TWMS J. App. and Eng. Math. V.15, N.6, 2025, pp. 1389-1396

CLAW-DECOMPOSITION OF THE KNESER GRAPH $KG_{n,3}$

C. SANKARI[1], R. SANGEETHA[1*], K. ARTHI[1], §

ABSTRACT. The Kneser graph $KG_{n,3}$ is the graph whose vertices are the 3-element subsets of n-elements, in which two vertices are adjacent if and only if their intersection is empty. A claw is a star with three edges. In this paper, we prove that $KG_{n,3}$ is claw-decomposable if and only if $n \geq 9$ and $n \equiv 0, 1, 2, 3, 4, 5 (mod\ 9)$.

Keywords: Decomposition, Kneser Graph, Star, Complete Bipartite Graph, Induced Subgraph.

AMS Subject Classification: 05C70.

1. INTRODUCTION

All the graphs considered in this paper are finite. If a graph G has no edges, then it is called a *null graph.* Let $K_{m,n}$ denote a *complete bipartite graph* with m and n vertices in the parts. A *star* with k edges is denoted by S_k and $S_k \cong K_{1,k}$. If $k = 3$, then the graph S_3 is called a *claw.* A *path* with k edges is denoted by P_k and a *cycle* with k edges is denoted by C_k. A *Hamilton cycle* of G is a cycle that contains every vertex of G. A graph G is *Hamiltonian* if it contains a Hamilton cycle. The degree of a vertex x of G, denoted by $d_G(x)$ is the number of edges incident with x in G. A graph G is said to be *k-regular*, if each vertex in G is of degree k. If $H_1, H_2, ..., H_l$ are edge disjoint subgraphs of a graph G such that $E(G) = E(H_1) \cup E(H_2) \cup ... \cup E(H_l)$, then we say that $H_1, H_2, ..., H_l$ *decompose* G and we denote it by $G = H_1 \oplus H_2 \oplus ... \oplus H_l$. If $H_i \cong S_k$ for $i = 1, 2, ..., l$, then we say that G is *S_k-decomposable* and we denote it by $S_k|G$. Let $X \subseteq V(G)$. The subgraph of G induced by X is denoted by $\langle X \rangle$. Let $A = \{1, 2, 3, ..., n\}$. Then $\mathcal{P}_k(A)$ denotes the set of all k-element subsets of A. The *Kneser Graph* $KG_{n,3}$ is defined as follows: $V(KG_{n,3})= \mathcal{P}_3(A)$ and $E(KG_{n,3}) = \{XY | X, Y \in \mathcal{P}_3(A)$ and $X \cap Y = \emptyset\}$. The *Generalized Kneser Graph*, $GKG_{n,k,r}$ is the graph whose vertices are the k-element subsets of A, in which two vertices are adjacent if and only if they intersect in precisely r elements.

In 1955, Kneser [3] introduced the Kneser graph. In 2000, Chen [1] proved that $KG_{n,2}$

[1] Department of Mathematics, A. V. V. M. Sri Pushpam College (Affiliated to Bharathidasan University), Poondi, Thanjavur, Tamil Nadu, India.
e-mail: sankari9791@gmail.com; ORCID: https://orcid.org/0000-0003-3325-7163.
e-mail: jaisangmaths@yahoo.com; ORCID: https://orcid.org/0000-0003-2726-0956.
e-mail: arthi1505@gmail.com; ORCID: https://orcid.org/0000-0002-0399-1343.
* Corresponding author.

§ Manuscript received: : March 16, 2024; accepted: August 02, 2024.
TWMS Journal of Applied and Engineering Mathematics, Vol.15, No.6;

is Hamiltonian, when $n \geq 3k$, $k \geq 1$. In 2004, Shields and Savage [7] proved that all connected Kneser graphs (except $KG_{5,2}$) have Hamilton cycles, when $n \leq 27$ and the problem $KG_{n,2}$ (except $KG_{5,2}$) is Hamiltonian is still open. In 2015, Rodger and Whitt [4] established the necessary and sufficient conditions for a P_3-decomposition of the Kneser graph $KG_{n,2}$ and the Generalized Kneser Graph $GKG_{n,3,1}$. In 2015, Whitt and Rodger [8] proved that the Kneser graph $KG_{n,2}$ is P_4-decomposable if and only if $n \equiv 0, 1, 2, 3(mod\ 16)$. In 2018, Ganesamurthy and Paulraja [2] proved that if $n \equiv 0, 1, 2, 3(mod\ 8k), k \geq 2$, then the Kneser graph $KG_{n,2}$ can be decomposed into paths of length $2k$. In the same paper they also proved that, for $k = 2^l$, $l \geq 1$, $KG_{n,2}$ has a P_{2k}-decomposition if and only if $n \equiv 0, 1, 2, 3(mod\ 2^{l+3})$. Recently, the authors [5, 6] proved that $KG_{n,2}$ is claw-decomposable, for all $n \geq 6$ and $KG_{n,2}$ is S_5-decomposable if and only if $n \geq 7$ and $n \equiv 0, 1, 2, 3(mod\ 5)$. In this paper, we prove that the Kneser graph $KG_{n,3}$ is claw-decomposable if and only if $n \geq 9$ and $n \equiv 0, 1, 2, 3, 4, 5(mod\ 9)$.

