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EXISTENCE OF SOLUTION FOR IMPULSIVE FRACTIONAL q_r-DIFFERENCE EQUATION OF IMPLICIT FORM WITH NONLOCAL BOUNDARY CONDITION

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Abstract. This study examines the conditions needed for the existence of solutions to an impulsive fractional q_r-difference equation with the implicit form. The fractional derivative we analyze in the problem is of the Caputo type, which involves a q-shifting operator of the form ${}_a\phi_q(u) = qu + (1-q)a$. Here, nonlocal conditions are the boundary conditions we take into account. Regarding the existence of solutions for the given problem, the result is obtained by means of Krasnoselskii's fixed point theorem. In addition, circumstances required for the Ulam-Hyers and Generalized Ulam-Hyers stability of the impulsive problem are explored. Finally, we provide an example to demonstrate our findings.

Keywords: quantum calculus, implicit, impulsive fractional q_r-difference equation, nonlocal boundary condition, Ulam-Hyers stability.

AMS Subject Classification: 39A13, 26A33, 34A37, 34A09, 34B10

1. Introduction

Fractional differential calculus has garnered significant attention from scholars in recent times, primarily owing to its proven utility across several domains ([1–3]). The existence and uniqueness of solutions for fractional differential equations have been the subject of considerable research in various academic domains, see ([4–6]) and the references therein. The q-difference equation was initiated in the twentieth century ([7], [8]) and has got significant attention in the recent years. The origin of the fractional q-difference calculus can be found in the works by Al-Salam [9] and Agarwal [10]. Afterwards, plenty of study has already been done on the existence and uniqueness of solutions to fractional

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q-difference equations for initial value and boundary value problems ([11–17]).
Tariboon *et al.* [21] developed a concept in fractional q calculus in terms of a shifting operator ${}_a\phi_q(u) = qu + (1-q)a$ and the terms q-integral and q-derivative were redefined. In addition, the Riemann-Liouville q-integral as well as q-derivative were defined, and the existence of solutions to the impulsive fractional q differential equation of first and second order was discovered. Depending upon the q-shifting operator Ahmad *et al.* [22] proposed the expression for the Caputo fractional q-derivative and found the conditions for existence of the solution of the following impulsive problem with anti-periodic boundary condition:

$$\begin{cases} ({}^c_{t_r}D^{\sigma_r}_{q_r}x)(t) = F(t, x(t)) \;\; t \in J_r \subseteq [0, \mathcal{T}], \;\; t \neq t_r \\ x(t_r^+) = x(t_r) + R_r({}_{t_{r-1}}I^{\rho_{r-1}}_{q_{r-1}}x(t_r)) \;\; r = 1, 2, \ldots, n \\ {}_{t_r}D_{q_r}x(t_r^+) = {}_{t_{r-1}}D_{q_{r-1}}x(t_r) + R^*_r({}_{t_{r-1}}I^{\nu_{r-1}}_{q_{r-1}}x(t_r)) \;\; r = 1, 2, \ldots, n \\ x(0) = -x(\mathcal{T}), \quad ({}_0D_{q_0}x)(0) = ({}_{t_n}D_{q_n}x)(\mathcal{T}), \end{cases}$$

where $0 = t_0 < t_1 < t_2 < \ldots < t_n < t_{n+1} = \mathcal{T}$, $1 < \sigma_r \leq 2$, $0 < q_r < 1$, $0 < \rho_r, \nu_r < 1$ and $J_0 = [0, t_1]$, $J_r = (t_r, t_{r+1}]$ for r = 1,2,. . .,n.
Ahmad *et al.* [23] obtained the existence result of fractional q-difference equation with Riemann- Liouville fractional derivative involving nonlocal boundary condition of the following problem:

$$\begin{cases} ({}_{t_r}D^{\sigma_r}_{q_r}x)(t) = F(t, x(t)) \;\; t \in J_r \subset [0, \mathcal{T}], t \neq t_r \\ ({}_{t_r}I^{\sigma_r}_{q_r}x)(t_r^+) = x(t_r) + R_r(x(t_r)) \\ a_{t_0}(I^{1-\alpha_0}_{q_0}x)(0) = bx(\mathcal{T}) + \sum_{l=0}^{n} c_l({}_{t_l}I^{\gamma_l}_{q_l}x(t_{l+1})), \end{cases}$$

where $0 = t_0 < t_1 < t_2 < \ldots < t_n < t_{n+1} = \mathcal{T}$, $0 < \sigma_r, \gamma_r \leq 1$, $0 < q_r < 1$ and $J_0 = [0, t_1]$, $J_r = (t_r, t_{r+1}]$ for r = 1,2,. . .,n. Also, existence result for nonlinear implusive q-difference equation can be find on Yu *et al.* [24], Jang *et al.* [25], Agarwal *et al.* [26] etc.
In recent years, the Ulam-Hyers stability anaysis of nonlinear q-fractional differential equation gets a lot of attention from the researchers. Ulam-Hyers stability can be defined as an exact solution near the approximate solution of the differential equation with minimal error. For the recent work on Ulam-Hyers stability on fractional q-difference equation one can see ([27–29]) and reference therein. Also, several research works in the field of implicit fractional difference equations are currently being conducted ([19, 20]). Abbas *et al.* [30] investigate conditions for existence, uniqueness and Ulam-Hyers-Rassias stability of solution of the initial value problem

$$\begin{aligned} ({}^cD^{\sigma}_qx)(t) &= F(t, x(t), ({}^cD^{\sigma}_qx)(t)), \; t \in [0, T] \\ x(0) &= x_0. \end{aligned}$$

