tilde{A}}$ *is given as follows:*
$ICV = \left[\frac{1}{9}\left(\widehat{a}_1 + \breve{a}_1 + \frac{5}{4}\left(\widehat{a}_2 + \breve{a}_2\right) + \frac{7}{4}\left(\widehat{a}_3 + \breve{a}_3\right) + \frac{1}{2}\left(\widehat{a}_4 + \breve{a}_4\right)\right); \frac{11}{36}\left(\widehat{h} + \breve{h}\right)\right]$ , $\tilde{\breve{A}} = \tilde{\widehat{A}} = \tilde{\tilde{A}}$.

*Proof.* Theorem 2 describes an efficient approach to order of level $\left[\breve{h}, \widehat{h}\right]$-interval-valued trapezoidal fuzzy numbers based on the concept of comparison of fuzzy numbers by the help of Index Vectorial Centroid is generated by Euclidean distance ranking.. □

**Theorem 6.3.** *The basic solution* (10) *is an optimal solution for the auxiliary problem* (1) *if satisfies the interval fuzzy trapezoidal number .*

**Theorem 6.4.** *If for a basic feasible solution with basis B and objective value* $\tilde{\tilde{Z}}$*, it holds for Index Vectorial Centroid is generated by Euclidean distance ranking. for some non-basic variable* $x_k$ *while* $\lambda \leq 0$*, then the optimal solution of the auxiliary problem* (1) *is unbounded..*

## 7. Proposed Method

**STEP1**
Formulate the issue as (1)
**STEP2**
The defuzzification is employing into the given model.
The extended vectorial centroid de-fuzzification for Interval valued Trapezoidal fuzzy numbers $\tilde{\tilde{A}} = \left\langle \left(\breve{a}_1, \breve{a}_2, \breve{a}_3, \breve{a}_4; \breve{h}\right), \left(\widehat{a}_1, \widehat{a}_2, \widehat{a}_3, \widehat{a}_4; \widehat{h}\right)\right\rangle$

The three parts of centroid first triangle , rectangle and the other triangle is defined as

$$\begin{array}{l} \lambda_{\left(\widehat{\lambda},\breve{\lambda}\right)}(x,y)=\left[\frac{1}{3}\left(\widehat{a}_1+\widehat{a}_2\right)+\frac{1}{3}\left(\breve{a}_1+\breve{a}_2\right),\frac{1}{6}\left(\widehat{h}+\breve{h}\right)\right] \\ \kappa_{\left(\widehat{\kappa},\breve{\kappa}\right)}(x,y)=\left[\frac{1}{4}\left(\widehat{a}_2+\widehat{a}_3+\breve{a}_2+\breve{a}_3\right),\frac{1}{4}\left(\widehat{h}+\breve{h}\right)\right] \\ \eta_{\left(\widehat{\eta},\breve{\eta}\right)}(x,y)=\left[\frac{1}{3}\left(\widehat{a}_3+\widehat{a}_4\right)+\frac{1}{3}\left(\breve{a}_3+\breve{a}_4\right),\frac{1}{6}\left(\widehat{h}+\breve{h}\right)\right] \end{array}$$

Therefore,

$$\text{IVC}=\frac{1}{3}\left(\lambda_{\left(\widehat{\lambda},\breve{\lambda}\right)}(x,y)+\kappa_{\left(\widehat{\kappa},\breve{\kappa}\right)}(x,y)+\eta_{\left(\widehat{\eta},\breve{\eta}\right)}(x,y),\kappa_{\left(\widehat{\kappa},\breve{\kappa}\right)}(x,y)\left[\frac{2}{3}\left(\frac{\lambda_{\left(\widehat{\lambda},\breve{\lambda}\right)}{}^{(x,y)+\eta}{}_{\left(\widehat{\eta},\breve{\eta}\right)}{}^{(x,y)}}{2}-\kappa_{\left(\widehat{\kappa},\breve{\kappa}\right)}(x,y)\right)\right]\right)$$

$$\text{ICV}=\left[\frac{1}{9}\left(\widehat{a}_1+\breve{a}_1+\frac{5}{4}\left(\widehat{a}_2+\breve{a}_2\right)+\frac{7}{4}\left(\widehat{a}_3+\breve{a}_3\right)+\frac{1}{2}\left(\widehat{a}_4+\breve{a}_4\right)\right);\frac{11}{36}\left(\widehat{h}+\breve{h}\right)\right]$$

