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.

A STUDY ON $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ UNIFORM SPECTRAL SPACES

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Abstract. A conceptual way of approach to sober space is explored by the irreducibility of closed sets and their components in topological spaces. Sober space has been defined by both generic points and the irreducibility of closed sets. From this, the extension of a novel space which is known as spectral space is developed. Spectral space is one of the inventive extensions of sober space. Spectral space, can also be studied along with compact spaces, T_0 space and sober space in topological space. T_0 space has played a major role in spectral space. Quasi-spectral space and Semi-spectral space are also probed in addition to spectral space. In this article, the author introduces a new concept called $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set. By using it the new space called $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space is introduced and studied. The extension of $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space is studied as $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform spectral space. Moreover, $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform semi-spectral space and $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral space are also introduced and some of its properties are discussed.

Keywords: $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed sets, $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space, $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform spectral space, $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral space and $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform semi-spectral space.

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1. Introduction

Zadeh .L [28] in 1965, proposed the novel set is called Fuzzy set from universal set X. It studies about fuzzy sset the uncertainity, vagueness etc. Each elements in a fuzzy set is represented as a membership value. The membership values are defined by membership functions from fuzzy set to [0,1]. It plays an important role in fuzzy sets. Later, in 1986, Atanassov K. T. [2, 3] extended fuzzy set as a new form called intuitionistic fuzzy set. An intuitionistic fuzzy set deals with both belongingness and non-belongingness of each elements in the intuitionistic fuzzy set. Topology deals with the geometrical objects. Otherwise it is called as Rubber sheet-geometry. In topological space many of the new

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ideas, geometrical interpretations has been evolved. So, one of the prominent author Chang C. L. [8] enhanced with a new idea called fuzzy topology in 1968. He framed the definition of fuzzy topological space, continuity, closed set, compactness etc. Followed by Chang, Lowen R. [23] in 1976, proposed an another defintion for fuzzy topological space. In 1977, Hutton B. [18] gives an detailed analysis on quasi uniformities in topological spaces and later it was extended to fuzzy topological spaces. The author's aim is to give an detailed explanation on quasi-uniformizable, uniformizability, complete regularity etc. Kotze W. [22] in 1986, had developed an idea of quasi-coincidence to fuzzy hausdorff space. The authors Kim Y. C. and Lee S. J. [20] in 1998, proposed a new idea called fuzzy quasi-uniform spaces amd explored some of its properties. Followed by above authors, Kim Y. C. and Ko J. M. [21] in 2006, extended to L-fuzzy quasi uniformity structure in L-fuzzy topology and some of its results has been proved. The concept of fuzzy quasi-uniformity was extended to intuitionistic fuzzy quasi-uniform topological space and it was studied by Revathi G. K. and et. al [25] in 2012. Functors, plays a huge impact in intuitionistic fuzzy quasi uniform space. Irreducibility in topological space leads to an extension called sober space along with spectral spaces.

2. Literature Review

Connectedness, compactness and continuous functions made a huge impact in topological spaces. Many of the novel ideas had been enhanced from these three C's. Among the three, compactness will play a major role for defining open cover from the collection of all family of open sets. Compactness has a vital role in dimension of a topological spaces. Among the authors, Chang C. L. [8], Lowen R. [23] and Gantner T. E.[15] gave the defintions for compactness in fuzzy topological spaces. Later that Coker D. [9] and Abbas S. E. [1] formed definition of intuitionistic fuzzy topological space and intuitionistic fuzzy compactness. Hochster M. [17] introduced the prime ideals structure in commutative rings in 1968. The author Johnstone P. T. [19] studied about the stone spaces in topological spaces. Later that Kotze W. [22] defined quasi-coincidence and quasi Hausdorff in topological space. David E. [11] investigated the irreduciblity components in topological sober space and also the soberfications concept are also studied. In addition the author defined soberity and explored its expansions along with the subcategories. In this article establishing, soberfications in topological space is the author's primary goal. Girez G., and et. al [16] introduced a concept called spectral theory of continuous lattices in which the author discussed about the irreduciblity, complete lattices etc. Echi O. and et. al [12, 13, 14] extended the concept to Gold-Spectral and Jacspectral spaces using quasi-homeomorphism. Additionally sober spaces and sober sets are also discussed. Belaid K. and et. al [5], extended the spectral space idea to Alexandorff space. The author gives an detailed analysis on extension theorems in that space. Bouacida E., and etal [7] gave extension theorems for sober spaces along with Goldman topology. Later in 2009 Xu L.[26, 27] discussed sober space and its decomposition by using functors and also via posets a scott topology and its soberfications had been defined. The author David Chen [10] introduced and studied sober topological spaces in Zariski topologies. The author gave a special emphasis to soberity in Zariski topology. Also David Chen [10], generalised Zarisiki topology as a new concept called spectral spaces and its functors. The sober space in topological space will make a new extension called spectral space. In spectral space, compactness, sober space, T_0 space will give an important results and ideas. In this paper, the authors expressed a new idea $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform spectral space. Further, an $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform spectral space can be induced from $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF} - \mathcal{Q}$ uniform topological space and make the article more interesting which studying the properties. In this article the author discussed $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform spectral space in $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform topological space. The ideas will give new results and some of the types of $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform spectral space is also defined.

3. Motivation and Contribution of the Study

The structure of $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform spectral space is quite interesting and hence various fruitful results had been established already. The study of spectral space is an important extension of sober space and also it is one of the main part in topological space. Also the $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform space has deep conceptual ideas in intuitionistic fuzzy topological space. Spectral space is an extension of sober space. The sober space can be studied with the functors. Likewise, spectral space can also dealt with the properties of sober space in topological space. $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober spaces and $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ spectral spaces have novel properties which were listed and studied in this work. The article is to framework new concept called $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform spectral space and which paved the way to find some interesting and essential topological properties. $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform spectral space portrays the new forms of $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform semi-spectral space, $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral space,$(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact spaces, $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform Γ_q-compact sets and $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform ξ-sets etc., Also various results are proved.

4. Preliminaries

Throughout this paper, $(\mathfrak{c},\mathfrak{d})$ intuitionistic fuzzy quasi uniform topological space is represented as $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform topological space.

Definition 4.1. [2, 3] Let X be a universal set. An intuitionistic fuzzy Set $\mathcal{A}$ in X is defined as $\mathcal{A}=\{\langle x,\mu_{\mathcal{A}(x)},\nu_{\mathcal{A}(x)}\rangle : x\in X\}$ where $\mu_{\mathcal{A}(x)}: X\to[0,1]$ and $\nu_{\mathcal{A}(x)}: X\to[0,1]$ are the membership and non-membership functions respectively for every $x\in X$, with the condition $0\leq\mu_{\mathcal{A}(x)}+\nu_{\mathcal{A}(x)}\leq 1$. The collection of intuitionistic fuzzy set is denoted as IF(X)

Definition 4.2. [9] An intuitionistic fuzzy topology τ on a non-empty set X is a collection of intuitionistic fuzzy sets with the following properties.

(i) $0_\sim, 1_\sim \in\tau$,
(ii) $\mathcal{A}_1\cap\mathcal{A}_2\in\tau$ for any $\mathcal{A}_1,\mathcal{A}_2\in\tau$,
(iii) $\cup\,\mathcal{A}_i\in\tau$ for any arbitrary family $\{\mathcal{A}_i$:i $\in$ J$\}\subseteq\tau$.

An element in τ and its complement with respect to τ are called intuitionistic fuzzy open and closed sets respectively. In this case, (X,τ) is called a intuitionistic fuzzy topological space.

Definition 4.3. [9] Let $\mathcal{A}=\langle x,\mu_{\mathcal{A}(x)},\nu_{\mathcal{A}(x)}\rangle$ and $\mathcal{B}=\langle x,\mu_{\mathbb{B}(x)},\nu_{\mathbb{B}(x)}\rangle$ be two intuitionistic fuzzy sets in X, then $\mathcal{A}$ and $\mathcal{B}$ are said to be q-coincident ($\mathcal{A}$ q $\mathcal{B}$ respectively) iff there exists an element $x\in X$ such that either $\mu_{\mathcal{A}(x)}>\nu_{\mathcal{B}(x)}$ or $\nu_{\mathcal{A}(x)}<\mu_{\mathcal{B}(x)}$.

Definition 4.4. [4, 27] Let (X,τ) be a topological space. Let $\mathcal{A},\mathcal{B}$ and $\mathcal{C}$ be a non-empty closed sets in (X,τ). Then $\mathcal{A}$ is said to be irreducible closed set in (X,τ), if either $\mathcal{A}\subseteq\mathcal{B}$ or $\mathcal{A}\subseteq\mathcal{C}$.

Definition 4.5. [13, 19] Let (X,τ) be a topological space. A topological space X is said to be a sober space, if for each irreducible closed set $\mathcal{C}$ there is a unique element $x\in X$ such that $\{\bar{x}\}=\mathcal{C}$, where $\{\bar{x}\}$ is the closure of x in X.