Motivated by the above results, we work on the following problem

$$\begin{cases} ({}^c_{t_r}D^{\sigma_r}_{q_r}x)(t) = F(t, x(t), ({}^c_{t_r}D^{\sigma_r}_{q_r}x)(t)), \;\; t \in J_r \subset J = [0, \mathcal{T}], t \neq t_r \\ x(t_r^+) = x(t_r) + \rho_r(x(t_r)), \;\; r = 1, 2, \ldots, n. \\ ax(0) + bx(\mathcal{T}) = \sum_{i=0}^{n} c_ix(\eta_i), \;\; \eta_0 \in [t_0, t_1], \eta_r \in (t_r, t_{r+1}] \;\; r = 1, 2, \ldots, n, \end{cases} \tag{1}$$

where $0 = t_0 < t_1 < t_2 < \ldots < t_n < t_{n+1} = \mathcal{T}$, $J_0 = [0, t_1]$ and $J_r = (t_r, t_{r+1}]$ for all r=1,2,. . .,n.
${}^c_{t_r}D^{\sigma_r}_{q_r}$ is the Caputo q_r-fractional derivative with α_r order, $a, b, c_0, c_1, \ldots, c_m$ are real constants and $n \in \mathbb{N}$, $0 < \sigma_r \leq 1$ and $0 < q_r < 1$ for $r = 0, 1, 2, \ldots, n$, $F : J \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ and $\rho_r : \mathbb{R} \to \mathbb{R}$ $r = 1, 2, \ldots, n$.
We try to find the conditions for existence of solution of the problem and also try to find conditions for Ulam-Hyers and Generalized Ulam-Hyers stability of the problem.

The research paper is structured as follows: In section 2, we covered some fundamental definitions, characteristics, and lemmas. The criteria for the existence of the solution of the problem (1) are covered in the next section. In section 4, conditions for Ulam-Hyers and Generalized Ulam-Hyers stability are derived. In section 5, a example is provided to verify the outcomes.

2. Preliminaries

We begin this section by reviewing some fundamental q-calculus properties [21].
The q-shifting operator define as ${}_a\phi_q(u) = qu + (1-q)a$, satisfy the following relations

(1) ${}_a\phi_q^0(u) = u,\ {}_a\phi_q^{(n)}(u) ={}_a \phi_q^{n-1}({}_a\phi_q(u)) \quad$ for $n \in \mathbb{N}$;
(2) ${}_a(u-v)_q^{(0)} = 1,\ {}_a(u-v)_q^{(p)} = \prod_{i=0}^{p-1}(u -{}_a \phi_q^i(v)), \quad p \in \mathbb{N}\cup\{\infty\}$;
(3) ${}_a(u-v)_q^{(\alpha)} = u^{(\alpha)} \prod_{i=0}^{\infty} \frac{1-{}_{a/u}\phi_q^i(v/u)}{1-{}_{a/u}\phi_q^{i+\alpha}(v/u)}, \quad \alpha \in \mathbb{R},\ n \neq 0.$

In the interval [a,b], the q_r-derivative of a function f is defined as

$$({}_aD_{q_r}f)(t) = \frac{f(t) - f({}_a\phi_{q_r}(t))}{(1-q_r)(t-a)},\ t \neq a \ and\ ({}_aD_{q_r}f)(a) = lim_{t\to a}({}_aD_{q_r}f)(t).$$

The higher order q_r-derivative is defined as

$$({}_aD^0_{q_r}f)(t) = f(t) \quad and \quad ({}_aD^n_{q_r}f)(t) = {}_aD_{q_r}{}^{n-1}({}_aD_{q_r}f)(t), \quad n \in \mathbb{N}.$$

For two functions, f and g, on the interval [a,b], the q_r-derivative of their product and division is

$$\begin{aligned} {}_aD_{q_r}(fg)(t) &= f(t){}_aD_{q_r}g(t) + g({}_a\phi_{q_r}(t)){}_aD_{q_r}f(t) \\ &= g(t){}_aD_{q_r}f(t) + f({}_a\phi_{q_r}(t)){}_aD_{q_r}g(t) \end{aligned}$$

and

$${}_aD_{q_r}(\frac{f}{g})(t) = \frac{g(t){}_aD_{q_r}f(t) + f(t){}_aD_{q_r}g(t)}{g(t)g({}_a\phi_{q_r}(t))}\ ,$$

where $g(t)g({}_a\phi_{q_r}(t)) \neq 0$.
A function f on the interval $[a,b]$ has a q_r-integral that is defined as

$$({}_aI_{q_r}f)(t) = \int_a^t f(s){}_ad_{q_r}s = (t-a)(1-q_r)\sum_{n=0}^{\infty} f({}_a\phi^i_{q_r}(t))q_r^n, \quad t \in [a,b].$$

The higher order q_r-integration is defined as

$$({}_aI^0_{q_r}f)(t) = f(t) \quad and \quad ({}_aI^n_{q_r}f)(t) = {}_aI^{n-1}_{q_r}({}_aI_{q_r}f(t)) \quad for \quad n \in \mathbb{N}.$$

For the interval [a,b], the q_r-integration by parts formula is,

$$\int_a^b f(s){}_aD_{q_r}g(s){}_ad_{q_r}s = (fg)(t)|_a^b - \int_a^b g({}_a\phi_{q_r}(s)){}_aD_{q_r}f(s){}_ad_{q_r}s.$$

These operators, ${}_aI_{q_r}$ and ${}_aD_{q_r}$, are covered by the calculus fundamental theorem, which is,

$$({}_aD_{q_r a}I_{q_r}f)(t) = f(t),$$

furthermore, if f is continuous at $x = a$, we get

$$({}_aI_{q_r a}D_{q_r}f)(t) = f(t) - f(a).$$

Now we recall the definition of Riemann-Liouville q_r-integral, q_r-derivative and Caputo fractional q_r-derivative and the properties satisfied by these operators [22].