The Index Vectorial Centroid is generated by Euclidean distance is defined as

$$R(\tilde{\tilde{A}})=\sqrt{x^2+y^2} \tag{8}$$

**STEP 3** Using the proposed ranking method , the model is formulated as
Max $\tilde{\tilde{Z}}\approx\tilde{\tilde{c}}x$
Subject to constraints

$$\begin{array}{l} \tilde{\tilde{A}}x_j\le,=,\ge\tilde{\tilde{b}},j=1,2,..n \\ \tilde{\tilde{l}}\le x_j\le\tilde{\tilde{u}} \end{array} \tag{9}$$

$$x_j\ge 0,j=1,..,n$$

Subject to constraints

$$\begin{array}{l} R\left(\tilde{\tilde{A}}x_j\right)\le,=,\ge R\left(\tilde{\tilde{b}}\right) \\ R\left(\tilde{\tilde{l}}\right)\le x_j\le R\left(\widehat{\tilde{u}}\right),j=1,..,n \\ x_j\ge 0 \end{array}$$

**STEP 4**
Utilizing the proposed ranking method , the model is converted into crisp BVP
Max $Z\approx cx$
Subject to constraints

$$\left.\begin{array}{l} Ax_j\le,=,\ge b \\ l\le x_j\le u \\ x_j\ge 0 \end{array}\right\},j=1,..,n \tag{10}$$

**STEP 5** The transformation occurs during the defuzzification process, which is implemented using the Zimmerman technique and is modeled based on the bounds of the basic variable through membership function.

$$\mu_Z=\begin{cases} 0,\ Z_{obj}\prec L_Z \\ \frac{Z_{obj}-L_Z}{U_Z-L_Z},\ U_Z\prec Z_{obj}\prec L_Z \\ 1,\ Z_{obj}\succ L_Z \end{cases} \tag{11}$$

The model is framed as

$$\begin{array}{l} Max\ \lambda \\ Subject\, to\, constraints \\ \mu_Z\left(x\right) \geq \lambda \\ \left.\begin{array}{l} Ax_j \leq, =, \geq b \\ l \leq x_j \leq u \\ x_j \geq 0 \end{array}\right\}, j = 1, .., n \end{array} \tag{12}$$

**STEP 6** From step 5 , we use LINGO to solve the model (12) and attain the optimistic value with satisfactory level.

TABLE 2. Pseudocode

| **Pseudocode** |
|---|
| 1.Function Solve_IVTFBVP(Input_Data): |
| 2. Initialize Solution_Set |
| 3. For each Interval Valued Trapezoidal Fuzzy Bounded Variable IVTFBV in Input_Data: |
| 4. Compute_Centroid(ICV) |
| 5. Compute_Index_Vectorial_Centroid |
| 6. Compute Membership Functions for IVTFBV Perform Fuzzy Arithmetic Operations on IVTFBV |
| 7. Determine Optimal Solution using Aggregation Techniques |
| 8. Add Solution to Solution_Set |
| 9. Perform Fuzzy Arithmetic Operations on IVTFBV |
| 10. Return Solution_Set |
| 11. End Function |
| |
| 12. Function Compute_Membership_Functions(IVTFBV): |
| 13. Compute membership functions for the intervalvalued trapezoidal fuzzy numbers |
| 14. Determine_Optimal_Solution(IVTFBV): |
| 15. End function |
| |
| Function Perform_Zimmerman technique: |
| Implement defuzzification transformation using the Zimmerman technique |
| 16. End Function |
| |
| 17. Function Determine_Optimal_Solution(IVTFBV): |
| 18. Apply aggregation techniques to determine the optimal solution |
| 19. Return the optimal solution |
| 20. End Function |
| |
| Real_Life_Application(Solution_Set): |
| 21. Define a real-life scenario where the IVTFBVP solution is applicable |
| 22. Apply the solution to the real-life scenario |
| Return the results of the application |
| 23. End Function |
| |
| Main Function Function Main(): |
| 24. Define input data for the IVTFBVP Input_Data = Initialize_Input_Data |
| 25. Solve the IVTFBVP Solution_Set = Solve_IVTFBVP(Input_Data) |
| 26. Apply the solution to a real-life scenario Real_Life_Application(Solution_Set) |
| 27. End Function |