2. Preliminaries

Let G be a graph on n vertices and $\{1, 2, ..., k\} \subset V(G)$. The notation $(1; 2, 3, ..., k)$ denotes a star with a center vertex 1 and $k-1$ pendent edges $12, 13, ..., 1k$. Let X and Y be two disjoint subsets of $V(G)$. Then $E(X, Y)$ denotes the set of edges in G, whose one end vertex is in X and the other end vertex is in Y. The notation $\langle E(X, Y)\rangle$ denotes the graph induced by the edges of $E(X, Y)$. To prove our results we use the following:

Theorem 2.1. *(Yamamoto et al. [9]) Let k, m and $n \in \mathbb{Z}_+$ with $m \leq n$. There exists an S_k-decomposition of $K_{m,n}$ if and only if one of the following holds:*

(i) $k \leq m$ and $mn \equiv 0(mod\ k)$;
(ii) $m < k \leq n$ and $n \equiv 0(mod\ k)$.

3. Claw-decomposition of $KG_{n,3}$

Note that, the cardinality of the edges of $KG_{n,3}$ is $\frac{1}{2}\binom{n}{3}\binom{n-3}{3}$ and is divisible by 3, if $n \geq 9$ and $n \equiv 0, 1, 2, 3, 4, 5(mod\ 9)$. Now, we prove that these obvious necessary conditions are also sufficient for the existence of a claw-decomposition of $KG_{n,3}$.

Lemma 3.1. *$KG_{9,3}$ is claw-decomposable.*

Proof. We know that $|V(KG_{9,3})|$=84. Note that, the addition in the subscripts are taken modulo 3 with residues 1,2,3. We partition $V(KG_{9,3})$=$A \cup B \cup C \cup D$, where

$$A = \bigcup_{i=1}^{3} A_i, \text{ where } A_1 = \{1, 2, 3\},\ A_2 = \{4, 5, 6\} \text{ and } A_3 = \{7, 8, 9\}$$

$$B = \bigcup_{i=1}^{3} B_i, \text{ where } B_i = \{\{a, b, c\} | a \in A_i,\ b, c \in A_{i+1}\ and\ b < c\}$$

$$C = \bigcup_{i=1}^{3} C_i, \text{ where } C_i = \{\{a, b, c\} | a, b \in A_i,\ a < b\ and\ c \in A_{i+1}\}$$

$$D = \bigcup_{i=1}^{3} D_i, \text{ where } D_i = \{\{a, b, c\} | a = i,\ b \in A_2\ and\ c \in A_3\}$$

Therefore, $|A_i|=3$, $|B_i|=|C_i|=|D_i|=9$, $1 \leq i \leq 3$. Observe that, the graph $KG_{9,3}=\langle A\rangle \cup \langle B\rangle \cup \langle C\rangle \cup \langle D\rangle \cup \langle E(A,B)\rangle \cup \langle E(A,C)\rangle \cup \langle E(A,D)\rangle \cup \langle E(B,C)\rangle \cup \langle E(B,D)\rangle \cup \langle E(C,D)\rangle$. Now, we show that each subgraph $\langle A\rangle$, $\langle B\rangle$, $\langle C\rangle$, $\langle D\rangle$, $\langle E(A,B)\rangle$, $\langle E(A,C)\rangle$, $\langle E(A,D)\rangle$, $\langle E(B,C)\rangle$, $\langle E(B,D)\rangle$ and $\langle E(C,D)\rangle$ is claw-decomposable.

We write $\langle C\rangle=\bigcup_{i=1}^{3}\langle E(C_i,C_{i+1})\rangle$. Note that $d_{\langle E(C_i,C_{i+1})\rangle}(\{a,b,c\})=3$, for all $\{a,b,c\} \in C_i$, $1 \leq i \leq 3$. Hence, $\langle C\rangle$ is claw-decomposable.

We write $\langle D\rangle=\bigcup_{i=1}^{3}\langle E(D_i,D_{i+1})\rangle$. Let $l \in \{1,2\}$ and $t \in \{0,1,2\}$. Take $1 \leq i \leq 3$, $j=i+t+3$ and $k=j+3$. Let $x \in \{i,j,k\}$

$$x+l=\begin{cases} i+l-3 & \text{if } i+l>3 \\ j+l-3 & \text{if } j+l>6 \\ k+l-3 & \text{if } k+l>9 \end{cases}$$

We choose the set of stars $\mathcal{S}^i=\{(\{i,j,k\};\{i+1,j+1,k+1\},\{i+1,j+1,k+2\},\{i+1,j+2,k+1\})\}$. Note that, $d_{\langle E(D_i,D_{i+1})\rangle \setminus E(\mathcal{S}^i)}(\{a,b,c\})=3$, for all $\{a,b,c\} \in D_{i+1}$, $1 \leq i \leq 3$. Hence, $\langle D\rangle$ is claw-decomposable.