Definition 2.1. *Given a function f defined on $[a,b]$ and $\sigma \geq 0$. The q_r Riemann Liouville ingtegral of fractional order σ is defined as*

$$({}_aI^{\sigma}_{q_r}f)(t) = \int_a^t \frac{{}_a(t-{}_a\phi_{q_r}(s))^{(\sigma-1)}_{q_r}}{\Gamma_{q_r}(\sigma)} f(s){}_ad_{q_r}s \text{ and } ({}_aI^0_{q_r}f)(t) = f(t) \;\; t \in [a,b].$$

Definition 2.2. *Consider a function f defined on $[a,b]$ with $\sigma \geq 0$. The definition of the order σ of Riemann Liouville fractional q_r-derivative is*

$$({}_aD^{\sigma}_{q_r}f)(t) = ({}_aD^{[\sigma]}_{q_r}{}_aI^{[\sigma]-\sigma}_{q_r}f)(t) \;\; \text{and} \;\; ({}_aD^0_{q_r}f)(t) = f(t) \;\; t \in [a,b]$$

provided that the lowest integer larger than or equal to σ is $[\sigma]$.

Definition 2.3. *Consider a function f defined on $[a,b]$ with $\sigma \geq 0$. The definition of the order σ of Caputo fractional q_r-derivative is*

$$({}^c_aD^{\sigma}_{q_r}f)(t) = ({}_aI^{[\sigma]-\sigma}_{q_r}({}_aD^{[\sigma]}_{q_r}f))(t)$$

provided that the lowest integer larger than or equal to σ is $[\sigma]$.

Lemma 2.1. *Given a function f defined on [a,b], let $\sigma, \nu \geq 0$. Then*
1. ${}_aI^{\nu}_{q_r}({}_aI^{\sigma}_{q_r}f)(t) = ({}_aI^{\sigma+\nu}_{q_r}f)(t)$;
2. ${}_aD^{\sigma}_{q_r}({}_aI^{\sigma}_{q_r}f)(t) = f(t)$.

Lemma 2.2. *Assume that $n \in \mathbb{N}$ and $\sigma > 0$. Then, the equality stated below is true:*

$${}_aI^{\sigma}_{q_r}({}_aD^n_{q_r}f)(t) = {}_aD^n_{q_r}({}_aI^{\sigma}_{q_r}f)(t) - \sum_{k=0}^{n-1} \frac{(t-a)^{\sigma-n+k}}{\Gamma_{q_r}(\sigma+k-n+1)}({}_aD^k_{q_r}f)(a).$$

Lemma 2.3. *Let $\sigma \in \mathbb{R}^+ \setminus \mathbb{N}$ and $n \in \mathbb{N}$. Then, the equality stated below is true:*

$${}_aI^{\sigma}_{q_r}({}^c_aD^{\sigma}_{q_r}f)(t) = f(t) - \sum_{k=0}^{[\sigma]-1} \frac{(t-a)^k}{\Gamma_{q_r}(k+1)}({}_aD^k_{q_r}f)(a).$$

The operator ${}_aI^{\sigma}_{q_r}$ and ${}_aD^{\sigma}_{q_r}$, satisfy the following two equation:

$${}_aD^{\sigma}_{q_r}(t-a)^{\nu} = \frac{\Gamma_{q_r}(\nu+1)}{\Gamma_{q_r}(\nu-\sigma+1)}(t-a)^{\nu-\sigma}.$$

$${}_aI^{\sigma}_{q_r}(t-a)^{\nu} = \frac{\Gamma_{q_r}(\nu+1)}{\Gamma_{q_r}(\nu+\sigma+1)}(t-a)^{\nu+\sigma}.$$

We define PC(J,$\mathbb{R}$)=$\{x : J \to \mathbb{R} \mid$ x is a continuous map everywhere, with the exception of a certain t_r, where $x(t_r^+)$, $x(t_r^-)$ exist and $x(t_r^-) = x(t_r)$ for each r=1,2,. . .,n.$\}$. Define $\|x\|_\infty = \sup_{t\in J}|x(t)|$, then (PC(J,$\mathbb{R}$),$\|.\|_\infty$) is a Banach space.

Theorem 2.1 (Krasnoselskii's fixed point theorem)**.** *Let M be a subset of a Banach space X that is closed, bounded, convex, and nonempty. Assume that P and Q are two operators. Then,*
(a) $Pt+Qs \in M$ whenever $t, s \in M$
(b) P is compact and continuous map
(c) Q is a contraction mapping
then there exist $u \in M$ such that $u = Pu + Qu$.

Let for $y \in$ PC(J,$\mathbb{R}$) and $\epsilon > 0$, consider the following inequalities,

$$\begin{cases} |({}^c_{t_r}D^{\sigma_r}_{q_r}y)(t) - f(t, y(t), ({}^c_{t_r}D^{\sigma_r}_{q_r}y)(t))| \leq \epsilon \\ |y(t_r^+) - y(t_r) - \rho_r(y(t_r))| \leq \epsilon. \end{cases} \tag{2}$$

where $t \in [t_0, t_1]$ or $t \in (t_r, t_{r+1}]$ for $r = 1, 2, \ldots, n$.

Definition 2.4. *If there is a real number $C > 0$ such that for every $\epsilon > 0$ and every solution $y \in PC(J,\mathbb{R})$ of inequality* (2)*, there exists a unique solution $x \in PC(J,\mathbb{R})$ of the problem* (1) *such that, for $t \in J$ it satisfy*

$$|y(t) - x(t)| \leq C\epsilon.$$

then the problem (1) *is said to be Ulam-Hyers stable.*

Definition 2.5. *If there is a real function $c \in C(\mathbb{R}^+,\mathbb{R}^+)$ with $c(0)=0$ such that for every $\epsilon > 0$ and every solution $y \in PC(J,\mathbb{R})$ of inequality* (2)*, there exists a unique solution $x \in PC(J,\mathbb{R})$ of the problem* (1) *such that, for $t \in J$ it satisfy*

$$|y(t) - x(t)| \leq c(\epsilon).$$

then the problem (1) is said to be Generalized Ulam-Hyers stable.