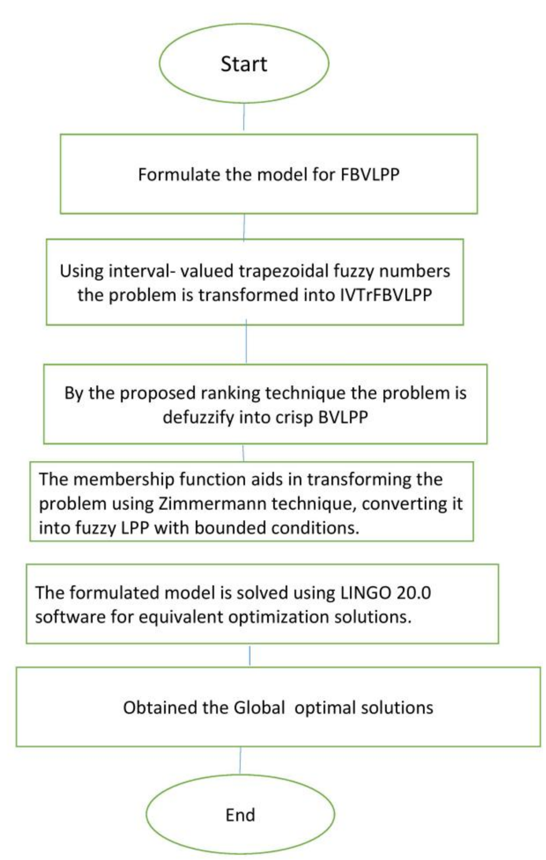


FIGURE 1. Graphically representation for proposed work

## 8. NUMERICAL EXAMPLE [13]

**Step 1**

$$Max\tilde{\tilde{Z}} = \widetilde{\widetilde{110}}x_1 + \widetilde{\widetilde{150}}x_2 + \tilde{\tilde{30}}x_3$$

Subject to constraints

$$\begin{aligned}
&\tilde{\tilde{4}}x_1 + \tilde{\tilde{2}}x_2 + \tilde{\tilde{1}}x_3 \leq \widetilde{\widetilde{100}}\\
&\tilde{\tilde{1}}x_1 + \tilde{\tilde{3}}x_2 + \tilde{\tilde{1}}x_3 \leq \tilde{\tilde{80}}\\
&\tilde{\tilde{10}} \leq x_1 \leq \tilde{\tilde{20}}\\
&\tilde{\tilde{15}} \leq x_2 \leq \tilde{\tilde{25}}\\
&\tilde{\tilde{15}} \leq x_3 \leq \tilde{\tilde{20}}\\
&x_1, x_2, x_3 \geq 0
\end{aligned}$$