Definition 4.6. [25] Let Ω_X be family of intuitionistic fuzzy mappings, $\mathfrak{g}$: IF$(X) \rightarrow$ IF(X) then the following are:

(i) $\mathfrak{g}(0_\sim)$ $=0_\sim$
(ii) $\mathcal{A} \subseteq \mathfrak{g}(\mathcal{A}), \forall \mathcal{A} \in$ IF(X)
(iii) $\mathfrak{g}(\cup \mathcal{A}_i)= \cup \mathfrak{g}(\mathcal{A}_i), \forall \mathcal{A}_i \in$ IF(X), i $\in$ J

For $\mathfrak{g} \in \Omega_X$, the function $\mathfrak{g}^{-1}(\mathcal{A})= \cap\{\mathcal{B} : \mathfrak{g}(\bar{\mathcal{B}}) \subseteq \bar{\mathcal{A}}\} \in \Omega_X$, given for all $\mathcal{A} \in$ IF(X), $\mathfrak{g} \sqcap \mathfrak{g}^{'}(\mathcal{A})$=$\cap\{\mathfrak{g}(\mathcal{A}_1) \cup \mathfrak{g}^{'}(\mathcal{A}_2) : \mathcal{A}_1 \cup \mathcal{A}_2 = \mathcal{A}\}$, $(\mathfrak{g} \circ \mathfrak{g}^{'})$ $(\mathcal{A})$=$\mathfrak{g}(\mathfrak{g}^{'}(\mathcal{A}))$.

Definition 4.7. [25] Let υ: $\Omega_X \rightarrow \mathcal{I} \times \mathcal{I}$ be a intuitionistic fuzzy mapping. Then υ is said to be a intuitionistic fuzzy quasi uniformity on X, if it satisfies the following conditions:

(i) $\upsilon(\mathfrak{g}_1 \sqcap \mathfrak{g}_2) \supseteq \upsilon(\mathfrak{g}_1) \cap \upsilon(\mathfrak{g}_2)$ for $\mathfrak{g}_1$, $\mathfrak{g}_2 \in \Omega_X$
(ii) $\mathfrak{g} \in \Omega_X$ we have $\cup\{\upsilon(\mathfrak{g}_1) : \mathfrak{g}_1 \circ \mathfrak{g}_1 \subseteq \mathfrak{g}\} \supseteq \upsilon(\mathfrak{g})$
(iii) $\mathfrak{g}_1 \supseteq \mathfrak{g}$ then $\upsilon(\mathfrak{g}_1) \supseteq \upsilon(\mathfrak{g})$
(iv) $\mathfrak{g} \in \Omega_X$ then $\upsilon(\mathfrak{f})$=$1_\sim$.

The pair (X, υ) is said to be an $\mathcal{IF}-\mathcal{Q}$ uniform space.

Definition 4.8. [25] Let (X, υ) be a $\mathcal{IF}-\mathcal{Q}$ uniform space. Let $\mathfrak{c} \in (0,1]$ $=\mathcal{I}_0$ and $\mathfrak{d} \in [0,1)$ $=\mathcal{I}_1$ with $\mathfrak{c}+\mathfrak{d} \leq 1$ and $\mathcal{A} \in$ IF(X).
Define $(\mathfrak{c}, \mathfrak{d})$ $IFQI_\upsilon(\mathcal{A}) = \cup\{\mathcal{B} : \mathfrak{f}(\mathcal{B}) \subseteq \mathcal{A}$ for some cases $\mathfrak{f} \in \Omega(X)$ with $\upsilon(\mathfrak{f}) > (\mathfrak{c}, \mathfrak{d})\}$

Definition 4.9. [25] Let (X, υ) be a $\mathcal{IF}-\mathcal{Q}$ uniform space. The function τ_υ : IF$(X) \rightarrow$ $\mathcal{I} \times \mathcal{I}$ is defined as
$\tau_\upsilon(\mathcal{A}) = \cup\{(\mathfrak{c}, \mathfrak{d}) : (\mathfrak{c}, \mathfrak{d})$ $IFQI_\upsilon(\mathcal{A})$=$\mathcal{A}$, $\mathfrak{c} \in \mathcal{I}_0$, $\mathfrak{d} \in \mathcal{I}_1$ with $\mathfrak{c}+\mathfrak{d} \leq 1\}$.Hence, a pair (X, τ_υ) is called $\mathcal{IF}-\mathcal{Q}$ uniform topological space. The elements of (X, τ_υ) is called $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open sets and its complement is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed sets.

Definition 4.10. [25] Let (X, τ_υ) be a $\mathcal{IF}-\mathcal{Q}$ uniform topological space and $\mathcal{A}$ be an intuitionistic fuzzy set. Then the $\mathcal{IF}-\mathcal{Q}$ uniform interior of $\mathcal{A}$ is defined as $(\mathfrak{c}, \mathfrak{d})$ IFQ int$_\upsilon(\mathcal{A})$=$\cup\{\mathcal{B} : \mathcal{B} \subseteq \mathcal{A}$ and $\mathcal{B}$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open set where $\mathfrak{c} \in \mathcal{I}_0$, $\mathfrak{d} \in \mathcal{I}_1$, and $\mathfrak{c}+\mathfrak{d} \leq 1\}$.

Definition 4.11. [25] Let (X, τ_υ) be an $\mathcal{IF}-\mathcal{Q}$ uniform topological space and $\mathcal{A}$ be an intuitionistic fuzzy set. Then the $\mathcal{IF}-\mathcal{Q}$ uniform closure of $\mathcal{A}$ is defined as $(\mathfrak{c}, \mathfrak{d})IFQcl_\upsilon(\mathcal{A})$ $\cap\{\mathcal{B} : \mathcal{B} \supseteq \mathcal{A}$ and $\mathcal{B}$ is an $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed set where $\mathfrak{c} \in \mathcal{I}_0$, $\mathfrak{d} \in \mathcal{I}_1$ with $\mathfrak{c}+\mathfrak{d} \leq 1\}$

Definition 4.12. [8] Let (X, τ) be a fuzzy topological space. A family $\mathbf{A}$ of fuzzy sets is a cover of a fuzzy set $\mathcal{B}$ iff $\mathcal{B} \subset \cup$ $\{$ $\mathcal{A}$: $\mathcal{A} \in \mathbf{A}$ $\}$. It is an open cover iff each member of $\mathcal{A}$ is an open fuzzy set. A subcover of $\mathbf{A}$ is a subfamily of $\mathbf{A}$ which is also a cover.

Definition 4.13. [9] Let (X, τ) be an intuitionistic fuzzy topological space. A family $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\}$ of intuitionistic fuzzy open sets in X satisfies the condition $\cup$ $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\} = \tilde{1}$ then it is called a fuzzy open cover $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\}$ of X. A finite subfamily of a fuzzy open cover $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\}$ which is also a fuzzy open cover of X is called a finite subcover of $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\}$.

Definition 4.14. [9] An intuitionistic fuzzy topological space (X, τ) is called intuitionistic fuzzy compact space, if for each open cover of X there exists a finite subcover of X.

Definition 4.15. [9] Let (X_1, τ_{υ_1}) and (X_2, τ_{υ_2}) be two $\mathcal{IF}-\mathcal{Q}$ uniform topological spaces and f:$(X_1, \tau_{\upsilon_1}) \rightarrow (X_2, \tau_{\upsilon_2})$ be a function. Then f is said to $(\mathfrak{c}.\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform continuous function if the pre-image of each $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open set in (X_2, τ_{υ_2}) is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open set in (X_1, τ_{υ_1}).

5. Structure of the Paper

In this article, a new conceptual idea called $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform spectral space is defined. The major extension of $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform spectral space. $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space deals with the irreducibility of closed sets and its generic points in $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform topological space. Likewise, in $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform spectral space gives a detailed version of $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform compactness and $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform T_0-space. In Section-6, $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space is discussed and some of its results are proved. In Section-7, an extension from $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space called $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform spectral space and $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral space is introduced. In this section, an $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact space has been discussed. Section-7.1, $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform semi-spectral space and new results are discussed.

6. $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space

In this section, gives an detailed version of $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed sets, $\mathcal{IF}-\mathcal{Q}$ uniform generic set are discussed. The definition of $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space is introduced.

Definition 6.1. Let (X,τ_v) be a $\mathcal{IF}-\mathcal{Q}$ uniform topological space. Let $\mathcal{A},\mathcal{B}$ and $\mathcal{C}$ be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed sets on (X,τ_v). Then $\mathcal{A}$ are said to be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set, if $\mathcal{A}\subseteq\mathcal{B}\cup\mathcal{C}$ such that either $\mathcal{A}\subseteq\mathcal{B}$ or $\mathcal{A}\subseteq\mathcal{C}$. The complements of a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed sets are said to be $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible open sets.

Remark 6.1. Every $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed set. The converse need not be true.