Remark: By definition it is obvious that definition (2.4) implies definition (2.5).

3. Existence of solutions

The integral form of the equation (1) and the conditions needed for the existence of the problem's solution will be determined in this section.

Theorem 3.1. *Existence of solution for impulsive fractional q_r-difference equation of implicit form with nonlocal boundary condition If $F \in AC$ $(J \times \mathbb{R} \times \mathbb{R}, \mathbb{R})$, then a solution of* (1) *is $x \in PC$ $(J,\mathbb{R})$ if and only if for $t \in [t_0, t_1]$ or $t \in (t_r, t_{r+1}]$, r= ,1,2,...,n ,*

$$x(t) = \sum_{j=1}^{n} d_j ({}_{t_{j-1}}I_{q_{j-1}}^{\sigma_{j-1}} G)(t_j) + \sum_{j=1}^{n} d_j \rho_j(x(t_j)) + \sum_{j=0}^{n} \frac{c_j}{A} ({}_{t_j}I_{q_j}^{\sigma_j} G)(\eta_j)$$
$$- \frac{b}{A} ({}_{t_n}I_{q_n}^{\sigma_n} G)(\mathcal{T}) + \sum_{j=0}^{r-1} ({}_{t_j}I_{q_j}^{\sigma_j} G)(t_{j+1}) + \sum_{j=1}^{r} \rho_j(x(t_j)) + ({}_{t_r}I_{q_r}^{\sigma_r} G)(t) \quad (3)$$

where $G(t) = F(t, x(t), G(t))$, A = a+b-$\sum_{j=0}^{n} c_j$ and $d_j = \frac{\sum_{i=j}^{n} c_i - b}{A}$ for all j = 0,1,2, ..., n.

Proof. In the interval $[t_0, t_1]$, let $x(t)$ satisfy the equation (1) then, consider $G(t) = ({}_{t_0}^{c}D_{\sigma_0}^{q_0} x)(t)$ integrating both side of the equation (1), we get

$$x(t) = x(0) + ({}_{t_0}I_{q_0}^{\sigma_0} G)(t)$$

and $G(t) = F(t, x(t), G(t))$.
At $t = t_1$, we get

$$x(t_1) = x(0) + ({}_{t_0}I_{q_0}^{\sigma_0} G)(t_1).$$

Using the equation (1), we get

$$x(t_1^+) = x(0) + ({}_{t_0}I_{q_0}^{\sigma_0} G)(t_1) + \rho_1(x(t_1)).$$

Similarly, in the next interval $(t_1, t_2]$, we have,

$$x(t) = x(t_1^+) + ({}_{t_1}I_{q_1}^{\sigma_1} G)(t).$$

Putting the value of $x(t_1^+)$ in the above equation we get,

$$x(t) = x(0) + ({}_{t_0}I_{q_0}^{\sigma_0} G)(t_1) + \rho_1(x(t_1)) + ({}_{t_1}I_{q_1}^{\sigma_1} G)(t).$$

then we have,

$$x(t_2^+) = x(0) + \sum_{i=0}^{1} ({}_{t_i}I_{q_i}^{\sigma_i} G)(t_{i+1}) + \sum_{i=1}^{2} \rho_i(x(t_i)).$$

So, for $t \in (t_2, t_3]$, we have

$$x(t) = x(0) + \sum_{i=0}^{1} ({}_{t_i}I_{q_i}^{\sigma_i} G)(t_{i+1}) + \sum_{i=1}^{2} \rho_i(x(t_i)) + ({}_{t_2}I_{q_2}^{\sigma_2} G)(t).$$

For $t \in (t_r, t_{r+1}]$, the integral from will be,

$$x(t) = x(0) + \sum_{i=0}^{r-1} ({}_{t_i}I_{q_i}^{\sigma_i} G)(t_{i+1}) + \sum_{i=1}^{r} \rho_i(x(t_i)) + ({}_{t_r}I_{q_r}^{\sigma_r} G)(t) \tag{4}$$

where $G(t) = F(t, x(t), G(t))$. So, for $t = \mathcal{T}$,

$$x(\mathcal{T}) = x(0) + \sum_{i=0}^{n-1} ({}_{t_i}I_{q_i}^{\sigma_i} G)(t_{i+1}) + \sum_{i=1}^{n} \rho_i(x(t_i)) + ({}_{t_n}I_{q_n}^{\sigma_n} G)(\mathcal{T}).$$

Putting the value of $x(\eta_0)$, $x(\eta_1)$, ..., $x(\eta_n)$ in the equation (1), we have,

$$\begin{aligned}
&ax(0) + bx(0) + b\sum_{i=0}^{n-1} ({}_{t_i}I_{q_i}^{\sigma_i} G)(t_{i+1}) + b\sum_{i=1}^{n} \rho_i(x(t_i)) + b({}_{t_n}I_{q_n}^{\sigma_n} G)(\mathcal{T}) \\
&= (\sum_{i=0}^{n} c_i)x(0) + \sum_{j=1}^{n}(\sum_{i=j}^{n} c_i)({}_{t_{j-1}}I_{q_{j-1}}^{\sigma_{j-1}} G)(t_j) + \sum_{j=1}^{n}(\sum_{i=j}^{n} c_i)\rho_j(x(t_j)) \\
&\quad + \sum_{j=0}^{n} c_j({}_{t_j}I_{q_j}^{\sigma_j} G)(\eta_j) \\
\Rightarrow x(0) &= \sum_{j=1}^{n} d_j({}_{t_{j-1}}I_{q_{j-1}}^{\sigma_{j-1}} G)(t_j) + \sum_{j=1}^{n} d_j\rho_j(x(t_j)) + \sum_{j=0}^{n} \frac{c_j}{A}({}_{t_j}I_{q_j}^{\sigma_j} G)(\eta_j) \\
&\quad - \frac{b}{A}({}_{t_n}I_{q_n}^{\sigma_n} G)(\mathcal{T}).
\end{aligned}$$