$1\tilde{\tilde{1}}0 = \left\langle \left(40, 45, 65, 70; \frac{2}{3}\right), (35, 40, 70, 75; 1) \right\rangle$
$1\tilde{\tilde{5}}0 = \left\langle \left(60, 65, 85, 90; \frac{2}{3}\right), (55, 60, 90, 95; 1) \right\rangle$
$1\tilde{\tilde{0}}0 = \left\langle \left(25, 35.7, 62.5, 75; \frac{2}{3}\right), (12.5, 25, 75, 87.5; 1) \right\rangle,$
$\tilde{\tilde{3}}0 = \left\langle \left(12, 13, 17, 18; \frac{2}{3}\right), (11, 12, 18, 19; 1) \right\rangle$
$\tilde{\tilde{4}} = \left\langle \left(1, 1.5, 2.5, 3; \frac{2}{3}\right), (0.5, 1, 3, 3.5; 1) \right\rangle, \tilde{\tilde{8}}0 = \left\langle \left(20, 30, 50, 60; \frac{2}{3}\right), (10, 20, 60, 70; 1) \right\rangle$
$\tilde{\tilde{1}} = \left\langle \left(0.25, 0.375, 0.625, 0.75; \frac{2}{3}\right), (0.125, 0.25, 0.75, 0.875; 1) \right\rangle$
$\tilde{\tilde{3}} = \left\langle \left(0.75, 1, 25, 1.875, 2.25; \frac{2}{3}\right), (0.375, 0.75, 2.25, 2.625; 1) \right\rangle$
$\tilde{\tilde{2}} = \left\langle \left(0.5, 0.75, 1.25, 1.5; \frac{2}{3}\right), (0.25, 0.5, 1.5, 1.75; 1) \right\rangle,$
$\tilde{\tilde{1}}5 = \left\langle \left(6, 6.5, 8.5, 9; \frac{2}{3}\right), (5.5, 6, 9, 9.5; 1) \right\rangle$
$\tilde{\tilde{2}}0 = \left\langle \left(5, 7.5, 12.5, 15; \frac{2}{3}\right), (2.5, 5, 15, 17.5; 1) \right\rangle,$
$\tilde{\tilde{1}}0 = \left\langle \left(2.5, 3.75, 6.25, 7.5; \frac{2}{3}\right), (1.25, 2.5, 7.5, 8.75; 1) \right\rangle$

**Step 2**

Using the ICV the interval valued trapezoidal fuzzy parameters are converted into the form of step2 of proposed algorithm in eqn .(7) and (8)

**Step 3** Applying the proposed ranking function and performing the transformation are steps that occur in subsequent processes

$$Max\tilde{\tilde{Z}} = R\left(1\tilde{\tilde{1}}0\right)x_1 + R\left(1\tilde{\tilde{5}}0\right)x_2 + R\left(\tilde{\tilde{3}}0\right)x_3$$

Subject to constraints

$$\begin{aligned}
&R\left(\tilde{\tilde{4}}\right)x_1 + R\left(\tilde{\tilde{2}}\right)x_2 + R\left(\tilde{\tilde{1}}\right)x_3 \le R\left(1\tilde{\tilde{0}}0\right)\\
&R\left(\tilde{\tilde{1}}\right)x_1 + R\left(\tilde{\tilde{3}}\right)x_2 + R\left(\tilde{\tilde{1}}\right)x_3 \le R\left(\tilde{\tilde{8}}0\right)\\
&R\left(\tilde{\tilde{1}}0\right) \le x_1 \le R\left(\tilde{\tilde{2}}0\right)\\
&R\left(\tilde{\tilde{1}}5\right) \le x_2 \le R\left(\tilde{\tilde{2}}5\right)\\
&R\left(\tilde{\tilde{1}}5\right) \le x_3 \le R\left(\tilde{\tilde{2}}0\right)\\
&x_1, x_2, x_3 \ge 0
\end{aligned}$$

**Step 4** By utilizing the proposed ranking function, the transformation is converted into a conventional linear bounded variable problem.

Max Z = $54.4423x_1 + 74.446x_2 + 14.8976x_3$
Subject to constraints
$2.0099x_1 + 1.0975x_2 + 0.7040x_3 \le 48.61377$
$0.7040\ x_1 + 1.0975x_2 + 0.7040x_3 \le 38.8923$
$4.8877 \le x_1 \le 9.7355$
$7.4618 \le x_2 \le 11.73326$
$7.4618 \le x_3 \le 9.7355$
$x_1, x_2, x_3 \ge 0$