In $\langle E(A,C)\rangle$, $d_{\langle E(A,C)\rangle}(\{a,b,c\})=9$, for all $\{a,b,c\} \in A$. Now, by fixing each vertex of A as a center vertex (3 times), we get a claw-decomposition in $\langle E(A,C)\rangle$. The graph $\langle E(A,D)\rangle$ is a null graph. In $\langle E(B,C)\rangle$, we choose a set of stars, say $\mathcal{S}'$ as follows:
$(\{1,2,4\};\{5,7,8\},\{5,7,9\},\{5,8,9\})$, $(\{1,2,5\};\{6,7,8\},\{6,7,9\},\{6,8,9\})$,
$(\{1,2,6\};\{4,7,8\},\{4,7,9\},\{4,8,9\})$, $(\{4,5,7\};\{8,1,2\},\{8,1,3\},\{8,2,3\})$,
$(\{4,5,8\};\{9,1,2\},\{9,1,3\},\{9,2,3\})$, $(\{4,5,9\};\{7,1,2\},\{7,1,3\},\{7,2,3\})$,
$(\{7,8,1\};\{2,4,5\},\{2,4,6\},\{2,5,6\})$, $(\{7,8,2\};\{3,4,5\},\{3,4,6\},\{3,5,6\})$
and $(\{7,8,3\};\{1,4,5\},\{1,4,6\},\{1,5,6\})$. We observe that, $d_{\langle E(B,C)\rangle \setminus E(\mathcal{S}')}(\{a,b,c\})=6$, for all $\{a,b,c\} \in B$. Now, by fixing each vertex of B as a center vertex (2 times), we get a claw-decomposition in $\langle E(B,C)\rangle \setminus E(\mathcal{S}')$.

In $\langle E(B,D)\rangle$ and $\langle E(C,D)\rangle$, the degree of each vertex of B and C respectively, is exactly 6. Now, by fixing each vertex of B and C as a center vertex (2 times), we get a claw-decomposition in $\langle E(B,D)\rangle$ and $\langle E(C,D)\rangle$ respectively.

Let $H=\langle A\rangle \cup \langle B\rangle \cup \langle E(A,B)\rangle$. In H, we choose a set of stars, say $\mathcal{S}$ as follows:
S^1:$(\{1,2,3\};\{4,5,6\},\{4,7,8\},\{4,7,9\})$, S^2:$(\{4,5,6\};\{7,8,9\},\{7,1,2\},\{7,1,3\})$,
S^3:$(\{7,8,9\};\{1,2,3\},\{1,4,5\},\{1,4,6\})$, S^4:$(\{4,8,9\};\{1,2,3\},\{7,1,2\},\{7,1,3\})$,
S^5:$(\{7,2,3\};\{4,5,6\},\{4,8,9\},\{1,4,6\})$, S^6:$(\{8,2,3\};\{1,4,5\},\{1,4,6\},\{1,5,6\})$,
S^7:$(\{9,2,3\};\{1,4,5\},\{1,4,6\},\{6,7,8\})$, S^8:$(\{1,4,5\};\{7,2,3\},\{6,7,9\},\{6,8,9\})$,
S^9:$(\{1,4,6\};\{5,7,8\},\{5,7,9\},\{5,8,9\})$ and S^{10}:$(\{1,5,6\};\{7,8,9\},\{4,7,8\},\{4,7,9\})$.

Let F be a subgraph of H such that $F=(\langle A\rangle \cup \langle E(A,B)\rangle) - (E(S^1 \cup S^2 \cup S^3) \cup \{1,2,3\}\{4,8,9\} \cup \{4,5,6\}\{7,2,3\} \cup \{7,8,9\}\{1,5,6\})$. We observe that $H \setminus E(\mathcal{S}) = F \cup \langle E(B_1 \setminus N_1, B_2 \cup B_3)\rangle \cup \langle E(B_2 \setminus \{4,8,9\}, B_1 \cup B_3)\rangle$, where $N_1=\{\{1,4,5\},\{1,4,6\}\} \subset B_1$. Now, we prove that each of the subgraph in the above union is claw-decomposable. Note that, $d_F(\{a,b,c\})=6$, for all $\{a,b,c\} \in A$, hence we get a claw-decomposition in F, see Figure 3.1. In $\langle E(B_1 \setminus N_1, B_2 \cup B_3)\rangle$, the degree of each vertex of $B_1 \setminus N_1$ is exactly 6 except $\{1,5,6\}$ and the degree of a vertex $\{1,5,6\}$ is exactly 3, hence we get a claw-decomposition in $\langle E(B_1 \setminus N_1, B_2 \cup B_3)\rangle$, see Figure 3.2. Finally, we consider the subgraph $\langle E(B_2 \setminus \{4,8,9\}, B_1 \cup B_3)\rangle$, the degree of each vertex of $B_2 \setminus \{4,8,9\}$ is exactly 3 and hence we get a claw-decomposition in $\langle E(B_2 \setminus \{4,8,9\}, B_1 \cup B_3)\rangle$, see Figure 3.3.