Putting the value of $x(0)$ in the equation (4), we get the integral form (3).
The converse part of the theorem is followed by direct computation. □

We consider the following assumptions:
(A1) F is a continuous function.
(A2) There exist $m, p, d \in$ C(J,$\mathbb{R}^+$) such that,

$$|F(t, x, y)| \leq d(t) + p(t)|x| + m(t)|y|$$

and $p^* = max_{t\in J}|p(t)|$, $d^* = max_{t\in J}|d(t)|$, $m^* = max_{t\in J}|m(t)|$.
(A3) There exist a constant $M' > 0$ such that for $t, s \in J$,

$$|\rho_r(t) - \rho_r(s)| \leq M'|t - s|$$

and $\rho_r(0) = 0$ for r = 1,2,3,. . .,n.
(A4)

$$(\sum_{i=0}^{n} |d_i| + n)M' < 1.$$

(A5) Let

$$\Lambda = \sum_{j=1}^{n} |d_j| \frac{(t_j - t_{j-1})^{\sigma_{j-1}}}{\Gamma_{q_j}(\sigma_{j-1}+1)} + \sum_{j=0}^{n} \frac{|c_j|}{|A|} \frac{(t_{j+1}-t_j)^{\sigma_j}}{\Gamma_{q_j}(\sigma_j+1)} + \frac{|b|}{|A|} \frac{(\mathcal{T}-t_n)^{\sigma_n}}{\Gamma_{q_n}(\sigma_n+1)}$$
$$+ \sum_{j=0}^{n-1} \frac{(t_{j+1}-t_j)^{\sigma_j}}{\Gamma_{q_j}(\sigma_j+1)} + \lambda.$$

where $\lambda = \max \{\frac{(t_{r+1}-t_r)^{\sigma_r}}{\Gamma_{q_r}(\sigma_r+1)}\}$ for all r = 0,1,2,..., n. Then,

$$p^*\Lambda < \{1 - (\sum_{i=1}^{n} |d_i| + n)M'\}(1 - m^*).$$

(A6)There exist constant $N, Q > 0$ such that,

$$|F(t, x_1, y_1) - F(t, x_2, y_2)| \leq N|x_1 - x_2| + Q|y_1 - y_2|.$$

(A7)

$$\Lambda \frac{N}{1-Q} + (\sum_{i=1}^{n} |d_i| + n)M' < 1.$$

Theorem 3.2. *If the equation* (1) *satisfies the assumptions (A1)-(A6), then the interval* $[0, \mathcal{T}]$ *contains the solution to the equation* (1).

Proof. Let the operator $\mathcal{J}$ defined from PC(J,ℝ) to PC(J,ℝ) such that,

$$(\mathcal{J}x)(t) = (Sx)(t) + (Tx)(t)$$

where S, T are two operator from from PC(J,ℝ) to PC(J,ℝ) which are defined as

$$(Sx)(t) = \sum_{j=1}^{n} d_j \rho_j(x(t_j)) + \sum_{j=1}^{r} \rho_j(x(t_j))$$
$$(Tx)(t) = \sum_{j=1}^{n} d_j ({}_{t_{j-1}}I_{q_{j-1}}^{\sigma_{j-1}} G)(t_j) + \sum_{j=0}^{n} \frac{c_j}{A} ({}_{t_j}I_{q_j}^{\sigma_j} G)(\eta_j) - \frac{b}{A} ({}_{t_n}I_{q_n}^{\sigma_n} G)(\mathcal{T})$$
$$+ \sum_{j=0}^{r-1} ({}_{t_j}I_{q_j}^{\sigma_j} G)(t_{j+1}) + ({}_{t_r}I_{q_r}^{\sigma_r} G)(t)$$

where $G(t) = F(t, x(t), G(t))$ for $t \in [t_0, t_1]$ or $t \in (t_r, t_{r+1}]$ for all r = 1,2,...,n.
Step 1: S is a contraction map.
Let $x, y \in$ PC(J,ℝ), $t \in [t_0, t_1]$ or $t \in (t_r, t_{r+1}]$ for r=1,2,...,n. Then we get,

$$|(Sx)(t) - (Sy)(t)| \leq \sum_{j=1}^{n} |d_j||\rho_j(x(t_j)) - \rho_j(y(t_j))| + \sum_{j=1}^{r} |\rho_j(x(t_j)) - \rho_j(y(t_j))|$$
$$\leq (\sum_{j=1}^{n} |d_j| + n)M'\|x - y\|_\infty.$$

Using the assumption (A4) we get, S is a contraction map.
Step 2: T is a compact map.
(i) T map bounded set to bounded set.