From step 4 , we use LINGO to solve the crisp BVP and to get the optimistic value as Max Z = 1548.553, $x_1$ = 9.735500, $x_2$= 11.73326, $x_3$ =9.735500 **Step 5** The range is defined by the upper bound denoted as $U_Z$ and the lower bound represented as $L_Z$and the values are $U_Z = 1548.533$, $L_Z = 946.150742$ . The membership function formulated as characterizes this range.

$$\mu_Z = \begin{cases} 0,\ Z_{obj} \prec 946.150742 \\ \frac{Z_{obj} - 946.150742}{U_Z - 946.150742},\ U_Z \prec Z_{obj} \prec 946.150742 \\ 1,\ Z_{obj} \succ 946.150742 \end{cases}$$

The Zimmermann technique is developed as ,

$$Max\,\lambda$$

Subject to constraints,

$$\begin{aligned}
&54.4423x_1 + 74.446x_2 + 14.8976x_3 - 946.150742 \geq \lambda\,(1548.533 - 946.150742)\\
&2.0099x_1 + 1.0975x_2 + 0.7040x_3 \leq 48.61377\\
&0.7040\,x_1 + 1.0975x_2 + 0.7040x_3 \leq 38.8923\\
&4.8877 \leq x_1 \leq 9.7355\\
&7.4618 \leq x_2 \leq 11.73326\\
&7.4618 \leq x_3 \leq 9.7355\\
&x_1, x_2, x_3 \geq 0
\end{aligned}$$

**Step 6**

The problem is solved through the lingo 20.0 and the satisfaction level is $\lambda = 1.0003$ and solutions are $x_1 = 9.735500,\ x_2 = 11.73326,\ x_3 = 9.735500$ , $Max\,\tilde{\tilde{Z}} = \left\langle \begin{array}{l} \left(703.99, 762.6619, 997.3271, 1055.99; \frac{2}{3}\right), \\ (645.32, 703.99, 1055.99, 1114.65; 1) \end{array} \right\rangle$

## 9. Real Life Application Problem [9]

A farmer who raises chickens would like to determine the amounts of the available ingredients that would meet certain nutritional requirements. The available ingredients and their cost per serving, along with the units of nutrients per serving in the ingredients are summarized in Table 2. The minimum daily requirements generally are imprecise numbers with the level $(w\ L,\ wU\ )$-intervalvalued trapezoidal possibility distributions over the planning horizon due to incomplete or unobtainable information. For example, the maximum, daily requirement of the protein and carbohydrates are $\left\langle\left(40, 45, 65, 70; \frac{2}{3}\right), (35, 40, 70, 75; 1)\right\rangle, \left\langle\left(60, 65, 85, 90; \frac{2}{3}\right), (55, 60, 90, 95; 1)\right\rangle$ , respectively. The objective is to determine which mix will meet certain nutritional requirements at a maximize cost. This problem is evidently an uncertain optimization problem due to variations in maximize daily requirements. So the amount of each unit of ingredients will be uncertain. Hence, we will model the problem as a level $(w_{\ L},\ w_U\ )$-interval-valued trapezoidal fuzzy variables linear programming problem. Let $\tilde{x}_1, \tilde{x}_2$ are the uncertain daily amount of protein and carbohydrates to determine the optimal combination, respectively. The daily amount of protein, measured by the weighted sum of asset volatilities, must not exceed a $\tilde{\tilde{10}}$and ateleast$\tilde{\tilde{40}}$ predefined target risk level. The daily amount of protein, measured by the weighted sum of asset volatilities, must not exceed a $\tilde{\tilde{15}}$ and atleast $\tilde{\tilde{48}}$ predefined . This constraint aims to control the overall certain nutritional requirements . In this case, the problem is formulated as follows:"

**Step 1**

$$Max\,\tilde{\tilde{Z}} = \tilde{\tilde{80}}x_1 + \tilde{\tilde{60}}_2$$

Subject to constraints

$$\begin{aligned}
&\tilde{\tilde{4}}x_1 + \tilde{\tilde{2}}x_2 \leq \tilde{\tilde{80}}\\
&\tilde{\tilde{1}}x_1 + \tilde{\tilde{3}}x_2 \leq \tilde{\tilde{60}}\\
&\tilde{\tilde{10}} \leq x_1 \leq \tilde{\tilde{40}}\\
&\tilde{\tilde{15}} \leq x_2 \leq \tilde{\tilde{48}}\\
&x_1, x_2 \geq 0
\end{aligned}$$