Example 6.1. Let $X=\{\mathbb{a},\mathbb{b}\}$ be a non empty set,
$\mathcal{A}=\left\langle x,\left(\frac{\mathbb{a}}{0.6},\frac{\mathbb{b}}{0.7}\right),\left(\frac{\mathbb{a}}{0.4},\frac{\mathbb{b}}{0.1}\right)\right\rangle$, $\mathcal{B}=\left\langle x,\left(\frac{\mathbb{a}}{0.1},\frac{\mathbb{b}}{0.2}\right),\left(\frac{\mathbb{a}}{0.5},\frac{\mathbb{b}}{0.6}\right)\right\rangle$,
$\mathcal{C}=\left\langle x,\left(\frac{\mathbb{a}}{0.3},\frac{\mathbb{b}}{0.2}\right),\left(\frac{\mathbb{a}}{0.4},\frac{\mathbb{b}}{0.3}\right)\right\rangle$ and $\mathcal{D}=\left\langle x,\left(\frac{\mathbb{a}}{0.2},\frac{\mathbb{b}}{0.2}\right),\left(\frac{\mathbb{a}}{0.5},\frac{\mathbb{b}}{0.4}\right)\right\rangle$

be intuitionistic fuzzy sets on X and $\mathcal{E}$ be also a intuitionistic fuzzy set on X.
Let Ω_X: IF$(X)\to$ IF(X) be an intuitionistic fuzzy mapping. Let $\mathfrak{g}_1,\mathfrak{g}_2$, $\mathfrak{g}_3$, $\mathfrak{g}_4$ and $\mathfrak{g}_5$ $\in\Omega_X$ be defined as:

$$\mathfrak{g}_1(\mathcal{E})=\begin{cases}0_\sim, & if\mathcal{E}=0_\sim\\ 1_\sim, & otherwise.\end{cases}$$

$$\mathfrak{g}_2(\mathcal{E})=\begin{cases}0_\sim, & if\mathcal{E}=0_\sim\\ \mathcal{A}, & if\mathcal{E}\subseteq\mathcal{A}\\ 1_\sim, & otherwise.\end{cases}$$

$$\mathfrak{g}_3(\mathcal{E})=\begin{cases}0_\sim, & if\mathcal{E}=0_\sim\\ \mathcal{B}, & if\mathcal{E}\subseteq\mathcal{B}\\ 1_\sim, & otherwise.\end{cases}$$

$$\mathfrak{g}_4(\mathcal{E})=\begin{cases}0_\sim, & if\mathcal{E}=0_\sim\\ \mathcal{C}, & if\mathcal{E}\subseteq\mathcal{C}\\ 1_\sim, & otherwise.\end{cases}$$

$$\mathfrak{g}_5(\mathcal{E}) = \begin{cases} 0_\sim, & if \mathcal{E} = 0_\sim \\ \mathcal{D}, & if \mathcal{E} \subseteq \mathcal{D} \\ 1_\sim, & otherwise. \end{cases}$$

$$\upsilon(\mathfrak{g}) = \begin{cases} (1,0), & if \mathfrak{g} = \mathfrak{g}_1 \\ (2/7,5/8), & if \mathfrak{g} = \mathfrak{g}_2 \\ (3/6,1/5), & if \mathfrak{g} = \mathfrak{g}_3 \\ (3/7,1/6), & if \mathfrak{g} = \mathfrak{g}_4 \\ (3/8,1/8), & if \mathfrak{g} = \mathfrak{g}_5 \\ (4/9,1/9), & if \mathfrak{g} = \mathfrak{g}_2 \sqcap \mathfrak{g}_3 \\ (5/7,5/9), & if \mathfrak{g} = \mathfrak{g}_2 \sqcap \mathfrak{g}_4 \\ (6/7,6/9), & if \mathfrak{g} = \mathfrak{g}_2 \sqcap \mathfrak{g}_5 \\ (3/7,8/9), & if \mathfrak{g} = \mathfrak{g}_3 \sqcap \mathfrak{g}_4 \\ (4/9,1/10), & if \mathfrak{g} = \mathfrak{g}_3 \sqcap \mathfrak{g}_5 \\ (1/6,4/7), & if \mathfrak{g} = \mathfrak{g}_4 \sqcap \mathfrak{g}_5 \\ (0,1), & otherwise \end{cases}$$

Clearly, (X,υ) is an $\mathcal{IF}-\mathcal{Q}$ uniform space, for $\mathfrak{c}$=0.08 and $\mathfrak{d}$=0.03. define intuitionistic fuzzy mapping, τ_υ: IF$(X) \to \mathcal{I}\times\mathcal{I}$ as

$$\tau_\upsilon(\mathcal{E}) = \begin{cases} (0,1), & if \mathcal{E} = 0_\sim \\ (2/5,3/7), & if \mathcal{E} = \mathcal{A} \\ (1/8,2/7), & if \mathcal{E} = \mathcal{B} \\ (1/3,3/8), & if \mathcal{E} = \mathcal{C} \\ (2/5,3/5), & if \mathcal{E} = \mathcal{D} \\ (1,0), & otherwise \end{cases}$$

Then τ_υ=$\{\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},0_\sim,1_\sim\}$ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open sets. Clearly (X,τ_υ) is a $\mathcal{IF}-\mathcal{Q}$ uniform topological space. In this example, clearly $\bar{\mathcal{B}}$ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed set but it is not a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible set and $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set.

Definition 6.2. A pair (X,τ_υ) is $\mathcal{IF}-\mathcal{Q}$ uniform topological space and $\mathcal{A}\in$ IF(X) is said to be a $\mathcal{IF}-\mathcal{Q}$ uniform generic set if $\mathcal{A}\subseteq\mathcal{B}$ and $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_\upsilon}(\mathcal{A})$=$\mathcal{B}$.

Definition 6.3. The pair (X,τ_υ) is $\mathcal{IF}-\mathcal{Q}$ uniform topological space is said to be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space, if for every $(\mathfrak{c},\mathfrak{d})$ $(\mathcal{IF}-\mathcal{Q})$ uniform irreducible closed set $\mathcal{A}$, there exists an $\mathcal{IF}-\mathcal{Q}$ uniform generic set $\mathcal{B}\in IF_G(X)$ of $\mathcal{A}$ such that $\mathcal{B}\subseteq\mathcal{A}$.

Definition 6.4. Let the pairs (X_1,τ_{υ_1}) and (X_2,τ_{υ_2}) be a $\mathcal{IF}-\mathcal{Q}$ uniform topological spaces and f: $(X_1,\tau_{\upsilon_1})\to$ (X_2,τ_{υ_2}) be a $\mathcal{IF}-\mathcal{Q}$ uniform continuous function. Then f is called as $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform homeomorphism, if for every $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open set (resp. closed set) $\mathcal{A}\in(X_1,\tau_{\upsilon_1})$, there exists a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open set (resp.closed set) $\mathcal{B}\in(X_2,\tau_{\upsilon_2})$ such that $\mathcal{A}$= $f^{-1}(\mathcal{B})$.

Remark 6.2. Let (X_1,τ_{υ_1}) and (X_2,τ_{υ_2}) be $\mathcal{IF}-\mathcal{Q}$ uniform topological spaces and the mapping f : $(X_1,\tau_{\upsilon_1})\to(X_2,\tau_{\upsilon_2})$ be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-homeomorphisms. For any two intuitionistic fuzzy sets $\mathcal{A},\mathcal{B}\in$ IF(X_2), if $f^{-1}(\mathcal{A})$= $f^{-1}(\mathcal{B})$, then $\mathcal{A}$=$\mathcal{B}$.

Definition 6.5. Let (X_1, τ_{υ_1}) and (X_2, τ_{υ_2}) be $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform topological spaces. A function f:$(X_1, \tau_{\upsilon_1}) \to (X_2, \tau_{\upsilon_2})$ is said to be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform injective function (one-to-one), if for each intuitionistic fuzzy set $\mathcal{A}$ in (X_1, τ_{υ_1}) there exists an intuitionistic fuzzy set $\mathcal{B}$ in (X_2, τ_{υ_2}) such that $f(\mathcal{A})$=$f(\mathcal{B})$ then $\mathcal{A}$=$\mathcal{B}$.

Definition 6.6. Let (X_1, τ_{υ_1}) and (X_2, τ_{υ_2}) be $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform topological spaces. A function f:$(X_1, \tau_{\upsilon_1}) \to (X_2, \tau_{\upsilon_2})$ is said to be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform surjective function (onto), if for every $\mathcal{B} \in (X_2, \tau_{\upsilon_2})$, there exists atleast one $\mathcal{A} \in (X_1, \tau_{\upsilon_1})$ such that $f(\mathcal{B})$=$\mathcal{A}$.

Definition 6.7. Let (X_1, τ_{υ_1}) and (X_2, τ_{υ_2}) be $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform topological spaces. A map g: $(X_1, \tau_{\upsilon_1}) \to (X_2, \tau_{\upsilon_2})$ is said to be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform bijective function, if it is both $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform injective function and $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform surjective function.

Proposition 6.1. The pairs (X_1, τ_{υ_1}) and (X_2, τ_{υ_2}) be any two $\mathcal{IF}-\mathcal{Q}$ uniform topological spaces and f:$(X_1, \tau_{\upsilon_1}) \to (X_2, \tau_{\upsilon_2})$ be an $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform surjective mapping and $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-homeomorphism. If $\mathcal{A}$ is a intuitionistic fuzzy set in (X_2, τ_{υ_2}), then the following statements are equivalent:

(i) $\mathcal{A}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X_2, τ_{υ_2});
(ii) $f^{-1}(\mathcal{A})$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X_1, τ_{υ_1}).