□

Theorem 3.1. *If $n \equiv 0 (mod\ 9)$, then the graph $KG_{n,3}$ is claw-decomposable.*

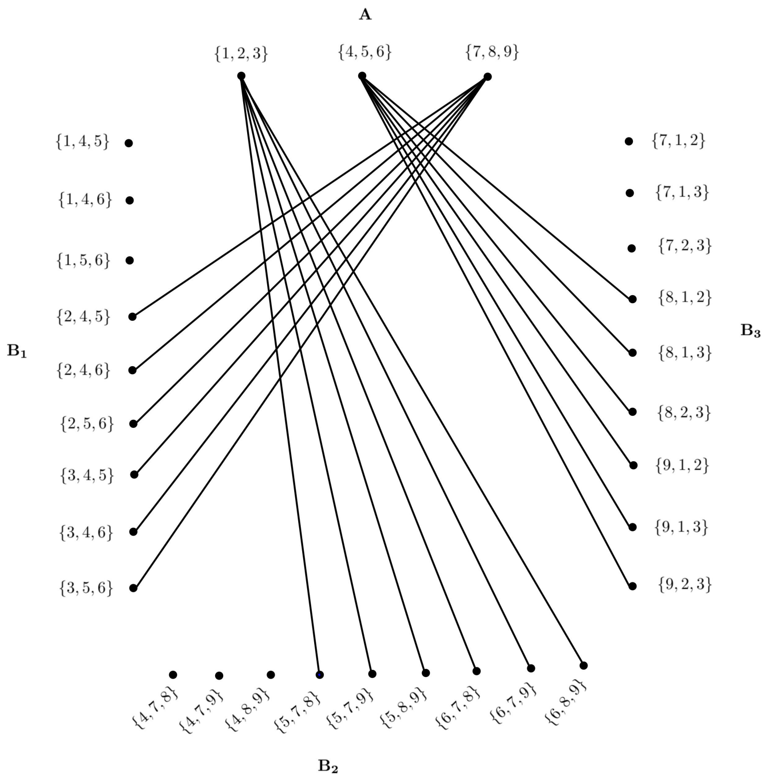

FIGURE 3.1. The subgraph F of H

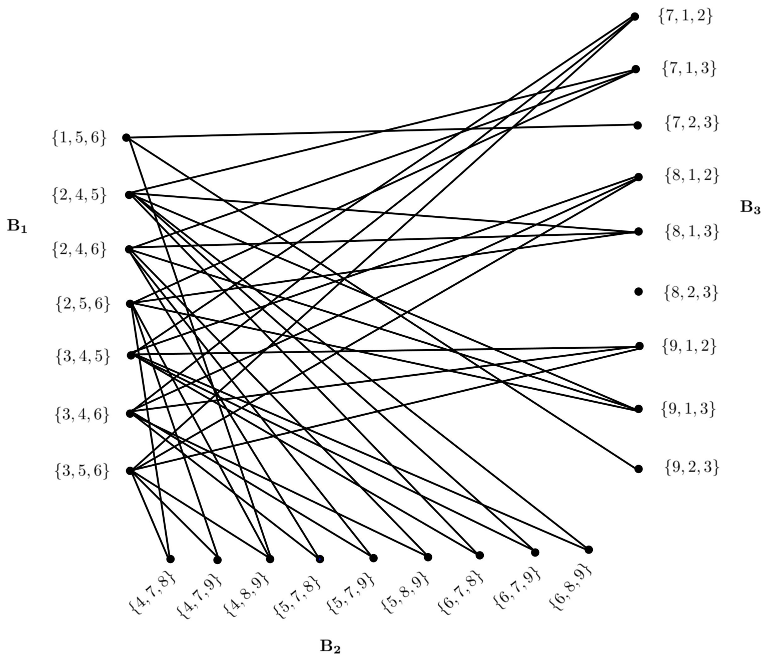

FIGURE 3.2. The subgraph $\langle E(B_1 \setminus N_1, B_2 \cup B_3)\rangle$

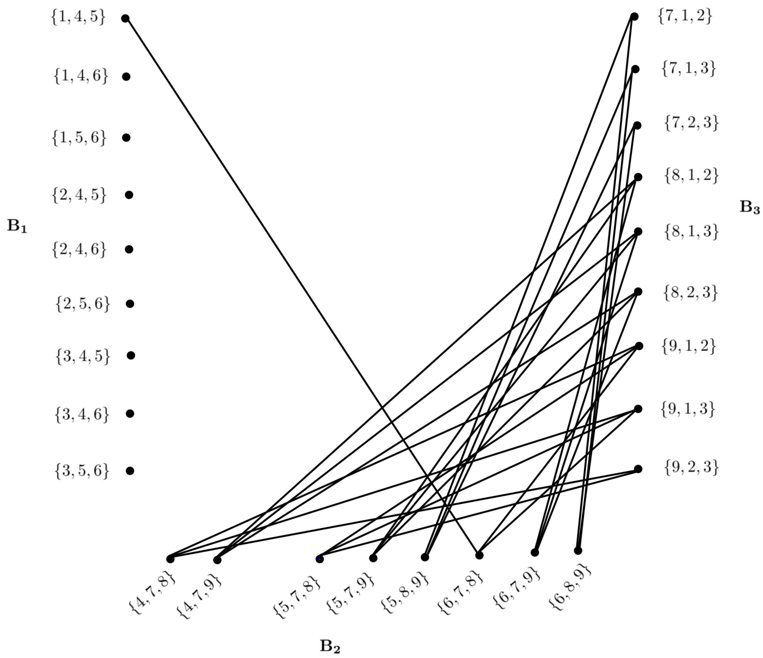

FIGURE 3.3. The subgraph $\langle E(B_2 \setminus \{4,8,9\}, B_1 \cup B_3)\rangle$