TABLE 3. The data of application

| Nutrient | Ingredient | |
|---|---|---|
| | Corn | Lime |
| Protein | 4 | 1 |
| Carbohydrates | 2 | 3 |
| Cost | 80 | 60 |

$\tilde{\tilde{80}} = \left\langle \left(40, 45, 65, 70; \frac{2}{3}\right), (35, 40, 70, 75; 1) \right\rangle$
$\tilde{\tilde{60}} = \left\langle \left(60, 65, 85, 90; \frac{2}{3}\right), (55, 60, 90, 95; 1) \right\rangle$
$\tilde{\tilde{4}} = \left\langle \left(1, 1.5, 2.5, 3; \frac{2}{3}\right), (0.5, 1, 3, 3.5; 1) \right\rangle,$
$\tilde{\tilde{1}} = \left\langle \left(0.25, 0.375, 0.625, 0.75; \frac{2}{3}\right), (0.125, 0.25, 0.75, 0.875; 1) \right\rangle$
$\tilde{\tilde{3}} = \left\langle \left(0.75, 1, 25, 1.875, 2.25; \frac{2}{3}\right), (0.375, 0.75, 2.25, 2.625; 1) \right\rangle$
$\tilde{\tilde{2}} = \left\langle \left(0.5, 0.75, 1.25, 1.5; \frac{2}{3}\right), (0.25, 0.5, 1.5, 1.75; 1) \right\rangle$
$\tilde{\tilde{15}} = \left\langle \left(6, 6.5, 8.5, 9; \frac{2}{3}\right), (5.5, 6, 9, 9.5; 1) \right\rangle$
$\tilde{\tilde{10}} = \left\langle \left(2.5, 3.75, 6.25, 7.5; \frac{2}{3}\right), (1.25, 2.5, 7.5, 8.75; 1) \right\rangle$
$\tilde{\tilde{40}} = \left\langle \left(12, 18, 22, 28; \frac{2}{3}\right), (10, 16, 24, 30; 1) \right\rangle$
$\tilde{\tilde{48}} = \left\langle \left(15, 23, 25, 33; \frac{2}{3}\right), (10, 22, 26, 38; 1) \right\rangle$

**Step 2** Using the ICV the interval valued trapezoidal fuzzy parameters are converted into the form of step 2 proposed algorithm

**Step 3** Applying the proposed ranking function and performing the transformation are steps that occur in subsequent processes

$$Max\ \tilde{\tilde{Z}} = R\left(\tilde{\tilde{80}}\right) x_1 + R\left(\tilde{\tilde{60}}\right) x_2$$

Subject to constraints

$$R\left(\tilde{\tilde{4}}\right) x_1 + R\left(\tilde{\tilde{2}}\right) x_2 \leq R\left(\tilde{\tilde{100}}\right)$$
$$R\left(\tilde{\tilde{1}}\right) x_1 + R\left(\tilde{\tilde{3}}\right) x_2 \leq R\left(\tilde{\tilde{80}}\right)$$
$$R\left(\tilde{\tilde{10}}\right) \leq x_1 \leq R\left(\tilde{\tilde{40}}\right)$$
$$R\left(\tilde{\tilde{15}}\right) \leq x_2 \leq R\left(\tilde{\tilde{48}}\right)$$
$$x_1, x_2, x_3 \geq 0$$

**Step 4** By utilizing the proposed ranking function, the transformation is converted into a conventional linear bounded variable problem.