Proof. **(i) $\Longrightarrow$ (ii)** Let $\mathcal{A} \in$ IF(X_2) be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X_2, τ_{υ_2}). Let $\mathcal{B}_1$, $\mathcal{B}_2 \in$ IF(X_1) be any two $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed set in (X_1, τ_{υ_1}) such that $f^{-1}(\mathcal{A}) \subseteq \mathcal{B}_1 \cup \mathcal{B}_2$. As f is an $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-homeomorphism, there exists two $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed sets $\mathcal{C}_1$ and $\mathcal{C}_2 \in$ IF(X_2) such that $\mathcal{B}_1 = f^{-1}(\mathcal{C}_1)$ and $\mathcal{B}_2 = f^{-1}(\mathcal{C}_2)$ Thus, $f^{-1}(\mathcal{A}) \subseteq \mathcal{B}_1 \cup \mathcal{B}_2$=$f^{-1}(\mathcal{C}_1) \cup f^{-1}(\mathcal{C}_1)$=$f^{-1}(\mathcal{C}_1 \cup (\mathcal{C}_2))$. Hence $f^{-1}(\mathcal{A}) \subseteq f^{-1}(\mathcal{C}_1 \cup \mathcal{C}_2)$. Also, $f(f^{-1}(\mathcal{A})) \subseteq f(f^{-1}(\mathcal{C}_1 \cup \mathcal{C}_2))$. Since f is $(\mathfrak{c}, \mathfrak{d})$ intuitionistic fuzzy quasi-uniform surjective with $\mathcal{A} \subseteq \mathcal{C}_1 \cup \mathcal{C}_2$, $\mathcal{A}$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform space irreducible closed set in (X_2, τ_{υ_2}), and $\mathcal{A} \subseteq \mathcal{C}_1$ or $\mathcal{A} \subseteq \mathcal{C}_2$. Thus $\mathcal{B}_1 = f^{-1}(\mathcal{C}_1) \subseteq f^{-1}(\mathcal{A})$ and hence $f^{-1}(\mathcal{A}) \subseteq \mathcal{B}_1$. Similarly $\mathcal{B}_2 = f^{-1}(\mathcal{C}_2) \subseteq f^{-1}(\mathcal{A})$ implies that $f^{-1}(\mathcal{A}) \subseteq \mathcal{B}_2$. Therefore $f^{-1}(\mathcal{A}) \subseteq \mathcal{B}_1$ or $f^{-1}(\mathcal{A}) \subseteq \mathcal{B}_2$. Hence f^{-1} is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X_1, τ_{υ_1}).

(ii) $\Longrightarrow$ (i)

Suppose that f^{-1} is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X_1, τ_{υ_1}). Let $\mathcal{D}_1$ and $\mathcal{D}_2 \in$ IF(X_2) be an $(\mathfrak{c}, \mathfrak{d})\mathcal{IF}-\mathcal{Q}$ uniform closed sets in (X_2, τ_{υ_1})such that $\mathcal{A} \subseteq \mathcal{D}_1 \cup \mathcal{D}_2$. Then $f^{-1}(\mathcal{A}) \subseteq f^{-1}(\mathcal{D}_1 \cup \mathcal{D}_2)$=$f^{-1}(\mathcal{D}_1) \cup f^{-1}(\mathcal{D}_2)$. Since f^{-1} is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set, $f^{-1}(\mathcal{A}) \subseteq f^{-1}(\mathcal{A}_1)$ or $f^{-1}(\mathcal{A}) \subseteq f^{-1}(\mathcal{A}_2)$. Also $f(f^{-1}(\mathcal{A})) \subseteq f(f^{-1}(\mathcal{D}_1))$ or $f(f^{-1}(\mathcal{A})) \subseteq f(f^{-1}(\mathcal{D}_2))$. Since f is an $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform surjective function , $\mathcal{A} \subseteq \mathcal{D}_1$ or $\mathcal{A} \subseteq \mathcal{D}_2$. Thus $\mathcal{A}$ is an $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X_2, τ_{υ_2}). □

Corollary 6.1. Let (X, τ_υ) be $\mathcal{IF}-\mathcal{Q}$ uniform topological space. If $\mathcal{A} \in$ IF(X) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X, τ_υ), then $(\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_\upsilon}(\mathcal{A})$ is also a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X, τ_υ).

Proof. Let $\mathcal{A} \in$ IF(X) be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X, τ_υ) and let $\mathcal{B}$,$\mathcal{C} \in$ IF(X) be any two $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed sets in (X, τ_υ). Let $(\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_\upsilon}(\mathcal{A}) \subseteq \mathcal{B} \cup \mathcal{C}$ and also $\mathcal{B} \subseteq (\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_\upsilon}(\mathcal{A})$, $\mathcal{C} \subseteq (\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_\upsilon}(\mathcal{A})$. Thus $\mathcal{A}\cap$ $(\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_\upsilon}(\mathcal{A}) \subseteq \mathcal{A} \cap (\mathcal{B} \cup \mathcal{C})$. So, $\mathcal{A}\cap$ $(\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_\upsilon}(\mathcal{A}) \subseteq (\mathcal{A} \cap \mathcal{B}) \cup (\mathcal{A} \cap \mathcal{C}) \Longrightarrow$ $\mathcal{A} \subseteq (\mathcal{A} \cap \mathcal{B}) \cup (\mathcal{A} \cap \mathcal{C})$. Since $\mathcal{A}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed sets, either $\mathcal{A}$ $\subseteq (\mathcal{A} \cap \mathcal{B})$ or $\mathcal{A} \subseteq (\mathcal{A} \cap \mathcal{C})$. Thus $\mathcal{A} \subseteq \mathcal{B}$ or $\mathcal{A} \subseteq \mathcal{C}$. Therefore $(\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_\upsilon}(\mathcal{A}) \subseteq (\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_\upsilon}(\mathcal{B})$ or $(\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_\upsilon}(\mathcal{A}) \subseteq (\mathfrak{c}, \mathfrak{d})$ $IFQcl_{\tau_\upsilon}(\mathcal{C})$. Since $\mathcal{B}$ and $\mathcal{C}$ are $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$

uniform closed sets such that $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_v}(\mathcal{A}) \subseteq \mathcal{B}$ or $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_v}(\mathcal{A}) \subseteq \mathcal{C}$. Hence $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_v}(\mathcal{A})$ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X,τ_v). □

7. $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ UNIFORM SPECTRAL SPACE AND $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ UNIFORM QUASI-SPECTRAL SPACE

The concept of $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compactness is defined in this section. Also the $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open cover and $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact space also discussed. This section defines the concept termed as $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform spectral space and $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral space. In addition, it explores a few of its important properties.

Definition 7.1. Let (X,τ_v) be $\mathcal{IF}-\mathcal{Q}$ uniform topological space. A family $\Im$=$\{\mathcal{B}_i$:$\mathcal{B}_i$ q $\mathcal{A}_i$ where $\mathcal{B}_i \neq \tilde{1}$ and $\mathcal{A}_i$ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible open sets and i $\in$ I $\}$ is said to be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-open cover of an intuitionistic fuzzy sets $\mathcal{B}$, if $\mathcal{B} \subseteq \cup$ $\{\mathcal{A}$: $\mathcal{A} \in \Im\}$

Note 7.1. A $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-open subcover of $\mathcal{A}$ is a subfamily of $\Im$ which is also a subset of $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-open cover.

Definition 7.2. Let (X,τ_v) be an $\mathcal{IF}-\mathcal{Q}$ uniform topological space and $\mathcal{A} \in$ IF(X). Then $\mathcal{A}$ is said to be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact set, if for every $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-open cover of $\Im$, there exists a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-open subcover such that $\mathcal{A} \subseteq \cup$ $\{$ $\mathcal{B}_i$: $\mathcal{B}_i \in \Im$ where i=1,2,3,...,n $\}$

Definition 7.3. Let (X,τ_v) be a $\mathcal{IF}-\mathcal{Q}$ uniform topological space. Then (X,τ_v) is said to be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact space, if for each $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-open cover of $\Im$ there exists a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-finite subcover.

Note 7.2. From the Example 6.1, Then τ_v=$\{\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},0_\sim,1_\sim\}$ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform topology and (X,τ_v) is a $\mathcal{IF}-\mathcal{Q}$ uniform topological space. Here $\mathcal{A}$ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible open set. Clearly (X,τ_v) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ q-compact space.

Definition 7.4. Let (X,τ_v) be a $\mathcal{IF}-\mathcal{Q}$ uniform topological space. Then (X,τ_v) is said to be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform T_0 space, if $\mathbb{a},\mathbb{b} \in X$ and $\mathbb{a} \neq \mathbb{b}$, there exists $\mathcal{A}$= $(\mu_\mathcal{A}$,$\nu_\mathcal{A})$ and $\mathcal{B}$=$(\mu_\mathcal{B}$,$\nu_\mathcal{B}) \in \tau_v$ such that $\mu_\mathcal{A}(\mathbb{a})$=1, $\mu_\mathcal{B}$ $(\mathbb{b})$=1,$\nu_\mathcal{A}(\mathbb{a})$=0, $\nu_\mathcal{B}(\mathbb{b})$=0, and $\mathcal{A}\cap\mathcal{B} = 0_\sim$.