Proof. We define the vertex set $V(KG_{n,3})$ as follows: Let n_1=9 and n_2=n−n_1. As n=9t for some $t \geq 1$, we have n_2=9(t−1). Define V_1=$\{1, 2, ..., 9\}$ and V_2=$\{10, 11, ..., n\}$. Now define $V(KG_{n,3})$=$N_1 \cup N_2 \cup N_3 \cup N_4$, where N_1=$\{\{i, j, k\}|i, j, k \in \mathcal{P}_3(V_1)\}$, N_2=$\{\{i, j, k\}|i, j, k \in \mathcal{P}_3(V_2)\}$, N_3=$\{\{i, j, k\}|i, j \in \mathcal{P}_2(V_1), k \in V_2\}$ and N_4=$\{\{i, j, k\}|i, j \in \mathcal{P}_2(V_2), k \in V_1\}$. Therefore, $|N_1|$=$\binom{n_1}{3}$, $|N_2|$=$\binom{n_2}{3}$, $|N_3|$=$n_2\binom{n_1}{2}$ and $|N_4|$=$n_1\binom{n_2}{2}$. Now, we write $KG_{n,3}$=$\oplus_{i=1}^{8} G_i$ where $G_i, 1 \leq i \leq 8$ is as follows: G_1=$\langle N_1\rangle \cong KG_{n_1,3}$, G_2=$\langle N_2\rangle \cong KG_{n_2,3}$, G_3=$\langle E(N_1, N_2)\rangle \cong K_{|N_1|,|N_2|}$, $G_4 \cong \langle N_3\rangle$, $G_5 \cong \langle N_4\rangle$, $G_6 \cong \langle E(N_3, N_4)\rangle$, $G_7 \cong \langle E(N_1, N_3 \cup N_4)\rangle$ and $G_8 \cong \langle E(N_2, N_3 \cup N_4)\rangle$.

We apply mathematical induction on t, to prove the theorem. Now, we show that the graph $KG_{n,3}$=$\oplus_{i=1}^{8} G_i$ is claw-decomposable. If t=1, then the graph $KG_{9,3}$ is claw-decomposable, by Lemma 3.1. Hence, the result is true for t=1. Now, assume that the result is true for $t < k$. We prove that the result is true for all t=k, $t > 1$. The graph G_1 is claw-decomposable, by Lemma 3.1 and G_2 is claw-decomposable, by our assumption. The graph G_3 is claw-decomposable, by Theorem 2.1.

For $i, j \in V_1$ and $i < j$, let X_l=$\{i, j, l$+$9|1 \leq l \leq n_2\}$. Then $G_4 = \bigcup_{1\leq l<m\leq n_2} \langle E(X_l, X_m)\rangle$. Note that, the degree of a vertex $\{i, j, l + 9\}$ in $\langle E(X_l, X_m)\rangle$, $1 \leq l < m \leq n_2$ is exactly $\binom{9}{2} - [(9 - 1) + (9 - 2)]$=21. By fixing these vertices as center vertices, we get a claw-decomposition in $\langle E(X_l, X_m)\rangle$ and hence G_4 is claw-decomposable.

For $i, j \in V_2$ and $i < j$, let Y_l=$\{i, j, l|1 \leq l \leq 9\}$. Then G_5=$\bigcup_{1\leq l<m\leq 9} \langle E(Y_l, Y_m)\rangle$. Note that, the degree of a vertex $\{i, j, l\}$ in $\langle E(Y_l, Y_m)\rangle$, $1 \leq l < m \leq 9$ is exactly $\binom{n_2}{2} - [(n_2 - 1) + (n_2 - 2)]$=$\binom{n_2}{2} - (2n_2 - 3)$ which is a multiple of 3. By fixing these vertices as center vertices, we get a claw-decomposition in $\langle E(Y_l, Y_m)\rangle$ and hence G_5 is claw-decomposable.