Max Z = $78.612x_1+74.446x_2$
Subject to constraints
$2.0099x_1+1.0975x_2 \leq 78.612$
$0.7040\ x_1+1.0975x_2+0.7040x_3 \leq 74.446$
$4.8877 \leq x_1 \leq 17.341$

$7.4618 \leq x_2 \leq 39.86705$

$$x_1, x_2 \geq 0$$

From step 4 , we use LINGO to solve the crisp BVP and to get the optimistic value as Max Z = 1548.553, $x_1$ = 9.735500, $x_2$= 11.73326, $x_3$ =9.735500

**Step 5** The range is defined by the upper bound denoted as $U_Z$ and the lower bound represented as $L_Z$and the values are $U_Z = 4331.16$, $L_Z = 1062.0303$ . The membership function formulated as characterizes this range.

$$\mu_Z = \begin{cases} 0,\ Z_{obj} \prec 1062.030 \\ \frac{Z_{obj}-1062.030}{4331.16-1062.030},\ 4331.16 \prec Z_{obj} \prec 1062.030 \\ 1,\ Z_{obj} \succ 4331.16 \end{cases}$$

The Zimmermann technique is developed as ,

$$Max\,\lambda$$

Subject to constraints,

$$\begin{array}{l} 78.612x_1 + 74.446x_2 - 1062.030 \geq \lambda\,(4331.16 - 1062.030) \\ 2.0099x_1 + 1.0975x_2 \leq 78.612 \\ 0.7040\ x_1 + 1.0975x_2 \leq 74.446 \\ 4.8877\ \leq\ x_1 \leq\ 17.341 \\ 7.4618\ \leq\ x_2 \leq 39.86705 \\ x_1, x_2, \geq 0 \end{array}$$

**Step 6** The problem is solved through the lingo 20.0 and the satisfaction level is $\lambda = 1$ and solutions are $x_1\ = 17.3411,\ x_2 = 39.89705$,

$$Max\,\tilde{\tilde{Z}} = \left\langle \begin{array}{l} \left(1387.288, 1560.699, 2254.343, 2427.754; \frac{2}{3}\right), \\ (1213.877, 1387.288, 2427.75, 2601.16; 1) \end{array} \right\rangle$$

## 10. Result Analysis

It is emphasized that the suggested approach does not impose any restrictions on the variables or parameters, and the outcomes meet all requirements. Because we take into account every component of the decision-making process in our calculations, our model portrays reality more accurately than the existing one. This model is not time-consuming and difficult, but ours is not. Our model decreases the complexity of the problem.The comparison chart reveals that the crisp optimal solutions both Ebrahimnejad et al.[9] and Ebrahimnejad [13] offer solutions in figure2. However, both authors demonstrate sub-optimal outcomes in their proposed solutions. Ebrahimnejad et al.[9] despite addressing theoretical challenges, falls short in achieving the desired efficiency. On the other hand Ebrahimnejad [13] while exploring practical applications, also exhibits limitations in the quality of the solution. This emphasizes the necessity for further advancements and improvements in effectively addressing the complexities of interval-valued trapezoidal fuzzy bounded variable problems within the specified topic. In contrast to current methodologies, our suggested model is relatively simple and convenient to use in real-world applications in figure 2.

## 11. Limitations

- The proposed solution may face challenges in scalability when dealing with large-scale problems due to its computational complexity. This could hinder its application in real-time or resource-constrained environments.

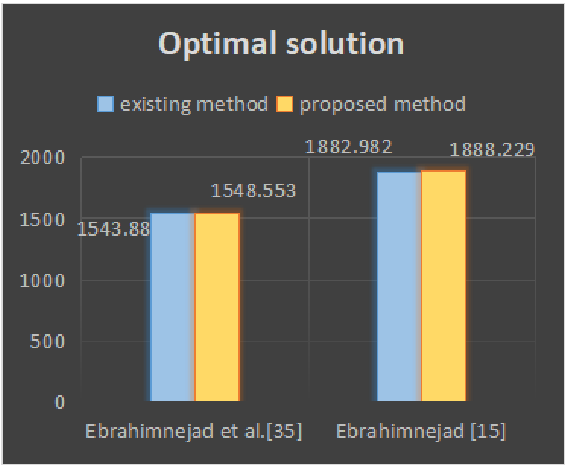


FIGURE 2. comparison for the Numerical example and application

- Sensitivity to parameter selection or initialization might exist, impacting the robustness and stability of the solution across various problem instances.
- The applicability of the proposed solution might be constrained to specific problem domains or scenarios due to assumptions made during the formulation, potentially limiting its wider practical utility.