Definition 7.5. Let (X,τ_v) be a $\mathcal{IF}-\mathcal{Q}$ uniform topological space. A base in (X,τ_v) is a subcollection **B** of τ_v such that each member of $\mathcal{A}$ of τ_v can be written as $\mathcal{A}$=$\cup_{j\in J}$ where each $\mathcal{A}_j \in$ **B**

Definition 7.6. Let (X,τ_v) be an $\mathcal{IF}-\mathcal{Q}$ uniform topological space and let $\Im$ be the collection of all $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open sets in (X,τ_v). Then (X,τ_v) is said to be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform spectral space, if the following axioms are to be satisfied:

(i) (X,τ_v) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform T_0 space.
(ii) (X,τ_v) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact space and it has a base, contains a collection of $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open sets.
(iii) If $\mathcal{A},\mathcal{B} \in \Im$ then $\mathcal{A}\cap\mathcal{B} \in \Im$.
(iv) (X,τ_v) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space.

Proposition 7.1. Let (X_1,τ_{v_1}) and (X_2,τ_{v_2}) be any two $\mathcal{IF}-\mathcal{Q}$ uniform topological spaces. Let the function φ: $(X_1,\tau_{v_1}) \to (X_2,\tau_{v_2})$ be $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-homeomorphism and $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform surjective function and $\mathcal{A}$ be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open set. Then the following conditions hold:

(i) $\mathcal{A}$ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact in (X_2,τ_{υ_2}).
(ii) $\varphi^{-1}(\mathcal{A})$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact in (X_1,τ_{υ_1})

Proof. **(i) $\implies$ (ii)**
Let $\mathcal{A}\in$ IF(X_2) be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact set in (X_2,τ_{υ_2}). The collection $\Im$ be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open covering of $\varphi^{-1}(\mathcal{A})\in$ IF(X_1). Then $\varphi^{-1}(\mathcal{A})\subseteq\cup_{i\in J}\ \mathcal{B}_i$. Since φ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-homeomorphism, and by Definition 6.4, For every $\mathcal{B}_i\in$ IF(X_1) where i∈J, there exists a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open set in $\mathcal{C}_i\in$ IF(X_2) such that $\mathcal{B}_i$= $\varphi^{-1}(\mathcal{C})$, i∈ J. Then $\varphi^{-1}(\mathcal{A})\subseteq\cup_{i\in J}\ \mathcal{B}_i$. $\varphi^{-1}(\mathcal{A})=\cup_{i\in J}\ \varphi^{-1}\ (\mathcal{C}_i)$= $\varphi^{-1}(\cup_{i\in J}\ \mathcal{C}_i)$. Thus $\varphi^{-1}(\mathcal{A})$= $\varphi^{-1}(\cup_{i\in J}\ \mathcal{C}_i)$. Since φ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-homeomorphism, $\mathcal{A}\subseteq\cup_{i\in J}\ \mathcal{C}_i$. Since $\mathcal{A}$ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact set, there exists a finite subset J_0 of J such that $\mathcal{A}\subseteq\cup_{i\in J_0}\mathcal{C}_i$. $\varphi^{-1}(\mathcal{A})\subseteq\varphi^{-1}(\cup_{i\in J_0}\mathcal{C}_i)\subseteq\cup_{i\in J_0}\ \varphi^{-1}(\mathcal{C}_i)\subseteq\cup_{i\in j_0}\ \mathcal{B}_i$. Therefore $\varphi^{-1}(\mathcal{A})\subseteq\cup_{i\in J_0}\ \mathcal{B}_i$. Hence $\varphi^{-1}(\mathcal{A})$ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact set in (X_1,τ_{υ_1}).
(ii) $\implies$ (i)
Assume that $\varphi^{-1}(\mathcal{A})\in$ IF(X_1) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact set in (X,τ_{υ_1}). The collection $\Im$ be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open covering of $\varphi^{-1}(\mathcal{A})\in$ IF(X_2). Then $\mathcal{A}\subseteq\cup_{i\in J}\ \mathcal{C}_i$. Thus $\varphi^{-1}(\mathcal{A})\subseteq\varphi^{-1}(\cup_{i\in J}\ \mathcal{C}_i)$= $\cup_{i\in J}\varphi^{-1}(\mathcal{C}_i)$. Therefore $\varphi^{-1}(\mathcal{A})\subseteq\varphi^{-1}(\mathcal{C}_i)$. Since φ^{-1} is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact set, there exists a finite subset $J_0\in J$ such that $\varphi^{-1}(\mathcal{A})\subseteq\cup_{i\in J_0}\ \varphi^{-1}\ (\mathcal{C}_i)$=$\varphi^{-1}\ (\cup_{i\in J_0}\ \mathcal{C}_i)$. $\varphi^{-1}\ (\mathcal{A})\subseteq\varphi^{-1}\ (\cup_{i\in J_0}\ \mathcal{C}_i)$.
$\varphi(\varphi^{-1}(\mathcal{A}))\subseteq\varphi(\varphi^{-1}(\cup_{i\in J_0}\mathcal{C}_i))$. Thus $\mathcal{A}\subseteq\cup_{i\in J_0}\ \mathcal{C}_i$. Since φ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform onto. Hence $\mathcal{A}$ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact set in (X_2,τ_{υ_2}). □

Definition 7.7. Let (X_1,τ_{υ_1}) and (X_2,τ_{υ_2}) be any two $\mathcal{IF}-\mathcal{Q}$ uniform topological space and φ: $(X_1,\tau_{\upsilon_1})\to(X_2,\tau_{\upsilon_2})$ be any intuitionistic fuzzy function. An intuitionistic fuzzy set $\mathcal{A}\in$ IF(X_1) is said to be $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform saturated under φ, if $\varphi^{-1}(\varphi(\mathcal{A}))$=$\mathcal{A}$

Remark 7.1. Let (X_1,τ_{υ_1}) and (X_2,τ_{υ_2}) be any two $\mathcal{IF}-\mathcal{Q}$ uniform topological space and φ: $(X_1,\tau_{\upsilon_1})\to(X_2,\tau_{\upsilon_2})$ be any intuitionistic fuzzy function. If $\mathcal{A}\in$ IF(X_1) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform saturated set in (X_1,τ_{υ_1}) under φ. $\mathcal{A}$ is also a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform saturated under φ. Then $\bar{\mathcal{A}}$ is also a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform saturated under φ.

Definition 7.8. Let (X_1,τ_{υ_1}) and (X_2,τ_{υ_2}) be any two $\mathcal{IF}-\mathcal{Q}$ uniform topological spaces. A mapping φ: $(X_1,\tau_{\upsilon_1})\to(X_2,\tau_{\upsilon_2})$ is said to be $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open function (resp. closed function), if for every $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open set (resp. closed set) $\mathcal{A}\in(X_1,\tau_{\upsilon_1})$ there exists a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open set(resp. closed set) $\varphi(\mathcal{A})$ in (X_2,τ_{υ_2}).

Proposition 7.2. Let (X_1,τ_{υ_1}) and (X_2,τ_{υ_2}) be any two $\mathcal{IF}-\mathcal{Q}$ uniform topological spaces. Let the mapping φ: $(X_1,\tau_{\upsilon_1})\to(X_2,\tau_{\upsilon_2})$ be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform surjective function. Then the following statements are equivalent:

(i) φ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open function and each $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open set in (X_1,τ_{υ_1}) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform saturated set under φ.
(ii) φ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed function and each $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed set in (X_1,τ_{υ_1}) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform saturated set under φ.

Proof. **(i) $\implies$ (ii)**
Let $\mathcal{A}\in$ IF(X_1) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed set in (X_1,τ_{υ_1}), then $\bar{\mathcal{A}}$ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open set in (X_1,τ_{υ_1}). Since each $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open set in (X_1,τ_{υ_1}) is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform saturated set under φ. By Remark 7.9, $\bar{\mathcal{A}}$ is also a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform saturated set under φ and φ ($\bar{\mathcal{A}}$)=$\mathcal{A}$. Further as φ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open function, $\varphi(\bar{\mathcal{A}})$=$\mathcal{A}$ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open set in (X_2,τ_{υ_2}). Thus $\varphi(\mathcal{A})$ is

a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed set in (X_2,τ_{υ_2}). Therefore φ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed function.

(ii) $\Longrightarrow$ (i)

This proof is simple. □

Definition 7.9. Let (X_1,τ_{υ_1}) and (X_2,τ_{υ_2}) be any two $\mathcal{IF}-\mathcal{Q}$ uniform topological spaces and φ: $(X_1,\tau_{\upsilon_1})\to(X_2,\tau_{\upsilon_2})$ be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform surjective and $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform continuous function. Then φ is a said to be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform special morphism, if φ satisfies one of the following axioms:

(i) φ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open function and each $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open set in (X_1,τ_{υ_1}) is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform saturated under φ.
(ii) φ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed function and each $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed set in (X_1,τ_{υ_1}) is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform saturated under φ.