Consider a bounded set B_R which defined as $B_R = \{x \in PC(J,\mathbb{R}) : \|x\|_\infty \leq R\}$. Let, $u \in B_R$ we have,

$$
\begin{aligned}
|Tu(t)| \leq & \sum_{j=1}^{n} |d_j| ({}_{t_{j-1}}I_{q_{j-1}}^{\sigma_{j-1}} |G(t_j)|) + \sum_{j=0}^{n} \frac{|c_j|}{|A|} ({}_{t_j}I_{q_j}^{\sigma_j} |G(\eta_j)|) + \frac{|b|}{|A|} ({}_{t_n}I_{q_n}^{\sigma_n} |G(\mathcal{T})|) \\
& + \sum_{j=0}^{r-1} ({}_{t_j}I_{q_j}^{\sigma_j} |G(t_j)|) + ({}_{t_r}I_{q_r}^{\sigma_r} |G(t)|).
\end{aligned}
$$

By using the assumption (A2) we have,

$$
\begin{aligned}
|F(t, x(t), G(t))| &\leq d(t) + p(t)\|x\|_\infty + m(t)\|G(t)\|_\infty \\
\|G(t)\|_\infty &\leq \frac{p^*\|x\|_\infty + d^*}{1 - m^*}.
\end{aligned}
$$

Using the above inequality we get,

$$
\begin{aligned}
|(Tx)(t)| \leq & \sum_{j=1}^{n} |d_j| (\frac{p^*\|x\|_\infty + d^*}{1 - m^*}) \frac{(t_j - t_{j-1})^{\sigma_{j-1}}}{\Gamma_{q_{j-1}}(\sigma_{j-1} + 1)} \\
& + \sum_{j=0}^{n} \frac{|c_j|}{|A|} (\frac{p^*\|x\|_\infty + d^*}{1 - m^*}) \frac{(t_{j+1} - t_j)^{\sigma_j}}{\Gamma_{q_j}(\sigma_j + 1)} \\
& + \frac{|b|}{|A|} (\frac{p^*\|x\|_\infty + d^*}{1 - m^*}) \frac{(\mathcal{T} - t_n)^{\sigma_n}}{\Gamma_{q_n}(\sigma_n + 1)} \\
& + \sum_{j=0}^{r-1} (\frac{p^*\|x\|_\infty + d^*}{1 - m^*}) \frac{(t_{j+1} - t_j)^{\sigma_j}}{\Gamma_{q_j}(\sigma_j + 1)} \\
& + \lambda (\frac{p^*\|x\|_\infty + d^*}{1 - m^*})
\end{aligned}
$$

which implies that, $|(Tx)(t)| \leq \Lambda(\frac{p^*R + d^*}{1 - m^*})$, So, T is a bounded map.
(ii) T is equicontinuous map.
Let $x \in$ PC(J,$\mathbb{R}$) and $\tau_1, \tau_2 \in [t_0, t_1]$ or $(t_r, t_{r+1}]$ for r =1,2,. . .,n. Also $\tau_1 \leq \tau_2$, then

$$
\begin{aligned}
|(Tx)(\tau_2) - (Tx)(\tau_1)| \leq & \frac{1}{\Gamma_{q_r}(\sigma_r)} (\int_{\tau_1}^{\tau_2} \{{}_{t_r}(\tau_2 - {}_{t_r}\phi_{q_r}(s))_{q_r}^{(\sigma_r - 1)} |G(s)|\}_{t_r} d_{q_r} s \\
& + \int_{t_r}^{\tau_1} \{{}_{t_r}(\tau_2 - {}_{t_r}\phi_{q_r}(s))_{q_r}^{(\sigma_r - 1)} - {}_{t_r}(\tau_1 - {}_{t_r}\phi_{q_r}(s))_{q_r}^{(\sigma_r - 1)}\} |G(s)|_{t_r} d_{q_r} s) \\
& \leq \frac{p^*\|x\|_\infty + d^*}{1 - m^*} (\frac{(\tau_2 - t_r)^{(\sigma_r)} - (\tau_1 - t_r)^{(\sigma_r)}}{\Gamma_{q_r}(\sigma_r)})
\end{aligned}
$$

as $\tau_1 \to \tau_2$ then, we have $(Tx)(\tau_1) \to (Tx)(\tau_2)$. So, T is a equicontinuous map.
Using Arzela-Ascoli's Theorem, we can determine that T is a compact map since it is bounded and equicontinuous.
Step 3: T is a continuous map.

Let $x_m, x \in$ PC(J,$\mathbb{R}$) for $m \in \mathbb{N}$ and $x_m \to x$ in PC(J,$\mathbb{R}$) as $m \to \infty$. So, we have,

$$\begin{aligned}\|(Tx_m)(t) - (Tx)(t)\| \leq \sum_{j=1}^{n} |d_j|({}_{t_{j-1}}I_{q_{j-1}}^{\sigma_{j-1}}|G_m - G|)(t_j) + \sum_{j=0}^{n} \frac{|c_j|}{|A|}({}_{t_j}I_{q_j}^{\sigma_j}|G_m - G|)(\eta_j) \\ + \frac{|b|}{|A|}({}_{t_n}I_{q_n}^{\sigma_n}|G_m - G|)(\mathcal{T}) + \sum_{j=0}^{r-1}({}_{t_j}I_{q_j}^{\sigma_j}|G_m - G|)(t_{j+1}) \\ + ({}_{t_r}I_{q_r}^{\sigma_r}|G_m - G|)(t)\end{aligned}$$

where $G_m(t) = F(t, x_m(t), G_m(t))$, $G(t) = F(t, x(t), G(t))$ and $G_m, G \in$ C(J,$\mathbb{R}$).

$$\begin{aligned}|G_m(t) - G(t)| &= |F(t, x_m(t), G_m(t)) - F(t, x(t), G(t))| \\ &\leq N|x_m(t) - x(t)| + Q|G_m(t) - G(t)|.\end{aligned}$$

So, we get,

$$\|G_m - G\|_\infty \leq \frac{N}{1-Q}\|x_m - x\|_\infty.$$

Using the above inequality we have,

$$\|Tx_m - Tx\|_\infty \leq \frac{N\Lambda}{1-Q}\|x_m - x\|_\infty.$$

So, as $x_m \to x$ we have $Tx_m \to Tx$.
So, T is a continuous map.
Step 4: If $x, y \in B_R$ then $Sx + Ty \in B_R$ where R such that,

$$\frac{d^*\Lambda}{(1-m^*)(1-\sum_{j=1}^{n}|d_j|M' - nM') - p^*\Lambda} \leq R.$$