## 12. ADVANTAGES

- The proposed solution offers a versatile framework to handle interval-valued trapezoidal fuzzy bounded variables, allowing for a more realistic representation of uncertainty in various decision-making scenarios.
- Medical Diagnosis in Healthcare Advantages: Employing extended vectorial centroid defuzzification enables healthcare professionals to interpret complex diagnostic data from multiple sources, facilitating accurate diagnosis and personalized treatment plans for patients, ultimately improving patient outcomes.
- It demonstrates superior performance in optimizing problems involving interval-valued trapezoidal fuzzy bounded variables, showcasing efficient convergence rates or computational efficiency compared to existing methods.
- The solution exhibits robustness by effectively managing uncertainty within variables, offering a reliable mechanism to address imprecise or ambiguous data prevalent in real-world applications.
- Its versatility extends across multiple domains such as engineering, finance, or logistics, providing a valuable tool for decision-making processes in a wide array of industries.
- Traffic Management in Smart Cities Advantages: Implementing the Zimmerman technique during defuzzification enables traffic management authorities to analyze and optimize traffic flow, reducing congestion, minimizing travel times, and improving overall transportation efficiency in urban areas.
- Its computational efficiency or reduced complexity may render it suitable for real-time decision-making scenarios, enhancing its practical utility in time-sensitive applications.

- It introduces a novel approach or methodology, contributing to the advancement of fuzzy logic-based optimization techniques, thereby expanding the theoretical and practical landscape in the field.

## 13. Conclusion and Future scope

This research work that we investigated the domain of FLP problems by assuming that all of the characteristics are expressed as levels IVTrFN and that the decision variables are restricted to lower and upper limits. To defuzzify the interval valued trapezoidal numbers, a novel index vectorial centoriod defuzzification using euclidean distance measure is proposed, along with a broader approach to solving BIVFNLP issues. We extract certain conclusions and the ideal circumstance for a workable solution to FLP difficulties. It is simple to improve upon the current ranking order of their works according to the proposed method . The disadvantages of the prior work 25, where decision variables are not in bounded variables, have been satisfactorily addressed by the given BIVTFNLP issue. In the future, we investigate how to solve fuzzy bounded variable problem in dynamic programming problem and FBLPs in more uncertain and ambiguous situations.By focusing on these future research directions, the field can advance towards a more comprehensive and practical understanding of efficient solutions for the Interval-Valued Trapezoidal Fuzzy Bounded Variable Problem, contributing to advancements in decision-making under uncertainty.

- Investigate and develop advanced algorithms that can further optimize the efficiency of solving the Interval-Valued Trapezoidal Fuzzy Bounded Variable Problem. This involves exploring computational techniques, heuristic methods, and machine learning approaches.
- Extend the application of the proposed solution across diverse fields such as finance, engineering, healthcare, and environmental science. Investigate how the methodology can adapt to the unique challenges presented by different domains, ensuring its versatility and effectiveness.
- Explore the integration of the solution with emerging technologies such as blockchain, Internet of Things (IoT), and artificial intelligence. Assess how these technologies can enhance the robustness and real-time applicability of the proposed solution in dynamic and complex systems.
- Develop methods for quantifying and characterizing uncertainty within the context of the Interval-Valued Trapezoidal Fuzzy Bounded Variable Problem. This involves devising measures and metrics to provide a clearer understanding of the uncertainty associated with the proposed solution.
- Investigate the integration of human-centric design principles in the development of the solution. This includes understanding user requirements, cognitive aspects, and usability factors to ensure that the solution is not only efficient but also user-friendly and accessible.

**Conflict of Interest.** The authors declare that they do not have any conflict of interest.

**Ethical approval.** This article does not contain any studies with human participants or animals performed by any of the authors.

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