We prove that, the graph G_6=$\langle E(\bigcup_{i,j\in\mathcal{P}_2(V_1)}\{i, j, k\}, N_4)\rangle$, $10 \leq k \leq n$ is claw-de

composable. For each k, $10 \leq k \leq n$ we prove that $\langle E(\bigcup_{i,j \in \mathcal{P}_2(V_1)}\{i,j,k\}, N_4)\rangle$ is claw-decomposable. Let $\mathcal{A}_k$=$\bigcup_{i,j \in \mathcal{P}_2(V_1)}\{i,j,k\}$, $10 \leq k \leq n$. Then $|\mathcal{A}_k|$=36. Note that, $d_{\langle E(\mathcal{A}_k,N_4)\rangle}(\{a,b,c\})=(n_1-2)[\binom{n_2}{2}-(n_2-1)]$, for all $\{a,b,c\} \in \mathcal{A}_k$, which is congruent to 1 modulo 3. Now, we partition each $\mathcal{A}_k$ into 12 subsets such that each subset has 3 vertices as follows:
For $1 \leq w \leq 12$, $T_{k,w}=\{\{i_{wt}, j_{wt}, k\} | 1 \leq t \leq 3\}$, where
$T_{k,1}=\{\{1,2,k\},\{1,3,k\},\{1,4,k\}\}$, $T_{k,2}=\{\{1,5,k\},\{1,6,k\},\{1,7,k\}\}$,
$T_{k,3}=\{\{1,8,k\},\{1,9,k\},\{2,3,k\}\}$, $T_{k,4}=\{\{2,4,k\},\{2,5,k\},\{2,6,k\}\}$,
$T_{k,5}=\{\{2,7,k\},\{2,8,k\},\{2,9,k\}\}$, $T_{k,6}=\{\{3,4,k\},\{3,5,k\},\{3,6,k\}\}$,
$T_{k,7}=\{\{3,7,k\},\{3,8,k\},\{3,9,k\}\}$, $T_{k,8}=\{\{4,5,k\},\{4,6,k\},\{4,7,k\}\}$,
$T_{k,9}=\{\{4,8,k\},\{4,9,k\},\{5,6,k\}\}$, $T_{k,10}=\{\{5,7,k\},\{5,8,k\},\{5,9,k\}\}$,
$T_{k,11}=\{\{6,7,k\},\{6,8,k\},\{6,9,k\}\}$ and $T_{k,12}=\{\{7,8,k\},\{7,9,k\},\{8,9,k\}\}$.
For $1 \leq w \leq 12$, define $s_w=max\{i_{wt}, j_{wt};\ 1 \leq t \leq 3\}$. Consider the set of stars as follows:

$$S = \begin{cases} (\{k+1, k+w+1, s_w+4\}; \{i_{w1}, j_{w1}, k\}, \{i_{w2}, j_{w2}, k\}, \{i_{w3}, j_{w3}, k\}) & \text{if } 1 \leq w \leq 7 \\ (\{k+2, k+3, s_w+1\}; \{i_{w1}, j_{w1}, k\}, \{i_{w2}, j_{w2}, k\}, \{i_{w3}, j_{w3}, k\}) & \text{if } w = 8 \\ (\{k+4, k+w-4, s_w+1\}; \{i_{w1}, j_{w1}, k\}, \{i_{w2}, j_{w2}, k\}, \{i_{w3}, j_{w3}, k\}) & \text{if } 9 \leq w \leq 12 \end{cases}$$

where addition in the first and second coordinates are taken modulo $n-n_1$ with residues $n_1+1, ..., n$ and third coordinate is taken modulo n_1 with residues $1, 2, ..., n_1$. The set of stars S of $\langle E(\bigcup_{i,j \in P_2(V_1)} \{i,j,10\}, N_4)\rangle$ in $KG_{18,3}$ is given in Figure 3.4. In $\langle E(\mathcal{A}_k, N_4)\rangle \setminus E(S)$, the degree of each vertex of $\mathcal{A}_k$ is $[\frac{(n_2-1)(n_2-2)(n_1-2)}{2}]-1$, which is congruent to 0 modulo 3. So, fix each vertex of $\mathcal{A}_k$ as a center vertex, we get a claw-decomposition in $\langle E(\mathcal{A}_k, N_4)\rangle \setminus E(S)$. By proceeding the same argument, we get a claw-decomposition in $\langle E(\bigcup_{i,j \in \mathcal{P}_2(V_1)}\{i,j,k\}, N_4)\rangle$, where $10 \leq k \leq n$. Hence, the graph G_6 is claw-decomposable. In $\langle E(N_1, N_3)\rangle$, the degree of each vertex of N_1 is $\frac{n_2}{2}[9^2-(7 \times 9)+12]=30\frac{n_2}{2}$. In $\langle E(N_1, N_4)\rangle$, the degree of each vertex of N_1 is $\frac{n_2(n_2-1)(9-3)}{2}=\frac{6n_2(n_2-1)}{2}$ and note that $n_2 \equiv 0(mod\ 9)$. In G_7, the degree of each vertex of N_1 is $\frac{n_2}{2}[30+6(n_2-1)]=3n_2(n_2+4)$. Now, fix each vertex of N_1 as a center vertex, we get a claw-decomposition in G_7. In $\langle E(N_2, N_3)\rangle$, the degree of each vertex of N_2 is $\frac{9(9-1)(n_2-3)}{2}=\frac{9}{2}[8(n_2-3)]$ and note that $n_2 \equiv 0(mod\ 9)$. In $\langle E(N_2, N_4)\rangle$, the degree of each vertex of N_2 is $\frac{9}{2}[n_2^2-7n_2+12]$. In G_8, the degree of each vertex of N_2 is $\frac{9}{2}[8(n_2-3)+(n_2^2-7n_2+12)]=\frac{9}{2}[n_2^2+n_2-12]$. Now, by fixing each vertex of N_2 as a center vertex, we get a claw-decomposition in G_8. By the principle of mathematical induction, we get the graph $KG_{n,3}$ is claw-decomposable. □