We have,

$$\begin{aligned}|(Sx+Ty)(t)| &\leq \sum_{j=1}^{n}|d_j|M'|x(t_j)| + \sum_{j=1}^{r}M'|x(t_j)| + \Lambda(\frac{p^*\|y\|_\infty + d^*}{1-m^*}) \\ &\leq R(\sum_{j=1}^{n}|d_j| + n)M' + \Lambda(\frac{p^*R + d^*}{1-m^*}) \\ &\leq R.\end{aligned}$$

So, $Sx + Ty \in B_R$.
So, by Krasnoselskii's fixed point theorem, there exist a solution x $\in$ PC(J,$\mathbb{R}$) of the equation (1). □

4. Ulam-Hyers Stability

Suitable conditions for Ulam-Hyers as well as Generalized Ulam-Hyers stability of the equation (1) is obtained in this section which is depicted through the following theorem.

Theorem 4.1. *If the problem* (1) *satisfies assumptions (A3), (A6), and (A7), then the problem is Ulam-Hyers stable, and as a result, it is also Generalized Ulam-Hyers stable.*

Proof. Consider x be a solution of the equation (1).
Let $\epsilon > 0$ and y be a solution of the following inequality

$$\begin{cases}|({}^{c}_{t_r}D_{q_r}^{\sigma_r}y)(t) - F(t, y(t), ({}^{c}_{t_r}D_{q_r}^{\sigma_r}y)(t))| < \epsilon, \;\; t \in J_r \subset J = [0, \mathcal{T}], t \neq t_r \\ |y(t_r^+) - y(t_r) - \rho_r(y(t_r))| < \epsilon \;\; r = 1, 2, \ldots, n.\end{cases}$$

Then, there exist $\theta : [0,\mathcal{T}] \to \mathbb{R}^+, \theta_r \in \mathbb{R}^+$ such that $|\theta(t)| < \epsilon$ and $|\theta_r| < \epsilon$ and it satisfy

$$\begin{cases} ({}^c_{t_r}D^{\sigma_r}_{q_r} y)(t) = F(t, y(t), ({}^c_{t_r}D^{\sigma_r}_{q_r} y)(t)) + \theta(t), \quad t \in J_r \subset J = [0,\mathcal{T}], t \neq t_r \\ y(t_r^+) = y(t_r) + \rho_r(y(t_r)) + \theta_r \quad r = 1, 2, \dots, n. \end{cases}$$

Then using theorem 3.1, the solution of the above problem is given by,

$$\begin{aligned} y(t) = &\sum_{j=1}^{n} d_j ({}_{t_{j-1}}I^{\sigma_{j-1}}_{q_{j-1}} H)(t) + \sum_{j=1}^{n} d_j \rho_j(y(t_j)) + \sum_{j=0}^{n} \frac{c_j}{A} ({}_{t_j}I^{\sigma_j}_{q_j} H)(\eta_j) \\ &- \frac{b}{A} ({}_{t_n}I^{\sigma_n}_{q_n} H)(\mathcal{T}) + \sum_{j=0}^{r-1} ({}_{t_j}I^{\sigma_j}_{q_j} H)(t_{j+1}) + \sum_{j=1}^{r} \rho_j(y(t_j)) + ({}_{t_r}I^{\sigma_r}_{q_r} H)(t) \\ &+ \sum_{j=1}^{n} d_j ({}_{t_{j-1}}I^{\sigma_{j-1}}_{q_{j-1}} \theta)(t_j) + \sum_{j=1}^{n} \theta_j d_j + \sum_{j=0}^{n} \frac{c_j}{A} ({}_{t_j}I^{\sigma_j}_{q_j} \theta)(\eta_j) \\ &- \frac{b}{A} ({}_{t_n}I^{\sigma_n}_{q_n} \theta)(\mathcal{T}) + \sum_{j=0}^{r-1} ({}_{t_j}I^{\sigma_j}_{q_j} \theta)(t_{j+1}) + \sum_{j=1}^{r} \theta_j + ({}_{t_r}I^{\sigma_r}_{q_r} \theta)(t) \end{aligned}$$

where $H(t) = F(t, y(t), H(t))$ and $t \in (t_r, t_{r+1}]$ for all $r = 1, 2, \dots, n$. Now

$$\begin{aligned} |y(t) - x(t)| \leq &\sum_{j=1}^{n} |d_j| ({}_{t_{j-1}}I^{\sigma_{j-1}}_{q_{j-1}} |H - G|)(t) + \sum_{j=1}^{n} |d_j| M' |y(t_j) - x(t_j)| \\ &+ \sum_{j=0}^{n} \frac{|c_j|}{|A|} ({}_{t_j}I^{\sigma_j}_{q_j} |H - G|)(\eta_j) + \frac{|b|}{|A|} ({}_{t_n}I^{\sigma_n}_{q_n} |H - G|)(\mathcal{T}) \\ &+ \sum_{j=0}^{r-1} ({}_{t_j}I^{\sigma_j}_{q_j} |H - G|)(t_{j+1}) \\ &+ \sum_{j=1}^{r} M' |y(t_j) - x(t_j)| + ({}_{t_k}I^{\sigma_r}_{q_r} |H - G|)(t) + \epsilon(\Lambda + \sum_{j=1}^{n} |d_j| + n) \\ \Rightarrow \|y - x\|_\infty \leq &\frac{\Lambda N}{1 - Q} \|y - x\|_\infty + (\sum_{j=1}^{n} |d_j| M' + nM') \|y - x\|_\infty \\ &+ (\Lambda + \sum_{j=1}^{n} |d_j| + n)\epsilon \\ \Rightarrow \|y - x\|_\infty \leq &C\epsilon \end{aligned}$$

where

$$C = \frac{(\Lambda + \sum_{j=1}^{n} |d_j| + n)}{1 - \Lambda \frac{N}{1-Q} - \sum_{j=1}^{n} |d_j| M' - nM'} .$$

So, the equation (1) is a Ulam-Hyers stable which also implies that the equation is also Generalized Ulam-Hyers stable. □

5. Example

We will provide an example in this part to show our results.