Proposition 7.3. Let (X_1,τ_{υ_1}) and (X_2,τ_{υ_2}) be any two $\mathcal{IF}-\mathcal{Q}$ uniform topological space. Let the mapping φ: $(X_1,\tau_{\upsilon_1})\to(X_2,\tau_{\upsilon_2})$ be a $(\mathfrak{c},\mathfrak{d})\mathcal{IF}-\mathcal{Q}$ uniform special morphism. If (X_1,τ_{υ_1}) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform T_0 space, then φ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform homeomorphism.

Proof. Since φ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform special morphism. By Definition 7.12, φ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed and continuous function. It is enough to prove that φ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform injective. Let $\mathcal{A},\mathcal{B}\in$ IF(X_1) be such that $\varphi(\mathcal{A})$=$\varphi(\mathcal{B})$. Then $(\mathfrak{c},\mathfrak{d})IFQcl_{\tau_{\upsilon_2}}(\varphi(\mathcal{A}))$= $(\mathfrak{c},\mathfrak{d})IFQcl_{\tau_{\upsilon_2}}(\varphi(\mathcal{B}))$. Since φ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed function.

$$(\mathfrak{c},\mathfrak{d})IFQcl_{\tau_{\upsilon_2}}(\varphi(\mathcal{A}))\subseteq\varphi((\mathfrak{c},\mathfrak{d})(IFQcl_{\tau_{\upsilon 1}}(\mathcal{A}))) \tag{1}$$

and

$$(\mathfrak{c},\mathfrak{d})IFQcl_{\tau_{\upsilon_2}}(\varphi(\mathcal{B}))\subseteq\varphi((\mathfrak{c},\mathfrak{d})(IFQcl_{\tau_{\upsilon_1}}(\mathcal{B}))). \tag{2}$$

Also, φ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform continuous function,

$$(\mathfrak{c},\mathfrak{d})IFQcl_{\tau_{\upsilon_2}}(\varphi(\mathcal{A}))\supseteq\varphi((\mathfrak{c},\mathfrak{d})(IFQcl_{\tau_{\upsilon_1}}(\mathcal{A}))) \tag{3}$$

and

$$(\mathfrak{c},\mathfrak{d})IFQcl_{\tau_{\upsilon_2}}(\varphi(\mathcal{B}))\supseteq\varphi((\mathfrak{c},\mathfrak{d})(IFQcl_{\tau_{\upsilon_1}}(\mathcal{B}))). \tag{4}$$

From equations (1) and (2). $(\mathfrak{c},\mathfrak{d})IFQcl_{\tau_{\upsilon_2}}(\varphi(\mathcal{A}))\subseteq$ φ $((\mathfrak{c},\mathfrak{d})$ $(IFQcl_{\tau_{\upsilon_1}}(\mathcal{A})))$ and the equations (3) and (4) $(\mathfrak{c},\mathfrak{d})IFQcl_{\tau_{\upsilon_2}}(\varphi(\mathcal{B}))\subseteq$ φ $((\mathfrak{c},\mathfrak{d})$ $(IFQcl_{\tau_{\upsilon_1}}(\mathcal{B})))$. Therefore $(\mathfrak{c},\mathfrak{d})IFQcl_{\tau_{\upsilon_2}}(\varphi(\mathcal{A}))$= $(\mathfrak{c},\mathfrak{d})IFQcl_{\tau_{\upsilon_2}}(\varphi(\mathcal{B}))$ implies that $\varphi((\mathfrak{c},\mathfrak{d})$ $(IFQcl_{\tau_{\upsilon_1}}(\mathcal{A})))$= φ $((\mathfrak{c},\mathfrak{d}))$ $(IFQcl_{\tau_{\upsilon_1}}(\mathcal{B}))$. By Proposition 7.2, it is known that every $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed set in (X_1,τ_{υ_1}) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform saturated set under φ. Since φ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform saturated, by the Definition 7.7, $(\mathfrak{c},\mathfrak{d})IFQcl_{\tau_{\upsilon_1}}(\varphi(\mathcal{A}))$= $(\mathfrak{c},\mathfrak{d})IFQcl_{\tau_{\upsilon_1}}(\varphi(\mathcal{B}))$. Since (X_1,τ_{υ_1}) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform T_0 space and also by $\mathcal{A}$=$\mathcal{B}$. Thus φ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform injective. Therefore φ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform homeomorphism. □

Definition 7.10. Let (X,τ_υ) be a $\mathcal{IF}-\mathcal{Q}$ uniform topological space. Then is said to be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral space, if it satisfies the following axioms:

(i) (X, τ_υ) be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact space and that has a base which contains a collection of $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open sets.
(ii) If $\mathcal{A}, \mathcal{B} \in \Im$, then $\mathcal{A} \cap \mathcal{B} \in \Im$
(iii) (X, τ_υ) be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space.

Proposition 7.4. Let (X_1, τ_{υ_1}) and (X_2, τ_{υ_2}) be any two $\mathcal{IF}-\mathcal{Q}$ uniform topological spaces. Let φ: $(X_1, \tau_{\upsilon_1}) \to (X_2, \tau_{\upsilon_2})$ be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-homeomorphims and (X_2, τ_{υ_2}) be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral. Then the following statments hold:

(i) (X_1, τ_{υ_1}) be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact and has a base of $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open sets.
(ii) If $\mathcal{B}, \mathcal{C} \in$ IF(X_1) are any two $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open sets in (X_1, τ_{υ_1}), then $\mathcal{B} \cap \mathcal{C}$ is also $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact in (X_1, τ_{υ_1}).

Proof. Let φ: $(X_1, \tau_{\upsilon_1}) \to (X_2, \tau_{\upsilon_2})$ be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-homeomorphism and let (X_2, τ_{υ_2}) be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral space.
(i) Given that (X_2, τ_{υ_2}) is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral space. The collection $\Im$ contains $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact sets. Since φ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-homeomorphism, there exists a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open sets $\mathcal{B}_i \in$ IF(X_1) where i$\in$ J such that $\mathcal{B}_i$=$\varphi^{-1}(\mathcal{C}_i)$, i$\in$ J. Let $\mathcal{B} \in$ IF(X_1) be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open set in (X_1, τ_{υ_1}). Since φ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-homeomorphism, then there exists a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open set $\mathcal{C} \in$ IF(X_2) in IF(X_2) in (X_2, τ_{υ_2}) such that $\mathcal{B}$=$\varphi^{-1}(\mathcal{C})$. Since $\mathcal{C}_i$ i $\in$ J is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact in (X_2, τ_{υ_2}) and by the Proposition 7.1, $\varphi^{-1}((C_i))$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact in (X_1, τ_{υ_1}). Thus $\varphi^{-1}(\mathcal{C}_i)$= $\mathcal{B}_i$, $i \in J$ is also $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact. Therefore there exists a finite subset J_0 of J such that $\mathcal{C} \subseteq \cup_{i\in J_0} \mathcal{C}_i$, hence $\mathcal{B}$= $\varphi^{-1}(\mathcal{C})$$\subseteq$$\varphi^{-1}(\cup_{i\in J_0} \mathcal{C}_i) \subseteq \cup_{i\in J_0} \varphi^{-1}(\mathcal{C}_i)$. Hence $\mathcal{B}$$\subseteq$ $\cup_{i\in J_0} \mathcal{B}_i$. Thus (X_1, τ_{υ_1}) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact. Therefore the collection $\Im$ is a base for (X_1, τ_{υ_1}) which is the collection of $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open sets in (X_1, τ_{υ_1}).
(ii): Let $\mathcal{B}, \mathcal{C} \in$ IF(X_1) be any two $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open sets in (X_1, τ_{υ_1}). Since φ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-homeomorphism, there exists two $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open sets $\mathcal{C}_1$,$\mathcal{B}_1 \in$ IF(X_2) such that $\mathcal{B}$=$\varphi^{-1}(\mathcal{B}_1)$ and $\mathcal{C}$=$\varphi^{-1}(\mathcal{C}_1)$. Since (X_2, τ_{υ_2}) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral, $\mathcal{B}_1 \cap \mathcal{C}_1$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact in (X_2, τ_{υ_2}). Also, $\mathcal{B} \cap \mathcal{C}$= $\varphi^{-1}(\mathcal{B}_1)$$\cap$ $\varphi^{-1}(\mathcal{C}_1)$=$\varphi^{-1}(\mathcal{B}_1 \cap \mathcal{C}_1)$. Since $\mathcal{B}_1 \cap \mathcal{C}_1$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact in (X_2, τ_{υ_2}) and by Proposition 7.1, $\varphi^{-1}(\mathcal{B}_1 \cap \mathcal{C}_1)$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact in (X_1, τ_{υ_1}). Therefore, $\mathcal{B} \cap \mathcal{C}$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact in (X_1, τ_{υ_1}). □

Proposition 7.5. Let (X_1, τ_{υ_1}) and (X_2, τ_{υ_2}) be any two $\mathcal{IF}-\mathcal{Q}$ uniform topological spaces. Let φ: $(X_1, \tau_{\upsilon_1}) \to (X_2, \tau_{\upsilon_2})$ be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-homeomorphism. If (X_1, τ_{υ_1}) is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral, then (X_2, τ_{υ_2}) $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral.