Theorem 3.2. *If $n \equiv 1, 2, 3, 4, 5(mod\ 9)$, then the graph $KG_{n,3}$ is claw-decomposable.*

Proof. We define the vertex set of $KG_{n,3}$ as follows: Let B=$\{1, 2, ..., n\}$, N_1=$\{\{i,j,k\}| i,j,k \in \mathcal{P}_3(B \setminus n)\}$ and N_2=$\{\{i,j,n\}|i,j \in \mathcal{P}_2(B \setminus n)\}$. Now, we write $KG_{n,3}$=$\langle N_1\rangle \cup \langle N_2\rangle \cup \langle E(N_1, N_2)\rangle$. Note that, the graph $\langle N_2\rangle$ is a null graph. If $n \equiv 1, 2, 4, 5(mod\ 9)$, the degree of each vertex of N_1 in $\langle E(N_1, N_2)\rangle$ is exactly $[\frac{(n-1)(n-2)}{2}]-3(n-3)$. Now, by fixing each vertex of N_1 as a center vertex, we get a claw-decomposition in $\langle E(N_1, N_2)\rangle$. If $n \equiv 3(mod\ 9)$, the degree of each vertex of N_2 in $\langle E(N_1, N_2)\rangle$ is exactly $\binom{n-1}{3}-(n-3)^2$. Now, by fixing each vertex of N_2 as a center vertex, we get a claw-decomposition in $\langle E(N_1, N_2)\rangle$. Note that, the graph $\langle N_1\rangle \cong KG_{n-1,3}$. If $n \equiv 1(mod\ 9)$, then the graph $KG_{n-1,3}$ is claw-decomposable by Lemma 3.1 and Theorem 3.1. If $n \equiv 2, 3, 4, 5(mod\ 9)$, then by repeatedly applying the above procedure, we get a claw-decomposition of $KG_{n,3}$. □