Example 5.1. *The impulsive fractional differential equation with implicit form below has a nonlocal boundary condition.*

$$\begin{cases} ({}^{c}_{t_r}D^{\frac{r^2+49}{r^2+50}}_{\frac{r+1}{r+2}}x)(t) = t + \frac{t}{2}x(t) + \frac{t}{50}sin(({}^{c}_{t_r}D^{\frac{r^2+49}{r^2+50}}_{\frac{r+1}{r+2}}x)(t)),\ t \in J = [0,1], t \notin \{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\} \\ x(t_r^+) = x(t_r) + \frac{(x(t_r))^r}{240},\ \ t_r = \frac{r}{4} \\ 100x(0) + x(1) = \frac{1}{4}x(\eta_0) + \frac{1}{4}x(\eta_1) + \frac{1}{4}x(\eta_2) + \frac{1}{4}x(\eta_3),\ \eta_0 \in [t_0, t_1], \eta_r \in (t_r, t_{r+1}], \end{cases} \tag{5}$$

comparing (5) *with the problem* (1) *we have* $\sigma_r = \frac{r^2+49}{r^2+50}$, $q_r = \frac{r+1}{r+2}$, $t_r = \frac{r}{4}$ *for* $r =$ *0,1,2,3.* $a = 100$, $b = 1$, $c_0 = \frac{1}{4}$, $c_1 = \frac{1}{4}$, $c_2 = \frac{1}{4}$, $c_3 = \frac{1}{4}$, $F(t,x,y) = t + \frac{t}{2}x + \frac{t}{50}sin(y)$ *and* $\rho_r(t) = \frac{t^r}{240}$ $r = 1, 2, 3$. *So* F *is a continuous function and satisfy the (A1) assumption. Again,*

$$\begin{aligned} |F(t,x,y)| &\leq |t| + |\frac{t}{2}||x| + |\frac{t}{50}||sin(y)| \\ &\leq t + \frac{t}{2}|x| + \frac{t}{50}|y|. \end{aligned}$$

So, assumption (A2) is satisfied where $d(t) = t$, $p(t) = t/2$, $m(t) = t/50$ *and* $d^* = 1$, $p^* = 1/2$, $m^* = 1/50$.
Since for k =1,2,3, $\rho_r(t)$ *satisfied* $\rho_k(0) = 0$ *and the Lipschitz condition with Lipschitz constant* $\mathcal{K} = \frac{1}{240}, \frac{1}{120}, \frac{1}{80}$ *respectively, so we choose* $M' = \frac{1}{80}$. *Now using the value of* $M', a, b,\ c_0,\ c_1,\ c_2,\ c_3$ *we get that* $(\sum_{i=0}^{3}|d_i| + 3)M' = 0.0377 < 1$ *which implies that assumption (A4) is also satisfied.*
Using the property of q-gamma function i.e., if $0 < q < 1$ *and* $1 < t \leq 2$, *then* $\Gamma(t) \leq \Gamma_q(t)$, *we get,*

$$\begin{aligned} \Lambda \leq &\sum_{j=1}^{3}|d_j|\frac{(t_j - t_{j-1})^{\sigma_{j-1}}}{\Gamma(\sigma_{j-1}+1)} + \sum_{j=0}^{3}\frac{|c_j|}{|A|}\frac{(t_{j+1}-t_j)^{\alpha_j}}{\Gamma(\sigma_j+1)} \\ &+ \frac{|b|}{|A|}\frac{(\mathcal{T} - t_n)^{\sigma_n}}{\Gamma(\sigma_n+1)} + \sum_{j=0}^{2}\frac{(t_{j+1}-t_j)^{\sigma_j}}{\Gamma(\sigma_j+1)} + \lambda \\ = &\ 1.0449, \end{aligned}$$

which implies, $p^*\Lambda \leq 0.52245 < 0.9431 = \{1 - (\sum_{i=1}^{3}|d_i| + 3)M'\}(1 - m^*)$. *Again for* $x_1, x_2, y_1, y_2 \in \mathbb{R}$ *and* $t \in [0,1]$ *we have,*

$$\begin{aligned} |F(t,x_1,y_1) - F(t,x_2,y_2)| &\leq \frac{t}{2}|x_1 - x_2| + \frac{t}{50}|sin(y_1) - sin(y_2)| \\ &\leq \frac{1}{2}|x_1 - x_2| + \frac{1}{50}|y_1 - y_2|. \end{aligned}$$

So, $N = \frac{1}{2}$ *and* $Q = \frac{1}{50}$. *Since* (5) *satisfy the assumption (A1), (A2), (A3), (A4), (A5), (A6), so there exist a solution of the problem* (5) *in [0,1].*
Also,

$$\Lambda\frac{N}{1-Q} + (\sum_{i=1}^{3}|d_i| + 3)M' \leq 0.52245 \times \frac{50}{49} + 0.0377 = 0.5708 \leq 1,$$

which implies that the problem (5) *satisfy the assumption (A7). So the solution is Ulam-Hyers stable as well as Generalized Ulam-Hyers stable.*

6. Conclusion

The conditions necessary for the existence of solutions to the impulsive fractional differential equation given in equation (1) are obtained in this study. We also derive the necessary conditions for the Ulam-Hyer stability and the Generalized Ulam-Hyer stability of the problem. we illustrate our results with a example.

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