Proof. It known that (X_1, τ_{υ_1}) is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral. Let the collection $\Im$ has $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open sets in (X_1, τ_{υ_1}). Since φ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-homeomorphism, there exists a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open set $\mathcal{C}_i \in$ IF(X_2) where i $\in$ J, such that $\mathcal{B}_i$=φ^{-1} $(\mathcal{C}_i)$, where each $\mathcal{B}_i \in$ IF(X_1) is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open sets in (X_1, τ_{υ_1}). Since $\mathcal{C}_i$ is $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact and $\mathcal{B}_i$= $\varphi^{-1}(\mathcal{C}_i)$. $\varphi^{-1}(\mathcal{C}_i)$ is also $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact in (X_1, τ_{υ_1}). By Proposition 7.1, $\mathcal{C}_i$ is a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact set in (X_2, τ_{υ_2}). Let $\mathcal{C} \in$ IF(X_2) be a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open set in (X_2, τ_{υ_2}). Since φ is also a $(\mathfrak{c}, \mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform continuous function,

φ^{-1} $(\mathcal{C})$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform open set in (X_1,τ_{υ_1}), there exists a finite subset J_0 of J such that $\varphi^{-1}(\mathcal{C}) \subseteq \cup_{i\in J_0}$ $\mathcal{B}_i \subseteq \cup_{i\in J_0}$ $\varphi^{-1}(\mathcal{C}_i)\subseteq$ $\varphi^{-1}(\cup_{i\in J_0}\mathcal{C}_i)$. Since φ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-homeomorphism and by Remark 6.2, $\mathcal{C} \subseteq \cup_{i\in J_0}$ $\mathcal{C}_i$. Then the collection $\Im$ has a base contains the collections of $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform compact open sets in (X_2,τ_{υ_2}). Let $\mathcal{B},\mathcal{C} \in$ IF(X_2) be any two $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open sets in (X_2,τ_{υ_2}). By Proposition 7.1, $\varphi^{-1}(\mathcal{C})$ and φ^{-1} $(\mathcal{B})$ are two $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open sets of (X_1,τ_{υ_1}). Also $\varphi^{-1}(\mathcal{B}) \cap \varphi^{-1}$ $(\mathcal{C})$ $=\varphi^{-1}(\mathcal{C}\cap\mathcal{B})$. Since (X_1,τ_{υ_1}) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral, $\varphi^{-1}(\mathcal{C}\cap\mathcal{B})$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact in (X_1,τ_{υ_1}) . Hence by Proposition 7.7, $\mathcal{B}\cap\mathcal{C}$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact set in (X_2,τ_{υ_2}). Let $\delta \in$ IF(X_2) be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X_2,τ_{υ_2}). Then by Proposition 6.9, $\varphi^{-1}(\delta)$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X_1,τ_{υ_1}). Since (X_1,τ_{υ_1}) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral space, (X_1,τ_{υ_1}) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space. Since $\varphi^{-1}(\delta)$ is (X_1,τ_{υ_1}) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X_1,τ_{υ_1}), there exists an $\mathcal{IF}-\mathcal{Q}$ uniform generic set $\mathcal{H} \in IF_G(X_1)$ of $\varphi^{-1}(\delta)$ such that $\mathcal{H} \subseteq \varphi^{-1}(\delta)$. Since $\mathcal{H}$ is an $\mathcal{IF}-\mathcal{Q}$ uniform generic set of $\varphi^{-1}(\delta)$, $\varphi^{-1}(\delta)=$ $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_{\upsilon_1}}(\mathcal{H})$. Such that δ $\subseteq$ $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_{\upsilon_1}}(\mathcal{H})$ and $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_{\upsilon_1}}(\mathcal{H}) \subseteq \delta$. This proves that $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_{\upsilon_1}}(\mathcal{H})=$ δ. Therefore for any $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set $\delta \in$ IF(X_2), there exists an $\mathcal{IF}-\mathcal{Q}$ uniform generic set $\varphi(\mathcal{H})$ of IF(X_2) of δ such that $\varphi(\mathcal{H}) \subseteq \delta$. Also since $\varphi(\mathcal{H})$ is a $\mathcal{IF}-\mathcal{Q}$ uniform generic set of δ, such that $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_{\upsilon_1}}(\mathcal{H})=$ δ. Hence (X_2,τ_{υ_2}) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space. Therefore (X_2,τ_{υ_2}) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral space. □

Proposition 7.6. Let (X_1,τ_{υ_1}) and (X_2,τ_{υ_2}) be any two $\mathcal{IF}-\mathcal{Q}$ uniform topological spaces. Let φ: $(X_1,\tau_{\upsilon_1})\rightarrow$ (X_2,τ_{υ_2}) be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-homeomorphism. If (X_1,τ_{υ_1}) is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral and (X_2,τ_{υ_2}) $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform spectral, then φ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform surjective function.

Proof. Given that (X_1,τ_{υ_1}) is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral and $\mathcal{B} \in$ IF(X_2) be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X_2,τ_{υ_2}). Then by the Corollary 6.1, $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_{\upsilon_2}}(\mathcal{B})$ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X_2,τ_{υ_2}). Since $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_{\upsilon_2}}(\mathcal{B})$ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set and by Proposition 6.1,$(\varphi^{-1}(\mathfrak{c},\mathfrak{d})(IFQcl_{\tau_{\upsilon_2}}(\mathcal{B})))$ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X_1,τ_{υ_1}). Since (X_1,τ_{υ_1}) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform spectral space, also it is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space. Then there exists an $\mathcal{IF}-\mathcal{Q}$ uniform generic set $\mathcal{H} \in IF_G(X_1)$ of φ^{-1} $((\mathfrak{c},\mathfrak{d})$ $(IFQcl_{\tau_{\upsilon_2}}(\mathcal{B})))$ such that $\mathcal{H} \subseteq \varphi^{-1}$ $((\mathfrak{c},\mathfrak{d})$ $(IFQcl_{\tau_{\upsilon_2}}(\mathcal{B})))$. Thus $\varphi(\mathcal{H})\subseteq$ φ $(\varphi^{-1}$ $(\mathfrak{c},\mathfrak{d})$ $(IFQcl_{\tau_{\upsilon_2}}(\mathcal{B}))) \subseteq$ $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_{\upsilon_2}}(\mathcal{B})$. Also that,

$$\varphi^{-1}((\mathfrak{c},\mathfrak{d})(IFQcl_{\tau_{\upsilon_2}})(\mathcal{B})) \supseteq (\mathfrak{c},\mathfrak{d})(IFQcl_{\tau_{\upsilon_2}})(\varphi(\mathcal{H})) \tag{5}$$

Since (X_1,τ_{υ_1}) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space and by Proposition 6.1 there exists an $\mathcal{IF}-\mathcal{Q}$ uniform generic set $\mathcal{H} \in IF_G(X_1)$ of $(\varphi^{-1}$ $(\mathfrak{c},\mathfrak{d})(IFQcl_{\tau_{\upsilon_2}}(\mathcal{B})))$ such that $(\varphi^{-1}$ $(\mathfrak{c},\mathfrak{d})$ $(IFQcl_{\tau_{\upsilon_2}}(\mathcal{B}))) \subseteq$ $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_{\upsilon_2}}(\mathcal{H})$. Thus $(\varphi^{-1}$ $(\mathfrak{c},\mathfrak{d})$ $(IFQcl_{\tau_{\upsilon_2}}(\mathcal{B}))) \subseteq$ $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_{\upsilon_2}}(\mathcal{H}) \subseteq \varphi^{-1}$ $(\varphi$ $(\mathfrak{c},\mathfrak{d})$ $(IFQcl_{\tau_{\upsilon_2}}(\mathcal{H})))$. Thus

$$(\varphi^{-1}(\mathfrak{c},\mathfrak{d})(IFQcl_{\tau_{\upsilon_2}}(\mathcal{B}))) \subseteq (\varphi^{-1}(\mathfrak{c},\mathfrak{d})(IFQcl_{\tau_{\upsilon_2}}))(\varphi(\mathcal{H})) \tag{6}$$

From the equations (5) and (6) $(\varphi^{-1}$ $(\mathfrak{c},\mathfrak{d})$ $(IFQcl_{\tau_{\upsilon_2}}(\mathcal{B})))$ $=$ (φ^{-1}) $(\mathfrak{c},\mathfrak{d})$ $(IFQcl_{\tau_{\upsilon_2}})$ $(\varphi(\mathcal{H}))$. Since φ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-homeomorphism. $(\mathfrak{c},\mathfrak{d})$ $(IFQcl_{\tau_{\upsilon_2}}(\mathcal{B})))$ $=$ $(\mathfrak{c},\mathfrak{d})$ $(IFQcl_{\tau_{\upsilon_2}})$ $(\varphi(\mathcal{H}))$. Since (X_2,τ_{υ_2}) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral space, it is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform T_0 space and so $\varphi(\mathcal{H})$=$\mathcal{B}$. Thus φ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform surjective. □

Proposition 7.7. Let (X_1,τ_{υ_1}) and (X_2,τ_{υ_2}) be any two $\mathcal{IF}-\mathcal{Q}$ uniform topological spaces. Let φ: $(X_1,\tau_{\upsilon_1})\rightarrow$ (X_2,τ_{υ_2}) be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed function and $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-homeomorphism. If (X_2,τ_{υ_2}) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform spectral and φ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform surjective, then (X_1,τ_{υ_1}) $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space.