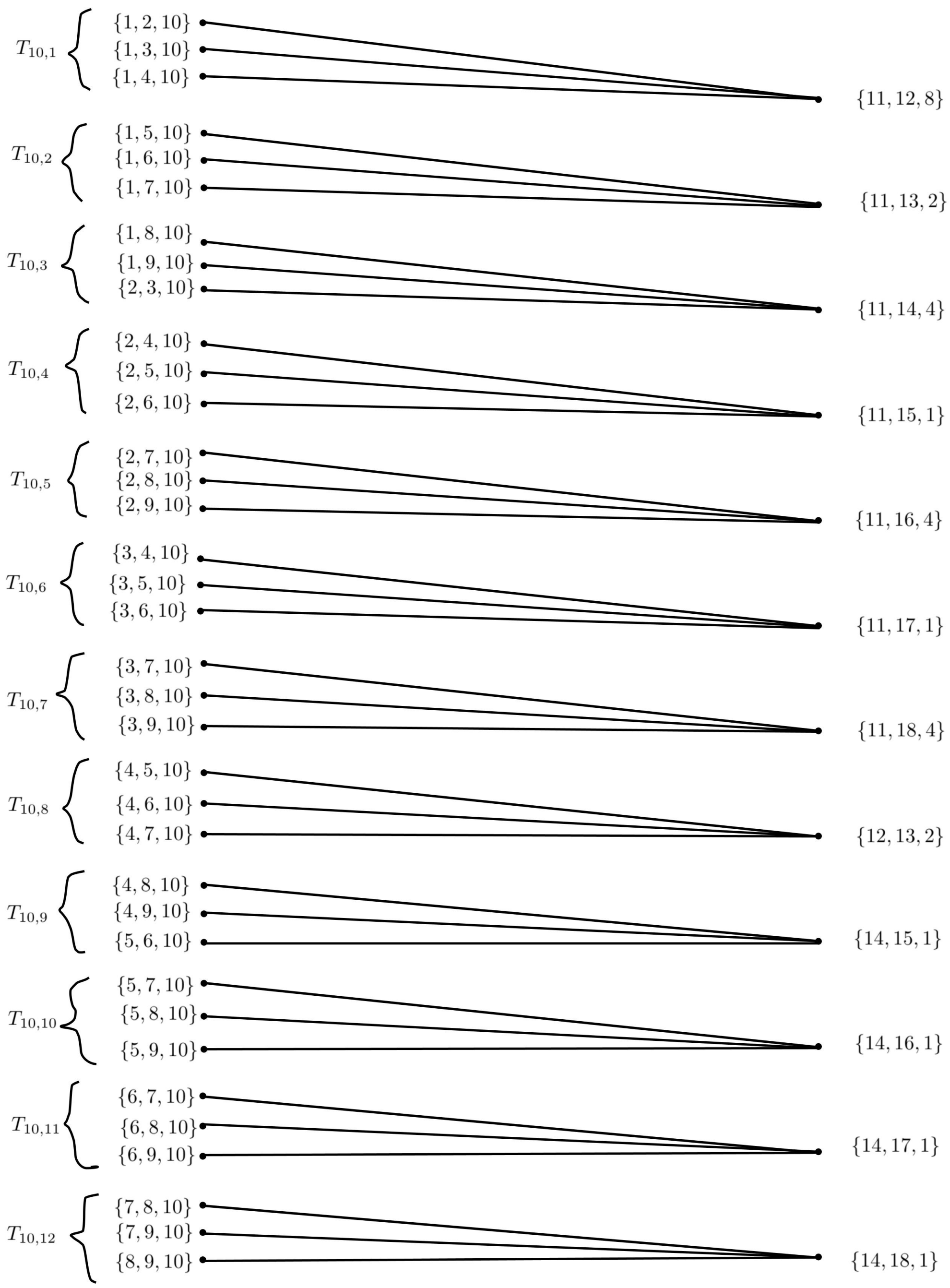

FIGURE 3.4. The set of stars in S when $n = 18$ and $k = 10$

By Combining Theorem 3.1 and 3.2, we get the following result:

Theorem 3.3. *$KG_{n,3}$ is claw-decomposable if and only if $n \geq 9$ and $n \equiv 0, 1, 2, 3, 4, 5 (mod\ 9)$.*

4. Conclusions

In this paper, we have proved that $KG_{n,3}$, the Kneser Graph is Claw-decomposable if and only if $n \geq 9$ and $n \equiv 0, 1, 2, 3, 4, 5 (mod\ 9)$.

Acknowledgment. The authors are grateful to the DST-FIST (SR/FST/College-222/ 2014) and DBT-STAR College scheme (HRD-11011/18/2022-HRD-DBT) for providing financial assistance for this research. The authors sincerely express their thanks to the management of A.V.V. M. Sri Pushpam College (Autonomous), Poondi, for providing the necessary facilities and support to carry out this work.

References

[1] Chen, Y., (2000), Kneser graphs are Hamiltonian for $n \geq 3k$, J. Combin. Theory Ser. B, 80, pp. 69-79.

[2] Ganesamurthy, S., Paulraja, P., (2018), Existence of a P_{2k+1}-decomposition in the Kneser graph $KG_{n,2}$, Discrete Math., 341, pp. 2113-2116.

[3] Kneser, M., (1955), Aufgabe 360. Jahresbericht der Deutschen Mathematiker-Vereinigung, 2, Abteilung, 58, pp. 27.

[4] Rodger, C. A., Whitt III, T. R., (2015), Path decompositions of Kneser and Generalized Kneser Graphs, Canad. Math. Bull., 58(3), pp. 610-619.

[5] Sankari, C., Sangeetha, R., and Arthi, K., (2022), Claw-decomposition of Kneser Graphs, Trans. Comb., 11(1), pp. 53-61 .

[6] Sankari, C., Sangeetha, R., and Arthi, K., (2022), S_5-decomposition of Kneser Graphs, South East Asian J. of Mathematics and Mathematical Sciences, Vol. 18(2), pp. 171-184.

[7] Shields, I., Savage, C.D., (2004), A note on Hamilton cycles in Kneser graphs, Bull. Inst. Combin. Appl., 40, pp. 13-22.

[8] Whitt, T. R., Rodger, C. A., (2015), Decomposition of the Kneser graph into paths of length four, Discrete Math., 338, pp. 1284-1288.

[9] Yamamoto, S., Ikeda, H., Shige-eda, S., Ushio, K., and, Hamada, N., (1975), On claw decomposition of complete graphs and complete bipartite graphs, Hiroshima Math. J., 5(1), pp. 33-42.

C. SANKARI is currently doing her doctoral research in the Department of Mathematics, A. V. V. M. Sri Pushpam College (Affiliated to Bharathidasan University), Poondi, Thanjavur(D.T), Tamil Nadu, India. She received her M.Sc degree from S.B.K. College (Affiliated to Madurai Kamaraj University), Aruppukottai. Her research interest is Graph decomposition problems..

R. SANGEETHA is currently working as an Assistant Professor in the Department of Mathematics, A. V. V. M. Sri Pushpam College (Affiliated to Bharathidasan University), Poondi, Thanjavur(D.T), Tamil Nadu, India. She received her M.Sc. degree from Bharathidasan University, Tiruchirapalli and her Ph.D. degree from Periyar University, Salem. Her research interest is Graph decomposition problems. She has completed a research project on "Decompositions of graphs into cycles" funded by University Grants Commission, New Delhi. She has published seven research articles in internationally reputed and refereed journals. She has delivered a number of talks and presented papers in national and international conferences.

K. ARTHI is currently doing her doctoral research in the Department of Mathematics, A. V. V. M. Sri Pushpam College (Affiliated to Bharathidasan University), Poondi, Thanjavur(D.T), Tamil Nadu, India. She received her M.Sc. degree from the same College. Her research interest is Graph decomposition problems