Proof. Let $\mathcal{B}\in$ IF(X_1) be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X_1,τ_{υ_1}). Since φ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-homeomorphism, for each $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed set $\mathcal{B}\in$ IF(X_1) in (X_1,τ_{υ_1}), there exists a unique $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed set $\mathcal{A}\in$ IF(X_2) in (X_2,τ_{υ_2}), such that $\varphi^{-1}(\mathcal{A})$=$\mathcal{B}$. Since $\varphi^{-1}(\mathcal{A})$=$\mathcal{B}$, where $\mathcal{B}$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X_1,τ_{υ_1}). So $\varphi^{-1}(\mathcal{A})$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X_1,τ_{υ_1}). By proposition 6.1, $\mathcal{A}\in$ IF(X_2) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X_2,τ_{υ_2}). Since (X_2,τ_{υ_2}) is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform spectral space, then it is well known that it is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space. Since $\mathcal{A}$ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X_2,τ_{υ_2}), there exists a intuitionistic fuzzy generic set $\mathcal{H}\in IF_G(X_2)$ of $\mathcal{A}$ such that $\mathcal{A}$=$(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_{\upsilon_2}}(\mathcal{H})$. Since φ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform surjective, for every $\mathcal{H}\in$ IF(X_2), there exists a $\mathcal{G}\in IF_G(X_1)$ such that $\varphi(\mathcal{G})$=$\mathcal{H}$.
$\mathcal{B}$=φ^{-1} $(\mathcal{A})$=φ^{-1} $((\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_{\upsilon_2}}(\mathcal{H}))$= φ^{-1} $((\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_{\upsilon_2}})$ $(\varphi(\mathcal{G}))$= φ^{-1} $(\varphi$ $(\mathfrak{c},\mathfrak{d})$ $(IFQcl_{\tau_{\upsilon_1}})$ $(\mathcal{G}))$. $\mathcal{B}$= $(\mathfrak{c},\mathfrak{d})$ $(IFQcl_{\tau_{\upsilon_1}})$ $(\mathcal{G})$. Therefore there exists a intuitionistic fuzzy generic set $\mathcal{G}$ of a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set $\mathcal{B}$ such that $\mathcal{B}$=$(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_{\upsilon_1}}$ $(\mathcal{G})$. Hence proves that (X_1,τ_{υ_1}) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space. □

8. $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform semi-spectral space

This section defines a new idea called $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform semi-spectral space. From this space, a new concepts called $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform Γ_q-compact open sets, $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform Γ_q-compact closed sets, $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform Γ_q-compact clopen sets, $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform ξ-sets are defined and some inter-relations are investigated.

Proposition 8.1. Let (X,τ_υ) be a $\mathcal{IF}-\mathcal{Q}$ uniform topological space and $Y\subseteq X$. Let (Y,τ_{υ_Y}) be a $\mathcal{IF}-\mathcal{Q}$ uniform topological subspace of (X,τ_υ). If $\mathcal{C}\in$ IF(Y) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (Y,τ_{υ_Y}), then $(\mathfrak{c},\mathfrak{d})$ $IFQcl_\tau(\mathcal{C})$ is also a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X,τ_υ).

Proof. Let (Y,τ_{υ_Y}) be a $\mathcal{IF}-\mathcal{Q}$ uniform topological subspace of (X,τ_υ). Since $\mathcal{C}\in$ IF(Y) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (Y,τ_{υ_Y}), for all $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed sets δ, $\gamma\in$ IF(Y) in (Y,τ_{υ_Y}) such that $\mathcal{C}\subseteq(\delta\cup\gamma)$, then either $\mathcal{C}\subseteq\gamma$ or $\mathcal{C}\subseteq\delta$. Thus $\mathcal{C}\subseteq(\delta\cup\gamma)$ implies that $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_\upsilon}$ $(\mathcal{C})\subseteq(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_\upsilon}$ $(\gamma\cup\delta)$ and so $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_\upsilon}$ $(\mathcal{C})\subseteq(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_\upsilon}(\gamma)\subseteq(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_\upsilon}(\delta)$. Thus either $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_\upsilon}$ $(\mathcal{C})\subseteq$ $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_\upsilon}(\gamma)$ or $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_\upsilon}$ $(\mathcal{C})\subseteq(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_\upsilon}$ (δ). Hence it proves that $(\mathfrak{c},\mathfrak{d})$ $IFQcl_{\tau_\upsilon}(\mathcal{C})$ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform irreducible closed set in (X,τ_υ). □

Definition 8.1. A pair (X,τ_υ) be $\mathcal{IF}-\mathcal{Q}$ uniform topological space is said to be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform semi-spectral space, if $\mathcal{A}_1$, $\mathcal{A}_2\in$ IF(X) are any two $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open sets in (X,τ_υ), then $\mathcal{A}_1\cap\mathcal{A}_2$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact in (X,τ_υ).

Definition 8.2. A pair (X,τ_υ) be a $\mathcal{IF}-\mathcal{Q}$ uniform topological space and $\delta\in$ IF(X). Then

(i) δ is said to be $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform Γ_q -compact open, if for each $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open set $\mu \in \mathrm{IF}(X)$, $\delta \cap \mu$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact.
(ii) δ is said to be $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform Γ_q -compact closed, if for each $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact closed set $\mu \in \mathrm{IF}(X)$, then $\delta \cap \mu$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact.
(iii) δ is said to be $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform Γ_q -compact clopen, if it is both $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact closed.
(iv) δ is said to be $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform co-compact if $\bar{\delta}$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform compact.

Definition 8.3. Let (X,τ_v) be $\mathcal{IF}-\mathcal{Q}$ uniform topological space. $\mathcal{A} \in \mathrm{IF}(X)$ is said to be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform ξ-set, if $\mathcal{A}$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed, $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact and $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform Γ_q-compact open in (X,τ_v).

Proposition 8.2. Let (X,τ_v) be a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform semi-spectral space with a base of $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ q-compact open sets and $\mathcal{A} \in \mathrm{IF}(X)$. Then the following conditions are equivalent:

(i) $\mathcal{A}$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact and $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed.
(ii) $\mathcal{A}$ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform ξ-set in (X,τ_v);
(iii) $\mathcal{A}$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed and there exists two $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open sets α, $\bar{\beta} \in \mathrm{IF}(X)$ such that $\mathcal{A}= \alpha \cap \bar{\beta}$.

Proof. **(i) $\implies$ (ii)**
Let $\mathcal{A} \in \mathrm{IF}(X)$ be $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact. Since (X,τ_v) has a base of $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open sets, $\mathcal{B} \in \mathrm{IF}(X)$ is also $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open. Since $\mathcal{A}$ and $\mathcal{B}$ are $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact and (X,τ_v) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact semi-spectral space, $\mathcal{A}\cap\mathcal{B}$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact. Since $\mathcal{B}$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open and $\mathcal{A}\cap\mathcal{B}$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact, by Defintion 8.2, $\mathcal{A}$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform Γ_q-compact open set. Therefore $\mathcal{A}$ is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform ξ set in (X,τ_v).
(ii) $\implies$ (iii)
Since $\mathcal{A}$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform ξ set, $\mathcal{A}$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed and $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ co-Γ_q-compact open. Thus $\bar{\mathcal{A}}$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform Γ_q-compact open. It known that in (X,τ_v), there exists a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform compact open set $\alpha \in \mathrm{IF}(X)$. Thus $\alpha\cap\bar{\mathcal{A}}=\beta$. Since $\bar{\mathcal{A}}$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform Γ_q-compact open and by Defintion 8.2, β is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact. Therefore $\alpha \cap \bar{\mathcal{A}}=\beta$ which implies that α - $\mathcal{A}$= β and so α - β =$\mathcal{A}$. Hence $\alpha \cap \bar{\beta}$= $\mathcal{A}$. Hence $\alpha \cap \bar{\beta}$=$\mathcal{A}$.
(iii) $\implies$ (i)
Suppose $\mathcal{A}$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed and there exists two $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact open sets α $\bar{\beta} \in \mathrm{IF}(X)$ such that $\mathcal{A}= \alpha \cap \bar{\beta}$. Since (X,τ_v) is a $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform semi-spectral and α, $\bar{\beta}$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact, $\mathcal{A}$ is also $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact. Hence $\mathcal{A}$ is $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform q-compact and $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform closed in (X,τ_v). □

9. Conclusion

In this article, $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform spectral space is introduced. $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform sober space and significant results are discussed. An $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform spectral space can redefines a new version it can be specified as $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform quasi-spectral space and $(\mathfrak{c},\mathfrak{d})$ $\mathcal{IF}-\mathcal{Q}$ uniform semi-spectral space are also discussed. This study pave the way to find some interesting extensions in Gold-spectral space and Jac-spectral topological space.

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